ON THE GRAVITY GRADIENT AT SATELLITE ALTITUDES

W. KÖHNLEIN

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ABSTRACT

The horizontal and vertical components of the gravity gradient are computed along equipotential surfaces at altitudes of 0, 500, 1000, 5000, 10 000, 50 000, and 100 000 km, from Kozai's (1964) and Gaposchkin's (1966) harmonic coefficients. Accuracy estimates for the gravity gradients, as obtained from satellite orbits, are derived from a comparison with Kaula's (1966) CA solution.

RÉSUMÉ

Les composantes horizontales et verticales de la gravité sont calculées le long de surfaces équipotentielles aux altitudes 0, 500, 1000, 5000, 10 000, 50 000, et 100 000 km, à partir des coefficients harmoniques de Kozai (1964) et Gaposchkin (1966).

Des évaluations sur la précision des gradients de gravité, obtenus à partir des orbites de satellites, sont dérivées par comparaison avec la CA solution de Kaula (1966).

КОНСПЕКТ

Вычислены по гармоническим коэффициентам Козая (1964) и Гапочкина (1966) горизонтальные и вертикальные составляющие градиента силы тяжести вдоль эквипотенциальных поверхностей на высотах в 0, 500, 1000, 5000, 10 000, 50 000 и 100 000 км. Оценки точности градиентов силы тяжести, полученных с помощью спутниковых орбит были выведены из сравнения с CA решением Каула (1966).
ON THE GRAVITY GRADIENT AT SATELLITE ALTITUDES

W. Köhnlein

1. INTRODUCTION

The earth's gravitational field is currently deduced from the perturbation of satellite orbits and the analysis of gravity anomalies, i.e., direct gravity measurements at the earth's surface. A third source may eventually become available, namely, the gravity gradients determined with gradiometers in orbiting satellites (Wollard, 1966). The purpose of this paper is to investigate the magnitude of the gravity gradient field components and their accuracies at present.*

However, one limitation should be mentioned: the gravity gradient field as derived from satellite orbits gives only a relatively smooth pattern because of the low-degree harmonics (up to 15, 14) used in the analysis. The actual disturbances near the surface of the earth can be several magnitudes larger, and it is that local short-periodic field that we would hope to measure by orbiting gradiometers. For example, let us consider a satellite altitude of about 160 km and a gravitational disturbance of about 100 mgal on the earth's surface in an area with a linear extension of a few hundred kilometers. Roughly speaking, this disturbance can be picked up by harmonic coefficients of degree 100 or so, resulting in a correspondingly larger (~100) effect than the low-order terms of, say, n below 10. At 160-km

*In one sense, this is an extension, to one higher derivative, of previous presentations of the geopotential (Köhnlein, 1966a) obtained at satellite altitudes.
elevation we get approximately $1/10$ of this magnitude, owing to the coefficient $(a/r)^n$ of equation (1) found in Section 2. At higher satellite altitudes (about 500 km and above) the measurable effect is well below the present accuracy of string gradiometers, i.e., $10^{-8}$ sec$^{-2}$. 
2. GRAVITY GRADIENT, COMPONENTS

Let us denote the earth's gravitational potential in free space by

\[ U = \frac{GM}{r} \left[ 1 + \sum_{n=2}^{\infty} \sum_{m=0}^{n} \left( \frac{a}{r} \right)^n (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\sin \phi) \right], \]  

where \( GM \) is the gravitational constant times the mass of the earth, \( a \) is the equatorial radius of the earth, \( r \) is the geocentric radius, \( \phi \) is the geocentric latitude, \( \lambda \) is the geocentric longitude, \( C_{nm}, S_{nm} \) are the harmonic coefficients, and \( P_{nm}(\sin \phi) \) are the associated Legendre functions. Then we obtain the gravitation

\[ g = \left[ \left( \frac{\partial U}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial U}{\partial \phi} \right)^2 + \frac{1}{r^2 \cos^2 \phi} \left( \frac{\partial U}{\partial \lambda} \right)^2 \right]^{1/2}. \]  

Hence, the gravity gradient in question is

\[ \nabla g = \frac{\partial g}{\partial x} i + \frac{\partial g}{\partial y} j + \frac{\partial g}{\partial z} k, \]  

where \( x, y, z \) are rectangular coordinates (with the origin at the mass center of the earth), and \( i, j, k \) denote the unit vectors along their axes. To get a picture of the magnitude of the field components, we will project \( \nabla g \) onto a local coordinate system that consists of the latitude and longitude curves of an equipotential surface and its normal, at the point \( \mathbf{r} \) under consideration. We can then derive the horizontal gravity gradient components along \( \phi = \text{const} \) and \( \lambda = \text{const} \), respectively, on \( U = \text{const} \) (see Kohnlein, 1967):
The approximate sign in equation (6) corresponds to the fact that the geocentric latitude and longitude curves on \( U = \text{const} \) do not, in general, intersect perpendicularly. However, the deviation is so small that for practical purposes the equal sign can be taken instead.

Analogously (Kohnlein, 1967), we obtain the vertical component of the gravity gradient:

\[
\nabla g_{\text{vert}} = - \frac{1}{g} \left( \frac{\partial U}{\partial r} \frac{\partial g}{\partial r} + \frac{1}{r} \frac{\partial U}{\partial \phi} \frac{\partial g}{\partial \phi} + \frac{1}{r^2 \cos^2 \phi} \frac{\partial U}{\partial \lambda} \frac{\partial g}{\partial \lambda} \right),
\]

i.e., the variation of gravitation along the gradient line (plumb line) of \( U = \text{const} \).

In the numerical analysis (Section 3) we treat the above expressions in two steps. First, we consider only the zonal rotationally symmetric part, and then we add to it a correction term that gives the combined zonal plus nonzonal contributions. This procedure has the advantage of handling mainly small numbers (see Number Maps).
The zonal part is obtained by setting all harmonic coefficients of order $m > 0$ to zero. Hence, we are dealing also with a local zonal coordinate system onto which the zonal gravity gradient is projected; the relation between the zonal and the combined zonal plus nonzonal gravitational fields is established by

$$U(r, \phi, \lambda, C_{nm}, S_{nm}) = \bar{U}(\bar{r}, \bar{\phi}, C_{n0}),$$

(8)

with the bar denoting the zonal part. Thence the expressions for the horizontal components of $\nabla g$ follow:

$$\nabla \bar{g}_\lambda = \text{const} = \left( \frac{\partial \bar{g}}{\partial \phi} - \frac{\partial \bar{g}}{\partial r} \frac{\partial \bar{U}}{\partial \phi} \right) \left[ \frac{1}{r^2} + \left( \frac{\partial \bar{U}}{\partial \phi} \right)^2 \right]^{-1/2},$$

(9)

with the anomalous meridional correction

$$\delta \nabla g_\lambda = \text{const} = \nabla g_\lambda = \text{const} - \nabla \bar{g}_\lambda = \text{const}$$

(10)

and the anomalous correction along the geocentric latitude curves

$$\delta \nabla g_\phi = \text{const} = \nabla g_\phi = \text{const},$$

(11)

since

$$\nabla \bar{g}_\phi = \text{const} = 0.$$  

(12)
From equations (10) and (11) we can compute an anomalous horizontal gravity gradient

\[ \delta \nabla g_{\text{horiz}} = \left[ (\delta \nabla g_\lambda = \text{const})^2 + (\delta \nabla g_\phi = \text{const})^2 \right]^{1/2}, \quad (13) \]

which gives the amount of deviation produced by the nonzonal harmonic coefficients \((m > 0)\).

The vertical component of \(\nabla g\) is also split up into a zonal part

\[ \nabla g_{\text{vert}} = -\frac{1}{g} \left( \frac{\partial U}{\partial \varphi} \frac{\partial g}{\partial \varphi} + \frac{1}{r^2} \frac{\partial U}{\partial \varphi} \frac{\partial g}{\partial \varphi} \right) \quad (14) \]

and a correction term

\[ \delta \nabla g_{\text{vert}} = \nabla g_{\text{vert}} - \nabla g_{\text{vert}}^- , \quad (15) \]

which follow directly from equation (7).
3. NUMERICAL ANALYSIS

The components of the gravity gradient are computed along equipotential surfaces $\bar{U} = \text{const}$ and $U = \text{const}$, at altitudes 0, 500, 1000, 5000, 10 000, 50 000, and 100 000 km above the earth's equator (zero elevation, for example, means only that $r$ of $\bar{U} = \text{const}$ has the value $r = 6,378,165$ m at $\phi = 0^\circ$). The harmonic coefficients used are Kozai's (1964) zonal and Gaposchkin's (1966) nonzonal sets, both of which are listed in Section 3.5. Kaula's (1966) CA solution, which is a combination of a satellite solution with gravity anomalies, serves for an accuracy estimate of the gravity gradients currently obtained at satellite altitudes.

3.1 Shapes of the Equipotential Surfaces

As mentioned in Section 2, we consider the zonal gravity gradient along the equipotential surface $\bar{U} = \text{const}$ and, analogously, the combined zonal plus nonzonal gravity gradients along $U = \text{const}$. Both surfaces have, in general, a different geocentric radius,

$$r(\phi, \lambda) \neq \bar{r}(\phi) \text{ for } U = \bar{U},$$

whose zonal part is given in Table 1 for the different altitudes. If we add the correction term

$$\Delta r(\phi, \lambda) = r(\phi, \lambda) - \bar{r}(\phi),$$

(16)

shown in Number Maps 1, we obtain, in the geocentric direction $\phi, \lambda$, the corresponding geocentric radius $r$ of $U = \text{const}$. 

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Table 1. Geocentric radius (zonal part) in meters

<table>
<thead>
<tr>
<th>$\phi^\circ$</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 km</td>
</tr>
<tr>
<td>90</td>
<td>6 367 812</td>
</tr>
<tr>
<td>80</td>
<td>6 368 122</td>
</tr>
<tr>
<td>70</td>
<td>6 369 016</td>
</tr>
<tr>
<td>60</td>
<td>6 370 389</td>
</tr>
<tr>
<td>50</td>
<td>6 372 076</td>
</tr>
<tr>
<td>40</td>
<td>6 373 871</td>
</tr>
<tr>
<td>30</td>
<td>6 375 561</td>
</tr>
<tr>
<td>20</td>
<td>6 376 940</td>
</tr>
<tr>
<td>10</td>
<td>6 377 846</td>
</tr>
<tr>
<td>0</td>
<td>6 378 165</td>
</tr>
<tr>
<td>-10</td>
<td>6 377 853</td>
</tr>
<tr>
<td>-20</td>
<td>6 376 954</td>
</tr>
<tr>
<td>-30</td>
<td>6 375 574</td>
</tr>
<tr>
<td>-40</td>
<td>6 373 879</td>
</tr>
<tr>
<td>-50</td>
<td>6 372 079</td>
</tr>
<tr>
<td>-60</td>
<td>6 370 381</td>
</tr>
<tr>
<td>-70</td>
<td>6 368 993</td>
</tr>
<tr>
<td>-80</td>
<td>6 368 088</td>
</tr>
<tr>
<td>-90</td>
<td>6 367 775</td>
</tr>
</tbody>
</table>
Example: The geocentric radius \( r \) in \( \phi = 30^\circ \), \( \lambda = 110^\circ \), and at the altitude 500 km is:

\[
\bar{r}(\phi) = 6,875,752 \text{ m} \quad \text{zonal part (Table 1)}
\]

\[
\Delta r(\phi, \lambda) = -10 \text{ m} \quad \text{correction term (Number Map 1)}
\]

\[
r(\phi, \lambda) = 6,875,742 \text{ m}
\]

The equatorial radii of Table 1 are obtained from the earth's equatorial radius \( a = 6,378,165 \text{ m} \) by adding the corresponding elevations. At 0-km elevation, the equipotential surface \( \bar{U} = \text{const} \) touches the earth only along the equator, while the polar regions (of \( \bar{U} = \text{const} \)) are about 11 km above the earth's surface. With increasing elevation, the flattening of \( \bar{U} = \text{const} \) decreases monotonically (as shown in Figure 3 of Köhnlein, 1966b).

Figure 1 gives the difference between the zonal radii and the ellipsoidal radii of an ellipsoidal potential field \( \bar{U}_e \) (see also Köhnlein, 1966a). The radii are related by the expression

\[
\bar{U}(\bar{r}, \phi, C_{n0}) = U_e (r_e, \phi, K_n)
\]

where the harmonic coefficients \( K_n \) are computed according to Köhnlein (1966a). This ellipsoidal field, although artificial, is a convenient tool for comparing an asymmetric field with a regular one. The anomalous differences of Figure 1, for example, immediately show the completely different pattern of the Northern Hemisphere from that of the Southern.

Example: The normalized difference \( \bar{r} - r_e \) (Figure 1) in \( \phi = 70^\circ \) at 1000-km elevation is 0.42, which we must multiply by the conversion factor \( k = 16 \) of Table 2 to get \( \bar{r} - r_e \) in meters:

\[
\bar{r} - r_e \approx 7 \text{ m} \quad \text{(at 1000-km elevation)}
\]
Figure 1. Geocentric radii: Normalized difference \( \langle r - r_e \rangle \) (see Table 2).
Table 2. Conversion factors $k$ (for zonal part) (see Figures 1, 2, and 3)*

<table>
<thead>
<tr>
<th></th>
<th>$k$</th>
<th>0 km</th>
<th>1000 km</th>
<th>10 000 km</th>
<th>100 000 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geocentric radii</td>
<td>23</td>
<td>16</td>
<td>2.7</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>$(F - r_e)$ meters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal gravity gradient</td>
<td>7.1 E-11</td>
<td>1.8 E-11</td>
<td>6.6 E-14</td>
<td>7.6 E-19</td>
<td></td>
</tr>
<tr>
<td>$\nabla g_\lambda = \text{const}$ $\nabla g_{e \lambda} = \text{const}$ $\text{sec}^{-2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical gravity gradient</td>
<td>1.0 E-10</td>
<td>3.6 E-11</td>
<td>2.3 E-13</td>
<td>2.6 E-18</td>
<td></td>
</tr>
<tr>
<td>$\nabla g_{vert} - \nabla g_{e \ vert}$ $\text{sec}^{-2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2 Horizontal Gravity Gradient

The zonal (meridional) component of $\nabla g$ is shown in Table 3 for the different altitudes. The negative sign in the Northern Hemisphere indicates that with positively increasing latitude $\phi$, the gravitation $\overline{g}$ along $\overline{U} = \text{const}$ decreases. Especially noteworthy is the rapid decrease of $\nabla g_\lambda = \text{const}$ with higher elevations; compared to an ellipsoidal field $U_e$, we see from Figure 2 the strong anomalous pattern at zero elevation and the rather monotonic structure beyond 5000 km.

The meridional correction term $\delta \nabla g_\lambda = \text{const}$ is given in Number Map 2 as a function of the geocentric latitude and longitude at different elevations. This value, added to the meridional zonal part $\nabla g_{e \lambda} = \text{const}$, leads to the horizontal meridional component of $\nabla g$ on $U = \text{const}$.

*The normalized amplitudes of Figures 1, 2, and 3 have to be multiplied by the corresponding conversion factor $k$ to obtain the actual differences.
### Table 3. Horizontal gravity gradient (meridional, zonal part) in sec$^{-2}$

<table>
<thead>
<tr>
<th>$\phi^\circ$</th>
<th>$\nabla g_h = \text{const}$</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 km</td>
<td>500 km</td>
</tr>
<tr>
<td>90</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>80</td>
<td>-82.5 E-11</td>
<td>-57.2 E-11</td>
</tr>
<tr>
<td>70</td>
<td>-158.8 E-11</td>
<td>-109.1 E-11</td>
</tr>
<tr>
<td>60</td>
<td>-217.0 E-11</td>
<td>-148.2 E-11</td>
</tr>
<tr>
<td>50</td>
<td>-244.6 E-11</td>
<td>-168.0 E-11</td>
</tr>
<tr>
<td>40</td>
<td>-245.9 E-11</td>
<td>-168.4 E-11</td>
</tr>
<tr>
<td>30</td>
<td>-215.4 E-11</td>
<td>-147.9 E-11</td>
</tr>
<tr>
<td>20</td>
<td>-160.4 E-11</td>
<td>-110.5 E-11</td>
</tr>
<tr>
<td>10</td>
<td>-91.2 E-11</td>
<td>-61.0 E-11</td>
</tr>
<tr>
<td>0</td>
<td>0.4 E-11</td>
<td>0.2 E-11</td>
</tr>
<tr>
<td>-10</td>
<td>87.2 E-11</td>
<td>59.1 E-11</td>
</tr>
<tr>
<td>-20</td>
<td>158.5 E-11</td>
<td>109.7 E-11</td>
</tr>
<tr>
<td>-30</td>
<td>220.1 E-11</td>
<td>149.7 E-11</td>
</tr>
<tr>
<td>-40</td>
<td>245.2 E-11</td>
<td>168.6 E-11</td>
</tr>
<tr>
<td>-50</td>
<td>244.1 E-11</td>
<td>168.4 E-11</td>
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<tr>
<td>-60</td>
<td>223.3 E-11</td>
<td>151.4 E-11</td>
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<tr>
<td>-70</td>
<td>164.0 E-11</td>
<td>112.3 E-11</td>
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<tr>
<td>-80</td>
<td>84.6 E-11</td>
<td>58.7 E-11</td>
</tr>
<tr>
<td>-90</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*E-11 means $10^{-11}$, etc.*
Figure 2. Horizontal gravity gradient: Normalized difference ($\nabla g_\lambda = \text{const} - \nabla g_{e \lambda} = \text{const}$) (see Table 2).
Example: The horizontal meridional component of the gravity gradient at 1000-km elevation in \( \phi = 30^\circ, \lambda = 70^\circ \) amounts to:

\[
\nabla g_\lambda = \text{const} = -1042.3 \times 10^{-12} \text{ sec}^{-2} \text{ (Table 3)}
\]

\[
\delta \nabla g_\lambda = \text{const} = 13 \times 10^{-12} \text{ sec}^{-2} \text{ (Number Map 2)}
\]

\[
\nabla g_\lambda = \text{const} = -1029 \times 10^{-12} \text{ sec}^{-2}
\]

If we take the lows and the peaks at the various altitudes of Number Map 2, we get the pattern shown in Table 4. The amplitudes decrease by the power 7 from zero elevation to 100 000-km altitude; this decrease is much greater than that for the corrections \( \Delta r \) in Number Map 1. However, the steep drop of \( \delta \nabla g_\lambda = \text{const} \) clearly occurs below 1000 km, although the magnitude remains quite the same between 0- and 500-km altitudes.

A similar pattern is found from the horizontal components along latitude curves. In Number Map 3 we have about the same magnitudes between 0- and 500-km altitudes and an analogous decrease from about 1000 km on upward. Table 4 gives the limits between which the amplitudes vary; at 100 000-km elevation the gradient components closely resemble the pattern of a triaxial field structure.

The nonzero (positive and negative) amplitudes at the poles are produced by the displacement of the astronomical pole relative to the geocentric pole (see Köhnlein, 1966a). However, the sum of the squares of \( \delta \nabla g_\lambda = \text{const} \) and \( \delta \nabla g_\phi = \text{const} \), hence \( \delta \nabla g_{\text{horiz}} \), leads to a constant amount, as expected from a point value. The greatest amounts of the horizontal anomalous component \( \delta \nabla g_{\text{horiz}} \) (see Number Map 3) are at places where the slopes of the gravity anomalies \( \Delta g = g - \bar{g} \) are steepest. This is particularly the case in low altitudes, between Canada and Greenland, around the Himalayan area, the Indian Ocean, and the Antarctic. In higher altitudes the pattern again tends toward the triaxial field structure.
Table 4. Anomalous range of the geocentric radius and the gravity gradient

<table>
<thead>
<tr>
<th>Elevation</th>
<th>0 km</th>
<th>500 km</th>
<th>1000 km</th>
<th>5000 km</th>
<th>10 000 km</th>
<th>50 000 km</th>
<th>100 000 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δr (m)</td>
<td>-93</td>
<td>73</td>
<td>-76</td>
<td>63</td>
<td>-64</td>
<td>55</td>
<td>-30</td>
</tr>
<tr>
<td>δνg λ = const</td>
<td>E-11</td>
<td>-12</td>
<td>E-12</td>
<td>-22</td>
<td>E-13</td>
<td>-22</td>
<td>E-14</td>
</tr>
<tr>
<td></td>
<td>-27</td>
<td>27</td>
<td>-12</td>
<td>13</td>
<td>-59</td>
<td>68</td>
<td>-22</td>
</tr>
<tr>
<td>δνg φ = const</td>
<td>E-11</td>
<td>-14</td>
<td>E-12</td>
<td>-20</td>
<td>E-13</td>
<td>-23</td>
<td>E-14</td>
</tr>
<tr>
<td></td>
<td>-34</td>
<td>27</td>
<td>-14</td>
<td>12</td>
<td>-67</td>
<td>71</td>
<td>-20</td>
</tr>
<tr>
<td>δνg horiz (sec^-2)</td>
<td>34</td>
<td>E-11</td>
<td>14</td>
<td>E-11</td>
<td>72</td>
<td>E-12</td>
<td>30</td>
</tr>
</tbody>
</table>
3.3 **Vertical Gravity Gradient**

This component is of special interest because it resembles almost the total amount of $\nabla g$. The maximum angle included by $\nabla g$ and the gradient line (plumb line) is, at zero elevation and middle latitudes, slightly smaller than 3 arcmin and drops at 100 000 km to about 0.6 arcsec (as shown in Köhnlein, 1966b). The zonal part of the vertical gravity gradient is listed in Table 5 for the different altitudes; because the gravitation $g$ is decreasing with increasing height, the sign of $\nabla g_{\text{vert}}$ is negative throughout. Figure 3 demonstrates the asymmetry between the Northern and Southern Hemispheres: we have a low at the North Pole and a peak at the South Pole. These irregularities decrease, as usual, with increasing altitude, as seen from the size of the conversion factors in Table 2.

Number Map 5 gives the anomalous vertical components $\delta \nabla g_{\text{vert}}$; if these are added to the above zonal part, we obtain the vertical component of $\nabla g$ at $U = \text{const.}$.

**Example:** The vertical gravity gradient in $\phi = 60^\circ$, $\lambda = 20^\circ$ at altitude 500 km amounts to:

$$\nabla g_{\text{vert}} = -0.244907 \times 10^{-5} \text{ sec}^{-2} \quad \text{(Table 5)}$$

$$\delta \nabla g_{\text{vert}} = -0.000009 \times 10^{-5} \text{ sec}^{-2} \quad \text{(Number Map 5)}$$

$$\nabla g_{\text{vert}} = -0.244916 \times 10^{-5} \text{ sec}^{-2}$$

The anomalous gravity gradients are used in geophysics for the analysis of complex geological structures. Mass defects in the earth's crust are more easily recognized from these studies than from purely gravimetric data. According to Number Map 5 (zero elevation), the vertical anomalies $\delta \nabla g_{\text{vert}}$ are mostly positive above the continents and take negative values along the worldwide mountain chains. The biggest amounts are found near Hudson Bay, in Central Asia, at the south coast of India, and above the Antarctic (Köhnlein, 1967).
Table 5. Vertical gravity gradient (zonal part) in sec$^{-2}$

<table>
<thead>
<tr>
<th>$\phi^\circ$</th>
<th>$V_\phi^\text{vert}$</th>
<th>0 km</th>
<th>500 km</th>
<th>1000 km</th>
<th>5000 km</th>
<th>10 000 km</th>
<th>50 000 km</th>
<th>100 000 km</th>
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<tbody>
<tr>
<td>90</td>
<td>-0.30 6754 E-5</td>
<td>-0.24 4655 E-5</td>
<td>-0.198 246 E-5</td>
<td>-0.540 9206 E-6</td>
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<td>-0.24 4771 E-5</td>
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<td>-0.541 0169 E-6</td>
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<td>-0.541 4059 E-6</td>
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<td>-0.4448 6571 E-8</td>
<td>-0.5622 32904 E-9</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3. Vertical gravity gradient: Normalized difference \((\overline{\nabla g_{\text{vert}}} - \nabla g_{\text{e vert}})\) (see Table 2).
3.4 Accuracy of the Gravity Gradient

Theoretically, for the accuracy determination, we could use the uncertainty of the harmonic coefficients obtained from a least-squares adjustment. But such a procedure gives mostly internal accuracies biased by unknown systematic errors. Hence, we compute only the spread of the Kozai-Gaposchkin field toward the Kaula field; the latter was obtained by a combination of satellite solutions with free-air gravity anomalies.

The horizontal and vertical components of the gravity gradients are compared in the same points \( r \) along \( \bar{U} = \text{const} \) and \( U = \text{const} \) (as shown in Table 1 and Number Map 1). However, we restrict our computations to zero altitude and 10 000-km elevations because of the limited meaning of the accuracy estimates.

Table 6 shows the difference of the zonal gravity gradients along a meridian. The vertical component has the greatest discrepancy at the poles, while the horizontal component shows most of its spread around the equatorial zones. Of course, parts of the deviations are due to the fact that we compare gravitational fields with different degrees and orders (harmonic coefficients). However, Kaula's gravitational field, although only a full 7,2 solution, has the advantage of having all available free-air gravity anomalies included; it is, therefore, best suited for this procedure.

If we consider the discrepancies as errors, we can derive a standard deviation for

1. the horizontal gravity gradient (zonal):

\[
\bar{m}_\lambda = \text{const} \begin{cases} 
\pm 2.3 \text{ E-11 sec}^{-2} & \text{at } 0 \text{ km} \\
\pm 5.9 \text{ E-16 sec}^{-2} & \text{at } 10 000 \text{ km};
\end{cases}
\]

2. the vertical gravity gradient (zonal):

\[
\bar{m}_{\text{vert}} \begin{cases} 
\pm 4.0 \text{ E-11 sec}^{-2} & \text{at } 0 \text{ km} \\
\pm 1.4 \text{ E-15 sec}^{-2} & \text{at } 10 000 \text{ km}.
\end{cases}
\]
Table 6. Spread between the zonal results (Kozai/Gaposchkin) — (Kaula CA) in sec$^{-2}$

<table>
<thead>
<tr>
<th>$\phi^\circ$</th>
<th>Elevation</th>
<th>0 km</th>
<th>10 000 km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal gravity gradient</td>
<td>Vertical gravity gradient</td>
<td>Horizontal gravity gradient</td>
</tr>
<tr>
<td>90</td>
<td>0.0</td>
<td>-5.2 E-11</td>
<td>0.0</td>
</tr>
<tr>
<td>80</td>
<td>2.1 E-11</td>
<td>-2.7 E-11</td>
<td>-0.8 E-15</td>
</tr>
<tr>
<td>70</td>
<td>0.8 E-11</td>
<td>2.4 E-11</td>
<td>-0.8 E-15</td>
</tr>
<tr>
<td>60</td>
<td>-1.3 E-11</td>
<td>0.2 E-11</td>
<td>-0.2 E-15</td>
</tr>
<tr>
<td>50</td>
<td>0.7 E-11</td>
<td>-0.5 E-11</td>
<td>0.4 E-15</td>
</tr>
<tr>
<td>40</td>
<td>-0.8 E-11</td>
<td>0.7 E-11</td>
<td>0.7 E-15</td>
</tr>
<tr>
<td>30</td>
<td>0.5 E-11</td>
<td>-2.0 E-11</td>
<td>0.8 E-15</td>
</tr>
<tr>
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<td>3.0 E-11</td>
<td>0.6 E-15</td>
</tr>
<tr>
<td>10</td>
<td>-3.4 E-11</td>
<td>-0.6 E-11</td>
<td>-0.1 E-15</td>
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<tr>
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<td>-0.3 E-15</td>
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<td>-2.5 E-11</td>
<td>0.5 E-15</td>
</tr>
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<td>3.9 E-11</td>
<td>0.5 E-15</td>
</tr>
<tr>
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<td>-4.1 E-11</td>
<td>0.1 E-15</td>
</tr>
<tr>
<td>-60</td>
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<td>0.0 E-15</td>
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<td>5.7 E-11</td>
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</tr>
<tr>
<td>-80</td>
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<td>0.6 E-15</td>
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<tr>
<td>-90</td>
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<td>-11.8 E-11</td>
<td>0.0</td>
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</table>
Hence, the vertical zonal component seems to be less accurate than the meridional zonal gravity gradient.

Analogous considerations are applied to the combined zonal and nonzonal fields. In Number Maps 6 and 7 we give the differences of the Kozai-Gaposchkin sets relative to Kaula's gravity gradients. If we again compute standard deviations, considering the area represented by the individual numbers, we obtain for

1. the horizontal, meridional component:
   \[ m_\lambda = \text{const} \begin{cases} 
   \pm 7.1 \times 10^{-11} \text{ sec}^{-2} & \text{at 0 km} \\
   \pm 1.0 \times 10^{-14} \text{ sec}^{-2} & \text{at 10 000 km};
   \end{cases} \]

2. the horizontal component along latitude curves:
   \[ m_\phi = \text{const} \begin{cases} 
   \pm 7.2 \times 10^{-11} \text{ sec}^{-2} & \text{at 0 km} \\
   \pm 1.2 \times 10^{-14} \text{ sec}^{-2} & \text{at 10 000 km};
   \end{cases} \]

3. the vertical component:
   \[ m_{\text{vert}} \begin{cases} 
   \pm 1.4 \times 10^{-10} \text{ sec}^{-2} & \text{at 0 km} \\
   \pm 4.2 \times 10^{-14} \text{ sec}^{-2} & \text{at 10 000 km}.
   \end{cases} \]

Again, the vertical component appears to have greater uncertainty than the horizontal components; however, the relative accuracy referred to the full range of Table 4 gives for all components the same percentage, namely, approximately 12%.

3.5 Constants and Coefficients Used

In the above analysis, we used the following constants:

\[ a = 6 \, 378 \, 165 \, \text{m}, \text{ the earth's equatorial radius}, \]
\[ GM = 3.986032 \times 10^{20} \, \text{cm}^3 \, \text{sec}^{-2}, \]
and
\[ K_2 = -0.108264 \times 10^{-2} \]
\[ K_4 = 0.21 \times 10^{-5}. \]
Also, the following zonal harmonic coefficients by Kozai and Kaula and nonzonal harmonic coefficients by Gaposchkin and Kaula were used:

Zonal Harmonic Coefficients (normalized)

<table>
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<tr>
<th></th>
<th>Kozai (1964) $\times 10^6$</th>
<th>Kaula (1966) $\times 10^6$</th>
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### Nonzonal Harmonic Coefficients (normalized)

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<th>(C_{nm}) (\times 10^6)</th>
<th>(S_{nm}) (\times 10^6)</th>
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Kaula CA (1966)
4. ACKNOWLEDGMENTS

I am much indebted to Dr. C. A. Lundquist, who suggested this numerical analysis to me.
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REMARKS TO NUMBER MAPS 1 TO 7

Geocentric latitude $\phi$: first column on the left of each table; Northern Hemisphere $+$, Southern Hemisphere $-$.

Geocentric longitude $\lambda$: first line on the top of each table (to be multiplied by 10); east of Greenwich $+$, west of Greenwich $-$.
### Number Map 1. Shapes of the equipotential surfaces

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<th>Δr in meters.</th>
<th>0-km elevation.</th>
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<th>Δr in meters.</th>
<th>0-km elevation.</th>
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<td>-200 - 1000</td>
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</table>
```

Ar in meters. 500-km elevation.
\[ \Delta r \text{ in meters.} \quad 10,000\text{-km elevation.} \]

\[ \Delta r \times 10^{-1} \text{ meters.} \quad 50,000\text{-km elevation.} \]
\[ \Delta r \times 10^{-1} \text{ meters.} \quad 100 \text{ 000-km elevation.} \]
Number Map 2. Horizontal anomalous gravity gradient along \( \lambda = \text{const} \)

\[
\delta \nabla g_\lambda = \text{const} \times 10^{-11} \text{ sec}^{-2}, \quad \text{0-km elevation.}
\]
\[ \Delta V_{g,\lambda} = \text{const} \times 10^{-12} \text{ sec}^{-2} \]. 1000-km elevation.

\[ \Delta V_{g,\lambda} = \text{const} \times 10^{-13} \text{ sec}^{-2} \]. 5000-km elevation.
ΔV₀ = const \times 10^{-14}\text{ sec}^{-2}. 10000-km elevation.

ΔV₀ = const \times 10^{-17}\text{ sec}^{-2}. 50000-km elevation.
**Table**

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<th>Gap</th>
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<td>Health Care</td>
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<td>10%</td>
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</table>

**Diagram**

- **Legend**:
  - Red: Aspirational Target
  - Blue: Current Status
  - Green: Gap

---

### Equation

\[
\Delta v = \text{const} \times 10^{-12} \text{ sec}^{-2}
\]

500-km elevation.

--

### Equation

\[
\Delta v_{\phi} = \text{const} \times 10^{-13} \text{ sec}^{-2}
\]

5000-km elevation.

---

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\[ \delta V_{g_\phi} = \text{const} \times 10^{-14} \text{ sec}^{-2}, \ 50 \text{ 000-km elevation.} \]

\[ \delta V_{g_\phi} = \text{const} \times 10^{-17} \text{ sec}^{-2}, \ 50 \text{ 000-km elevation.} \]
$\delta \nabla g_{\phi} = \text{const} \times 10^{-18} \text{ sec}^{-2}$. 100 000-km elevation.
### Number Map 4. Total horizontal anomalous gravity gradient

| X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   | X   | Y   | Z   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  | 22  | 23  | 24  | 25  | 26  | 27  | 28  | 29  | 30  | 31  | 32  | 33  | 34  | 35  | 36  | 37  | 38  | 39  | 40  | 41  | 42  | 43  | 44  | 45  | 46  | 47  | 48  | 49  | 50  | 51  | 52  | 53  | 54  | 55  | 56  | 57  | 58  | 59  | 60  | 61  | 62  | 63  | 64  | 65  | 66  | 67  | 68  | 69  | 70  | 71  | 72  | 73  | 74  | 75  | 76  | 77  | 78  | 79  | 80  | 81  | 82  | 83  | 84  | 85  | 86  | 87  | 88  | 89  | 90  | 91  | 92  | 93  | 94  | 95  | 96  | 97  | 98  | 99  | 100 |

\[ \delta V_{\text{horiz}} \times 10^{-11} \text{ sec}^{-2} \cdot 0.5 \text{km elevation.} \]
$\delta V_{\text{horiz}} \times 10^{-18}$ \text{sec}^{-2}. 100 000-km elevation.
\[ \delta V_{\text{vert}} \times 10^{-14} \text{ sec}^{-2} \cdot 10000\text{-km elevation.} \]
\[ 5 \nabla g_{\text{vert}} \times 10^{-18} \text{ sec}^{-2}, \ 100 \ 000\text{-km elevation}. \]
Number Map 6. Horizontal and vertical spread of the gravity gradient; 0-km elevation

\[
\left( \nabla g_{\text{Kozai Gaposchkin}} \right) - \left( \nabla g_{\text{Kaula CA}} \right) \times 10^{-11} \text{ sec}^{-2}.
\]

\[\lambda = \text{const}\]

<table>
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<td>1</td>
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</table>

\[\left( \nabla g_{\text{Kozai Gaposchkin}} \right) - \left( \nabla g_{\text{Kaula CA}} \right) \times 10^{-11} \text{ sec}^{-2}.\]

\[\phi = \text{const}\]

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\[
\left( \nabla g_{\text{Kozai}} - \nabla g_{\text{Kaula CA}} \right) \times 10^{-11} \text{ sec}^{-2}. \\
\text{Gaposchkin vert}
\]
Number Map 7. Horizontal and vertical spread of the gravity gradient; 10,000 km elevation

\[
\left( \frac{\nabla g_{\text{Kozai}}}{\nabla g_{\text{Kaula}}} \right)_\Lambda = \text{const}
\]

\[
\left( \frac{\nabla g_{\text{Kozai}} - \nabla g_{\text{Kaula}}}{\nabla g_{\text{Kaula}}} \right)_\Phi = \text{const}
\]

\( \times 10^{-15} \text{ sec}^{-2} \)
\[
\left( \nabla_{g_{\text{Kozai}}} - \nabla_{g_{\text{Kaula CA}}} \right)_{\text{vert}} \times 10^{-14} \text{ sec}^{-2}
\]
BIIOGRAPHICAL NOTE

WALTER KÖHNLEIN received his degrees from the Technological Institute in Munich and the Technological Institute in Braunschweig, Germany, in 1958 and 1962, respectively.

He worked for a year at Ohio State University, Colombus, Ohio, before joining SAO in 1963. In 1965 he became an associate of the Harvard College Observatory.

Dr. Kohnlein's work at Smithsonian includes studies of the figure of the earth and its gravitational field by means of artificial satellites.