A SEARCH FOR UNBOUND HELIUM 3 LEVELS WITH ISOBARIC SPIN 1/2

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

An attempt was made to excite the unbound $T = 1/2$ states in helium 3, recently reported by Kim, Bunch, Devins, and Forster, by $(\alpha - \text{He}^3)$ inelastic scattering. No states in $\text{He}^3$ between 3- and 13-MeV excitation were found. The shape of the inelastic alpha-particle spectrum was calculated with the assumption that phase space alone determines the shape. Precise fits to the observed spectra were not obtained, but the similarity between calculated and observed spectra gave evidence that a broad maximum in the 17.5$^0$ spectrum was caused by phase space rather than a level in $\text{He}^3$. Other calculations with Baldin's formalism showed that no enhancement of the inelastic cross section near 8-MeV excitation in $\text{He}^3$ should be expected.

INTRODUCTION

Interest in possible excited states of the three-nucleon system has been stimulated by the recent observation by Ajdacić et al. (ref. 1) of a peak in the energy spectrum of protons from the $(n + t)$ reaction. They attributed this peak to the bound trineutron. In contrast, Thornton et al. (ref. 2) repeated this experiment with 20.8- rather than 14.4-MeV incident neutrons and with improved accuracy and obtained a negative result. Similarly, both Anderson et al. (ref. 3) and Cookson (ref. 4) have measured neutron spectra from the $(p + \text{He}^3)$ reaction and failed to observe the corresponding, slightly unbound state of the triproton.

Kim et al. (ref. 5) have examined the inelastic proton spectrum from the $(p + \text{He}^3)$ reaction and observed structure corresponding to unbound $\text{He}^3$ levels at 7.7 MeV (threshold energy for $\text{He}^3 \rightarrow p + p + n$) and at 10.2 and 12.6 MeV. It is unlikely that any of these

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levels is the $T = 3/2$ isobaric spin analog state of the trineutron, since most experimental evidence now indicates that it does not exist. However, a $T = 1/2$ level has been predicted by Baldin (ref. 6), who used effective range theory to correctly predict the 20-MeV excited state ($S = 0$, $T = 0$) of He$^4$.

Energy spectra of inelastic alpha particles from the $(\alpha + \text{He}^3)$ reaction, which show no structure corresponding to any unbound states in He$^3$, are presented herein. Calculations which show that there should be no enhancement of the cross section as predicted by Baldin are also presented.

### SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>normalizing factor for computed cross sections</td>
</tr>
<tr>
<td>$a, b, c$</td>
<td>curve-fitting parameters</td>
</tr>
<tr>
<td>$a(k)$</td>
<td>nucleon-deuteron scattering length at propagation number $k$</td>
</tr>
<tr>
<td>$d\Omega$</td>
<td>laboratory solid-angle element into which alpha particle is emitted</td>
</tr>
<tr>
<td>$d\Omega_\alpha$</td>
<td>center-of-mass solid-angle element into which alpha particle is emitted</td>
</tr>
<tr>
<td>$d\Omega_d$</td>
<td>center-of-mass solid-angle element into which deuteron is emitted</td>
</tr>
<tr>
<td>$d^2\sigma/d\Omega , dE_{\alpha l}$</td>
<td>differential inelastic scattering cross section for detection of final alpha particle in solid angle $d\Omega$ and energy interval $dE_{\alpha l}$</td>
</tr>
<tr>
<td>$E$</td>
<td>kinetic energy of nucleon plus deuteron in their center-of-mass system</td>
</tr>
<tr>
<td>$E_c$</td>
<td>laboratory total initial kinetic energy</td>
</tr>
<tr>
<td>$E_d$</td>
<td>center-of-mass kinetic energy of final deuteron</td>
</tr>
<tr>
<td>$E_o$</td>
<td>laboratory kinetic energy of incident alpha particle</td>
</tr>
<tr>
<td>$E_p$</td>
<td>center-of-mass kinetic energy of final proton</td>
</tr>
<tr>
<td>$E_\alpha$</td>
<td>center-of-mass kinetic energy of final alpha particle</td>
</tr>
<tr>
<td>$E_{\alpha l}$</td>
<td>laboratory kinetic energy of final alpha particle</td>
</tr>
<tr>
<td>$E_{\alpha 0}$</td>
<td>value of $E_\alpha$ when 10.2-MeV level in He$^3$ is excited</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck constant</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>$h/2\pi$</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian</td>
</tr>
<tr>
<td>$k$</td>
<td>propagation number</td>
</tr>
<tr>
<td>$m$</td>
<td>nucleon mass</td>
</tr>
<tr>
<td>$2$</td>
<td></td>
</tr>
</tbody>
</table>
The 42-MeV alpha-particle beam from the 1.5-m cyclotron was focused at the center of a 10-centimeter-diameter gas cell containing He³ gas at one-fourth of atmospheric pressure. This gas was analyzed in a mass spectrometer before and after the experiment; its contaminants, mostly He⁴, were less than 5 parts per thousand. The scattered alpha particles were detected by an 800-micron silicon surface-barrier detector. Inelastic alpha-particle energy spectra were accumulated at laboratory scattering angles of 17.5°, 20°, 22°, and 25°. An energy calibration was made by observing both He³ and
alpha particles from elastic ($\alpha - \text{He}^3$) scattering at several angles. The linearity was better than 1 percent, and the energy resolution (typically 0.7-MeV full width at half maximum (FWHM)) was largely accounted for by kinematic broadening.

**ANALYSIS AND DISCUSSION**

Measured absolute inelastic cross sections are shown in figure 1. The sharp rise in cross section just below $E_{\alpha\ell} = 16$ MeV is caused by the detection of deuterons. These particles can lose a maximum of 15.6 MeV in the detector. A similar rise just below 12 MeV (not shown) indicates the onset of proton detection. Therefore, the data are valid only for $E_{\alpha\ell} \geq 16$ MeV.

The arrows in figure 1 indicate the expected locations of peaks caused by unbound levels at 10.2- and 12.6-MeV excitation in He$^3$, as well as the highest energies at which breakup of He$^3$ into $(d + p)$ and into $(p + p + n)$ can take place. No peaks were observed at these energies. In general, the spectra rise monotonically from the threshold energy for $(\text{He}^3 \rightarrow p + d)$, except for the small, rather broad peak at 17.5° and about 13-MeV excitation. In the next section, it is suggested that this peak and the plateau at 20° near $E_{\alpha\ell} = 16$ MeV are phase space effects.

**Search for 10.2-MeV State**

The most prominent of the states reported by Kim et al. (ref. 5) was the 10.2-MeV state of 1-MeV width FWHM, which was excited with a cross section of 1 or 2 millibarns ($1 \times 10^{-31}$ or $2 \times 10^{-31}$ m$^2$). The following procedure was employed to set an upper limit on the intensity of a possible 10.2-MeV peak in the 20°, 22°, and 25° spectra. First, the continuum cross section in the region from 1.8 MeV below to 1.8 MeV above the presumed peak, but excluding the region from 0.6 MeV below to 0.6 MeV above the peak, was fitted by a straight line

$$\frac{d^2\sigma}{d\Omega\, dE_{\alpha\ell}} = aE_{\alpha\ell} + b$$

for which $a$ and $b$ were calculated by the method of least squares. Next, a test was made of the hypothesis that the data in the seven channels in the peak region (from 0.6 MeV below to 0.6 MeV above the presumed peak) were fitted by the expression

\[ \text{E-3747} \]
\[
\frac{d^2\sigma}{d\Omega\ dE_{\alpha\ell}} = aE_{\alpha\ell} + b + c \left\{ \exp \left[ \frac{-(E_{\alpha} - E_{\alpha\ell})^2}{\Gamma^2} \right] \right\} \]

(2)

At each angle \( \chi^2 \) was calculated as a function of \( c \), which in turn determines the differential cross section for excitation of the level \( \sigma \). The terms \( E_{\alpha\ell} \) and \( \Gamma \) were chosen to be consistent with the excitation energy and width reported by Kim et al. (ref. 5). Values of \( \chi^2 \) for the yield in the seven channels nearest 10.2-MeV excitation energy in He\(^3\), as a function of the laboratory cross section for excitation of the 10.2-MeV level are as follows:

<table>
<thead>
<tr>
<th>Total cross section, ( \sigma )</th>
<th>Laboratory scattering angle, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>mb</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Statistical chi-square sum</td>
<td></td>
</tr>
<tr>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( 0.1 \times 10^{-3} )</td>
<td>20.3</td>
</tr>
<tr>
<td>( 0.4 )</td>
<td>25.2</td>
</tr>
<tr>
<td>( 0.3 )</td>
<td>31.5</td>
</tr>
<tr>
<td>( 0.5 )</td>
<td>39.5</td>
</tr>
<tr>
<td>( - )</td>
<td>----</td>
</tr>
</tbody>
</table>

For \( \sigma = 0 \) (c = 0), \( \chi^2 \) ranged from 4 (80 percent probability) at 22° to 15 (5 percent probability) at 20°. There is no a priori reason why the spectra should be linear, but it was felt that the fits thus obtained were sufficiently good to serve as a reference in searching for the peak. In general, the probability of occurrence of \( \chi^2 \approx 9.8 \) for seven data points fitted by a curve with no free parameters is 20 percent. Therefore, rather arbitrarily, the upper limit of the cross section \( \sigma \) for excitation of the 10.2-MeV state was determined from the relation

\[
\chi^2(\sigma) - \chi^2(0) = 9.8
\]

Thus, upper limits of about 0.4, 0.2, and 0.15 millibarn (0.04, 0.02, and 0.015 m\(^2\)b) were obtained at 20°, 22°, and 25°, respectively. At 17.5°, the procedure was similar except that a smooth curve, drawn without referring to the data near the peak, was required for a satisfactory fit to the continuum channels. An upper limit of about 0.2 millibarn (0.02 m\(^2\)b) was obtained for the excitation of the 10.2-MeV level. These cross-section values are an order of magnitude lower than those quoted by Kim et al.
Calculation of Phase Space Factor for \((\alpha + \text{He}^3 \rightarrow \alpha + p + d)\)

The shape of the alpha-particle energy spectrum is calculated under these assumptions:

(1) The yield of \((\alpha + \text{He}^3 \rightarrow \alpha + 2p + n)\) is negligible in comparison with that for \((\alpha + \text{He}^3 \rightarrow \alpha + d + p)\). This assumption is reasonable since the measured spectra show no appreciable change in slope at the threshold for \((\alpha + \text{He}^3 \rightarrow \alpha + 2p + n)\) and since the increases in yield at the onset of proton and deuteron detection in the detector are about equal.

(2) The effects of nuclear interactions on the spectra are ignored, so that they are entirely determined by the phase space factor (i.e., by kinematic effects). The spectral shapes thus calculated are quite similar to the observed spectra and predict broad maxima at \(E_{\alpha f} \approx 15\) or 20 MeV and thus account for the broad peak in the 17.5\(^\circ\) spectrum.

First, the phase space factor \(\rho\) is calculated in the center-of-mass system, starting from the following equation:

\[
\rho = \left(\frac{V}{\hbar^3}\right)^2 \int \int d^3 \vec{P}_\alpha \ d^3 \vec{P}_d \ d^3 \vec{P}_p \ \delta(E_\alpha + E_d + E_p + Q - E_C) \delta(\vec{P}_\alpha + \vec{P}_d + \vec{P}_p) \tag{4}
\]

where \(E_\alpha\), \(E_d\), and \(E_p\) are the kinetic energies of the three particles in the final state; \(\vec{P}_\alpha\), \(\vec{P}_d\), and \(\vec{P}_p\) are their linear momenta. \(E_C\) is the total energy in the initial state; \(Q\) is the threshold center-of-mass energy for the reaction \(\alpha + \text{He}^3 \rightarrow \alpha + d + p\). The three-dimensional momentum-conserving \(\delta\)-function is integrated over all proton momenta and yields

\[
\rho = \left(\frac{V}{\hbar^3}\right)^2 \int P_\alpha^2 d\Omega_\alpha P_d^2 d\Omega_d \delta(E_\alpha + E_d + E_p + Q - E_C) \tag{5}
\]

Momentum conservation is used to eliminate \(E_p\) in the argument of the energy-conserving \(\delta\)-function, so that

\[
\rho = \left(\frac{V}{\hbar^3}\right)^2 \int P_\alpha^2 d\Omega_\alpha P_d^2 d\Omega_d \delta\left(E_\alpha + Q - E_C + \frac{P_d^2 + P_\alpha^2 + 2P_d P_\alpha \cos \theta_d}{2m}\right) \tag{6}
\]
(The z-axis is taken to coincide with the direction of $\vec{P}_\alpha$, and the masses of $d$ and $\alpha$ are assumed to be exactly two and four times the nucleon mass $M$.) In carrying out the integration over $dP_d$, the rule is used that

$$\delta[f(P_d)] = \left(\frac{df}{dP_d}\right)^{-1} \delta(P_d - P'_d)$$

where

$$f(P'_d) = 0$$

The result is

$$\frac{\rho}{d\Omega_\alpha dE_\alpha d(-\cos \theta_d)} = \frac{16\pi M^2 V^2}{h^6} \frac{P_\alpha P^2_d}{3P_d + 2P_\alpha \cos \theta_d}$$

The overall phase space factor for the detection of alpha particles is found by integrating equation (8) over all deuteron scattering angles between $0$ and $\pi$. But equation (8) is greatly simplified by changing the variable from $\cos \theta_d$ to $E_d$, which results in

$$\frac{\rho}{d\Omega_\alpha dE_\alpha dE_d} = \frac{16\pi M^3 V^2}{h^6} = A$$

Equation (9) is readily integrated over deuteron energies to give

$$\frac{\rho}{d\Omega_\alpha dE_\alpha} = A(E_{d2} - E_{d1})$$

where $E_{d2}$ and $E_{d1}$ are the deuteron energies when $\theta_d$ is $180^0$ and $0^0$, respectively. In turn, it can be shown that equation (10) is equivalent to

$$\frac{\rho}{d\Omega_\alpha dE_\alpha} = A \sqrt{E_\alpha (E_{\alpha}^{\max} - E_\alpha)}$$

where $E_{\alpha}^{\max}$ is the maximum energy the alpha particle can have in the center-of-mass system.
The laboratory to center-of-mass transformation equations are

\[ \tan \theta = \frac{\sin \theta_l}{\cos \theta_l - \frac{4}{7} \sqrt{\frac{E_0}{E_{\alpha l}}}} \] (12)

\[ E_{\alpha} = E_{\alpha l} + \frac{16}{49} E_0 - \frac{8}{7} \sqrt{E_0 E_{\alpha l}} \cos \theta_l \] (13)

where the scattering angles in laboratory and center-of-mass systems are \( \theta_l \) and \( \theta \), and the energies in these systems are \( E_{\alpha l} \) and \( E_{\alpha} \), and \( E_0 \) is the laboratory kinetic energy of the incident particle. By straightforward calculations,

\[ J = \frac{\partial (\cos \theta, E_{\alpha l})}{\partial (\cos \theta_l, E_{\alpha l})} = \sqrt{\frac{E_{\alpha l}}{E_{\alpha}}} \] (14)

Finally,

\[ \left( \frac{d^2\sigma}{d\Omega \, dE_{\alpha l}} \right)_{\text{lab}} = J \left( \frac{d^2\sigma}{d\Omega_{\alpha} \, dE_{\alpha}} \right)_{\text{cm}} \alpha \frac{J\rho}{d\Omega_{\alpha} \, dE_{\alpha}} = A \sqrt{E_{\alpha l} (E_{\alpha}^{\text{max}} - E_{\alpha})} \] (15)

From the last equation \( d^2\sigma/d\Omega \, dE_{\alpha l} \) was calculated as a function of \( E_{\alpha l} \), and the results were plotted in figure 2. The 22° curves are similar in shape to those for 20° and 25° and, for clarity, were omitted from figure 2. At each scattering angle, the normalization factor \( A \) was chosen to make the calculated and measured cross sections equal at the energy at which the former reached its maximum value. The values of \( A \) used were 0.95, 1.84, and 2.72 at 25°, 20°, and 17.5°, respectively. At 25° the phase space curve gives a fairly good fit to the experimental data. At 20° and 17.5° the experimental curves are more sharply peaked near \( E_{\alpha l} \approx 20 \) MeV. This peak is expected, since near the highest allowed values of \( E_{\alpha l} \), the relative momentum of \((p + d)\) is small, and Coulomb effects presumably inhibit the reaction. Even though the phase space curves do not exactly reproduce the shape of the observed spectra, their prediction of broad maxima near \( E_{\alpha l} = 20 \) MeV suggests that these features of the observed spectra should be attributed to phase space effects rather than unbound levels in \( \text{He}^3 \).
Discussion of $T = 1/2$ State Predicted by Baldin

A theoretical paper by Baldin (ref. 6) uses effective range theory to correctly predict the 20-MeV excited state ($S = 0, T = 0$) of He$^4$. It also predicts a level of $S = 1/2$ and $T = 1/2$ at about 8 MeV in the three-nucleon system; this is the energy at which the nucleon-deuteron doublet phase shift passes through $\pi/2$. Kim et al. (ref. 5) note that the small peak which they observed near the threshold energy for $(p + \text{He}^3 - 3p + n)$ is at about the excitation energy of the Baldin state.

However, calculations presented herein show that when nucleons are scattered by deuterons, there should be no enhancement of the cross section at the energy of the Baldin level. Baldin's equation (3) was used to calculate the phase shift for the nucleon-deuteron system $\delta$ as a function of $k$:

$$-\frac{1}{a(k)} = k \cot \delta = -\beta + \frac{1}{2} \rho_0^2 + \frac{1}{2} \rho_0^2 k^2$$  \hspace{1cm} (16)

The propagation number $k$ and energy $E$ are related through $\hbar^2 k^2 = 2\mu E$. Therefore, $\hbar \beta = \sqrt{2\mu \epsilon}$, where $\mu$ is the reduced mass of the nucleon-deuteron system and $\epsilon$ is its binding energy; $\beta = 0.42$ fermi$^{-1}$. By using the zero-energy scattering length $a(0) = 8.26$ fermis recommended by Baldin and setting $k = 0$ in equation (16), the effective range $\rho_0$ was calculated to be 3.4 fermis. Then, $\delta$ was computed as a function of $k$, and $\delta$ was plotted as a function of $E$ in figure 3.

This figure also shows the total cross section $\sigma$ plotted as a function of $E$ where $\sigma$ is proportional to $\sin^2 \delta/k^2$ and is given in arbitrary units. Note that $E$ is the kinetic energy of nucleon plus deuteron in their center-of-mass system, so that $E$ equals the excitation energy in He$^3$ minus $\epsilon$, where $\epsilon = 5.5$ MeV. Therefore, at $E = 2.5$ MeV (the energy at which $\delta = \pi/2$), the excitation energy in He$^3$ is 8 MeV. It is clear that the nucleon-deuteron scattering cross section is not enhanced at this energy, and this effect would not be expected to cause peaks in the spectra of particles from the breakup reaction.

The n-p system behaves similarly. Baldin points out that the n-p triplet phase shift is $\pi/2$ at 15 MeV; therefore, in his formalism, at this energy the n-p system has a level which is the reflection of the deuteron ground state. However, as is well known, the n-p phase shift varies so slowly that the cross section decreases monotonically with increasing energy in this region.

An additional objection to Baldin's work is that van Oers and Seagrave (ref. 7) have recently found that the nucleon-deuteron scattering length $a(0)$ which he used is in error.
SUMMARY OF RESULTS

Inelastic alpha-particle energy spectra from the \((\alpha + \text{H}^3)\) reaction were measured, and no \(T = 1/2\) levels in \(\text{H}^3\) between 3- and 13-MeV excitation were found. Effective range calculations employing the Baldin formalism indicate that no prominent state should be observed near the threshold for \((\alpha + \text{He}^3 \rightarrow \alpha + 2p + n)\).

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 8, 1967,
129-02-04-06-22.

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1. Ajdacić, V.; Cerineo, M.; Lalovic, B.; Paić, G.; Slaus, I.; and Tomas, P.: Reactions \(\text{H}^3(n,p)3n\) and \(\text{H}^3(n,\text{H}^4)\gamma\) at \(E_n = 14.4\) MeV. Phys. Rev. Letters, vol. 14, no. 12, Mar. 22, 1965, pp. 444-446.


Figure 1. - Inelastic cross sections for \((\alpha + \text{He}^3)\) reaction. Laboratory scattering angles are specified; for clarity, elastic peaks at \(20^\circ\), \(22^\circ\), and \(25^\circ\) are omitted; at \(17.5^\circ\) elastic alpha- and \(\text{He}^3\)-particle groups differ in energy by only 0.4 MeV; elastic peak shown consists of these two unresolved groups. Insert shows effect on \(17.5^\circ\) spectrum of level at 10.2-MeV excitation, total cross section, 0.4 milli-barn (0.04 fm\(^2\)).
Figure 2. - Inelastic spectra at 17.5°, 20°, and 25°. (Calculated spectra were arbitrarily normalized and were computed with the assumption that phase space alone determines spectral shape.)
Figure 3. - Phase shift and total cross section for nucleon-deuteron scattering, calculated in effective range approximation. Units of total cross section are arbitrary. Zero energy corresponds to nucleon and deuteron at rest at infinite separation; on this energy scale, the proposed Baldin state would appear at about 2.5 MeV.
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