PERFORMANCE POTENTIAL OF A RADIANT-HEAT-TRANSFER LIQUID-CORE NUCLEAR ROCKET ENGINE

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SUMMARY

Parametric calculations were conducted to establish the kind of performance available from a liquid-core nuclear rocket engine. The heat source is a liquid carbide containing uranium. Rotating the tube holds the molten fuel in place on the inner surface of the tube. The hydrogen gas flows axially down the tube and is heated by thermal radiation. The Reynolds number is between 26,000 and 121,000 for all cases calculated; therefore, the flow is turbulent. Mass transfer of the carbide solvent and the uranium into the flowing hydrogen occurs because of the relatively high vapor pressure of the liquid carbide mixture. The tube length was 5 feet (1.524 m) for all calculations.

Engine performance is expressed in terms of specific impulse, thrust to core-weight ratio, and hydrogen-to-uranium-flow-rate ratio. (The reactor core weight includes the carbide solvent and the water moderator. It does not include an external reflector, a pressure shell, or a shield.) These quantities were calculated for tube surface temperatures between 9000° and 11,000° R (5000° and 6110° K), for reactor pressures between 200 and 1000 atmospheres (2020 and 10,100 N/cm²), for tube diameters between 1 and 4 inches (2.54 and 10.16 cm), and for hydrogen flow rates from 0 to 1 pound per second (0.45 kg/sec).

The calculations indicate that this liquid-core engine is potentially capable of producing a maximum specific impulse of 1660 seconds. The thrust to core-weight ratio can be as high as 19. The hydrogen-to-uranium-flow-rate ratio will not exceed 100, unless the surface temperature is less than 9000° R (5000° K). The upper limits of these three parameters cannot be obtained together. One or two of them can be maximized at the expense of the other one or two. A compromise, or average, engine has a specific impulse of 1400 seconds, a thrust to core-weight ratio of 4, and a hydrogen-to-uranium-flow-rate ratio of 50.
INTRODUCTION

The primary goal of an advanced nuclear rocket engine is to produce a substantially higher specific impulse than the 850 seconds available from a solid-core reactor. It is generally proposed to use a higher temperature heat source than solid fuel elements to achieve the necessary increase in exhaust hydrogen enthalpy. In a liquid-core reactor, the fuel element consists of a rotating tube whose inner surface is a liquid-carbide solution containing uranium. Typically, the molten fuel would be a low-vapor-pressure system such as uranium carbide ($\text{UC}_2$) dispersed in niobium carbide (NbC) or zirconium carbide (ZrC).

In a "bubble-through" concept, hydrogen is introduced at the outer surface of the tube. The hydrogen first cools the solid-structure portion of the element and then bubbles radially inward through the liquid fuel. Finally, it flows axially down the center of the tube to a nozzle. As a bubble of hydrogen proceeds radially inward through the molten fuel, heat is conducted into it. Unfortunately, mass is also transferred to the bubble because of the vapor pressure exerted by the liquid carbide. Thus, when the bubble emerges at the inner surface, it is essentially at the fuel temperature and is also saturated with heavy vapor, which increases the average molecular weight of the exhaust mixture. An increase in fuel temperature increases both the hydrogen enthalpy and the amount of heavy vapor in the hydrogen. Because vapor pressure increases exponentially with temperature, the latter effect eventually becomes dominant. Thus, the increasing molecular weight finally causes a decrease in specific impulse, even as temperature is increased.

It has been shown that for the bubble-through system, there is a maximum thrust (refs. 1 and 2). Reference 1 reports that a single, large cylinder operating in such a bubble-through mode would be limited to a specific impulse of about 1200 seconds and a thrust to core-weight ratio between $10^{-2}$ and $10^{-1}$. Nelson, et al. (ref. 2) investigated a multitube version of the same flow pattern and concluded that a specific impulse greater than 1500 seconds and a thrust to core-weight ratio of 1 are possible.

An alternate concept, to be studied in this report, is discussed in reference 3. A molten fuel is still held on the inner surface of a rotating tube, but the hydrogen flows axially down the tube and is heated by radiation. There are two potential advantages to such a configuration:

1. Higher thrust may be obtained because the hydrogen flow rate is not limited by bubble velocity.

2. The loss of heavy vapor may be reduced because the concentration of heavy vapor in the bulk hydrogen stream will not be in equilibrium with the liquid surface.

Reference 4 presents some heat- and mass-transfer characteristics of this radiation-heat-transfer flow pattern. The general conclusion was that the major advantage was
increased thrust; the maximum specific impulse was 1400 or 1500 seconds, about the same as in a bubble-through system.

The final results of reference 4 are illustrated with a sample radiation-heat-transfer engine. The engine uses 4-inch-inside-diameter (10.16-cm-i.d.) tubes; the thrust to core-weight ratio is 3.2 (31 N/kg); the specific impulse is 1400 seconds. The heavy-gas loss rate is characterized in terms of the weight-flow-rate ratio of the hydrogen to the uranium flowing from the engine. This ratio is 49 for the sample engine, which operates at a pressure of 200 atmospheres (2020 N/cm²). It was pointed out (ref. 4) that trade-offs exist among thrust to core-weight ratio, specific impulse, and uranium loss rate. These trade-offs can be obtained by changing tube diameter, reactor pressure, or fuel surface temperature.

This analysis was performed to examine the interdependence of these variables, in order to establish the performance potential of a radiant-heat-transfer liquid-core nuclear rocket engine. In presenting results, it is difficult to maintain a clear picture of just which quantities are fixed and which varying, because of the large number of variables involved. For this reason, the general pattern of the calculations is discussed briefly here.

First, a number of engine characteristics are constant for all of the results to be presented. The fuel-element flow passage is shown in figure 1(a). The hydrogen inlet temperature is 3500° R (1945° K), the specific heat is 6 Btu per pound per °R (6 J/(g)(°K)), and the viscosity is 275×10⁻⁷ pound per foot per second (40.2×10⁻⁷ N/(cm)(sec)) The opacity (absorption coefficient times tube diameter) of the seeded hydrogen is

(a) Flow-passage model. Entire system at reactor pressure P_r; tube length, 5 feet (1.524 m); tube diameter, 1, 2, or 4 inches (2.54, 5.08, or 10.16 cm); ratio of outside diameter to tube diameter, D_o/D_t; 2; density of structural region, 146 pounds per cubic foot (2.34 g/cc cm³); inlet temperature, 3500 °R (1945° K).

(b) Structural configuration used to estimate engine weight. All dimensions are in inches (cm).
assumed to be 3. The fuel mixture is NbC with a UC₂ mole fraction of 0.02. The inside diameter of the tube is one half of the outside diameter. The average density of the various core materials comprising the structural region is 146 pounds per cubic foot (2.34 g/cm³). This average density is based on the configuration shown in figure 1(b). Weight estimates in this report are based on the reactor core only. The total weight would include the weight of a pressure shell, a shield, a turbopump, and other engine components. The tube length is assumed to be 5 feet (1.524 m). This length is considered to be reasonable for the reactor power levels of interest, 2000 to 5000 megawatts.

The three parameters of the calculations are surface temperature, reactor pressure, and tube diameter. The four values of fuel surface temperature considered ranged from 9000° to 11 000° R (5000° to 6110° K). The reactor pressures investigated were 200, 500, and 1000 atmospheres (2020, 5050, and 10 100 N/cm²). The tube diameter was assumed to be 1, 2, or 4 inches (2.54, 5.08, or 10.16 cm).

The independent variable was the flow rate of hydrogen gas through the tube, which for all calculations ranged from 0 to 1 pound per second (0.454 kg/sec). For a given hydrogen flow rate and set of parametric values, three dependent variables were calculated: (1) specific impulse, (2) thrust to core-weight ratio, and (3) hydrogen-to-uranium-flow-rate ratio leaving the tube. The results of these basic calculations were then cross-plotted to show the trade-offs among the three dependent variables for various selections of the parameters.

The general objective of the calculations is to establish what kind of performance could be obtained from a liquid-core engine using radiation heat transfer. Thus, the following criteria are to be established: (1) the range of specific impulse, (2) the thrust to core-weight ratio for reasonable values of other parameters, and (3) the probable ratio of hydrogen to uranium in the rocket exhaust. It is not sufficient just to give ranges for these dependent variables, because obviously not all combinations are possible. Therefore, some specific engines were chosen to illustrate how one variable, such as thrust to core-weight ratio, can be maximized at the expense of either specific impulse or uranium loss rate. Finally, a set of engine parameters was selected to represent a compromise of the available trade-offs.

CALCULATION PROCEDURE

Model for Calculation

The model of the fuel-element passage used as a basis for the calculation is shown in figure 1(a). Pure hydrogen enters the tube at a temperature of 3500° R (1945° K). The inner surface of the tube is at a constant temperature $T_s$, and the reactor pressure
is $P_r$. At the surface of the tube, the hydrogen is saturated with a mass fraction $C_s$ of heavy vapor that is in equilibrium with the liquid fuel. As the hydrogen flows down the 5-foot-long (1.524-m-long) tube, heat is transferred by radiation, and mass is transferred by forced convection. The hydrogen exits from the tube at a temperature $T_2$ that is greater than $T_1$ but less than $T_s$. The hydrogen contains some mass fraction $C_b$ of heavy vapor that is less than $C_s$.

The flow is assumed to be turbulent, with a fully developed velocity profile. Hydrogen is transparent at the conditions considered herein; therefore, the absorbing ability of the hydrogen is presumed to be the result of seeding with solid particles, liquid droplets, or other gases. The heavy vapor from the molten fuel is assumed to be transparent, and the amount of seeding is adjusted so that the product of the absorption coefficient and the tube diameter, or opacity, is three for all tube sizes considered. This opacity can be achieved without affecting the specific impulse (ref. 4). The constituents of the molten fuel, NbC and UC$_2$, are assumed to vaporize as molecules that do not dissociate or react with the hydrogen. The average density of the tube structure surrounding the flow passage is assumed to be 146 pounds per cubic foot ($2.34 \text{ g/cm}^3$). This value includes the weight of the liquid fuel and is based on the configuration given in reference 4. For all calculations, the mole fraction of UC$_2$ in the NbC solvent is assumed to be 0.02. This value should provide a critical mass for an assembly of the 5-foot-long (1.524-m-long) tubes that gives an engine length to diameter ratio reasonably close to 1 (e.g., between 1/2 and 2).

**Variables Considered**

The surface temperature of the liquid fuel, the flow-passage diameter, and the reactor pressure were treated as parameters in the calculation. The four values used for surface temperature were 9000°, 9500°, 10 000°, and 11 000° R (5000°, 5280°, 5580°, and 6110° K). Tube diameters investigated were 1, 2, and 4 inches (2.54, 5.08, and 10.16 cm). Three reactor pressures were used: 200, 500, and 1000 atmospheres (2020, 5050, and 10 100 N/cm$^2$). These values bound the region of interesting engine performance.

For example, a tube diameter less than 1 inch (2.54 cm) would give an excessive uranium loss rate, and a diameter larger than 4 inches (10.16 cm) would give a low thrust to core-weight ratio, at least for the 5-foot (1.524-m) tube length used. Similarly, a surface temperature below 9000° R (5000° K) yields a low specific impulse, while a temperature above 11 000° R (6110° K) causes too much uranium loss because of the strong dependence of vapor pressure on temperature. A reactor pressure below 200 atmospheres (2020 N/cm$^2$) also causes a high uranium loss in proportion to the hydrogen
flow rate. A pressure greater than 1000 atmospheres (10 100 N/cm$^2$) would probably lead to excessive engine weight (because of the turbopump and pressure shell), although this effect is not indicated in the calculation because only the core weight is included. At any rate, from the results obtained, a high pressure does not seem desirable; for a pressure above 1000 atmospheres (10 100 N/cm$^2$), either the specific impulse or the thrust to core-weight ratio would have to be sacrificed to obtain a high value of the other.

The hydrogen flow rate was the independent variable. The dependent variables were specific impulse, thrust to core-weight ratio, and hydrogen- to uranium-flow-rate ratio. The calculational procedure was as follows. First, surface temperature, reactor pressure, and tube diameter were specified. Then a relatively low hydrogen flow rate was selected (generally about 0.05 lb/sec or 0.0227 kg/sec), and the specific impulse, the thrust to core-weight ratio, and the hydrogen- to uranium-flow-rate ratio were determined. At the low flow rate, the loss rate was high, and the specific impulse was low. As the flow rate increased, the loss rate dropped, and the specific impulse passed through a maximum. The hydrogen flow rate was increased until the specific impulse dropped to about 1200 seconds. This procedure was followed for nearly all combinations of surface temperature, reactor pressure, and tube diameter. From these basic curves, crossplots were constructed to depict the trade-offs among specific impulse, thrust to core-weight ratio, and hydrogen- to uranium-flow-rate ratio.

**Equations Used**

Basically, there are only two quantities to calculate: the outlet temperature and the concentration of NbC and UC$_2$ in the exit hydrogen. These values determine the specific impulse, the thrust to core-weight ratio, and the hydrogen- to uranium-flow-rate ratio. The order of the calculation was to assign values to the three parameters (surface temperature, reactor pressure, and tube diameter), to select a hydrogen flow rate, next to determine the outlet gas temperature, and finally to calculate the mass fractions of NbC and UC$_2$ in the exhaust hydrogen. The outlet temperature determines the maximum specific impulse of the engine, that of pure hydrogen. This relation is shown in figure 2 for four reactor pressures; the arbitrary nozzle coefficient 0.9 is used to account for effects such as a finite area ratio, incomplete recombination, and a nonvacuum rocket environment. Figure 2 is based on enthalpy values taken from reference 5.

The outlet temperature is fixed by specifying the tube length and diameter, the surface temperature, the hydrogen flow rate, the specific heat, the inlet temperature, and the opacity. These calculations were reported in reference 4, and the results are shown in figure 3. These curves account for radiative heat transfer only; if forced-convective heat transfer becomes significant, figure 3 will give a low estimate of outlet
temperature. For surface temperatures of 9000° R (5000° K) and above, radiative heat transfer should be dominant (ref. 4); below this value, the contribution of forced convection could become important, depending on the Reynolds number, the gas properties, etc.

The mass fractions of UC₂ and NbC at the surface are the boundary values required for the mass-transfer calculation. Basically, these values are calculated from Raoult's law, which equates the partial pressure of a component to the product of its vapor pressure and its mole fraction in the liquid:

\[ P_i = X_i P^0 \]  

(1)

Figure 4 gives the vapor-pressure curves of NbC and UC₂ used herein. The NbC values are taken from reference 6. The UC₂ curve is taken from reference 1. Although the NbC curve is based on extrapolated data (ref. 6), thermodynamic calculations (refs. 7 and 8) indicate that the vapor pressure may be higher than indicated in figure 4. This possibility is discussed in reference 6. The point of view in this report is that if the
engine performance based on this rather optimistic vapor-pressure estimate is uninteresting, then the matter need not be pursued. If the performance does appear worthwhile, a second look at the approximations used in the calculations is justified.

Equation (1) and the curves of figure 4 give the partial pressures of NbC and UC$_2$ at the surface of the tube. The difference between the sum of these two pressures and the reactor pressure is contributed by the hydrogen. These partial pressures are readily changed to mass fractions. For example, the mass fraction of UC$_2$ at the surface is

$$C_{UC_2,s} = \frac{M_{UC_2} P_{UC_2}}{M_{UC_2} P_{UC_2} + M_{NbC} P_{NbC} + 2(P_r - P_{UC_2} - P_{NbC})}$$  \hspace{1cm} (2)$$

In all the calculations of mass fractions, the hydrogen molecular weight was assumed to be 2; dissociation was taken into account in the calculation of specific impulse. The mass fractions of the various components at the surface are shown in figure 5 for reactor pressures of 200, 500, and 1000 atmospheres (2020, 5050, and 10 100 N/cm$^2$).
For fully developed turbulent flow, reference 4 shows that the ratio of bulk to surface mass fraction can be written as

\[
\frac{C_{1,b}}{C_{1,s}} = \frac{1}{1 + C_{H_2,s} (e^{4j L/D} - 1)^{-1}}
\]

where \( j \), the mass-transfer factor, is a function of the hydrogen Reynolds number. The curve for \( j \) (fig. 6) was taken from reference 9. For all the calculations in this report, the Reynolds number was between 26 000 and 121 000. For the tube length to diameter
ratios considered (15, 30, and 60), the assumption of a fully developed turbulent velocity profile should not be seriously in error. Equation (3) was used to calculate the bulk mass fractions of UC\(_2\) and NbC in the reactor exhaust. The hydrogen bulk mass fraction is

\[
C_{H_2, b} = 1 - C_{UC_2, b} - C_{NbC, b}
\]  

(4)

The ratio of hydrogen to uranium flow rates is then obtained from

\[
\frac{\dot{w}_{H_2}}{\dot{w}_U} = \frac{C_{H_2, b}}{C_{UC_2, b}} \frac{M_{UC_2}}{M_U}
\]  

(5)

The specific impulse of the exhaust mixture is calculated by correcting the specific impulse of pure hydrogen for the increase in molecular weight caused by the heavy vapor:

\[
I_{sp} = \left( \frac{2}{\bar{M}} \right)^{1/2} I_{sp^*}
\]  

(6)

The average molecular weight \(\bar{M}\) is obtained from the individual mole fractions and molecular weights:

\[
\bar{M} = \sum Y_{1,b} \bar{M}_1
\]  

(7)

The engine thrust to core-weight ratio can be written in the following form (ref. 4):

\[
\frac{F}{W_c} = \left( \frac{\bar{A}}{1 - \bar{A}} \right) \left( \frac{G I_{sp}}{\beta_r L} \right)
\]  

(8)
RESULTS AND DISCUSSION

First, the results of the basic calculations are presented to show how specific impulse, thrust to core-weight ratio, and hydrogen-to-uranium-flow-rate ratio vary with hydrogen flow rate. Sets of curves are presented for various combinations of surface temperature, reactor pressure, and tube diameter. Next, these results are cross-plotted to show the trade-offs between thrust to core-weight ratio and hydrogen-to-uranium-flow-rate ratio at constant specific impulses of 1300, 1400, and 1500 seconds. Trade-off curves are used to show the effects of tube diameter, surface temperature, and reactor pressure. Finally, four specific examples are presented to illustrate the kinds of performance possible.

Effect of Hydrogen Flow Rate

The influence of hydrogen flow rate is shown in figure 7(a), for a reactor pressure of 200 atmospheres (2020 N/cm²), a surface temperature of 9000° R (5000° K), and all three tube diameters. These curves are typical of those for other surface temperatures and reactor pressures. As hydrogen flow rate increases from zero, specific impulse increases to a maximum and then decreases. Thrust to core-weight ratio and hydrogen-to-uranium-flow-rate ratio monotonically increase with hydrogen flow rate. The specific impulse decreases to the left of the maximum because the average molecular weight is too high, and decreases to the right of the maximum because of insufficient heat transfer. The dashed portions of the curves indicate extrapolations of the calculated values. These extensions are easily made; for example, the maximum possible specific impulse is that of pure hydrogen at the surface temperature, and the specific impulse at zero hydrogen flow is that of saturated hydrogen at the surface temperature. The curves for thrust to core weight are extended to the origin. Those for hydrogen to uranium flow rate are extended to lower hydrogen flow rates with a constant slope until a flow rate is reached that corresponds to a Reynolds number of about 2000. This end point is connected to the ratio for a saturated mixture at a flow rate of zero. Nearly all the results and conclusions are based on the solid, calculated portions of the curves. The dashed portions are presented to afford a more complete picture of engine behavior.

The reactor pressure for figure 7(b) is the same as for figure 7(a) (200 atm or 2020 N/cm²), but the surface temperature is 9500° R (5280° K), 500° R (278° K) higher. The major effect of using a higher surface temperature is to increase the value of the maximum specific impulse and to decrease the ratio of hydrogen to uranium flow rate. Figure 7(c) shows the same set of curves for a surface temperature of 10 000° R (5580° K). No higher surface temperatures were investigated at this pressure (200 atm
Figure 7. Effect of hydrogen flow rate.

(a) Reactor pressure, 200 atmospheres (2020 N/cm²), surface temperature, 9000° R (5280° K).
(b) Reactor pressure, 200 atmospheres (2020 N/cm²), surface temperature, 9500° R (5280° K).
Figure 7. - Continued.

(c) Reactor pressure, 200 atmospheres (2020 N/cm²); surface temperature, 10 000° R (5580° K).
or 2020 N/cm²), because either the thrust to core-weight ratio or the hydrogen-to uranium-flow-rate ratio is too low, depending on the tube diameter. A 1-inch (2.54-cm) tube gives a high thrust to core-weight ratio but a low hydrogen-to uranium-flow-rate ratio; a 4-inch (10.16-cm) tube results in the other extreme.

The hydrogen-to uranium-flow-rate ratio can be increased by operating at a higher reactor pressure. Of course, this increased pressure tends to depress the specific impulse because of less dissociation. This effect can be offset by operating at a higher surface temperature, which, in turn, tends to increase the uranium loss rate. Figure 7(d) shows the basic curves for a 2-inch (5.08-cm) tube for a reactor pressure of 500 atmospheres (5050 N/m²). Surface temperatures of 10 000° and 11 000° R (5580° and 6110° K) are shown. The curves for 11 000° R (6110° K) show a decided advantage in impulse, little influence on thrust to core-weight ratio, but a severe increase in
uranium loss rate.

The same set of curves are shown in figure 7(e) for a reactor pressure of 1000 atmospheres (10 100 N/cm²). The specific impulse maximums are decreased by about 20 or 30 seconds, the thrust to core-weight ratio curves are practically unaffected, and the uranium loss rate is cut in half.

The general influence of the hydrogen flow rate indicated in figure 7 is the same for all surface temperatures, reactor pressures, and tube diameters. There is some "best" flow rate that maximizes the specific impulse. The thrust to core-weight ratio increases almost linearly with hydrogen flow rate. The ratio of hydrogen to uranium flow rate is nearly independent of hydrogen flow rate. These basic curves are cross-plotted (figs. 8 to 10) to show the trade-offs available between thrust to core-weight ratio and hydrogen-to-uranium-flow-rate ratio at constant specific impulse. The effects of tube diameter, surface temperature, and reactor pressure on these secondary curves are shown.

Effect of Tube Diameter

Figure 8(a) shows the effect of tube diameter. The reactor pressure is fixed at 200 atmospheres (2020 N/cm²). The trade-off between thrust to core-weight ratio and hydrogen-to-uranium-flow-rate ratio is shown in the form of a parallelepiped; the top and bottom sides are for 1300 and 1500 seconds specific impulse, respectively, and the left and right sides are for surface temperatures of 10 000° and 9000° R (5580° and 5000° K), respectively. There are three such parallelepipeds, one for each tube diameter: 1, 2, and 4 inches (2.54, 5.08, and 10.16 cm).

The general effect shown in figure 8(a) is that a large tube diameter favors the hydrogen-to-uranium-flow-rate ratio at the expense of thrust to core-weight ratio. It is also apparent from figure 8(a) that tube diameters from 1 to 4 inches (2.54 to 10.16 cm) encompass the range of interesting performance, at least for a tube of fixed length (5 ft or 1.524 m) and a reactor pressure of 200 atmospheres (2020 N/cm²). One effect not shown in figure 8 is that a smaller tube diameter would necessitate more tubes per engine. For a fixed engine frontal area, the total number of tubes would vary inversely as the square of the tube diameter. This effect would probably influence the choice of a large tube diameter rather than a small one.

Effect of Surface Temperature

The effect of surface temperature on the trade-off between thrust to core-weight
Figure 8. - Thrust to core-weight ratio for reactor pressure of 200 atmospheres (7020 N/cm²).

(a) Effect of tube diameter.
Figure 8 - Concluded.

(i) Effect of surface temperature.

- Hydrogen-to-uranium-flow-rate ratio, \( \frac{w_{H2}}{w_U} \)

- Thrust to core-weight ratio, F/Wc

- Tube diameter, D, in. (cm)

- Surface temperature, T

- Specific impulse, Isp, sec

- Chamber pressure, P, lb/in²

- Reaction, 9.500 kg/sec

- Mass flow rate, q, kg/sec

- Flow rate, w, lb/sec

- Tube diameter, D, in. (cm)

- Surface temperature, T

- Specific impulse, Isp, sec

- Chamber pressure, P, lb/in²

- Reaction, 9.500 kg/sec

- Mass flow rate, q, kg/sec

- Flow rate, w, lb/sec
ratio and hydrogen- to uranium-flow-rate ratio is shown in figure 8(b). There are three solid areas, one for each surface temperature: 9000°, 9500°, and 10 000° R (5000°, 5280°, and 5580° K). The top and bottom boundaries of each region are for specific impulses of 1300 and 1500 seconds, respectively. The left and right edges are for tube diameters of 1 and 4 inches (2.54 and 10.16 cm), respectively. The reactor pressure is 200 atmospheres (2020 N/cm²).

A high surface temperature gives a high thrust to core-weight ratio and a low hydrogen- to uranium-flow-rate ratio. The uranium flow rate increases with increasing surface temperature because the vapor pressure increases. The thrust increases because the increase in wall temperature at a constant specific impulse is accomplished by increasing the hydrogen flow rate through the fixed-length tube.

**Effect of Reactor Pressure**

The effect of an increase in reactor pressure is to increase the ratio of hydrogen to uranium flow rates at the expense of a decrease in either specific impulse or thrust to core-weight ratio. This effect is shown in figure 9 for a 2-inch-diameter (5.08-cm-diam) tube and for surface temperature ranging from 9000° to 11 000° R (5000° to 6110° K). Figure 9 also shows that a higher surface temperature is necessary in order
to exploit a performance advantage from an increase in pressure. It also appears that
an increase in reactor pressure is more beneficial at a specific impulse of 1300 seconds
than at 1500 seconds. This condition is illustrated in figure 9 at the point corresponding
to a thrust to core-weight ratio of about 4 and a hydrogen-to-uranium-flow-rate ratio
of 36. There is no advantage to operating at a pressure of 500 atmospheres (5050 N/cm²)
instead of 200 atmospheres (2020 N/cm²) for a specific impulse of 1500 seconds. In fact,
the higher pressure engine must operate at a higher surface temperature to maintain the
specific impulse at a value of 1500 seconds.

In general, it seems reasonable to expect an advantage from operation at high re­
actor pressure. An increase in the ratio of hydrogen to uranium should certainly result,
since the hydrogen density is proportional to pressure and the uranium density is fixed
by its vapor pressure at a given temperature. However, the surface temperature must
be increased along with the pressure to keep a constant specific impulse. This increase
in surface temperature increases the partial pressure of the uranium. The vapor pres­
sure variation with temperature is strong enough to override most or all of the potential
advantage, at least at a relatively high specific impulse.

It is possible to exploit the increase in pressure to reduce the uranium loss rate,
if the surface temperature is held constant, as is shown in figure 10. In this case, the
specific impulse is held constant by decreasing the hydrogen flow rate as the reactor
pressure is increased. Eventually the thrust to core-weight ratio of the 1500-second
curve would drop to zero, because even pure hydrogen at the surface temperature of
10 000° R (5580° K) will not give a 1500-second impulse if the pressure is much over
1000 atmospheres (10 100 N/cm²). Figures 9 and 10 are for a tube diameter of 2 inches
(5.08 cm), but the general conclusions and trends would also apply to tube diameters of
Figure 11. - Effect of reactor pressure on thrust to core-weight ratio and uranium loss rate at constant specific impulse; surface temperature, 10 000° R (5580° K); tube diameter, 2 inches (5.08 cm).

Figure 11 shows the direct influence of reactor pressure on the thrust to core-weight ratio and the hydrogen-to-uranium-flow rate ratio for a constant specific impulse, for a tube diameter of 2 inches (5.08 cm) and a surface temperature of 10 000° R (5580° K). The hydrogen flow rate decreases as reactor pressure increases in order to keep the specific impulse constant. These curves show again that there is an advantage to high pressure only for a low specific impulse engine. Thus, a high-pressure reactor would be employed to achieve an engine characterized by high thrust to core-weight ratio, low uranium loss rate, and low specific impulse.

### Combined Effects

Various combinations of tube diameter, reactor pressure, and surface temperature can be selected to achieve engines with certain performance characteristics. There are practical or absolute limitations, however, which restrict engine performance to a finite range of each of its important parameters.

The limits of the assigned parameters are (1) tube surface temperature, 9000° to 11 000° R (5000° to 6110° K), (2) reactor pressure, 200 to 1000 atmospheres (2020 to
10,000 N/cm²), (3) tube diameter, 1 to 4 inches (2.54 to 10.16 cm), and (4) hydrogen flow rate per tube, 0.05 to 0.5 pound per second (0.023 to 0.23 kg/sec).

The results of the calculations indicate that for all reasonable combinations of the variables considered, engine performance will fall within the following ranges:
(1) specific impulse, 1300 to 1500 seconds, (2) thrust to core-weight ratio, 1 to 15, and (3) ratio of hydrogen to uranium in the engine exhaust, 10 to 100.

The preceding limits include a complete spectrum of engines, as characterized by thrust to core-weight ratio, hydrogen-to-uranium-flow-rate ratio, and specific impulse. It is therefore impossible to designate any one engine as the best, simply because "best" is not definable without considering a particular mission, engine reliability, engine components such as pressure vessel, pump, nozzle, reactor control, and a host of other factors not included in this study.

Four engines were selected to illustrate the combinations of performance criteria that are available. These engines are described in table I. The first three engines maximize one of the major performance parameters at the expense of one or more of the others. The fourth engine represents compromise or intermediate values of specific impulse, thrust to core-weight ratio, and hydrogen-to-uranium-flow-rate ratio.

Engine 4 is not greatly different from the sample engine described in reference 4. The specific impulse is 1400 seconds, and the hydrogen to uranium flow rate is about 50 for both engines. The thrust to core-weight ratio is slightly higher for the engine in this report than for that in reference 4, 4 to 3 (49 to 29 N/kg). The reactor pressure is

TABLE I. SAMPLE LIQUID-CORE ENGINES

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Type of engine:</td>
<td>High</td>
</tr>
<tr>
<td>Specific impulse</td>
<td>High</td>
</tr>
<tr>
<td>Thrust to core-weight ratio</td>
<td>Low</td>
</tr>
<tr>
<td>Hydrogen-to-uranium-flow-rate ratio</td>
<td>High</td>
</tr>
<tr>
<td>Specific impulse, sec</td>
<td>1500</td>
</tr>
<tr>
<td>Thrust to core-weight ratio</td>
<td>1.4</td>
</tr>
<tr>
<td>Hydrogen-to-uranium-flow-rate ratio</td>
<td>92</td>
</tr>
<tr>
<td>Reactor pressure, atm</td>
<td>200</td>
</tr>
<tr>
<td>Reactor pressure, N/cm²</td>
<td>2020</td>
</tr>
<tr>
<td>Surface temperature, °R</td>
<td>9000</td>
</tr>
<tr>
<td>Surface temperature, °K</td>
<td>5000</td>
</tr>
<tr>
<td>Tube diameter, in.</td>
<td>4</td>
</tr>
<tr>
<td>Tube diameter, cm</td>
<td>10.16</td>
</tr>
</tbody>
</table>
200 atmospheres (2020 N/cm²) in both cases. The two engines differ in tube diameter and in surface temperature. The engine of reference 4 used a tube diameter of 4 inches (10.16 cm); the tube surface temperature was 9500° R (5280° K). Engine 4 of this report uses smaller tubes, 2 inches (5.08 cm) in diameter, and a lower surface temperature, 9100° R (5060° K). Neither engine has a clear advantage over the other. A lower surface temperature would probably be easier to achieve. But a smaller tube diameter requires more tubes, for the same frontal area, and this would probably increase the mechanical complexity of the engine.

SUMMARY OF RESULTS

Specific impulse, thrust to core-weight ratio, and hydrogen-to-uranium-flow-rate ratio were calculated for a radiation-heat-transfer liquid-core nuclear rocket engine. The calculations were performed for the following assigned values of the indicated parameters:

<table>
<thead>
<tr>
<th>Hydrogen flow rate</th>
<th>Reactor pressure</th>
<th>Surface temperature</th>
<th>Tube diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>lb/sec  kg/sec</td>
<td>atm N/cm²</td>
<td>°R</td>
<td>°K</td>
</tr>
<tr>
<td>0 to 1 0 to 0.45</td>
<td>0 to 500</td>
<td>1000</td>
<td>11000</td>
</tr>
<tr>
<td>200</td>
<td>500</td>
<td>1000</td>
<td>11000</td>
</tr>
<tr>
<td>500</td>
<td>1000</td>
<td>11000</td>
<td>110000</td>
</tr>
</tbody>
</table>

The results of this study for these ranges of conditions are summarized as follows:
1. The highest specific impulse calculated was 1660 seconds.
2. The maximum thrust to core-weight ratio calculated was 19.
3. The highest ratio of hydrogen to uranium flow rates was 100.
(Surface temperature must be lower than 9000° R (5000° K) to increase this ratio above 100.)
4. These maximums can not be obtained together. One or two performance characteristics can be maximized at the expense of the others.
5. The following "compromise engine" illustrates the kind of performance available when intermediate values are chosen for each major parameter:
Specific impulse, sec ................................................................. 1400
Thrust to core-weight ratio ......................................................... 4
Hydrogen-to uranium-flow-rate ratio ........................................... 50

This performance is obtained by using a 2-inch (5.08-cm) tube, a surface temperature of 9100° R (5060° K), and a reactor pressure of 200 atmospheres (2020 N/cm²).

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, May 25, 1967,
122-28-02-16-22.
### APPENDIX - SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{A}$</td>
<td>free-flow factor (flow area divided by frontal area)</td>
</tr>
<tr>
<td>C</td>
<td>mass fraction</td>
</tr>
<tr>
<td>D</td>
<td>diameter</td>
</tr>
<tr>
<td>F</td>
<td>thrust</td>
</tr>
<tr>
<td>G</td>
<td>weight flow rate per unit flow area</td>
</tr>
<tr>
<td>g</td>
<td>gravitational constant</td>
</tr>
<tr>
<td>H</td>
<td>enthalpy</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>specific impulse of exhaust mixture</td>
</tr>
<tr>
<td>$I_{sp}^*$</td>
<td>specific impulse of pure hydrogen</td>
</tr>
<tr>
<td>J</td>
<td>mechanical equivalent of heat</td>
</tr>
<tr>
<td>j</td>
<td>mass-transfer factor, eq. (3)</td>
</tr>
<tr>
<td>L</td>
<td>length</td>
</tr>
<tr>
<td>M</td>
<td>molecular weight</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>average molecular weight of mixture</td>
</tr>
<tr>
<td>P</td>
<td>partial pressure</td>
</tr>
<tr>
<td>$P^0$</td>
<td>vapor pressure</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>W</td>
<td>weight</td>
</tr>
<tr>
<td>$\dot{w}$</td>
<td>weight flow rate</td>
</tr>
<tr>
<td>X</td>
<td>mole fraction in liquid phase</td>
</tr>
<tr>
<td>Y</td>
<td>mole fraction in vapor phase</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
</tbody>
</table>

**Subscripts:**
- b: bulk
- c: core
- H$_2$: hydrogen
- i: component i
- NbC: niobium carbide
- o: outside
- r: reactor
- s: surface
- t: tube
- U: uranium
- UC$_2$: uranium carbide
REFERENCES


6. Masser, C. C.: Vapor-Pressure Data Extrapolated to 1000 Atmospheres (1.01x10^8 N/m^2) for 13 Refractory Materials With Low Thermal Absorption Cross Sections. NASA TN D-4147, 1967.


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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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