HYDROGEN AND HELIUM ABUNDANCES IN NEUTRON STAR ATMOSPHERES

Leonard C. Rosen

Institute for Space Studies
Goddard Space Flight Center, NASA
New York, N. Y.

Lawrence Radiation Laboratory
Livermore, California

and

Columbia University
New York, N. Y.

Abstract

The question whether one may identify a neutron star by observing hydrogen and helium lines in its spectra is examined. An analysis is presented of the burning and diffusion of hydrogen and helium in a neutron star atmosphere. An initial two-component atmosphere of hydrogen and carbon and of helium and carbon is examined utilizing various photospheric temperatures. The hydrogen burns through the proton-proton reaction and the CN cycle and the helium by the triple alpha process and alpha capture on carbon. It is found that only a negligible proportion of hydrogen or helium can exist after a few years in the neutron star atmosphere. Therefore spectra which include hydrogen and helium lines cannot be attributed to neutron stars.

I. INTRODUCTION

There is at present a great deal of uncertainty as to the chemical composition of a neutron star atmosphere. This is regrettable since it disallows one to associate with any degree of certainty an optical source with a neutron star. Work has been performed on the statistical equilibrium composition of neutron star atmospheres (Tsuruta and Cameron 1965). However, because of the extremely small scale heights in the neutron star envelope, diffusion effects may drastically change the composition. The density changes by about eight orders of magnitude and the temperature by about two orders of magnitude in a depth of approximately one to twenty meters depending upon the choice of model (Morton 1964, Tsuruta and Cameron 1966). Chiu and Salpeter (1964) have estimated that hydrogen and helium will not be present in the envelope of a neutron star whose age is greater than a few years. The hydrogen and helium will have diffused down to the interior regions and have been completely burned due to the high temperatures prevailing there. It is the purpose of this work to examine this idea in detail: to analyse the degree of burning and diffusion of hydrogen and helium in a neutron star with the end in mind of noting whether any optical sources containing hydrogen or helium lines may be associated with neutron stars.

II. APPROACH TO THE PROBLEM

A typical neutron star model of $1.0M_{\odot}$ and 10 km. radius has been chosen for this problem Initially, the atmosphere of the star is composed entirely of carbon with a layer of hydrogen or helium superimposed on the carbon for the two cases. This initial configuration is chosen in order to maximize the time required to burn any fraction of the original mass of hydrogen or helium. The hydrogen burns through a combination of the proton-proton reaction and the CN cycle; the helium reacts through the triple alpha process and alpha capture on carbon. Carbon burning is neglected in this study. Although one may expect a neutron star atmosphere to contain nuclei up to the iron peak (Tsuruta and Cameron 1965, Cameron 1965), the proton and alpha capture rates will be lowest for carbon at the relevant temperatures. At a temperature of 1.0×10^9 °K, $< \sigma v >$ for the CN cycle is 1.88×10^{-23} cm^3/sec (Reeves 1965) whereas $< \sigma v >$ for the reactions: $^{23}Na(p,\gamma)^{24}Mg$, $^{31}P(p,\gamma)^{32}S$ and $^{27}Al(p,\gamma)^{28}Si$ is given by 1.02×10^{-21} cm^3/sec, 2.93×10^{-22} cm^3/sec and 4.24×10^{-22} cm^3/sec respectively (Truran, Hansen, Cameron and Gilbert 1966). At this temperature, $< \sigma v >$ for the reaction $^{12}C(\alpha,\gamma)^{16}O$ is 3.66×10^{-30} cm^3/sec compared to 1.51×10^{-27} cm^3/sec, 1.30×10^{-26} cm^3/sec and 2.28×10^{-27} cm^3/sec for the reactions $^{16}O(\alpha,\gamma)^{24}Mg$, and $^{24}Mg(\alpha,\gamma)^{28}Si$ respectively (Truran et al 1965). Therefore the presence of advanced products of evolution in the neutron star atmosphere would speed up the burning of hydrogen and helium.

Since the particle diffusion velocity is proportional to the gradient of the number density, an intermediate, continuously varying strata of carbon plus hydrogen or helium is inserted between the purely carbon atmosphere and the overlying layer of pure hydrogen or helium. The diffusion takes place under the influence of external forces. These forces include gravity and the electric field due to the charge separation of ions and electrons in the atmosphere.

The time-dependent coupled set of equations of stellar structure, nuclear reactions and diffusion are solved numerically by computer. The entire atmosphere is divided into zones and the equations solved for each zone at each epoch. The resulting variables are then used to solve the equations for each zone at the next epoch. This procedure is continued until at least .95 of the original mass of the hydrogen or helium layer has been burned.

III. EQUATIONS OF STELLAR STRUCTURE

The non-equilibrium stellar structure equations for a spherically symmetric diffusing system are (Chapman and Cowling 1952, Chandrasekhar 1939)

$$\frac{1}{\rho}\frac{dP}{d\rho} + \frac{GM}{r^2} + \frac{\partial c_o}{\partial t} + c_o\frac{\partial c_o}{\partial r} = 0 \qquad (1)$$

$$c_o\rho = \sum_i \rho_i c_i \qquad (2)$$

$$\frac{dT}{dr} = -\frac{3\kappa L\rho}{16\pi a c r^2 T^3} \qquad (3)$$

$$\frac{dM}{dr} = 4\pi r^2\rho \qquad (4)$$

$$P = P(\rho, T) \qquad (5)$$

where P, M, ρ, T, L and κ are the pressure, mass, density, temperature, luminosity and opacity respectively. ρ_i and c_i are the density and diffusion velocity of the i^{th} component. The radiative energy transfer equation (3) is utilized since the neutron star is stable against convection (Morton 1964, Tsuruta and Cameron 1966).

Because of the special nature of neutron star atmospheres, certain simplifying approximations to the equations of stellar structure may be made. Previous studies of equilibrium neutron star atmospheres have shown that the envelope is only about one to ten meters thick (Tsuruta 1964, Tsuruta and Cameron 1966). The mass in equation (1) may then be set equal to the total mass of the star, thereby decoupling the mass equation (4). Further, energy generation in such a thin atmosphere is very small compared with interior processes so that the luminosity in equation (3) may be taken as a constant. For a neutron star of one solar mass and a radius, R_E, of 10 km., $g = \frac{GM}{R_E^2} \approx 10^{10}\, g_\odot$. Therefore the last two terms in equation (1) are extremely small compared with the first two so that a "quasi-static" equation may be used Since the depth of the atmosphere constitutes less than one per cent of the radius of the star, the radial distance in equation (1) is taken as constant and set equal to the radius of the star. Equations (1,3) then reduce to

$$\frac{dP}{dr} = - \rho g \tag{6}$$

$$\frac{dT}{dr} = - B \frac{\kappa P}{r^2 T^3} \tag{7}$$

where B is a constant equal to $\frac{3L}{16\pi ac}$.

The equation of state, represented as a function of density and temperature in equation (4), is that for a perfect gas of electrons and ions plus radiation pressure.

$$P = P_i + P_e + \frac{1}{3} aT^4 \tag{8}$$

where P_i and P_e are the pressures of the ions and electrons respectively and a is the Stefan-Boltzmann constant. The general expression for the pressure of a fermion of spin 1/2 is given by (Landau and Lifshitz 1958)

$$P = \frac{8\pi}{3h^3} \int_0^{\infty} \frac{p^3}{e^{(E-U)/kT} + 1} \frac{\partial E}{\partial p} dp \tag{9}$$

where U is the chemical potential of the particle. The contribution to pressure by the ions is only important in regions in which degeneracy effects are unimportant. Therefore the ion pressure may be represented throughout the atmosphere by the non-degenerate perfect gas equation of state

$$p_i = n_i kT = \frac{\rho kT}{\mu_i H} \tag{10}$$

where H is the mass of a proton and $\mu_i = \rho/(n_i H)$.

It is necessary to evaluate the integral in equation (9) for the electrons, however, at each density and temperature in the atmosphere. This is done most easily by tabulating the equation of state for the electrons in terms of a degeneracy parameter λ (Handbuch der Physik 1958)

$$P_e = n_e kT \frac{(2/3) F_{3/2}(\lambda)}{F_{1/2}(\lambda)} = \frac{\rho kT}{\mu_e H} \frac{(2/3) F_{3/2}(\lambda)}{F_{1/2}(\lambda)} \tag{11}$$

$$n_e = (2/h^3)(\pi\ m_e kT)^{3/2}\ \ F_{1/2}(\lambda) \tag{12}$$

where the functions $F_{3/2}(\lambda)$ and $F_{1/2}(\lambda)$ are defined by

$$F_s(\lambda) = \int_0^\infty \frac{t^s\, dt}{e^{t-\lambda}+1} \tag{13}$$

and $\mu_e = \rho/(n_e H)$. In the non-degenerate regions of the envelope, the ratio$(2/3)F_{3/2}(\lambda)/F_{1/2}(\lambda)$ approaches unity and the equation of state (11) reduces to the non-degenerate perfect gas law. As the density increases for a given temperature, the parameter λ increases. When λ exceeds 20 , the electron gas may be considered to be completely degenerate (Handbuch der Physik 1958). At this point, the Fermi energy of the electron is sufficiently high compared with kT such that a zero temperature Fermi gas well approximates the statistical distribution of electrons. The equation of state is then given by (Chandrasekhar 1939)

$$P_e = \frac{\pi m_e{}^4 c^5\ f(x)}{3h^3} \tag{14}$$

$$n_e = \frac{8\pi}{3}\left(\frac{m_e c}{h}\right)^3 x \tag{15}$$

where $$x = p_o/mc \tag{16}$$

and $$f(x) = x(2x^2 - 3)(x^2 + 1)^{\frac{1}{2}} + 3 \sin h^{-1} x \tag{17}$$

IV. PARTICLE DIFFUSION

A two particle diffusion approximation will be utilized in this calculation to find the diffusion velocity of hydrogen and helium in the neutron star atmosphere. The relative diffusion velocity of such a two particle gas mixture is given by (Chapman and Cowling 1952)

$$\vec{c}_1 - \vec{c}_2 = - \frac{n^2}{n_1 n_2} D_{12} \left[\vec{d}_{12} + \frac{K_T}{T} \nabla T \right] \tag{18}$$

where $n = n_1 + n_2$, D_{12} is the coefficient of diffusion, K_T is the thermal-diffusion ratio and $\vec{d}_{12}$ is defined for a gas with zero mass flux to be

$$\vec{d}_{12} = \frac{1}{nKT} \left[K \nabla(n_1 T) - \rho_1 \vec{F}_1 / m_1 \right] \tag{19}$$

$\vec{F}_1$ is the resultant external force acting on species 1. For the problem under consideration, this includes the gravitational force and electric field due to the charge separation of ions and electrons in the gravitational field.

Both the coefficient of diffusion and the thermal-diffusion ratio depend on the form of the particle interactions which in turn are a function of the degree of ionization in the envelope. The temperature in the atmosphere of a neutron star depends upon the model chosen and on the time which has elapsed since formation of the star. Tsuruta and Cameron (1966) have shown that for neutron stars of about one solar mass, it takes approximately 10^3 to 10^5 years for the photospheric temperature to cool from 10^7 °K to 10^6 °K. During this time, the neutron star may be observed as a point source black-body emitter with a maximum in the x-ray region. Since the significant temperature range of the envelope is at least 10^6 °K, complete ionization of the carbon-hydrogen-helium atmosphere will be assumed in this calculation. Thus the particle interaction is that of Coulomb repulsion between completely ionized particles and the coefficient of diffusion is given by (Chapman and Cowling 1952)

$$D_{12} = \frac{3}{16n}\left(\frac{2kT}{\pi m_R}\right)^{1/2}\left(\frac{2kT}{Z_1 Z_2 e^2}\right)^2 \Big/ \ln[1 + \mathrm{ctn}(\theta_o/2)] \tag{20}$$

where $m_R = m_1 m_2/(m_1 + m_2)$, Z_1 and Z_2 are the charges of the two ions and θ_o is the minimum scattering angle. Chapman and Cowling use the average for $\mathrm{ctn}(\theta_o/2)$ given by

$$\mathrm{ctn}(\theta_o/2) = \left(\frac{4dkT}{Z_1 Z_2 e^2}\right)^2 \tag{20a}$$

where d is the mean interparticle-spacing of ions taken in this work to be $[3/(4\pi n)]^{1/3}$. It is quite possible that this underestimates the number of small angle collisions thereby giving too large a diffusion coefficient and too large a relative diffusion velocity. It is the wish of the author to maximize the time required for diffusion and burning whenever a choice of variables may be made. Therefore, in accordance with the techniques of Mestel (1949), Lee (1950) and Spitzer (1956), the minimum scattering angle θ_o is defined as

$$\theta_o = \lambda_T/\lambda_D = \frac{h/[3kT\rho/n]^{1/2}}{[kT/(4\pi e^2)]^{1/2}/(Z_1^2 n_1 + Z_2^2 n^2)^{1/2}} \tag{20b}$$

where λ_T is the de Broglie wave length and λ_D is the Debye screening radius for the two ion mixture. The method employed in this analysis is to calculate $\mathrm{ctn}(\theta_o/2)$ through both equations (20a) and (20b) and choose the largest value in order to minimize D_{12}. The contribution of $K_T \nabla T$ to the diffusion equation (18) is small for a neutron star and will be neglected (Chapman and Cowling, Tsuruta and Cameron 1966).

Since the time scales for diffusion are very slow compared to the mean collision time of the gas mixture, the distribution of particles will differ only slightly from statistical equilibrium. Therefore the force $\vec{F}_1$ will be taken as that for the equilibrium configuration and is given by

$$\vec{F}_1 = - m_1 g\hat{r} + Z_1 e\vec{E} \tag{21}$$

where $g = MG/R_E^{\;2}$ as in equation (6). The gravitational field produces a separation of electrons and ions which produces the electric field $\vec{E}$ in equation (21). This field, which acts radially outwards and tends to reduce the effect of gravity on the ions, is given by (Spitzer 1956)

$$\vec{E} = \frac{m_1 g}{(Z_1 + 1)\cdot e}\;\hat{r} \tag{22}$$

so that

$$\vec{F}_1 = - \frac{m_1 g}{Z_1 + 1}\;\hat{r} \tag{23}$$

The electric field reduces the magnitude of the gravitational force by a factor of $1/(Z_1 + 1)$.

IV. NUMBER DENSITY NETWORK

The number density of a particular species can change in two ways: by diffusion and by nuclear burning. This may be represented for the i^{th} species by

$$\frac{\partial n_i}{\partial t} = Q(n_i) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 n_i c_i\right) \tag{24}$$

where $Q(n_i)$ is the nuclear burning and production rate of species i. Hydrogen burns to produce helium through the proton-proton reaction and the CN cycle and helium burns to produce carbon by the triple alpha process and through the reaction $^{12}C(\alpha,\gamma)^{16}O$. At any given time and position in the atmosphere, the number density network for the three nuclei: hydrogen, helium and carbon is given by

$$\frac{\partial n_H}{\partial t} = - \frac{1}{r^2}\frac{\partial}{\partial r}\,(r^2 n_H c_H) - \beta\, n_H{}^2 - \gamma\, n_H n_c \tag{25}$$

$$\frac{\partial n_{He}}{\partial t} = - \frac{1}{r^2}\frac{\partial}{\partial r}\,(r^2 n_{He} c_{He}) + \frac{\beta}{4}\, n_H{}^2 + \frac{\gamma}{4}\, n_H n_c - \delta n_{He}{}^3 - \epsilon n_{He} n_c \tag{26}$$

$$\frac{\partial n_c}{\partial t} = - \frac{1}{r^2}\frac{\partial}{\partial r}\,(r^2 n_c c_c) + \frac{\delta}{3}\, n_{Hc}{}^3 + \frac{\epsilon}{3}\, n_{He} n_c \tag{27}$$

The reaction probabilities; β, γ, δ and ϵ (in cm^3/sec) are given by (Reeves 1965).

$$\beta = (1.0+2.5\rho^{1/2}/T_6^{3/2})(1.0+1.23\times10^{-2}T_6^{1/3}+7.8\times10^{-3}T_6^{2/3}+6.73\times10^{-4}T_6)$$

$$\times\ (3.3\times10^{-13}H)\ \mathrm{Exp}\ (-33.81/T_6^{1/3})/T_6^{2/3} \tag{28}$$

$$\gamma = (1.0+1.75\rho^{1/2}/T_6^{3/2})(1.0+2.74\times10^{-3}T_6^{1/3} - 3.74\times10^{-3}T_6^{2/3} - 7.17\times10^{-5}T_6)$$

$$\times\ (4.79\times10^{9}H)\ \mathrm{Exp}\ (-152.31/T_6^{1/3})/T_6^{2/3} \tag{29}$$

$$\delta = \mathrm{Exp}\ (2.4\rho^{1/2}/T_6^{3/2})\ H^2[9.47\times10^{-6}\cdot 10^{-\left(\frac{18.9}{T_8}\right)}$$

$$+\ 5.0\times10^{-5}\cdot 10^{-\left(\frac{119.0}{T_8}\right)}]/\ T_8^{3} \tag{30}$$

$$\epsilon = \mathrm{Exp}(2.4\rho^{1/2}/T_6^{3/2})\cdot H\cdot\Big\{\Big[1.84\times10^{10}\cdot 10^{-(30.05/T_8^{1/3} - \delta)}\Big/$$

$$T_8^{2}(1-4\times10^{-3}T_8 + 0.2T_8^{-2/3})^2\Big] + \Big[4.3\times10^{3}\cdot10^{-122/T_8} + 2.35\times10^{4}\times10^{-135/T_8}\Big]\Big/T_8^{3/2}\Big\} \tag{31}$$

$$\delta = 0.06T_8^{2/3}(1 + 0.07T_8^{1/3}) \tag{31a}$$

where $T_6 = T/1.0\times10^6\ ^{o}K$ and $T_8 = T/1.0\times10^8\ ^{o}K$. The helium which is burned through the reaction $^{12}C(\alpha,\gamma)^{16}O$ is recycled into carbon. This is done to simplify the atmosphere to a two-component one for helium burning and more important to maximize the burning time. Reeves (1965) has shown that for temperatures greater than $3.4\times10^8\ ^{o}K$, the reaction $^{16}O(\alpha,\gamma)^{20}Ne$ will always be faster than the reaction $^{12}C(\alpha,\gamma)^{16}O$. Since all significant helium burning through alpha capture will take place above this temperature, the procedure utilized in this work will slow down the total time required to burn the helium.

V. COMPUTATIONAL PROCEDURE

The solution of the diffusion and burning of hydrogen and helium in an atmosphere of hydrogen, helium and carbon is a time-dependent problem which requires coupling the equations of stellar structure, equations (6, 7, 8), with the diffusion equations (18, 19) and the number density network, equations (25, 26, 27). This is accomplished with the use of a computer by differencing all equations, choosing boundary conditions at the photosphere of the neutron star and then at each time step solving for the variables downwards from the photosphere until the isothermal internal core temperature is reached. The process is then repeated for the next time step utilizing the variables computed from the preceding epoch. This procedure is continued until the mass of the hydrogen or helium is reduced to a specified fraction of its original value.

The boundary values of density and pressure at the photosphere are found in the following way. For a given radius and luminosity of the star, the effective temperature is defined by

$$T_E = \left(\frac{L}{4\pi R_E^2 \sigma}\right)^{1/4} \tag{32}$$

where σ is the Stefan's radiation constant. Coupling the definition of the optical depth τ for the medium: $\frac{d\tau}{dr} = - \kappa(\rho,T)\rho$ (33)

with equation (5) yields

$$\frac{dP}{d\tau} = \frac{g}{\kappa(\rho,T)} \tag{34}$$

Integrating from $\tau = \infty$ to the photosphere one finds

$$P_{ph} = \int_{\infty}^{\tau_{ph}} \frac{g d\tau}{\kappa(\rho, T)} + P_o \qquad (35)$$

The Eddington approximation gives $\tau_{ph} = 2/3$ and $P_o = aT_E^4/6$. For a constant g and assuming κ is independent of height, equation (35) yields

$$P_{ph} = \frac{2}{3} \frac{g}{\kappa(\rho_{ph}, T_E)} + \frac{a}{6} T_E^4 \qquad (36)$$

The equation of state gives a second relation for the photospheric pressure.

$$P_{ph} = \frac{1}{3} aT_E^4 + \frac{\rho_{ph} k T_E}{(\mu_e + \mu_i)H} \qquad (37)$$

Equations (36, 37) may be iterated to find P_{ph} and ρ_{ph}. An initial value is chosen for the opacity giving a value of P_{ph} from equation (36). This result is substituted into equation (37) to find a ρ_{ph}. This value of the density is used to compute a new κ which in turn is used to find a new P_{ph}. The procedure is continued until the values of P_{ph} and ρ_{ph} converge to within the desired accuracy, in this case one part in 10^4. The opacity is found from a table prepared from the code developed by A.N. Cox and his associates of Los Alamos Scientific Laboratory. The code includes contributions from bound-bound, bound-free, free-free absorption, electron scattering, negative ion absorption and electron conduction.

Equilibrium studies of neutron star atmospheres have shown that the density changes by about 8 orders of magnitude and the temperature by about 2 orders of magnitude from those values at the photosphere in a distance of less than 20 meters (Tsuruta and Cameron 1966). Because of these large changes in the variables with distance, the stellar structure equations are written in logarithmic form with pressure as the dependent variable in order to reduce inaccuracies for a given zone size. Equations (6, 7) transform to

$$\frac{d \ln r}{d \ln P} = - \frac{1}{g} \exp(\ln P - \ln r - \ln \rho) \qquad (38)$$

$$\frac{d \ln T}{d \ln P} = \frac{3.L}{16\pi \, acg} \exp(\ln P + \ln \kappa - 2 \ln r - 4 \ln T) \qquad (39)$$

All equations in this study are differenced implicitly. This is done to insure maximum stability in the coupled equations of stellar structure, nuclear reactions and diffusion. Severe problems of stability may arise should explicit or symmetrical differencing be used with a poor choice of time steps. Richtmeyer (1957) has shown that the linear second order diffusion equation is unconditionally stable for all time increments when differenced implicitly. However, the use of such an approach creates difficulties of solution in that the variables to be solved for at the updated time appear in all derivatives and in all functions. Thus, no simple recursion relations exist as they do for explicit differencing techniques between the variables at time $t + \Delta t$ and the variables at time t. The implicitly differenced equations in this study are non-linear and must be solved by some iterative method. The program developed first chooses the proper equation of state which is dependent upon the value of the degeneracy parameter λ, defined by equation (13). The choice of one of equations (10, 11, or 14) yields a relation between the density, temperature and pressure. The substitution of this into equations (39, 40) gives two non-linear equations in two unknowns, ln r and ln T at the updated time. For a given ln P, these may be written as

$$F\left[(\ln T)_{J+1}\ ,\ (\ln T)_J\ ,\ (\ln r)_{J+1}\ ,\ (\ln r)_J\right]^{t\,+\,\Delta t} = 0 \tag{40}$$

$$G\left[(\ln T)_{J+1}\ ,\ (\ln T)_J\ ,\ (\ln r)_{J+1}\ ,\ (\ln r)_J\right]^{t\,+\,\Delta t} = 0 \tag{41}$$

where the subscripts represent the zone at which the variables are to be found and the superscript indicates that these variables are evaluated at the updated time, $t + \Delta t$. The solution of these equations is begun with J=1 corresponding to the photosphere. The boundary values are therefore assigned to $(\ln T)_1$ and $(\ln r)_1$. Subsequent values of the variables are found from the values at the preceding zone by utilizing the Newton-Raphson method to solve equations (40, 41). That is for any zone, these equations are linearized to yield

$$F + \frac{\partial F}{\partial(\ln T)_{J+1}}\,\delta(\ln T)_{J+1} + \frac{\partial F}{\partial(\ln r)_{J+1}}\,\delta(\ln r)_{J+1} = 0 \tag{42}$$

$$G + \frac{\partial G}{\partial(\ln T)_{J+1}}\,\delta(\ln T)_{J+1} + \frac{\partial G}{\partial(\ln r)_{J+1}}\,\delta(\ln r)_{J+1} = 0 \tag{43}$$

$\delta(\ln T)_{J+1}$ and $\delta(\ln r)_{J+1}$ are found for an initial "guess" of $(\ln T)_{J+1}$ and $(\ln r)_{J+1}$. New values of the variables are then computed from

$$(\ln T)_{J+1} = (\ln T)_{J+1} + \delta(\ln T)_{J+1} \tag{44}$$

$$(\ln r)_{J+1} = (\ln r)_{J+1} + \delta(\ln r)_{J+1} \tag{45}$$

These values are then substituted into equations (42, 43) and the process is repeated until values of $(\ln T)_{J+1}$ and $(\ln r)_{J+1}$ are found to within the desired degree of convergence. This process is repeated for 40 zones. The evaluated temperature and density profile of the atmosphere is then substituted into the diffusion and reaction network equations to yield new mass fractions at the updated time. For a given initial layer of either hydrogen or helium, this process is repeated until at least .95 of the mass of the original layer is burned.

VI. RESULTS AND CONCLUSION

Figures 1 and 2 illustrate the initial density and temperature profile as a function of distance from the photosphere (R_E - R) of five models employed in this work: two of hydrogen and two of helium, with photospheric temperatures of 5.0×10^6 °K and 1.0×10^7 °K and one of hydrogen with a photospheric temperature of 2.0×10^7 °K. The layers are purely of hydrogen or helium for 50 cm. and then vary sinusoidally to zero for the next 100 cm. The initial mass fractions for the two component atmospheres are given by

$$x = 1.0, \qquad z = 0.0 \qquad R_E - R \leq A-B \tag{46a}$$

$$x = \sin^2 \frac{\pi}{2}\left(\frac{A-R}{B}\right), \quad z = 1.0 - x \quad A-B \leq R_E - R \leq A \tag{46b}$$

$$x = 0.0, \qquad z = 1.0 \qquad A \leq R_E - R \tag{46c}$$

where A and B are 150 cm. and 100 cm. and x and z represent the mass fractions of hydrogen (or helium) and carbon respectively. This layer will be referred to as the 150 cm. layer. Later in this section a 35 cm. layer will be discussed which is described by equations (46a, b, c) with A = 35 cm. and B = 10 cm. One notes in Figures 1 and 2 the typical behavior of the temperature and density of the neutron star atmosphere: the temperature varying appreciably in a distance of less than 10 meters and the density changing by four to five orders of magnitude in a distance of about 3 meters.

In Figure 3, the burning and diffusion of hydrogen layers with time is compared for the three different photospheric temperatures of 5.0×10^6 °K, 1.0×10^7 °K and 2.0×10^7 °K for the same thickness (150 cm.) layer. The ratio of the integrated mass of the hydrogen layer over the entire atmosphere at any time to its integrated mass initially is plotted with time. One half of the original masses of 3.28×10^{16} gm., 4.53×10^{15} gm. and 2.45×10^{13} gm. remains at 9.6×10^4 sec., 4.8×10^4 sec. and 2.5×10^4 sec. for the 5.0×10^6 °K, 1.0×10^7 °K and 2.0×10^7 °K photospheric temperatures respectively. The increase in the rate of burning with temperature is due to two effects: the diffusion velocity increases as the $T^{5/2}$ as noted in equation (20) and the nuclear burning rate increases exponentially with temperature. For all three cases 0.95 of the initial hydrogen layers are burned in less than 3.5×10^5 sec. Little difference in the rates of hydrogen burning are noted when two different thicknesses of hydrogen layers are compared in Figure 4. The 150 cm. layer is compared with the 35 cm. layer for the same photospheric temperature of 1.0×10^7 $^{\circ}$K. The 35 cm. layer corresponds to an initial integrated mass of 1.09×10^{14} gm. The two layers of hydrogen are 95% burned in less than 2.0×10^5 sec.

A time study of the mass fraction of hydrogen as a function of distance from the photosphere is shown in Figure 5. The decrease in mass fraction and the diffusion of hydrogen to points deeper within the neutron star is illustrated for the case of the 150 cm. initial layer of hydrogen at a photospheric temperature of 1.0×10^7 $^{\circ}$K. The four times of 0.0 sec., 4.0×10^4 sec., 1.0×10^5 sec. and 3.4×10^5 sec. correspond to integrated mass ratios of 1.0, 0.518, 0.120 and 0.023.

The burning and diffusion of the 150 cm. layer of helium is compared for the two photospheric temperatures of 1.0×10^7 $^{\circ}$K and 5.0×10^6 $^{\circ}$K in Figure 6; in Figure 7, the 150 cm. layer of helium is compared to the 35 cm. layer of helium at the same photospheric temperature of 1.0×10^7 $^{\circ}$K. The thickness of the layer is apparently unimportant as the two layers, initially of 1.38×10^{15} gm. and 1.55×10^{17} gm., are seen to burn at almost exactly the same rate: 95% burned in less than 8.0×10^6 sec. Temperature difference changes the rate of burning by a factor of less than three. The 150 cm. layer, initially 9.60×10^{17} gm., with a photospheric temperature of 5.0×10^6 $^{\circ}$K is 95% burned in approximately 1.8×10^7 sec.

These results all indicate that 0.95 of the original masses of hydrogen and helium are burned in less than one year for photospheric temperatures greater than or equal to 5.0×10^{6} °K. This is in general agreement with Chiu and Salpeter (1964) who estimated that any hydrogen or helium present in a neutron star atmosphere would diffuse inwards and be burned in a few years. Since the cooling time of the photosphere of a neutron star from $T_E = 1.0\times10^{7}$ °K to $T_E = 5.0\times10^{6}$ °K is of the order of 100 years (Tsuruta and Cameron 1966), it may be concluded that only a negligible proportion of either hydrogen or helium may exist in the atmosphere of a neutron star after a few years. Indeed the cooling times presented by Tsuruta and Cameron (1966) indicate that a medium mass neutron star, initially at an internal temperature of about 10^{10} °K will cool to a photospheric temperature of 2.0×10^{7} °K in 10^{4} - 10^{5} sec. and to 1.0×10^{7} °K in 10^{6} - 10^{7} sec. Since the burning time of hydrogen at $T_E = 2.0\times10^{7}$ °K is about 5×10^{4} sec. and the burning time of helium at $T_E = 1.0\times10^{7}$ °K is about 5×10^{6} sec., these photospheric temperatures would appear to be the appropriate ones to consider. Further, the analysis presented herein probably overestimates the burning time due to the choice of the slowest diffusion coefficient and the neglect of proton and alpha capture on nuclei heavier than carbon. Hence one would not expect to see any evidence of hydrogen or helium lines associated with a neutron star.

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FIGURE CAPTIONS

Figure 1. Initial density profiles as a function of distance from the photosphere, R_E. Five models of 150 cm. initial layers of hydrogen or helium are plotted for various photospheric temperatures, T_E, for a one solar mass, 10 Km. neutron star.

Figure 2. Initial temperature profiles as a function of distance from the photosphere, R_E. Five models of 150 cm. initial layers of hydrogen or helium are plotted for various photospheric temperatures, T_E, for a one solar mass, 10 Km. neutron star.

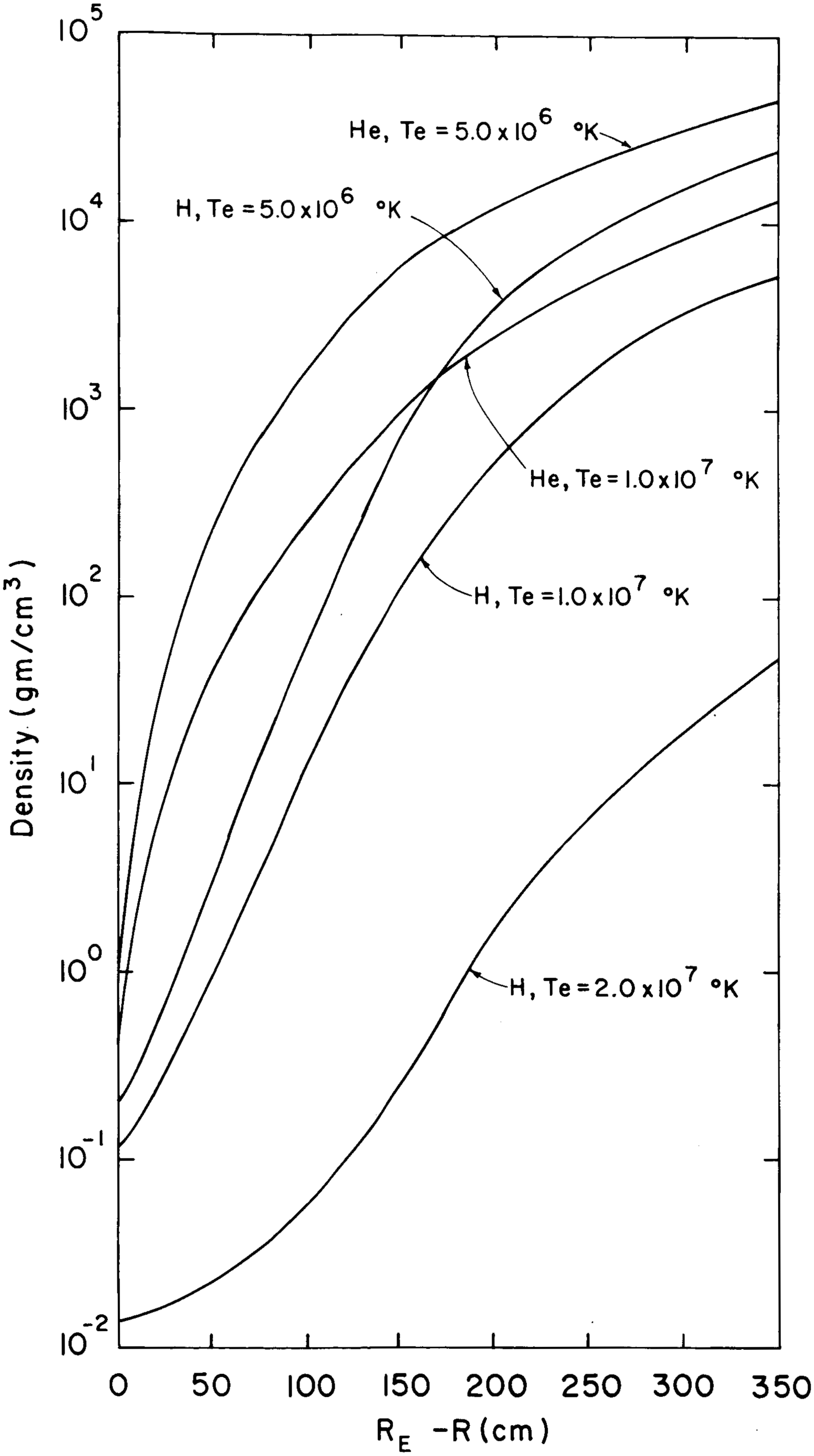
Figure 3. Hydrogen burning rate for three photospheric models with the same initial depth of layer. The ratio of the integrated mass of hydrogen in the neutron star atmosphere over the mass of the initial 150 cm. layer of hydrogen is plotted as a function of time.

Figure 4. Comparison of hydrogen burning rate of two different initial thicknesses of layer with the same photospheric temperature, $T_E = 1.0 \times 10^7$ °K. The ratio of the integrated mass of hydrogen in the neutron star atmosphere over the mass of the initial layer of hydrogen is plotted as a function of time.

Figure 5. Time history of 150 cm. initial layer of hydrogen in neutron star atmosphere with a photospheric temperature, $T_E = 1.0 \times 10^7$ °K. The mass ratio of hydrogen in the atmosphere is plotted as a function of distance from the photosphere, R_E, for four times.

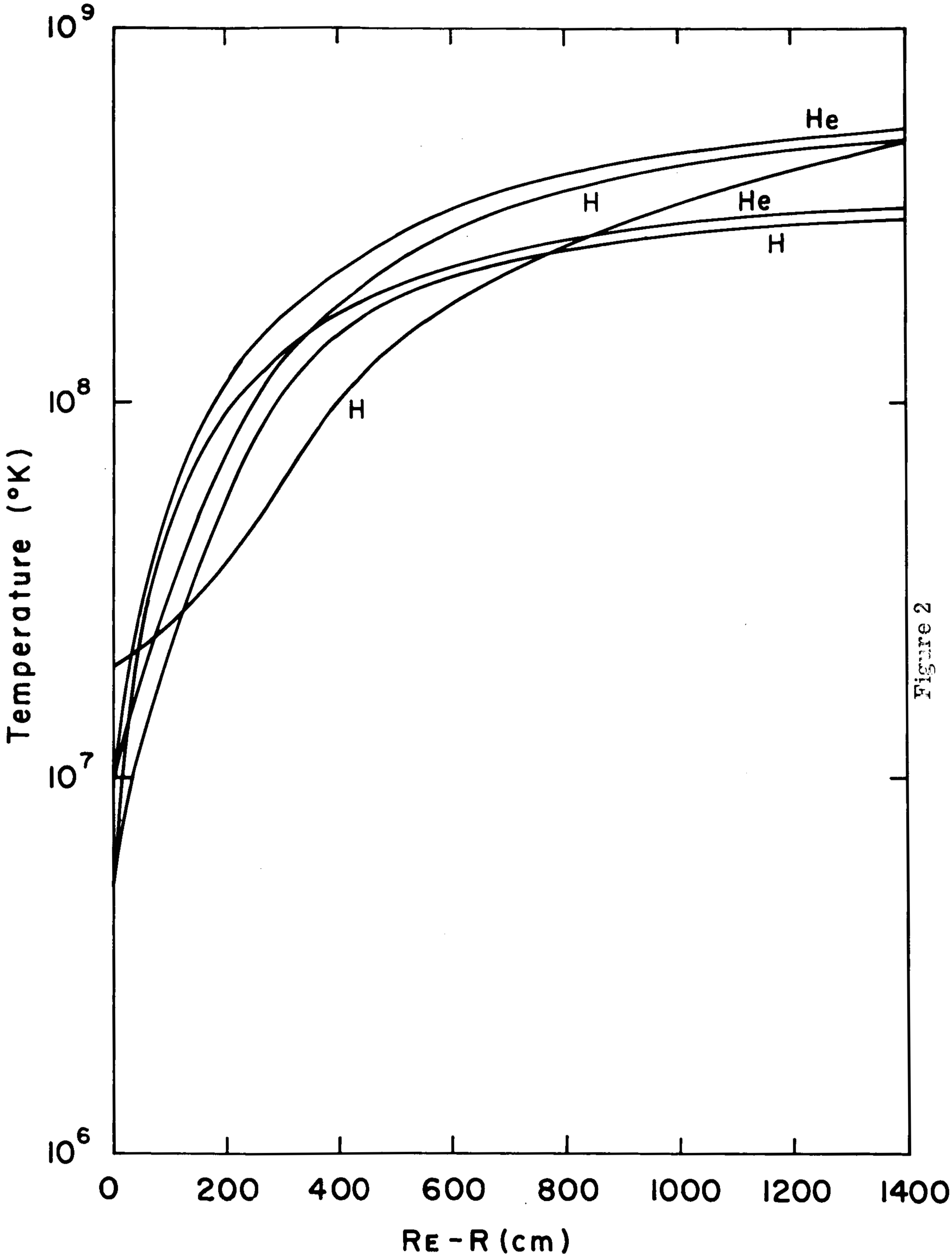
Figure 6. Helium burning rate for two photospheric models with the same initial depth of layer. The ratio of the integrated mass of helium in the neutron star atmosphere over the mass of the initial 150 cm. layer of helium is plotted as a function of time.

Figure 7. Comparison of helium burning rate of two different initial thicknesses of layer with the same photospheric temperature, $T_E = 1.0x10^7$ ^oK. The ratio of the integrated mass of helium in the neutron star atmosphere over the mass of the initial layer of helium is plotted as a function of time.



Density (gm/cm3)
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
50
100
150
200
250
300
350
R_E −R (cm)
He, Te = 5.0 x 10^6 °K
H, Te = 5.0 x 10^6 °K
He, Te = 1.0 x 10^7 °K
H, Te = 1.0 x 10^7 °K
H, Te = 2.0 x 10^7 °K

Figure 1



He
H
He
H
H
Temperature (°K)
10^9
10^8
10^7
10^6
0
200
400
600
800
1000
1200
1400
RE - R (cm)

Figure 2

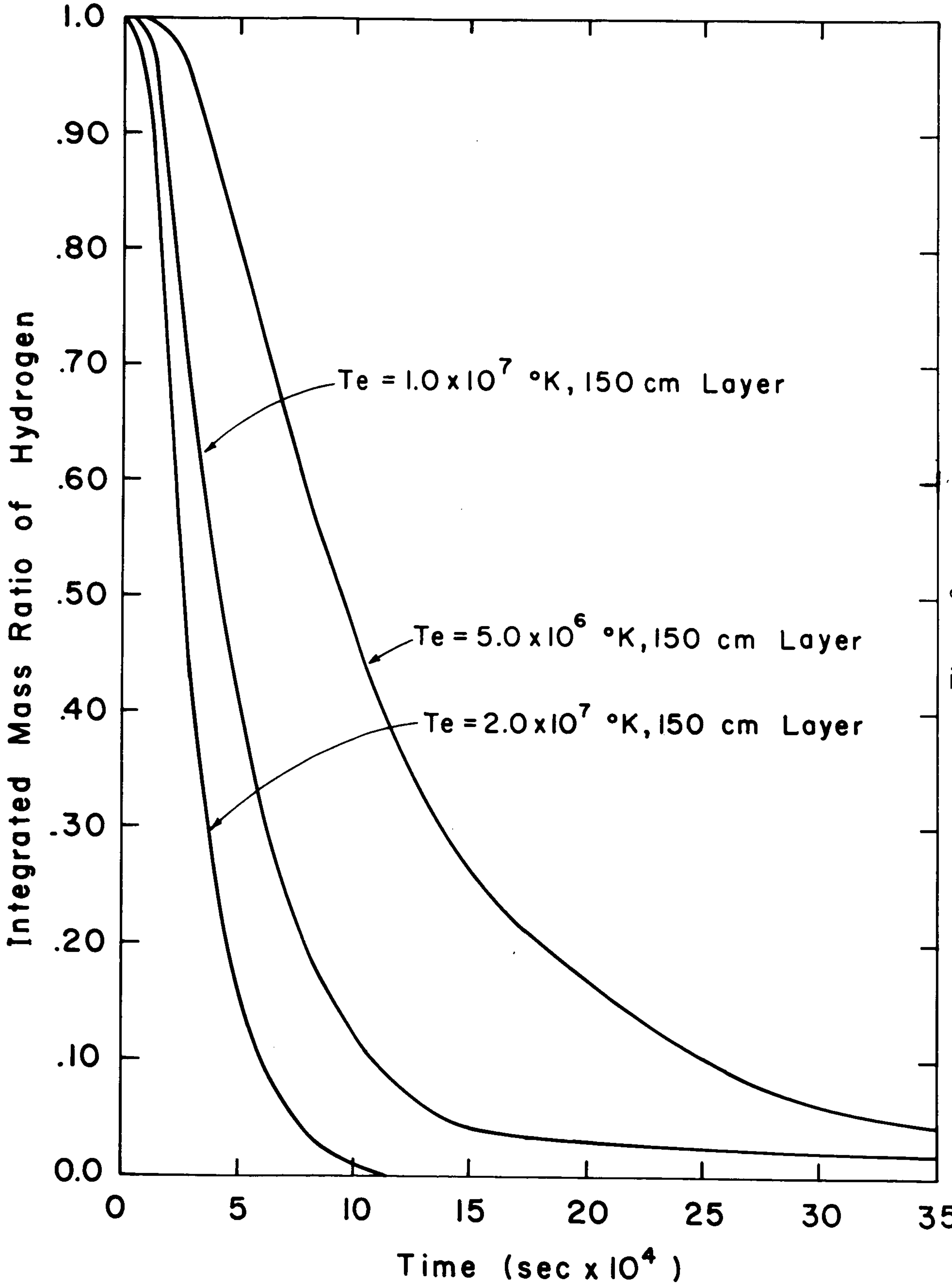
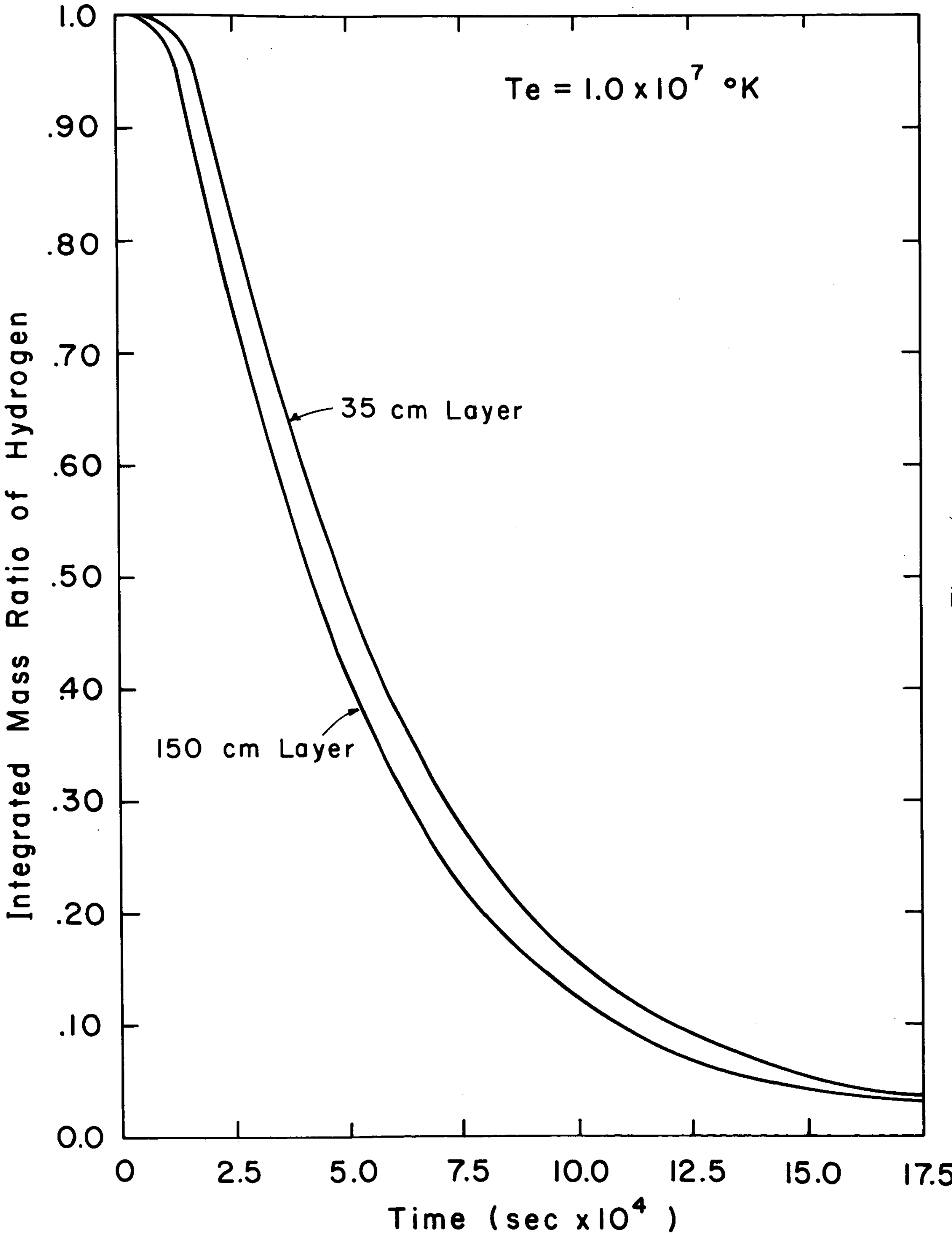

1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
0.0
Integrated Mass Ratio of Hydrogen
Te = 1.0 x 10^7 °K, 150 cm Layer
Te = 5.0 x 10^6 °K, 150 cm Layer
Te = 2.0 x 10^7 °K, 150 cm Layer
0
5
10
15
20
25
30
35
Time (sec x 10^4)

Figure 3



Te = 1.0 x 10^7 °K
35 cm Layer
150 cm Layer
Integrated Mass Ratio of Hydrogen
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
0.0
0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
Time (sec x 10^4)

Figure 4

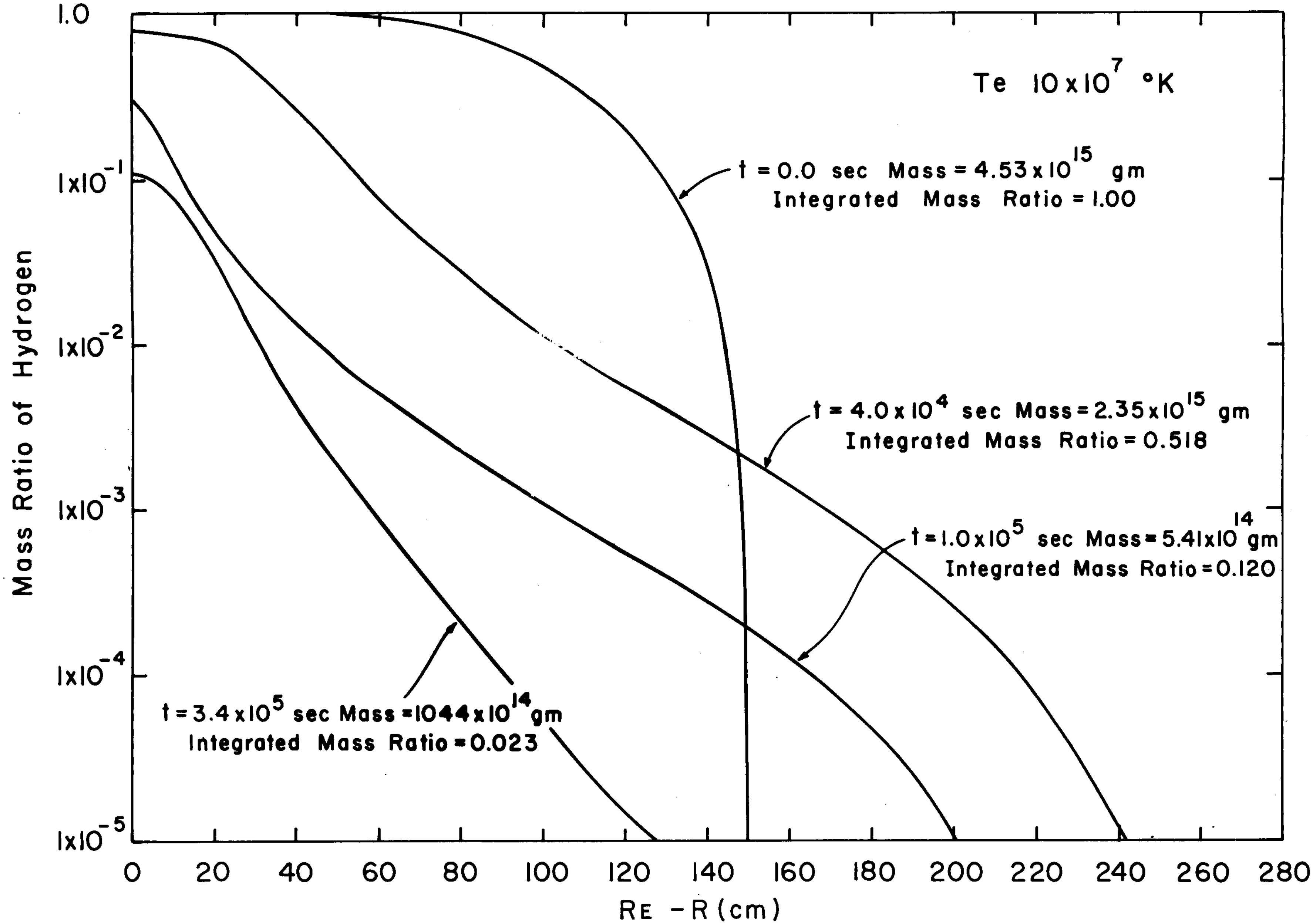
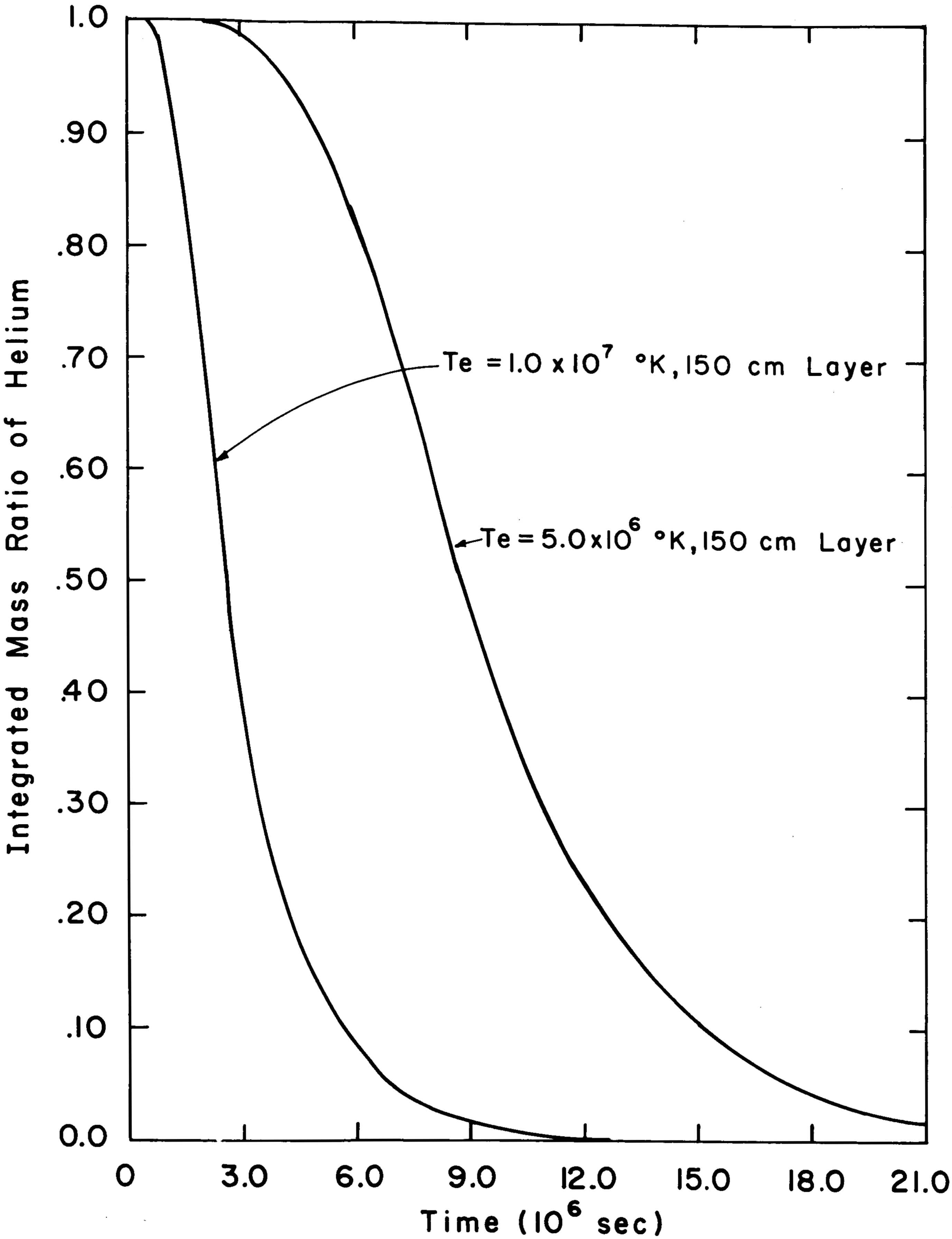

Te 10x10^7 °K
t = 0.0 sec Mass = 4.53x10^15 gm
Integrated Mass Ratio = 1.00
t = 4.0x10^4 sec Mass = 2.35x10^15 gm
Integrated Mass Ratio = 0.518
t = 1.0x10^5 sec Mass = 5.41x10^14 gm
Integrated Mass Ratio = 0.120
t = 3.4x10^5 sec Mass = 1044x10^14 gm
Integrated Mass Ratio = 0.023
Mass Ratio of Hydrogen
1.0
1x10^-1
1x10^-2
1x10^-3
1x10^-4
1x10^-5
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
RE - R (cm)

Figure 5



Integrated Mass Ratio of Helium
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
0.0
0
3.0
6.0
9.0
12.0
15.0
18.0
21.0
Time (10^6 sec)
Te = 1.0 x 10^7 °K, 150 cm Layer
Te = 5.0 x 10^6 °K, 150 cm Layer

Figure 6

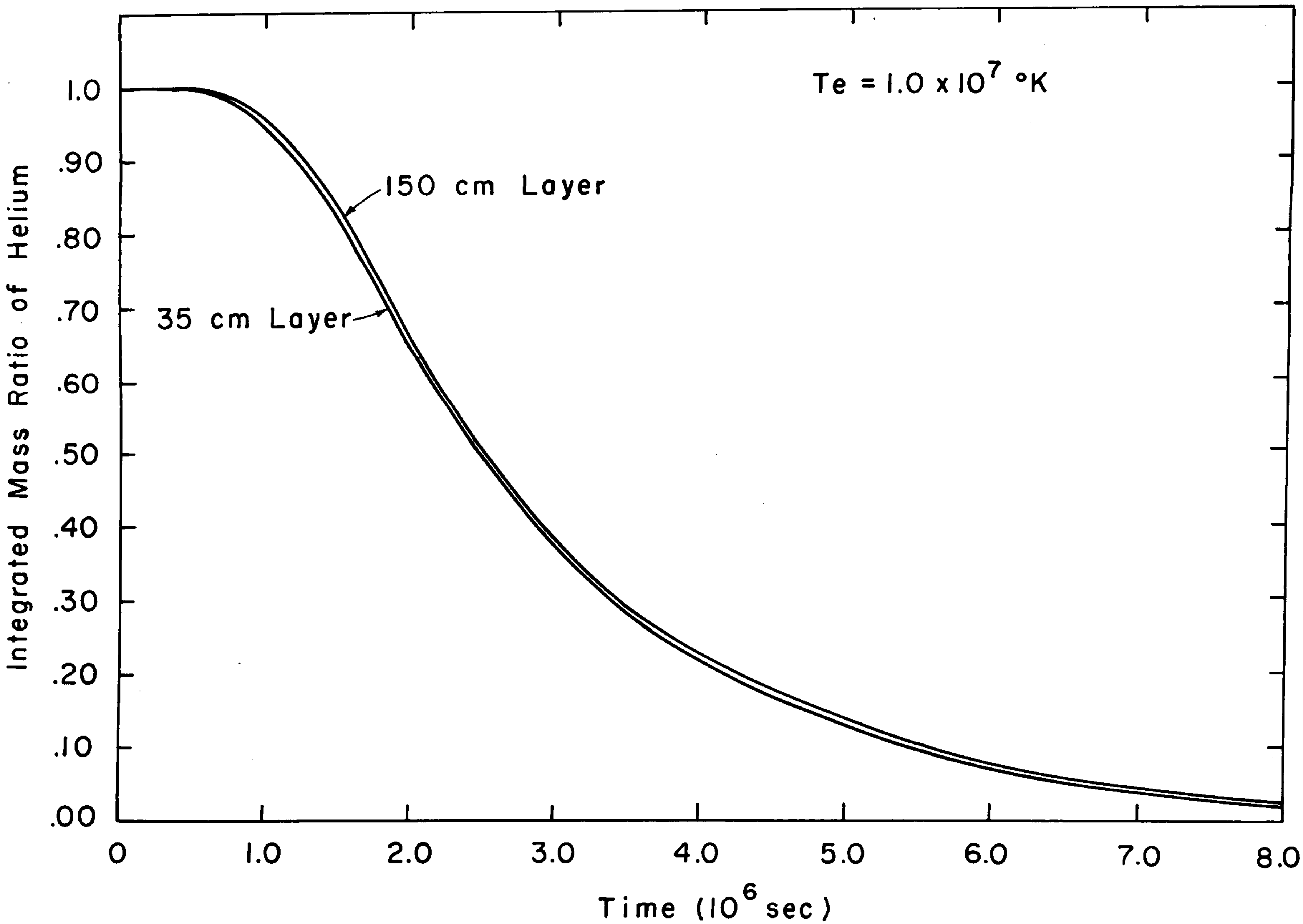

Te = 1.0 x 10^7 °K
150 cm Layer
35 cm Layer
Integrated Mass Ratio of Helium
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
Time (10^6 sec)

Figure 7