CORRELATION OF LOCAL HEAT-TRANSFER COEFFICIENTS FOR SINGLE-PHASE TURBULENT FLOW OF HYDROGEN IN TUBES WITH TEMPERATURE RATIOS TO 23

by Maynard F. Taylor

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JANUARY 1968
CORRELATION OF LOCAL HEAT-TRANSFER COEFFICIENTS FOR SINGLE-PHASE TURBULENT FLOW OF HYDROGEN IN TUBES WITH TEMPERATURE RATIOS TO 23

By Maynard F. Taylor

Lewis Research Center
Cleveland, Ohio
The existing methods of correlating and predicting single-phase turbulent heat-transfer coefficients with variable properties give coefficients which are sometimes in poor agreement with measured values. Each prediction method can be used only over a limited range of pressure, temperature, ratio of axial position to diameter, and ratio of surface to fluid bulk temperature ratio.

Local heat-transfer coefficients for hydrogen from 10 investigations using symmetrically heated straight tubes were used to determine a single equation that will predict heat-transfer coefficients over a wide range of conditions including surface to fluid bulk temperature ratios to 23.

The correlation equation predicts heat-transfer coefficients with acceptable accuracy over a wide range of conditions:

\[ \text{Nu}_b = 0.023 \, \text{Re}_b^{0.8} \, \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right) \exp \left( - \left( 0.57 - \frac{1.59}{x/D} \right) \right) \]

where \( \text{Nu}_b \) is the bulk Nusselt number, \( \text{Re}_b \) is the bulk Reynolds number, \( \text{Pr}_b \) is the bulk Prandtl number, \( T_s \) is surface temperature, and \( T_b \) is bulk temperature. This equation does not predict heat-transfer coefficients with acceptable accuracy in the near-critical pressure and temperature region where the inlet temperature is between \( 45^\circ \) R \( (25^\circ \) K) and the transposed critical temperature (the temperature at which the specific heat of a fluid at constant pressure reaches a maximum), and where the inlet pressure is between the critical pressure and 530 psia \((3.65 \text{ MN/m}^2)\). The correlation equation also accurately predicted variable property heat-transfer coefficients for helium gas and nitrogen gas.
INTRODUCTION

Hydrogen is attractive for use as a propellant for both advanced chemical and nuclear rockets because of its low molecular weight, which gives a high specific impulse \( (I_{sp} \propto \sqrt{T/m}) \), and because of its excellent heat-transfer properties. These propulsion system concepts produce extreme conditions which are a severe test for existing methods of predicting heat-transfer coefficients.

Most of the experimental heat-transfer data for hydrogen was taken in a tube at supercritical pressures near the critical temperature. There are some data for hydrogen flowing through tubes with surface temperatures near those encountered in a solid core nuclear rocket. At present, no data exist for fluid temperatures above 2800\(^{0}\)R (1556\(^{0}\)K) for flow through tubes.

In this investigation, all available single-phase heat-transfer data for hydrogen flowing turbulently through symmetrically heated straight circular tubes, along with all helium and nitrogen data with large variations in the physical properties, were studied. The range of conditions covered by the hydrogen data examined in this investigation is shown in table I. The data do not cover all possible combinations of the various conditions (e.g., large \( T_s/T_b \) at large \( x/D \)).

A new correlating equation is presented which correlates and predicts single-phase forced-convection heat-transfer coefficients for turbulent flow through tubes over a much

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Ratio of maximum to minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of distance from entrance of test section to inside diameter of test section, ( x/D )</td>
<td>2.0</td>
<td>252</td>
<td>126</td>
</tr>
<tr>
<td>Ratio of surface to bulk temperature, ( T_s/T_b )</td>
<td>1.1</td>
<td>27.6</td>
<td>25</td>
</tr>
<tr>
<td>Inlet temperature, ( T_i ), (^{0})R; (^{0})K</td>
<td>45; 25</td>
<td>573; 318</td>
<td>13</td>
</tr>
<tr>
<td>Absolute static pressure, ( p ), psia; ( MN/m^2 )</td>
<td>18; 0, 124</td>
<td>2500; 17.2</td>
<td>140</td>
</tr>
<tr>
<td>Heat flux, ( Q/S ), Btu/(sec)(in.(^2)); MW/m(^2)</td>
<td>0.036; 0.059</td>
<td>27.6; 45.6</td>
<td>767</td>
</tr>
<tr>
<td>Bulk Reynolds number, ( Re_b )</td>
<td>7500</td>
<td>13 800 000</td>
<td>1840</td>
</tr>
<tr>
<td>Surface temperature, ( T_s ), (^{0})R; (^{0})K</td>
<td>53; 29.5</td>
<td>5600; 3110</td>
<td>106</td>
</tr>
</tbody>
</table>
greater range of conditions than was previously possible. This single equation can be used in place of the several correlating equations, each of which is recommended for use over a limited range of conditions.

SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>constant from ref. 11 used in eq. (12)</td>
</tr>
<tr>
<td>$C_2$</td>
<td>exponent of $T_s/T_b$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat of gas at constant pressure</td>
</tr>
<tr>
<td>$D$</td>
<td>inside diameter of test section</td>
</tr>
<tr>
<td>$G$</td>
<td>mass flow rate per unit cross-sectional area</td>
</tr>
<tr>
<td>$h$</td>
<td>local heat-transfer coefficient</td>
</tr>
<tr>
<td>$h_{av}$</td>
<td>average heat-transfer coefficient</td>
</tr>
<tr>
<td>$h_S$</td>
<td>asymptotic heat-transfer coefficient</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>specific impulse</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity of gas</td>
</tr>
<tr>
<td>$L$</td>
<td>length of test section</td>
</tr>
<tr>
<td>$m$</td>
<td>molecular weight of gas</td>
</tr>
<tr>
<td>$Nu$</td>
<td>Nusselt number, $hD/k$</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number, $c_p \mu/k$</td>
</tr>
<tr>
<td>$p$</td>
<td>absolute static pressure</td>
</tr>
<tr>
<td>$Q$</td>
<td>rate of heat transfer to gas</td>
</tr>
<tr>
<td>$Re_b$</td>
<td>bulk Reynolds number, $GD/\mu_b$</td>
</tr>
<tr>
<td>$Re_f$</td>
<td>modified film Reynolds number, $\rho_f V_b D/\mu_f$</td>
</tr>
<tr>
<td>$Re_s$</td>
<td>modified surface Reynolds number, $\rho_s V_b D/\mu_s$</td>
</tr>
<tr>
<td>$Re_{0.4}$</td>
<td>modified Reynolds number, $\rho_{0.4} V_b D/\mu_{0.4}$</td>
</tr>
<tr>
<td>$S$</td>
<td>heat-transfer area</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$T^*$</td>
<td>transposed critical temperature (temperature at which specific heat of fluid at constant pressure reaches a maximum)</td>
</tr>
<tr>
<td>$V$</td>
<td>velocity</td>
</tr>
</tbody>
</table>
X  parameter for specifying reference temperature  \( T_X = X(T_s - T_b) + T_b \)
x  distance from entrance of test section
\( \mu \)  absolute viscosity of gas
\( \nu \)  kinematic viscosity of gas, \( \mu/\rho \)
\( \rho \)  density of gas

Subscripts:
\( \text{av} \)  average
\( \text{b} \)  bulk (when applied to properties, indicates evaluation at bulk temperature \( T_b \))
\( \text{c} \)  critical
\( \text{cal} \)  calculated
\( \text{ex} \)  experimental
\( \text{f} \)  film (when applied to properties indicates evaluation at film temperature \( T_f \))
\( \text{i} \)  inlet
\( \text{s} \)  surface (when applied to properties indicates evaluation at surface temperature \( T_s \))
\( 0.4 \)  reference (when applied to properties indicates evaluation at \( T_{0.4} = T_b + 0.4(T_s - T_b) \))

**DISCUSSION OF EXISTING CORRELATIONS**

For many years, heat-transfer coefficients were predicted by evaluating the physical properties and density of the fluid at a reference temperature \( T_X = X(T_s - T_b) + T_b \).

Humble, Lowdermilk, and Desmon (ref. 1) correlated average heat-transfer coefficients for heating and cooling of air flowing through a smooth tube with wall temperatures to 3050° R (1695° K) and surface to fluid bulk temperature ratios (\( T_s/T_b \)) from 0.46 to 3.5 using the following equation in which the physical properties are evaluated at the film reference temperature \( X = 0.5 \):

\[
\text{Nu}_f = 0.034 \, \text{Re}_f^{0.8} \, \text{Pr}_f^{0.4} \left( \frac{L}{D} \right)^{-0.1}
\]  \( \text{(1)} \)

Taylor and Kirchgessner (ref. 2) extended the range of wall temperature to 5900° R (3278° K) and the range of \( T_s/T_b \) to 3.9 and measured both local and average heat-
transfer coefficients for helium flowing through a tungsten tube. They found that the average heat-transfer data for helium along with some air data from reference 1 could best be correlated by

\[ \text{Nu}_{av,f} = 0.021 \text{Re}_f^{0.8} \text{Pr}_f^{0.4} \left[ 1 + \left( \frac{L}{D} \right)^{-0.7} \right] \] (2)

while the local heat-transfer coefficients for helium were correlated by

\[ \text{Nu}_f = 0.021 \text{Re}_f^{0.8} \text{Pr}_f^{0.4} \] (3)

Thompson and Geery (ref. 3) attained \( \frac{T_s}{T_b} \) as great as 16.5 by lowering the inlet temperature of the hydrogen to 55° R (30.6° K). At supercritical pressures, they reported two modes or regimes of heat transfer: A lower heat-flux regime A for which they recommend as the correlating equation

\[ \text{Nu}_b = 0.028 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right)^{-0.64} \] (4)

and high heat flux regime B with the correlating equation

\[ \text{Nu}_b = 0.0217 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right)^{-0.34} \] (5)

Unfortunately, there was considerable overlap of the two regimes, which makes the choice of whether equation (4) or (5) should be used exceedingly difficult.

McCarthy and Wolf (ref. 4) report heat-transfer data for both helium and hydrogen with \( \frac{T_s}{T_b} \) up to 11.1 and inlet temperatures from 135° to 540° R (75° to 300° K). They reported that their local heat-transfer coefficients near the exit of their test sections were best correlated by

\[ \text{Nu}_b = 0.025 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right)^{-0.55} \] (6)

but for some reason the following equation was recommended:
\[ \text{Nu}_S = 0.025 \text{Re}_S^{0.8} \text{Pr}_S^{0.4} \] (7)

An analysis by Deissler and Presler (ref. 5) showed that, in general, the best value for \( X \) in the reference temperature concept is \( 0.4 \), and the available data could be represented reasonably well by

\[ \text{Nu}_{0.4} = \frac{\text{Re}_0^{0.75}}{31} \] (8)

Szetela (ref. 6) reported a few heat-transfer coefficients for hydrogen and showed that equation (3) was not suitable for correlating that data.

Weiland (ref. 7) presented local heat-transfer data for helium and hydrogen flowing through a tube with an \( \text{L/D} \) of 252 and showed that equation (3) worked for \( x/D > 30 \) and \( 1.1 < T_s/T_b < 2.5 \); however, for \( 2.5 < T_s/T_b < 4.5 \), there was an effect of \( x/D \), and equation (3) worked only for \( x/D > 160 \).

Taylor (ref. 8) reported heat-transfer coefficients for helium and hydrogen in a study of the effect of dissociation of hydrogen at tube surface temperature to 5600° R (3110° K). It appeared that any dissociation of hydrogen at the tube wall had less effect on the heat-transfer characteristics than does \( T_s/T_b \). Equation (3) did not correlate the data with \( T_s/T_b \) greater than about 3.5.

Dalle Donne and Bowditch (ref. 9) measured heat-transfer coefficients for helium and air and recommended using the following equation for \( 26 \leq x/D \leq 166 \):

\[ \text{Nu}_b = 0.024 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right) \exp - \left[ 0.29 + 0.0056 \left( \frac{x}{D} \right) \right] \] (9)

Perkins and Worsoe-Schmidt (ref. 10) reported heat-transfer coefficients for nitrogen with \( T_s/T_b \) to 7.35 and recommended the following correlation based on bulk temperature

\[ \text{Nu}_b = 0.024 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right)^{-0.7} \left[ 1 + \left( \frac{x}{D} \right)^{-0.7} \left( \frac{T_s}{T_b} \right)^{0.7} \right] \] (10)

and based on wall temperature
They recommend that the entry parameter \( 1 + (x/D)^{-0.7}(T_s/T_b)^{0.7} \) be dropped from equation (10) for \( x/D \) greater than 40 and from equation (11) for \( x/D \) greater than 24.

Simoneau and Hendricks (ref. 11) through the use of a simplified equation

\[
h = C_1 G^{0.8} D^{-0.2} \left( \frac{T_w}{T_b} \right)^{-0.5}
\]

(12)
suggested that the variation of the transport and thermodynamic properties does not influence the computation of the heat-transfer coefficient greatly. The constant \( C_1 \) is 0.048 for hydrogen, 0.020 for helium, 0.0042 for air, and 0.00385 for carbon dioxide.

Taylor (ref. 12) measured heat-transfer coefficients for precooled helium and hydrogen flowing through a tungsten tube with wall temperatures to 5300° R (2945° K) and \( T_s/T_b \) to 8. The use of equations (3), (6), (9), and (12) were studied by using the data of both reference 8 and 12. Simultaneously, Miller and Taylor (ref. 13) investigated the use of equations (3), (6), (8), and (9) to correlate the experimental data of Weiland (ref. 7) which is for an L/D of 250 (which is in the range of L/D of a nuclear rocket reactor). For large \( x/D \), the exponent of \( T_s/T_b \) in equation (9) was influenced too strongly by \( x/D \). It was modified and resulted in the following equation:

\[
\text{Nu}_b = 0.021 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right)^{\text{exp} - \left( 0.29 + 0.0019 \left( \frac{x}{D} \right) \right)}
\]

(13)
The data of Weiland (ref. 7) and Taylor (refs. 8 and 12) were best correlated by equation (13).

Hess and Kunz (ref. 14) made a study of heat transfer to supercritical hydrogen and recommended the equation

\[
\text{Nu}_f = 0.0208 \text{Re}_f^{0.8} \text{Pr}_f^{0.4} \left( 1 + 0.01457 \frac{\nu_s}{\nu_b} \right)
\]

(14)

Hendricks, Simoneau, and Friedman (ref. 15) reported the heat-transfer characteristics for cryogenic hydrogen at pressures from 1000 to 2500 psia (6.9×10^6 to
17.5 MN/m²) and recommended the use of equation (3) for \( x/D \) from 19 to 29, 27 to 46, and 30 to 69 depending on the inside diameter of the test section.

Miller, Seader, and Trebes (refs. 16 and 17) measured hydrogen heat-transfer coefficients with high heat flux, \( T_s/T_b \) to 27.6, and supercritical pressures. They found that a modified form of equation (13)

\[
\text{Nu}_{0.4} = 0.0204 \text{Re}^{0.8} \text{Pr}^{0.4} \left( 1 + 0.00983 \frac{\nu_s}{\nu_b} \right)
\]

(15)

correlated their data best for \( x/D \) from 30 to 60. This range of \( x/D \) also limited the \( T_s/T_b \) to 12.6 or less.

Hendricks, Graham, Hsu, and Friedman (ref. 18) reported heat-transfer coefficients for hydrogen with inlet temperatures as low as 45° R (25° K) at supercritical pressures. They reported that in the supercritical pressure region the gross trend of success with which the data could be correlated with equation (3) increased directly with pressure. In general, however, all the supercritical data were used to test equations (3), (6), and (14), and no systematic attempt was made to study the effect of inlet temperature and inlet pressure.

The heat-transfer coefficients for hydrogen with heat fluxes to 27.6 Btu per second per square inch (45.1 MW/m²) are presented in reference 19 with the recommendation that equation (14) be used with the addition of a term \( C_L \), a function of the coolant temperature. The quantity \( C_L \) varies nonlinearly from 2.0 at 50° R (28° K) to 0.85 at 85° R (47° K):

\[
\text{Nu}_f = 0.0208 C_L \text{Re}_f^{0.8} \text{Pr}_f^{0.4} \left( 1 + 0.01457 \frac{\nu_s}{\nu_b} \right)
\]

(16)

A comprehensive study (ref. 20) of the recommended correlation equations (3) to (6), (9), (14), (15), and (16) was made with the conclusion that none of the equations showed any evidence of being greatly superior to the other correlation equations. Equation (16) was selected for film-temperature-based correlations, and equation (5) was selected for bulk-temperature-based correlations.

Petukhov, Kirillov, and Maidanik (ref. 21) measured heat-transfer coefficients for nitrogen gas with \( T_s/T_b \) to 6 and correlated the data with

\[
\text{Nu}_b = 0.0212 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right)^{-2}
\]

(17)
where $C_2$ is a nonlinear function of $x/D$. In a discussion of reference 21, Taylor presented the same type of correlation equation with different values for the exponent $C_2$. He pointed out that the highest $T_s/T_b$ data available should be used to determine the exponent, since small ratios are not greatly affected by the powers to which they are raised.

The range of temperature, pressure, $T_s/T_b$, and $x/D$ for the correlation equations recommended by references 3, 4, 15, 16, 19, and 20 are shown in figures 1(a), 1(b), 1(c), 1(d), 1(e), and 1(f), respectively. Using the temperature-entropy diagram is a convenient method of showing the location of measured inlet pressure and temperature in relation to the saturation lines and critical pressure and temperature.

METHOD OF CORRELATION

The possibility of success in correlating heat-transfer data by including the ratio of surface to fluid bulk temperature ratio raised to a power which is a function of $x/D$ was reported by references 9, 12, 13, and 21. Unfortunately, in all cases, the correlations were applicable only for large values of $x/D$. Obviously, a correlation which would be valid over the complete length of a tube (including the entrance region), as well as for wide variations in the physical properties, would be extremely useful.

Because of the great interest in hydrogen for use as a nozzle coolant and propellant, this gas has been the subject of a number of experimental investigations covering the extremely wide range of conditions shown in table II. This range of conditions affords not only an excellent test of any correlation but, in addition, provides a sufficiently wide range of parameter variation that their effects are clearly shown.

The procedure of this report was to use the conventional type of equation presented in references 9, 12, 13, and 21.

$$\text{Nu}_b = 0.023 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left(\frac{T_s}{T_b}\right)^{-C_2}$$

(18)

(where $C_2$ is a function of $x/D$) and to determine an exponent $C_2$ which would best correlate the heat-transfer coefficients presented in the investigations shown in table II. The data of each investigator were analyzed individually. All the heat-transfer data points reported in the 11 investigations listed in table II were used in the present investigation. Each investigator's data were analyzed by a digital computer program. At each $x/D$, the exponent of the surface to fluid temperature ratio was computed so that the heat-transfer coefficient calculated by equation (18) was equal to the experimental heat-transfer coefficient.
(a) Correlation from Thompson and Geery (ref. 3; equation (4)) where \( \frac{T_s}{T_b} \) ranges from 1.1 to 8.7 and \( x/f \) ranges from 10 to 30;
equation (5) where \( \frac{T_s}{T_b} \) ranges from 3.5 to 16.5 and \( x/D \) ranges from 10 to 30.

(b) Correlation from McCarthy and Wolf (ref. 4; equation (6)) where \( \frac{T_s}{T_b} \) ranges from 1.5 to 9.2 and \( x/D \) ranges from 15 to 50.

(c) Correlation from Hendricks, Simoneau, and Friedman (ref. 15; equation (3)) where \( \frac{T_s}{T_b} \) ranges from 1.4 to 9.1 and \( x/D \) ranges from 19 to 29, 27 to 46, and 30 to 69.

(d) Correlation from Miller, Seader, and Trebes (refs. 16 and 17; equation (15)) where \( \frac{T_s}{T_b} \) ranges from 1.6 to 12.6 and \( x/D \) ranges from 30 to 50.

(e) Correlation from reference 19; equation (16) where \( \frac{T_s}{T_b} \) ranges from 6.1 to 21 and \( x/D \) ranges from 6 to 34.

(f) Correlation from reference 20; equation (16) where \( \frac{T_s}{T_b} \) ranges from 1.8 to 21 and \( x/D \) ranges from 3.7 to 148.

Figure 1. - Range of hydrogen inlet temperature and inlet pressure covered by correlation equations recommended by various investigators.
### TABLE II. - TEST CONDITIONS FOR VARIOUS SOURCES OF HYDROGEN DATA

<table>
<thead>
<tr>
<th>Investigators</th>
<th>Reference</th>
<th>Ratio of distance from entrance of test section to inside diameter of test section, ( x/D )</th>
<th>Ratio of surface temperature to bulk temperature, ( T_s/T_b )</th>
<th>Inlet temperature, ( \theta_R ) ( \theta_K )</th>
<th>Pressure, psia</th>
<th>Heat input, MN/m(^2) ( Btu/(\text{in.}^2\text{sec}) )</th>
<th>Heat input, MW</th>
<th>Mass flow rate, lb/sec</th>
<th>Mass flow rate, kg/sec</th>
<th>Wall temperature, ( \theta_R ) ( \theta_K )</th>
<th>Inside diameter of test section, ( \text{in.} ) ( \text{cm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thompson and Geery</td>
<td>3</td>
<td>10.3 to 30.9</td>
<td>1.1 to 16.5</td>
<td>55 to 30.6 to 102</td>
<td>682 to 4.70 to 56.6</td>
<td>0.14 to 0.23 to 0.012</td>
<td>0.09 to 0.009 to 0.002</td>
<td>77 to 42.8 to 1.92</td>
<td>0.192 to 0.488</td>
<td>1594 to 885</td>
<td></td>
</tr>
<tr>
<td>McCarthy and Wolf</td>
<td>4</td>
<td>5.8 to 50.2</td>
<td>1.5 to 11.1</td>
<td>135 to 75 to 560</td>
<td>32 to 0.23 to 1354</td>
<td>0.035 to 0.059 to 0.013</td>
<td>0.001 to 0.0005 to 0.128</td>
<td>830 to 461 to 0.194</td>
<td>.493</td>
<td>2240 to 1245</td>
<td>.430</td>
</tr>
<tr>
<td>Weiland</td>
<td>7</td>
<td>2.7 to 252</td>
<td>1.1 to 4.5</td>
<td>222 to 123 to 548</td>
<td>250 to 1.72 to 304</td>
<td>0.24 to 0.392 to (a)</td>
<td>(a) to (a) to 550</td>
<td>294 to 188</td>
<td>0.188</td>
<td>2300 to 1278</td>
<td></td>
</tr>
<tr>
<td>Taylor</td>
<td>8</td>
<td>11.6 to 73.1</td>
<td>1.3 to 5.6</td>
<td>557 to 309 to 573</td>
<td>41 to 0.28 to 79</td>
<td>1.29 to 0.001 to 0.0005</td>
<td>738 to 410 to 0.116</td>
<td>.295</td>
<td>5630 to 3130</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor</td>
<td>12</td>
<td>11.6 to 73.1</td>
<td>1.4 to 8.0</td>
<td>263 to 146 to 304</td>
<td>37 to 0.26 to 68</td>
<td>0.84 to 1.37 to 0.002 to 0.0009</td>
<td>350 to 194 to 0.115</td>
<td>.292</td>
<td>5300 to 2945</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hendricks, Simoneau, and Friedman</td>
<td>15</td>
<td>3.4 to 78.2</td>
<td>1.5 to 11.0</td>
<td>53 to 29.4 to 300</td>
<td>1000 to 6.89 to 167</td>
<td>0.36 to 0.59 to 163</td>
<td>0.08 to 0.02 to 0.18</td>
<td>114 to 63.3 to 1200</td>
<td>.666</td>
<td>438</td>
<td>1.113</td>
</tr>
<tr>
<td>Miller, Seader, and Trebes</td>
<td>16, 17</td>
<td>4.7 to 47.4</td>
<td>1.6 to 27.6</td>
<td>51 to 28.3 to 69</td>
<td>458 to 3.16 to 38.3</td>
<td>1.37 to 2.4 to 39.2</td>
<td>0.160 to 0.737 to 1730</td>
<td>59.4 to 0.211</td>
<td>0.536</td>
<td>.211</td>
<td>0.536</td>
</tr>
<tr>
<td>Hendricks, Graham, Hsu, and Friedman</td>
<td>18</td>
<td>4.9 to 114</td>
<td>1.1 to 15.6</td>
<td>45 to 25 to 115</td>
<td>220 to 1.52 to 63.9</td>
<td>0.32 to 0.52 to 3.3</td>
<td>0.029 to 0.195 to 1311</td>
<td>25 to 0.188</td>
<td>.478</td>
<td>.507</td>
<td>1.288</td>
</tr>
<tr>
<td>Aerojet-General Corp.</td>
<td>19</td>
<td>6.7 to 33.9</td>
<td>6.1 to 21.4</td>
<td>53 to 29.4 to 69</td>
<td>696 to 4.80 to 38.3</td>
<td>6.4 to 10.5 to 45.1</td>
<td>0.001 to 0.0018 to 1681</td>
<td>934 to 0.147</td>
<td>.373</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gladden and Watt</td>
<td>(b)</td>
<td>2.0 to 127</td>
<td>1.4 to 6.3</td>
<td>65 to 36.1 to 170</td>
<td>18 to 0.12 to 94.5</td>
<td>0.12 to (a) to 0.011</td>
<td>0.004 to 0.0018 to 250</td>
<td>139 to .335</td>
<td>.651</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( ^a \)Not given.

\( ^b \)Unpublished data, Lewis Research Center.
The computed exponents determined for all the experimental data were curve fit by using first-, second-, third-, and fourth-order polynomials in $\frac{D}{x}$. Equal weight was given to each investigator's data. The fourth-order polynomial gave the best fit but was only slightly better than the more easily used and remembered first-order fit:

$$C_2 = 0.57 - \frac{1.59}{\frac{x}{D}}$$

Equations (18) and (19) are combined to give

$$\text{Nu}_b = 0.023 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left(\frac{T_s}{T_b}\right) \exp \left(-\left(0.57 - \frac{1.59}{\frac{x}{D}}\right)\right)$$

A plot of $C_2$ calculated from equation (19) is shown in figure 2. Exponents of Dalle Donne and Bowditch (ref. 9); Miller and Taylor (ref. 13); and Petukhov, Kirillov, and Maidanik (ref. 21) are also shown for comparison.

Equations (18) and (19) are combined to give
TABLE III. - PERCENT OF LOCAL HEAT-TRANSFER COEFFICIENT THAT DEVIATES LESS THAN ±25 PERCENT FROM EXPERIMENTAL HEAT-TRANSFER COEFFICIENTS FOR VARIOUS INVESTIGATORS

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Reference</th>
<th>Number of data points used</th>
<th>Percent of $h_{cal}$ that deviates less than ±25 percent from $h_{ex}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1, hydrogen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thompson and Geery</td>
<td>3</td>
<td>154</td>
<td>68</td>
</tr>
<tr>
<td>McCarthy and Wolf</td>
<td>4</td>
<td>249</td>
<td>86</td>
</tr>
<tr>
<td>Weiland</td>
<td>7</td>
<td>96</td>
<td>99</td>
</tr>
<tr>
<td>Taylor</td>
<td>8 and 12</td>
<td>208</td>
<td>97</td>
</tr>
<tr>
<td>Hendricks, Simoneau, and Friedman</td>
<td>15</td>
<td>934</td>
<td>88</td>
</tr>
<tr>
<td>Miller, Seader, and Trebes</td>
<td>16 and 17</td>
<td>560</td>
<td>73</td>
</tr>
<tr>
<td>Hendricks, Graham, Hsu, and Friedman</td>
<td>18</td>
<td>816</td>
<td>89</td>
</tr>
<tr>
<td>Aerojet-General Corp.</td>
<td>19</td>
<td>232</td>
<td>98</td>
</tr>
<tr>
<td>Gladden and Watt</td>
<td>(a)</td>
<td>425</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3674 Total</td>
<td>87 Average</td>
</tr>
<tr>
<td>Region 2, hydrogen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miller, Seader, and Trebes</td>
<td>16 and 17</td>
<td>60</td>
<td>42</td>
</tr>
<tr>
<td>Hendricks, Graham, Hsu, and Friedman</td>
<td>18</td>
<td>888</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>948 Total</td>
<td>40 Average</td>
</tr>
<tr>
<td>Helium gas</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>McCarthy and Wolf</td>
<td>4</td>
<td>111</td>
<td>96</td>
</tr>
<tr>
<td>Weiland</td>
<td>7</td>
<td>96</td>
<td>99</td>
</tr>
<tr>
<td>Taylor</td>
<td>8 and 12</td>
<td>152</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>359 Total</td>
<td>98 Average</td>
</tr>
<tr>
<td>Nitrogen gas</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Perkins and Worsoe-Schmidt</td>
<td>10</td>
<td>88</td>
<td>97</td>
</tr>
</tbody>
</table>

*Unpublished data, Lewis Research Center.*
A computer program by Goldberg and Haferd (ref. 22) was used to calculate physical properties of hydrogen for use in equation (20).

DISCUSSION OF RESULTS

The measured heat-transfer coefficients of the investigations shown in table II were compared with the heat-transfer coefficients calculated by equation (20) for the experimental conditions. The percent deviation was calculated by

\[
\frac{h_{\text{cal}}}{h_{\text{ex}}} - 1 \times 100 = \text{Percent deviation}
\]  

(21)

The total number of data points used from each investigation and the percentage of them which had deviations within ±25 percent are recorded in table III. The hydrogen data are separated into regions 1 and 2, as shown in figure 3, so that the effect of inlet temperature, inlet pressure, and transposed critical temperature could be studied. The results shown in table III indicate that equation (20) can be used with confidence over the wide range of conditions covered by region 1. In region 1, 87 percent of the 3674 calculated heat-transfer coefficients deviated less than ±25 percent from the measured values. Region 2 defined by \(45^\circ R < T_i < T^*\) and \(p_c < p_i < 530\) psia (3.65 MN/m\(^2\)) is often referred to as the near-critical region. In region 2, only 40 percent of 948 calculated heat-transfer coefficients deviated less than ±25 percent from the measured values. Equation (20) should be used with caution in this region. At present, however, there is no other equation which predicts heat-transfer coefficients with much more success. The inability of equation (20) to predict heat-transfer coefficients in this region may be a result of the lack of accurate transport properties or to a mode of heat transfer which is not approximated by single-phase heat transfer.

In addition to the hydrogen data, 88 nitrogen gas data points reported by Perkins and Worsoe-Schmidt (ref. 10) and 359 helium gas data points reported by McCarthy and Wolf (ref. 4), Weiland (ref. 7), and Taylor (refs. 8 and 12) were predicted by using equation (20) with 98 percent of the calculated heat-transfer coefficients deviating less than ±25 percent from the measured heat-transfer coefficients.

A fair comparison between the correlation equations (recommended by the investigators listed in table II) and equation (20) could not be made since these equations were not recommended for use at small values of \(x/D\), where the largest \(T_s/T_b\) usually occurs. A comparison could be made by using both the data from reference 19 with \(x/D\) from 6.7 to 33.9 and the recommended correlation, equation (16), from reference 20 for \(x/D\) from 5.7 to 148. Figure 4 shows the results of comparing equation (16) of reference 20 with equation (20) of the present investigation. As shown in fig-

14
Figure 3. - Range of hydrogen inlet temperature and inlet pressure for which equation (20) has been experimentally checked.
Figure 4, Variation of deviation of the calculated heat-transfer coefficient from measured heat-transfer coefficient with local fluid bulk temperature (232 data points from ref. 19). Ratio of distance from entrance of test section to inside diameter of test section, x/D, 6.7 to 33.9.

Equation (20) has predicted heat-transfer coefficients for hydrogen which are in good agreement with experimental values over a wide range of conditions including values of x/D from 2.0 to 252. However, the wide variation of the exponent $C_2$ with x/D for values of x/D less than about 5 indicates a need for caution in using equation (20) in this range. None of the investigations listed in table II were made to study heat transfer in the entrance region. Those with large $T_S/T_D$ usually have large surface temperature gradients which are not conducive to accurate heat-transfer data in the entrance region.
The ratio of the local heat-transfer coefficient \( h \) to the asymptotic heat-transfer coefficient \( h_s \) for water for several values of \( x/D \) from 0.19 to 6.3 and a \( T_s/T_b \) of 1.06 were taken from an experimental investigation by Harnett (ref. 23) and are shown in figure 5. The analytical curves of Deissler (ref. 24) and Sparrow, Hallman, and Siegel (ref. 25) for a Prandtl number of 10 and constant properties are shown with the curve predicted by equation (20) for a \( T_s/T_b \) of 1.06. For this particular case with \( T_s/T_b \) near 1, there is good agreement between the experimental data, the analytical results, and the prediction of equation (20). Equation (20) was also used to predict heat-transfer coefficients for air with \( T_s/T_b \) from 1.10 to 1.91, as presented by Magee (ref. 26). In figure 6, the ratio of the measured local heat transfer to the asymptotic heat-transfer coefficient is compared with curves predicted by equation (20) using the
local experimental values for $T_s/T_b$, $x/D$, and Reynolds number showing an effect of both $T_s/T_b$ and Reynolds number.

The need for caution in using heat-transfer equations for predicting wall temperatures has been demonstrated by Miller and Taylor (ref. 13). Since equation (20) is empirical, it is recommended that it not be extrapolated beyond the conditions for which it has been tested.

**CONCLUSIONS**

Variable-property single-phase hydrogen heat-transfer data from 10 investigations that used symmetrically heated straight tubes have been correlated over a wide range of conditions by the single equation

$$\text{Nu}_b = 0.023 \text{Re}_b^{0.8} \text{Pr}_b^{0.4} \left( \frac{T_s}{T_b} \right) \exp \left( -0.57 - \frac{1.59}{x/D} \right)$$

where $\text{Nu}_b$ is the bulk Nusselt number, $\text{Re}_b$ is the bulk Reynolds number, $\text{Pr}_b$ is the bulk Prandtl number, $T_s$ is surface temperature, and $T_b$ is bulk temperature. The same equation was also used to correlate helium and nitrogen data.

The range of conditions over which the correlation applies for hydrogen is
Ratio of surface to bulk temperature, $T_s/T_b$ .................. 1.1 to 23
Ratio of distance from entrance of test section to inside diameter of test section, $x/D$ .................... 2.0 to 252
Heat flux $Q/S$, Btu/(sec)(in.$^2$); MW/m$^2$ .................. 0.036 to 27.6; 0.059 to 45.7
Bulk Reynolds number, $Re_b$ ................................. 7 500 to 13 800 000
Surface temperature, $T_s$, °R; °K ............................ 114 to 5630; 63 to 3130

All possible combinations of these conditions are not available.

The hydrogen heat-transfer conditions were divided into regions 1 and 2 in the inlet temperature and inlet pressure plane. The deviation of predicted heat-transfer coefficients from measured values in each region is as follows:

1. In region 1, 87 percent of the 3674 calculated heat-transfer coefficients deviated less than ±25 percent from the measured heat-transfer coefficients. The correlation equation is recommended to predict heat-transfer coefficients in this region.

2. In region 2, 40 percent of the 948 calculated heat-transfer coefficients deviated less than ±25 percent from the measured heat-transfer coefficients. Since no other equation can predict heat-transfer coefficients much more successfully, the correlation equation is suggested for use with caution in this region.

The correlation equation was used to predict heat-transfer coefficients for helium gas and nitrogen gas. Of 447 heat-transfer coefficients that were calculated, 98 percent deviated less than ±25 percent from the measured heat-transfer coefficients.

Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio, September 27, 1967, 122-28-02-16-22.

REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C. 20546