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NUMERICAL DIGITAL COMPUTER METHOD FOR DETERMINING THE TRANSIENT RESPONSES OF NONLINEAR AUTOMATIC SYSTEMS BASED ON CALCULATION OF THE CONVOLUTION INTEGRAL

by

A. V. Vul'fson

Izvestiya Vysshikh Uchebnykh Zavedeniy, Elektromekhanika
(News of Higher Educational Institutions, Electromechanics),
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Translated from the Russian

August 1967

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NUMERICAL DIGITAL COMPUTER METHOD FOR DETERMINING THE TRANSIENT RESPONSES OF NONLINEAR AUTOMATIC SYSTEMS BASED ON CALCULATION OF THE CONVOLUTION INTEGRAL

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A. V. Vul'fson

Discussed is a technique for the digital computer calculation of transient processes for systems with one or more nonlinear characteristics, using an extension of the convolution technique developed by Carson for systems with one nonlinearity. The method does not require formulation of a system of first-order differential equations with subsequent programing of the right-hand sides for each problem. The output data are transfer functions of the linear part of the system. The procedure for programing the solution of a specific problem is simplified, reducing essentially to the mere input of numerical data. The nonlinearities may be given tabularly, and they may be discontinuous.

Transient processes of automatic systems are usually determined by solving numerically differential equations with the aid of digital computers. However, the investigation of a system with a complex linear part containing differentiating links, and of a system with one or several nonlinearities, encounters difficulties in reducing differential equations to a normal Cauchy form, particularly when the nonlinearities have breaks or discontinuities. Similar difficulties are also encountered in determining the transient processes of electrical circuits containing one or several nonlinear components. In many cases it is possible to avoid these difficulties by using the numerical methods of solving integral equations.

J. Carson was probably the first to use a convolution integral in recording an equation for the transient process of a system with a branched linear part and one nonlinearity [1]. A numerical-graphic method of calculating the transient processes of similar systems, based on the calculation of the convolution integral, was developed by N. I. Sokolov [2].

A method is offered in this article of using a digital computer for determining the transient process of a system containing one or several nonlinear characteristics, which is based on the idea of Carson. This method requires no composing of systems of differential equations of the first order to be solved for the derivatives, and no programing for each problem of the right parts of the differential equations. The initial data consist of the transfer functions of the linear part of a system; it simplifies the programing procedure for solving a specific problem and reduces it mostly to an introduction of the numerical material. The nonlinearities may be specified in tabular form and may have discontinuities (relay characteristics).

Numerical Determination of Transient Process of a System with One Nonlinearity

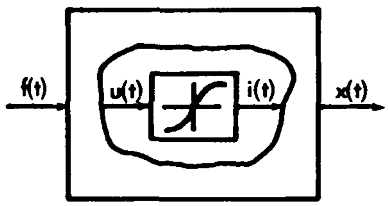

FIGURE 1. A SYSTEM WITH ARBITRARY STRUCTURE OF LINEAR PART AND WITH ONE NONLINEARITY

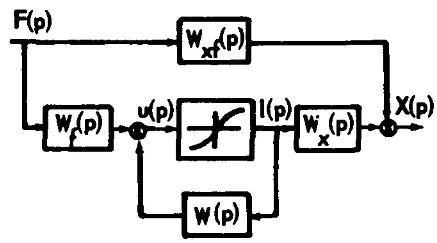

FIGURE 2. REDUCED STRUCTURAL PATTERN OF A SYSTEM WITH ONE NONLINEARITY

The system shown in Figure 1 consists of a linear part with an arbitrary structure and one inertialess link with a static characteristic $i = \psi(u)$ (it is shown separately on the drawing). The specified input action and the sought output response at the output of the system are, respectively, $f(t)$ and $x(t)$; $u(t)$ and $i(t)$ are, respectively, the input and output of the nonlinear link. If $f(t)$ has a Laplace representation $F(p)$ (it will be also assumed that this representation is a fractional-rational function), the following system of equations can be written to determine $x(t)$ [1]:

$$
\begin{aligned}
X(p) &= W_x(p)I(p) + W_{xf}(p)F(p), && \text{(a)} \\
U(p) &= W(p)I(p) + W_f(p)F(p), && \text{(b)} \qquad (1) \\
i(t) &= \psi[u(t)]. && \text{(c)}
\end{aligned}
$$

At $t < 0$, it is assumed that $x = u = i = 0$. A similar writing corresponds to a system reduced to the form shown in Figure 2. The terms $W_x(p)$, $W_{xf}(p)$, $W(p)$, and $W_f(p)$ are determined by the structure of the linear part of the system with the aid of the known methods of transforming the structures of linear systems. Let us assume that $w_x(t)$ and $w(t)$ are the originals of the Laplace representations $W_x(p)$ and $W(p)$

$$f_{xf}(t) \doteqdot W_{xf}(p)\,F(p),$$

$$f_f(t) \doteqdot W_f(p)\,F(p).$$

Using the theorem of convolution, Equations (1) are represented in the form of a system containing integral equations [1, 2]:

$$\begin{aligned} x(t) &= \int_0^t w_x(t-\tau)\,i(\tau)\,d\tau + f_{xf}(t), && \text{(a)} \\ u(t) &= \int_0^t w(t-\tau)\,i(\tau)\,d\tau + f_f(t), && \text{(b)} \qquad (2) \\ i(t) &= \psi[u(t)]. && \text{(c)} \end{aligned}$$

The first stage of the numerical calculation of the transient process consists of determining the transient characteristics $w_x(t)$, $f_{xf}(t)$, $w(t)$, and $f_f(t)$. A numerical method is described [5] which can be used for composing a program which requires no changes for determining the transient characteristics from a fractional-rational Laplace representation of any order. This method is based on the use of interpolating expressions of Adams and provides a solution with an accuracy of the second order.

It will be assumed that this method is used to determine the transient characteristics $w_x(t)$, $f_{xf}(t)$, $w(t)$, and $f_f(t)$. The latticed functions obtained as a result of the numerical calculation will be designated by $w_x[n]$, $f_{xf}[n]$, $w[n]$, and $f_f[n]$. Their quantization period is constant as is equal to the step Δt of the numerical determination of the transient characteristics. To find the numerical solution of the system (2), the integrals entering (2a) and (2b) are replaced with finite sums of the quadrature formulas which are obtained as a result of the piecewise-linear interpolation of each of the discrete sequences: $w_x[n]$, $w[n]$, and $i[n]$.

$$x[n] = \left(\frac{w_x[n-1]}{6} + \frac{w_x[n]}{3}\right) \Delta ti[0]$$

$$+ \sum_{m=1}^{n-1} \left(\frac{w_x[n-m-1]}{6} + \frac{w_x[n-m]}{1.5} + \frac{w_x[n-m+1]}{6}\right) \Delta ti[m]$$

$$+ \left(\frac{w_x[0]}{3} + \frac{w_x[1]}{6}\right) \Delta ti[n] + f_{xf}[n], \qquad (a)$$

$$u[n] = \left(\frac{w[n-1]}{6} + \frac{w[n]}{3}\right) \Delta ti[0] \qquad (3)$$

$$+ \sum_{m=1}^{n-1} \left(\frac{w[n-m-1]}{6} + \frac{w[n-m]}{1.5} + \frac{w[n-m+1]}{6}\right) \Delta ti[m]$$

$$+ \left(\frac{w[0]}{3} + \frac{w[1]}{6}\right) \Delta ti[n] + f_f[n], \qquad (b)$$

$$i[n] = \psi(u[n]). \qquad (c)$$

$$n = 1, 2, 3 \ldots$$

Using the first differences of (3a) for (3b), we reduce the system (3) to a form of:

$$\left\{\begin{array}{ll} \Delta x[n] = R_x[n] + S_x \Delta i[n], & (a) \\ \Delta u[n] = R[n] + S\Delta i[n], & (b) \\ i[n] + \Delta i[n] = \psi(u[n] + \Delta u[n]). & (c) \end{array}\right. \qquad (4)$$

$$n = 0, 1, 2 \ldots$$

In which case,

$$S_x = \left(\frac{w_x[0]}{2} + \frac{\Delta w_x[0]}{6}\right) \Delta t, \qquad (5)$$

$$S = \left(\frac{w[0]}{2} + \frac{\Delta w[0]}{6}\right) \Delta t: \qquad (6)$$

$$R_x[0] = \left(w_x[0] + \frac{\Delta w_x[0]}{2} \right) \Delta ti[0] + \Delta f_{xf}[0],$$

$$\begin{aligned} R_x[n] &= \left(\frac{\Delta w_x[n-1]}{6} + \frac{\Delta w_x[n]}{3} \right) \Delta ti[0] \\ &+ \sum_{\epsilon=1}^{n-1} \left(\frac{\Delta w_x[\epsilon-1]}{6} + \frac{\Delta w_x[\epsilon]}{1.5} + \frac{\Delta w_x[\epsilon+1]}{6} \right) \Delta ti[n-\epsilon] \\ &+ \left(w_x[0] + \frac{5}{6} \Delta w_x[0] + \frac{\Delta w_x[1]}{6} \right) \Delta ti[n] + \Delta f_{xf}[n]. \end{aligned} \tag{7}$$

$$n = 1, 2, 3 \ldots$$

$$R[0] = \left(w[0] + \frac{\Delta w[0]}{2} \right) \Delta ti[0] + \Delta f_f[0],$$

$$\begin{aligned} R[n] &= \left(\frac{\Delta w[n-1]}{6} + \frac{\Delta w[n]}{3} \right) \Delta ti[0] \\ &+ \sum_{\epsilon=1}^{n-1} \left(\frac{\Delta w[\epsilon-1]}{6} + \frac{\Delta w[\epsilon]}{1.5} + \frac{\Delta w[\epsilon+1]}{6} \right) \Delta ti[n-\epsilon] \\ &+ \left(w[0] + \frac{5}{6} \Delta w[0] + \frac{\Delta w[1]}{6} \right) \Delta ti[n] + \Delta f_f[n]. \end{aligned} \tag{8}$$

$$n = 1, 2, 3 \ldots$$

Note that for $R_x[n]$ and $R[n]$ the special expressions for $n = 0$ are obtained only when the signal at the input of the nonlinearity or at the output of the system changes jumpwise when a perturbation is applied at the input of the system. In such a case, it is first necessary to determine:

$$\begin{aligned} u[0] &= f_f[0], & \text{(a)} \\ i[0] &= \psi(u[0]), & \text{(b)} \\ x[0] &= f_{xf}[0]. & \text{(c)} \end{aligned} \tag{9}$$

Therefore, the second stage of the numerical calculation of the transient process consists of determining for the nth step ($n = 0, 1, \ldots$) the value of $R_x[n]$ and $R[n]$ and solving the system of equations (4) for the unknown $\Delta x[n]$, $\Delta u[n]$, $\Delta i[n]$, followed by the determination of:

$$\begin{aligned} x[n+1] &= x[n] + \Delta x[n], &\quad \text{(a)} \\ u[n+1] &= u[n] + \Delta u[n], &\quad \text{(b)} \\ i[n+1] &= i[n] + \Delta i[n]. &\quad \text{(c)} \end{aligned} \tag{10}$$

The latticed function with a quantization period of Δt - $x[n]$ is the one which represents the sought response at the output; if necessary, it can be interpolated, in which case it is expedient to use a parabolic interpolation when taking into account the order of accuracy of the expressions for the numerical determination of the transient characteristics and the determination of the convolution.

It was stated above that the action at the input $f(t)$ should have a fractional-rational Laplace representation.

In those cases where this is not observed (for example, when $f(t)$ is specified in tabular form), $f_{xf}[n]$ and $f_f[n]$ can be determined by using the expressions for numerical determination of the convolution integral, as in Eq. (3). In any other respects, the pattern of the solution remains unchanged.

Conditions under Which the Method Can Be Used

1. Each individual transfer function $W_x(p)$, $W(p)$, $W_{xf}(p)$, and $W_f(p)$ should correspond to a stable system. In other words, the linear system formed by opening a nonlinear link should be stable. An ultimate case of still being able to use the method is the presence of an integrating factor $1/p$ in the transfer functions. In such a case, a stable system should correspond to the transfer function remaining after separation of the integrating factor.

2. The transfer functions $w_x(t)$, $w(t)$, $f_{xf}(t)$, and $f_f(t)$ should be finite when $t = 0$. Note that, as a rule, this requirement is satisfied by automatic control systems and, by no means always, by electrical circuits with passive nonlinear components.

The subsequent determination of the numerical solution is similar to the one used for a case with one nonlinearity. The only thing to be noted is that, instead of calculating four transient characteristics in case of one nonlinearity, it is necessary to calculate $(d + 1)^2$ of such characteristics. A system of algebraic equations similar to (4), in case of d nonlinearities, will be written as:

$$
\begin{aligned}
\Delta x[n] &= R_x[n] + S_{x1}\Delta i_1[n] + S_{x2}\Delta i_2[n] + \dots + S_{xd}\Delta i_d[n], \\
\Delta u_1[n] &= R_1[n] + S_{11}\Delta i_1[n] + S_{12}\Delta i_2[n] + \dots + S_{1d}\Delta i_d[n], \\
\Delta u_2[n] &= R_2[n] + S_{21}\Delta i_1[n] + S_{22}\Delta i_2[n] + \dots + S_{2d}\Delta i_d[n], \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
\Delta u_d[n] &= R_d[n] + S_{d1}\Delta i_1[n] + S_{d2}\Delta i_2[n] + \dots + S_{dd}\Delta i_d[n], \\
i_1[n] + \Delta i_1[n] &= \psi_1(u_1[n] + \Delta u_1[n]), \\
i_2[n] + \Delta i_2[n] &= \psi_2(u_2[n] + \Delta u_2[n]), \\
&\dots\dots\dots\dots\dots \\
i_d[n] + \Delta i_d[n] &= \psi_d(u_d[n] + \Delta u_d[n]). \\
n &= 0, 1, 2 \dots
\end{aligned}
\tag{12}
$$

Here, remaining unknown, will be:

$$\Delta x[n],\ \Delta u_1[n],\ \Delta u_2[n],\ \dots,\ \Delta u_d[n],\ \Delta i_1[n],\ \Delta i_2[n],\ \dots,\ \Delta i_d[n].$$

It is obvious that the time-and-labor spent for the solution increases proportionally to the square of the number of nonlinearities.

The Experiment

This method was used for programing the "Ural-2" digital computer. The program served to determine the transient process of a system containing one nonlinearity and a linear part having an arbitrary structure. The preparation for the solution consists of composing Equations (1) for the system. This

Estimating the Accuracy of the Solution

The most practically convenient method is the well-known method of the reduced step of calculation. Since the step remains constant during the solution of a problem, the problem should be solved twice and, during the second time, the step should be reduced to one-half, for example.

In estimating the accumulated error, it should be assumed that the error is decreasing proportionally to the square of the step. It is expedient to estimate the accuracy only when the closed system is stable.

Numerical Determination of Transient Processes of Systems Containing Several Nonlinear Links

Under consideration is a system with d nonlinearities (their static characteristics are: $i_1 = \psi_1(u_1)$, $i_2 = \psi_2(u_2)$, ... $i_d = \psi_d(u_d)$ and whose linear part has an arbitrary structure (Figure 3). As it was done for a case with one nonlinearity (1), let us write for the transient process of the system of equations:

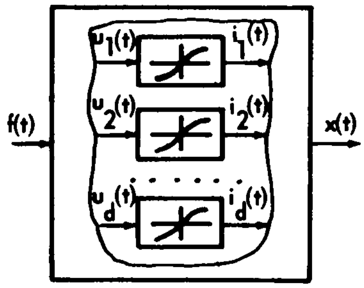

FIGURE 3. A SYSTEM WITH A LINEAR PART HAVING AN ARBITRARY STRUCTURE AND CONTAININ d NONLINEAF TIES

$$X(p) = W_{x1}(p)I_1(p) + W_{x2}(p)I_2(p) + \ldots + W_{xd}(p)I_d(p) + W_{xf}(p)F(p),$$

$$U_1(p) = W_{11}(p)I_1(p) + W_{12}(p)I_2(p) + \ldots + W_{1d}(p)I_d(p) + W_{1f}(p)F(p),$$

$$U_2(p) = W_{21}(p)I_1(p) + W_{22}(p)I_2(p) + \ldots + W_{2d}(p)I_d(p) + W_{2f}(p)F(p),$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$U_d(p) = W_{d1}(p)I_1(p) + W_{d2}(p)I_2(p) + \ldots + W_{dd}(p)I_d(p) + W_{df}(p)F(p),$$

$$i_1(t) = \psi_1[u_1(t)],$$

$$i_2(t) = \psi_2[u_2(t)],$$

$$\ldots\ldots\ldots\ldots$$

$$i_d(t) = \psi_d[u_d(t)]. \qquad (11)$$

can be accomplished by using any of the methods of composing the transfer functions of linear systems (the method of directional graphs, for example). Since following the reduction to the form of system (1) the algorithm for the solution is the same for any problem, no additional programing is required and the preparation for the counting is reduced to an introduction of numerical information: the numerator and denominator coefficients of the fractional-rational representations of $W_x(p)$, $W_{xf}(p)F(p)$, $W(p)$, $W_f(p)F(p)$, their order, their step, and number of steps. Additional programing of $i = \psi(u)$ is required only when a nonlinearity is specified analytically. When specified in tabular form, only the following numbers are introduced for the characteristics of the nonlinearities: the constant step u and the ordinates i. The program performs the sampling of the intermediate values by a parabolic interpolation. The system of algebraic equations (4) is solved by the method of iteration.

As a result of the performance by the program, we obtain:

$$x[n],\ u[n],\ i[n],\ n = 0,\ 1,\ 2,\ \ldots$$

The program was used to determine the transient processes of several automatic systems (including relay systems) and of electrical circuits. In cases of periodic steady-state conditions, the fluctuations (forced or natural) were obtained merely by continuing the counting of the transient process.

It was established that in case of linear parts having complex structures, only the composing of the system (1) becomes more time-consuming. The total time spent by the machine in solving the problem practically does not increase when the orders of the transfer functions in (1) increase; it is determined only by the number of steps.

Let us consider one of the solved examples. Shown in Figure 4 is a block-diagram of a primary astatic control of the absolute angle of a turbine unit connected with a high-power system, during the correction through the turbine [4]. The reduced transfer function of the regulator is:

$$W_p(p) = \frac{0.00915p^5 + 0.2305p^4 + 2.4319p^3 + 14.21p^2 + 30.575p + 15.03}{(0.00545p^4 + 0.1333p^3 + 1.581p^2 + 10.19p + 22.6)p}$$

The transfer function of the turbine unit is

$$W_o(p) = \frac{1}{0.019p^2 + 0.01p}\ .$$

LEGEND: φ = the deviation of the absolute angle
μ = the increment of the power-exchange between the system and the unit
μ = the increment in power of the working substance at the input of the turbine
μ_H = the load increment.

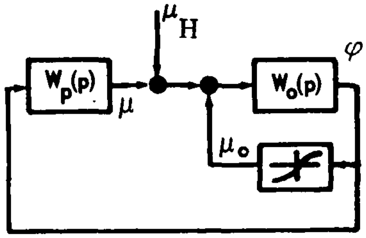

FIGURE 4. BLOCK-DIAGRAM OF A SYSTEM OF PRIMARY ASTATIC CONTROL OF ABSOLUTE ANGLE OF A TURBINE UNIT CONNECTED WITH A HIGH-POWER SYSTEM

The system is nonlinear, because the relationship for the increment in power-exchange between the unit and the system and the deviation of the absolute angle is specified by the expression

$$\mu_o = 0.3 \sin \varphi .$$

Under investigation is the transient process in the system when a nominal load $\mu_H = 1$ is added.

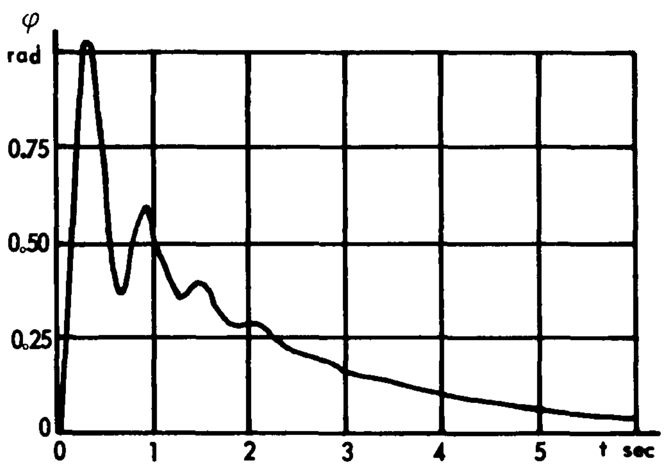

FIGURE 5. CURVE OF THE TRANSIENT PROCESS $\varphi(t)$ WITH LOAD ADDED

The system corresponding to Equations (1) will be written for this case (in Laplace representation), as follows:

$$\mu(p) = - \frac{W_o W_p}{1 + W_o W_p} \mu_o(p) + \frac{W_o W_p}{1 + W_o W_p} \cdot \frac{1}{p}, \quad \text{(a)}$$

$$\varphi(p) = - \frac{W_o}{1 + W_o W_p} \mu_o(p) + \frac{W_o}{1 + W_o W_p} \cdot \frac{1}{p}, \quad \text{(b)}$$

$$\mu_o(t) = 0.3 \sin \varphi t. \quad \text{(c)}$$

(13)

The selected step of solution was $\Delta t = 0.04$ seconds; the number of steps was 220. The time spent in introducing the information into the machine, after it was reduced to the form of system (13), ranged from 25 to 30 minutes. The problem was solved by the machine in 16 minutes (including the control). The curve of the transient process $\varphi(t)$ is shown in Figure 5. The accuracy of the solution (in percent of maximum amplitude) was 0.65 percent.

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LITERATURE CITED

1. J. R. Carson, THEORY AND CALCULATION OF VARIABLE ELECTRICAL SYSTEMS, Physical Review, No. 2, 1921.

2. N. I. Sokolov, ANALYTICAL METHOD OF APPROXIMATE CALCULATION OF TRANSIENT PROCESSES OF CERTAIN NONLINEAR AUTOMATIC CONTROL SYSTEMS, Trudy MAI (Trans. of Moscow Aviation Institute), No. 12, 1959.

3. I. S. Berezin and N. P. Zhidkov, METODY VYCHISLENIY (Methods of Computing), 1-2, Fizmatgiz, 1960.

4. Ye. I. Yurevich, DEVELOPMENT OF AUTOMATIC CONTROL SYSTEM OF SUPERPOWER COMBINED SYSTEMS BY CONTROLLING THE ANGLE, Dissertation for the Scientific Degree of Doctor of Technical Sciences, Leningrad, 1963.

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13. ABSTRACT

Discussed is a technique for the digital computer calculation of transient processes for systems with one or more nonlinear characteristics, using an extension of the convolution technique developed by Carson for systems with one nonlinearity. The method does not require formulation of a system of first-order differential equations with subsequent programing of the right-hand sides for each problem. The output data are transfer functions of the linear part of the system. The procedure for programing the solution of a specific problem is simplified, reducing essentially to the mere input of numerical data. The nonlinearities may be given tabularly, and they may be discontinuous.

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