ON THE UTILITY OF CONVENTIONAL TURBULENCE EXPERIMENTAL METHODS IN THE STUDY OF PLASMA FLUCTUATIONS

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ABSTRACT

Experimental methods common in studies of ordinary fluid-dynamic turbulence are found to have significant application in the studies of plasma fluctuations. The low-frequency fluctuations, believed to have connection with problems of anomalous diffusion, are easily measured with presently available turbulence-measuring equipment. In particular, the space-time correlations of plasma fluctuations are useful in revealing convective effects. The rejection of various portions of the frequency spectra of the signals prior to their being correlated allows a further insight into the relationships between low-frequency peak portion and the remaining spectral portion of the fluctuations. The techniques and their application are discussed for the case of an annular Hall-current plasma accelerator. A slightly ionized argon discharge is subjected to an axial electric field and a radial magnetic field. The fluctuations are measured using Langmuir probes. In addition to the r.m.s. magnitude and amplitude spectra, the space-time correlations of the fluctuations have been made with a correlation analyzer having a response up to 500 khz. The results are compared with those obtained in fluid-dynamic turbulence studies.

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INTRODUCTION

The study of fluid-dynamic turbulence has reached a high level of sophistication and understanding. Experimental procedures and techniques (for example, ref. 1) in this area are well developed and results obtained give a fairly clear picture of the turbulent regime. The study of turbulence or turbulent-like processes in plasmas, on the other hand, is in a rudimentary stage. Very little experimental work has been done and the techniques are not well-defined. The present paper describes an attempt to apply some of the well-developed techniques of fluid-dynamic turbulence to studies of the fluctuations or turbulent processes found in plasmas.

It is well-known that for a turbulent process the correlation and spectrum functions do not completely describe what is going on. However, because of their relative simplicity and utility, considerable use has been made of them in fluid-dynamic turbulence. In experiments, one must often deal with the one-dimensional spectrum expressed in terms of frequency and cross-correlation rather than the three-dimensional spectrum expressed in wave-number and correlation in which the theoretical results are usually expressed. Despite these limitations, considerable knowledge has resulted on the behavior of the fluctuating physical quantities at least phenomenologically and often fundamentally as well.
In the application of the fluid-dynamic techniques to the study of plasma fluctuations a number of similarities are found. The major differences are in the types of quantities to be measured, and in the instruments available for their measurement. In an ordinary fluid, pressure and velocity fluctuations are easily and directly measurable. Moreover, in combination these quantities carry most of the meaningful information about the process. In a plasma, fluctuations of density and velocity or temperature may occur for each charged species, in addition to electric and magnetic field fluctuations. These quantities are not all individually accessible to existing instrumentation.

The present study is limited to a single typical plasma, that in a low-density annular Hall-current plasma accelerator. It is also limited to one type of sensor of the fluctuations. Langmuir probes are used under various conditions of bias.

APPARATUS AND PROCEDURE

The plasma used was generated by the low-density Hall-current accelerator (ref. 2) shown in fig. 1. Argon gas was fed into the plasma chamber through holes in the annular anode. An emitting cathode, a radial arrangement of tantalum wires, was positioned as shown. The electric field was axial. The magnet poles as shown produce a radial magnetic field, perpendicular to the applied electric field. The plasma accelerator used for the present study has an inner diameter \( (d_I) \) of
5.08 cm and an outer diameter ($d_0$) of 10.16 cm. A cylindrical coordinate system (r,θ,z) will be used with the z = 0 plane being the end plane of the magnet. The anode is at $z = -7.62 \text{ cm}$. The plasma has an average electron temperature of about 10 ev and an average electron number density of about $5.0 \times 10^{10}/\text{cm}^3$. The neutral pressure is about 5 millitorr and the current is 0.60 amperes.

The fluctuations were measured with 0.025 mm diameter Langmuir probes at various conditions of probe bias voltage. The probes are located radially one-fifth of the annular plasma thickness from the inner wall. The two probes used in the correlation measurements are identified by the subscripts, 1 or 2, on coordinates describing the probe positions. The fluctuating signals were obtained across 1000 $\Omega$ resistors. A schematic of the instrumentation used for the correlation and spectrum measurements is given in fig. 2. The output signals of the two probes were fed into differential amplifiers. The outputs of the amplifiers could then be passed through filters which were capable of high- or low-pass frequency filtering with a fall-off of 18 db per octave. The frequency spectrum up through 600 khz was measured and recorded using a heterodyne-type spectrum analyzer. For the correlation work the fluctuations from the two probes after suitable amplification were fed into a Honeywell 9410 Correlator. This instrument was found to have a response flat up to 300 khz and at 500 khz was only 3 db down. The outputs of the correlator, namely the autocorrelation or cross-correlation as a function of the time-delay, were fed into an x-y recorder.
RESULTS AND DISCUSSION

Amplitude spectra for the plasma and for a typical fluid case are shown in fig. 3. The relative amplitudes should be considered arbitrary. The frequency coordinates for the two cases differ by a factor of 10, to facilitate comparison. The fluid-dynamic spectrum was obtained from measurements of the pressure fluctuations at the boundary of a fully-developed turbulent boundary layer as given in Reference 1. The plasma fluctuation spectrum was obtained using a Langmuir probe at ion saturation, positioned at $z = 0$ and $\theta = \pi$.

The shape of the spectrum for the fluid case is typical of those one-dimensional spectra found in turbulent flows, particularly boundary layers. The spectrum of the plasma fluctuations except for one striking feature has a general similarity to the spectrum for the fluid case. The difference is that a strong resonance peak and even a first harmonic peak occur at mid-frequencies. In this case, the peaks result from a helical disturbance rotating about the axis. (This helical phenomenon is common to many plasmas and its occurrence may be the result of one of several types of plasma instabilities.) As is seen, plasma fluctuations possess discrete frequency characteristics and turbulent-like characteristics at the same time. Such a spectrum is rarely encountered in studies of fluid-dynamic turbulence.
The plasma fluctuations represented by the spectrum of fig. 3 could consist primarily of fluctuations in density. As shown in Ref. 2, the Langmuir probe biased to collect either ions or electrons yielded a similar, peaked spectrum. When drawing no net current, and ostensibly measuring potential fluctuations, the probe gave a spectrum wherein the resonant peaks were greatly diminished. This spectrum bore an even closer resemblance to that of the fluid-dynamic turbulence. The similarity in shape of the spectra obtained at various probe biases—apart from the resonant peak—suggests that there are turbulent-like processes within the plasma which may involve fluctuations in potential, density, and/or velocity of the charged species. These variables are not cleanly separable at the present time.

For the fluid-dynamic spectrum the fall-off in magnitude at the higher frequencies is the consequence of viscous damping. For the spectrum of plasma fluctuations the fall-off in magnitude at the higher frequencies is not clearly understood. There are several possibilities even including the effect on the fluctuations of passing through the sheath surrounding the probe.

In Fig. 4 the cross-correlation coefficient, \( R \), is presented as a function of \( \tau \), the time displacement in microseconds, for the fluid-dynamic and plasma fluctuations. The \( \tau \) coordinates for the two
cases differ by a factor of 10. For the fluid-dynamic case from Ref. 1 the cross-correlations are for wall-pressure fluctuations of a fully-developed turbulent boundary layer. The transducer separations are indicated in the figure. The cross-correlations of the plasma fluctuations were obtained using two Langmuir probes at ion saturation positioned as indicated. The several curves for the fluid-dynamic fluctuations indicate that turbulence is being convected in the streamwise direction approximately at 0.80 times the free-stream velocity. This represents the average convection velocity of essentially an entire set of turbulent eddies having a spectrum such as that in Fig. 3. For the plasma fluctuations there exists a quite different situation. There is an apparent convection, but it is of different character. For the plasma there is not a convection of the entire spectrum of fluctuations as in the fluid case. The apparent convection affects only the peak portion of the spectrum which appears in the cross-correlation as a damped sinusoidal curve. This convection is seen as a displacement in $\tau$ of the peaks of the sinusoidal portion of the cross-correlation in going from one curve to the next. As pointed out in Ref. 2, this is associated with a helical disturbance rotating about the axis. The average convection velocity of this helical disturbance is $9 \times 10^5$ cm-sec$^{-1}$ in the axial direction. For convenience the turbulent-like fluctuations
(the entire spectrum of fluctuations excepting the peak portion of the spectrum) are designated as the non-coherent fluctuations. This designation does not imply that the cross-correlation curve is zero for all \( \tau \). The narrow peaks at \( \tau \) near 0 are the result of the correlation of the non-coherent or turbulent-like portion of the fluctuations. For all of the curves in the lower part of Fig. 4 the narrow peaks occur for values of \( \tau \) less than 0.06 microseconds. This implies that the fluctuations so represented have no convection velocity less than \( 10^8 \)cm-sec\(^{-1}\) (if convection occurs at all for this part of the fluctuation). Thus for the plasma fluctuations the convection occurring at \( 10^6 \)cm-sec\(^{-1}\) is definitely not the broad band convection of the entire set of fluctuations. The plasma operating parameters can be so adjusted that the helical disturbance is made weaker relative to the rest of the fluctuations. Then the peak near \( \tau = 0 \) increases in magnitude, while the sinusoidal part of the cross-correlation diminishes. It is possible to obtain a condition where the cross-correlation peak near \( \tau = 0 \) is substantially larger than the sinusoidal portion peaks representing the coherent oscillations. These results are consistent with a fluctuating field comprised of two parts, (1) a turbulent-like field with a significant degree of correlation near \( \tau = 0 \) existing throughout the plasma volume and (2) a coherent helical oscillation or
instability also existing throughout the plasma volume. The relationships between the helical coherent oscillation and the turbulent-like field needs further investigation. From the aforementioned differences in convection it would appear that these two are not directly connected. In other words, one may occur independently of the other and on this basis a transfer of energy from the peak portion to the higher frequencies would not be expected. This is not the situation in the fluid-dynamic turbulence where energy is transferred from the free-stream to the large-size eddies which in turn transfer energy to smaller-size eddies, the process continuing until ended by viscous dissipation. On the other hand the strong resemblance of the plasma fluctuation spectrum to the fluid-dynamic one along with the theoretical results of C.M. Tchen (ref. 3) would lead one to expect that there should be some relationship between the fluctuations over the entire spectrum as in fluid-dynamic turbulence.

In the case of fluid-dynamic turbulence filtering of the original signals prior to performing the cross-correlation gives results which are generally similar in nature to the full spectrum results. Because of the shape of the cross-correlation results of Fig. 4, filtering of the original signals should give results different from those for the full spectrum. The results of one such procedure are given in Fig. 5.
which compares the $R$ versus $\tau$ curves for no frequency filtering, 50 khz high-pass frequency band and 50 khz low-pass frequency band.

The two probes are at ion saturation with $z = 0$ and $\theta_2 - \theta_1 = -\pi$. The value of 50 khz for the band rejection was chosen to assure sufficient rejection of the resonance peak during the high-pass band analysis.

Comparison of the three curves shows that there is no general similarity between the full-spectrum and partial spectrum correlations as for fluid-dynamic turbulence. For the plasma fluctuations rejecting the higher frequency fluctuations yields an almost sinusoidal curve which shows a loss of coherence as $|\tau|$ increases. On the other hand removal of the resonance peak leaves a cross-correlation curve which peaks sharply at $\tau = 0$ and is essentially zero at other values of $\tau$. Thus, this result once again shows the separability of the two types of fluctuations.

Although the physical interpretation of the fluctuations sensed by the Langmuir probe remains subject to improvement, some insight may be gained by comparing cross and auto-correlations obtained at different probe biases. Such results are shown on Fig. 6. The probes are located at $z = 0$ and $\Delta \theta = -\pi$. The autocorrelations show that in going from ion saturation to floating potential there is a loss in the
coherent part of the fluctuations. The cross-correlation for both probes collecting ions is of the same general nature as the full-frequency curve of figure 5, although the coherent part is even stronger at this condition. The cross-correlation for both probes floating shows a dramatic loss of the coherent part, and resembles the curve of Fig. 5 for the 50 khz high-pass condition. If the floating probes are indeed sensing fluctuations in plasma potential, the similarity may imply that the turbulent-like portion of the fluctuations at ion saturation also results from potential fluctuations. Alternatively one could suggest that the turbulence-like phenomenon being sensed contains interrelated fluctuations in potential, density, and/or velocity.

The use of the conventional cross-correlation and spectral techniques for the plasma fluctuations has been shown to yield valuable information. The additional use of frequency filtering has been shown to be of particular value in studying the convection of coherent phenomena in a plasma such as the helical disturbance. There are several limitations to the use of the cross-correlation technique in analyzing the fluctuations. The first is that it is restricted in frequency range to that of the available correlators. This has not posed a problem in the present case. Another limitation of the correlation technique may be found in those plasma fluctuations which have spectra containing
multiple peaks of comparable magnitude. In some of these cases, however, it may be possible to handle this type of data by a judicious use of frequency filtering prior to cross-correlation.

The ultimate goal in using the cross-correlation and spectral techniques is to determine the effect of the plasma fluctuations on the bulk properties of the plasma. An example is the measurement of the diffusion occurring as a result of these fluctuations. The successful attainment of this ultimate goal is actually less hampered by the cross-correlation and spectral techniques than by the interpretation of the transducer signals. For the fluid-dynamic turbulence the hot-wire anemometers and piezoelectric ceramic pressure transducers have been developed into fairly sophisticated systems and they present useful and well-understood signals of the fluctuating velocities and pressures. For the plasma fluctuations the situation is by comparison in a much less desirable and more primitive state.

What can be done to provide further knowledge of the processes existing within plasma fluctuations and instabilities? The fluctuating quantities in a plasma may include the density, directed and thermal velocity of each species, as well as electric field or potential and magnetic field intensity. When the fluctuating magnetic fields are of sufficient intensity, magnetic coil probes or Hall effect probes can be used. However, in the present example the intensities of the fluctuating magnetic fields are apparently quite small and their
measurement has proven to be quite an undertaking not yet successfully achieved. The measurement of fluctuating plasma potential can be attempted using emissive probes, although this technique is not well-developed.

The use of the Langmuir probe in measuring plasma fluctuations can yield potential fluctuations as well as fluctuations in plasma density or velocity. However, the interpretation of the signals does involve some unverified assumptions. For example at ion saturation the fluctuation sensed by the probe is actually a fluctuation in $n\nu$, the product of density and velocity. Only by neglecting the velocity fluctuation can one call the measurement a fluctuation in density. In addition, the measurements at floating yield potential fluctuations, but these are about the mean floating potential and not the mean plasma potential. Where a sheath exists about the Langmuir probe, the dynamics of the sheath at the higher frequencies should be considered.

Another goal in the measurement of plasma fluctuations by the techniques discussed herein very obviously is the examination of plasma instabilities per se. For this the cross-correlation and spectral techniques may prove to be too cumbersome and less satisfactory than other simpler and faster methods. However, it may be desirable to determine the type and degree of interplay
of the turbulent-like fluctuations with the instabilities. Then, the use of the cross-correlations and spectral techniques may be not only valuable but essential.

CONCLUDING REMARKS

The utility of the cross-correlation and spectral techniques has been shown for a particular case of plasma fluctuations. By comparing with a particular example of fluid-dynamic turbulent flow, the cross-correlations and spectra for the plasma fluctuations were shown to be similar in some respects to the fluid turbulence. The principal difference arises from the coherent oscillations in the plasma. For these fluctuations frequency filtering of the signals before they are correlated allows a separation of the coherent from the turbulent-like part of the fluctuations.

The limitations in the application of these techniques are not fundamental ones. Two principal difficulties may be encountered in future work with the techniques as used herein. One is the availability of equipment with frequency range suitable for measuring the cross-correlations; the other is proper physical interpretation of the data obtained from the fluctuation-sensing transducers.
REFERENCES


Figure 2. - Schematic diagram of instrumentation used to obtain cross-correlations and spectra for plasma fluctuations.

Figure 3. - Comparison of amplitude spectrum for plasma fluctuations (Langmuir probe at ion saturation, $z \cdot 0$, $\theta \cdot \pi$) with that for fluid dynamic fluctuations (wall-pressure fluctuations of a fully-developed turbulent boundary layer Ref. 1).
Figure 4. - Comparison of cross-correlation coefficient versus the time displacement $\tau$ for plasma fluctuations (Langmuir probes at ion saturation) with those for fluid-dynamic fluctuations (wall-pressure fluctuations of a fully developed boundary layer, Ref. 1).

Figure 5. - Comparison of cross-correlations for full frequency band plasma fluctuations with those subjected to 50 kHz high-pass and low-pass frequency filtering; probe at ion saturation, $z_1 = -22$ cm, and $z_2 - \theta_1 = -\pi$. 
Figure 6. Comparison of correlations of plasma fluctuations for probes at ion saturation with those for probes floating; $z_1 = z_2 = 0$, $\theta_2 - \theta_1 = -\pi$. 