

# NASA TECH BRIEF



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## The $X^2$ Statistic and Goodness of Fit Test

The test for goodness of fit consists of testing the hypothesis that a given set of  $N$  observations constitutes values of a random variable with a specified frequency function  $f(x)$ . The  $X^2$  statistic has been found to be a useful measure of the discrepancy between the actual distribution of a set of data points and the theoretical distribution of a random variable of which the data points supposedly are values. Thus the  $X^2$  statistic is frequently used in goodness of fit tests. Unfortunately the probability density function of  $X^2$  is quite involved. In fact it has been only recently that explicit information, either in terms of a generating function or an analytical representation, on the distribution of  $X^2$  has been provided. This difficulty has been mitigated somewhat by the fact that for fairly large samples the distribution of the  $X^2$  statistic is closely approximated by the distribution of the well known chi-square variable. Hence in a goodness of fit test, although it is the  $X^2$  statistic which is calculated, it is from the appropriate table of the chi-square distribution that a critical value for the test is chosen. The subject of the chi-square approximation to the  $X^2$  statistic and the circumstances under which it properly can be used has had much discussion in the statistical literature. Authors treating this question frequently have been vague and have not been in general agreement. Thus when confronted with marginally small samples to be tested for goodness of fit,

the investigator discovers in the literature imprecise and contradictory advice as to how to proceed. This unsatisfactory situation is corrected in a report that provides tables of critical values of the  $X^2$  statistic which are applicable to those cases in which the use of the chi-square approximation is questionable. The  $X^2$  statistic and its distribution, its applicability to goodness of fit tests, and its approximation by the chi-square variable are discussed in some detail. An example is also provided to demonstrate that the proper application of the tables can add considerable precision to a goodness of fit test when small samples are involved.

### Note:

Copies of the report may be obtained from:  
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