PARAMETRIC MASS ANALYSIS AND COMPARISON OF TWO TYPES OF REACTANT COOLING-AND- STORAGE UNITS FOR A LUNAR-BASED HYDROGEN-OXYGEN REGENERATIVE FUEL-CELL SYSTEM

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SUMMARY

This study was to determine the effect of prestorage reactant cooling on the mass of a lunar-based hydrogen-oxygen regenerative fuel cell system. Consideration was given to cooling by radiators and by refrigerator-radiator combinations. Parametric mass analyses were performed for components of each type of cooling-and-storage unit and for the residual reactant in each tank.

A mass comparison of the two units established the mass limitations of the refrigerators, which were not included as part of the total mass of the radiative-plus-refrigerative units.

The parameters considered were the temperature to which the reactant was cooled, the pressure under which the cooling occurred, the final storage pressure of the reactant, the heat leak into the storage tanks, the sink temperature for the radiators, and the effective sink temperature for the surface of the tankage insulation. For the combined unit, consideration was also given to the refrigerator efficiency and the temperature difference between the reactant and refrigerant.

The combined radiative-plus-refrigerative unit has a minimum mass when the reactant is cooled to subcritical temperatures. The mass of the purely radiative unit minimizes when the reactant is cooled to within 0.5°C of the radiator sink temperature. All cooling which occurs above these optimum temperatures decreases, but does not minimize, the mass of these units.

Decreasing the final storage pressure leads to considerable mass increases when the reactants are stored at supercritical temperatures. There exists an optimum allowable heat leak into the storage tanks which minimizes the total mass of the unit.

For the combined unit there exists an optimum heat-rejection temperature which minimizes the sum of the masses of the solar cells and the radiator required by the refrigerator.

The minimum-mass purely radiative cooling-and-storage units are considerably heavier than the corresponding combined units, indicating that the actual refrigerators could be quite heavy and still offer a mass advantage over cooling by radiators alone.
INTRODUCTION

Expanded lunar exploration beyond the initial manned landings (the Apollo program) may include the establishment of permanent lunar stations. Such stations would require electric power for environment control, communications, experimentation and vehicular activity. One possible power supply would combine regenerative hydrogen-oxygen fuel cells and solar cells. The fuel cells would supply power during the lunar night. In the daytime, solar cells would be the power source for the station and for an electrolysis unit in which the fuel cell reaction product (water) would be electrolyzed back to hydrogen and oxygen.

One study (ref. 1) has presented the weight of an orbiting 500-watt regenerative hydrogen-oxygen fuel cell as a function of orbital altitude. A similar study (ref. 2) included consideration of the mass of such a system on the lunar surface. In neither of these studies was consideration given to prestorage cooling of the reactants.

To determine the effect of prestorage reactant cooling on the mass of the system, two types of cooling-and-storage units were examined. One utilized radiators alone, and the other a radiator-refrigerator combination.

Parametric mass analyses were performed for the storage tanks, insulation, plumbing, supports, radiators, and the solar cells necessary to power the refrigerators. Also determined was the mass of unused reactant remaining in each tank at the end of the lunar night.

Since it was not within the scope of this study to actually design refrigerators, no mass term was given to them. Instead, the limitations for the refrigerator masses were inferred from the final mass comparison of the two reactant cooling-and-storage units.

Those portions of the complete power system whose masses were independent of the reactant cooling method were not considered. Among these were the fuel cell unit, the electrolysis unit, and the solar cells necessary to power the manned station and the electrolysis unit.

The parameters assumed to influence the masses of the cooling-and-storage units were the temperature to which the reactant was cooled, the pressure under which the cooling occurred, the final storage pressure of the reactant, the heat leak into the storage tanks, the sink temperature seen by the radiators, and the effective sink temperature seen by the surface of the tank insulation. In addition, for the radiative-plus-refrigerative cooling-and-storage unit, the refrigerator efficiency and the temperature difference between the reactant and the refrigerant were considered.

The information compiled in this study offers some guidelines for the design of reactant cooling-and-storage units for a lunar-based regenerative hydrogen-oxygen fuel cell system.
SYMBOLS

\( A \) \hspace{1cm} \text{area, m}^2

\( C_p \) \hspace{1cm} \text{heat capacity at constant pressure, J/(kg)(}^{\circ}\text{K); } 1.423 \times 10^4 \text{ for hydrogen, } 0.92 \times 10^3 \text{ for oxygen}

\( F_A \) \hspace{1cm} \text{configuration factor}

\( F_\epsilon \) \hspace{1cm} \text{emissivity factor}

\( h \) \hspace{1cm} \text{enthalpy per unit mass, J/kg}

\( h_R \) \hspace{1cm} \text{heat-transfer coefficient, W/(m}^2\text{)(}^{\circ}\text{K); } 85.2 \text{ for hydrogen; } 17.2 \text{ for oxygen}

\( k \) \hspace{1cm} \text{thermal conductivity, W/(m)(}^{\circ}\text{K); } 3.64 \times 10^{-5} \text{ for superinsulation}

\( M \) \hspace{1cm} \text{component mass, kg}

\( m \) \hspace{1cm} \text{reactant flow rate, kg/sec; } 1.305 \times 10^{-4} \text{ for hydrogen, } 10.39 \times 10^{-4} \text{ for oxygen}

\( m \) \hspace{1cm} \text{reactant mass, kg}

\( P \) \hspace{1cm} \text{power, W}

\( p \) \hspace{1cm} \text{pressure, N/m}^2

\( Q \) \hspace{1cm} \text{heat-transfer rate, W}

\( \bar{Q}_L \) \hspace{1cm} \text{average heat-transfer rate through insulation, W}

\( r \) \hspace{1cm} \text{radius, m}

\( r_{T'} \) \hspace{1cm} \text{dummy variable used in calculation of reactant tankage radius}

\( S \) \hspace{1cm} \text{working strength, N/m}^2

\( S_R \) \hspace{1cm} \text{radiator specific mass, } 14.7 \text{ kg/m}^2

\( S_{sc} \) \hspace{1cm} \text{solar cell specific mass, } 45.5 \text{ kg/kW}

\( T \) \hspace{1cm} \text{temperature, } ^{\circ}\text{K}

\( \bar{T} \) \hspace{1cm} \text{average temperature, } ^{\circ}\text{K}

\( T_s \) \hspace{1cm} \text{effective sink temperature, } ^{\circ}\text{K}

\( t \) \hspace{1cm} \text{thickness, m}

\( u \) \hspace{1cm} \text{internal energy per unit mass, J/kg}

\( V \) \hspace{1cm} \text{volume, m}^3

\( v \) \hspace{1cm} \text{specific volume, m}^3/\text{kg}

\( \Delta \) \hspace{1cm} \text{refrigerant-to-reactant temperature difference, } ^{\circ}\text{K}
\[ \epsilon \quad \text{emissivity} \]
\[ \eta \quad \text{refrigerator efficiency} \]
\[ \sigma \quad \text{Stefan-Boltzmann constant, } 5.73 \times 10^{-8} \text{ W/(m}^2\text{)}(\text{K}^4) \]
\[ \tau \quad \text{electrolysis period (lunar daytime), } 1.21 \times 10^6 \text{ sec} \]
\[ \rho \quad \text{density, kg/m}^3 \]

Subscripts

A \quad \text{absorbed}

a \quad \text{actual}

E \quad \text{reactant condition leaving electrolyzer}

H \quad \text{heat rejection}

i \quad \text{insulation, insulation surface, ideal}

L \quad \text{reactant condition leaving cooling device}

L' \quad \text{reactant condition in storage tank}

Im \quad \text{log mean}

o \quad \text{reactant storage condition at end of lunar night}

P \quad \text{plumbing and supports}

R \quad \text{radiator}

Si \quad \text{insulation surface sink}

sc \quad \text{solar cell}

T \quad \text{storage tank}

ten \quad \text{total}

w \quad \text{wall}

w, E \quad \text{surface condition at radiator entrance}

w, L \quad \text{surface condition at radiator exit}

**METHOD OF ANALYSIS**

Two types of regenerative hydrogen-oxygen fuel cell systems were examined. These systems differed in the method used to cool the regenerated reactants prior to storage. One used simple space radiators (fig. 1); the other radiator-refrigerator combinations (fig. 2).
During the lunar night either system would consume the stored hydrogen and oxygen and thus provide useful power. The reaction product, water, would be removed from the fuel cells and stored. During the lunar day the water would be pumped to the electrolysis unit, where hydrogen and oxygen would be regenerated. Arrays of solar cells would provide power for the pump and electrolysis unit.

The regenerated reactants would leave the electrolyzer and pass through chemical filters and driers. In these, moisture and entrained electrolyte would be removed. The reactants, still at the pressure existing in the electrolysis unit, would then pass through individual cooling units prior to being throttled into their respective storage tanks. These spherical storage tanks would be insulated and surrounded by multiple thin, lightweight solar radiation shields. These shields would cause effective sink temperatures for the inner insulation surfaces to be lower than the lunar daytime surface equilibrium temperature of about $390^\circ K$.

Many components would be common to both systems. They were assumed to have the same mass. These included the fuel cell unit, electrolysis unit, water pump and storage tank, gas filters and driers, and the solar cell arrays to supply power to the pump.
and electrolysis unit. The solar radiation shields were assumed to have negligible mass (see ref. 3). These components were not included in the mass comparison of the two cooling-and-storage units. Furthermore, because the design of actual refrigerators was beyond the scope of this study, no mass term was given to them. However, attention was given to the masses of two major auxiliary components of the refrigerators: the solar cell arrays and the radiators. In order to estimate the power and heat-rejection requirements of the refrigerator, a hypothetical thermodynamic cycle was established for the refrigerant. For this study, the cycle was assumed to consist of isentropic expansion and compression of the working fluid, isothermal heat rejection and nonisothermal heat acceptance.

Equations were derived for the masses of the various components of the reactant cooling-and-storage units as functions of the parameters under consideration. These equations were solved on a digital computer.
Reference 4 shows that an ideal refrigerator with a nonisothermal load has the same coefficient of performance as a Carnot (ideal, isothermal) refrigerator which absorbs heat at the log-mean temperature of the nonisothermal load. The coefficient of performance of a Carnot cycle refrigerator is shown in reference 5 to be

\[
\frac{Q_A}{P_1} = \frac{(\text{Isothermal load temperature})}{(\text{Isothermal heat rejection temperature}) - (\text{Isothermal load temperature})}
\] (1)

Therefore, for the assumed refrigerator it was possible to write:

\[
\frac{Q_A}{P_1} = \frac{(\text{Log-mean load temperature})}{(\text{Isothermal heat rejection temperature}) - (\text{Log-mean load temperature})}
\] (2)

It was assumed that heat transfer in the refrigerator occurred in a counterflow heat exchanger, and that \( \Delta \), the temperature difference between reactant and refrigerant, was constant. It was then possible to express the refrigerant temperatures in terms of the reactant temperatures: When the reactant was refrigerated from \( T_E \) to \( T_L \), the refrigerant temperature rose from \( T_L - \Delta \) to \( T_E - \Delta \). The log-mean refrigerant temperature was then expressed, by definition, as

\[
\frac{(T_E - T_L)}{\ln \frac{T_E - \Delta}{T_L - \Delta}}
\]

Next, a refrigerator efficiency \( \eta \) was defined as

\[
\eta = \frac{P_1}{P_a}
\] (3)

Finally, the rate of heat absorption by the refrigerant, \( Q_A \) was expressed in terms of the mass flow rate of the reactant \( \dot{m} \) and its enthalpy change:

\[
Q_A = \dot{m}(h_E - h_L)
\] (4)
Equations (3) and (4) were substituted in equation (2), which was rearranged to express the actual refrigerator power requirement:

\[
P_a = \frac{\dot{m}(h_E - h_L) \left[ T_H \ln \left( \frac{T_E - \Delta}{T_L - \Delta} \right) - (T_E - T_L) \right]}{\eta(T_E - T_L)}
\]  

(5)

Solar cells. - The mass of the solar cell array necessary to meet this power requirement \( M_{sc} \) was determined by multiplying the power requirement by the specific mass of a solar cell array \( S_{sc} \). Therefore,

\[
M_{sc} = S_{sc} \frac{\dot{m}(h_E - h_L) \left[ T_H \ln \left( \frac{T_E - \Delta}{T_L - \Delta} \right) - (T_E - T_L) \right]}{\eta(T_E - T_L)}
\]

(6)

For the purposes of this study, \( S_{sc} \) was assumed to be 45.5 kilograms per kilowatt.

Radiator. - The rate of heat rejection from the reactant refrigerator \( Q_R \) would be the sum of the rate of heat absorption by the refrigerant \( Q_A \) and the power supplied to the refrigerator \( P_a \). It was assumed that this heat rejection would occur isothermally from a radiator at temperature \( T_H \). The radiator surface could be given a special contour so that it would not be much affected by reflected energy from the lunar surface. Redundant radiator sections could be deployed so that the working section would always be facing away from direct sunlight. In either case, the radiator would be made to see an effective sink temperature \( T_s \), which would be less than that of the lunar daytime surface equilibrium temperature of about 390°K.

The rate of heat rejection from such a radiator is

\[
Q_R = \sigma F \varepsilon F_A A_R \left( T_H^4 - T_s^4 \right)
\]

(7)

For this study, the product \( F_A \varepsilon \) was assumed to be 0.85 (ref. 6).

Since \( Q_R \) also equaled \( Q_A + P_a \), equations (4), (5), and (7) were combined and solved for the prime radiator area \( A_R \):
This assumes that the heat-transfer coefficient of the refrigerant is large enough so that
the temperature drop from the refrigerant to the radiator surface is insignificant.

A specific mass factor $S_R$ was then introduced which, when multiplied by $A_R$, gave
the total radiator mass

$$M_R = S_R \dot{m} (h_E - h_L) \left[ (\eta - 1)(T_E - T_L) + T_H \ln \left( \frac{T_E - \Delta}{T_L - \Delta} \right) \right]$$

$$= \frac{0.85 \eta (T_E - T_L)}{T_H - T_S} \left( \frac{T_E^4 - T_L^4}{T_H^4 - T_S^4} \right)$$

where $S_R$ was assigned a value of 14.7 kilograms per square meter.

Storage tanks. - At the end of the lunar night there would remain in a storage tank a
mass $m_o$ of unused reactant. This reactant would be at a pressure $p_o$ determined by
the minimum supply pressure requirement of the fuel cells. It would be at a temperature
$T_o$ resulting from a heat loss to (or heat gain from) the lunar nighttime surface equilib-
rium temperature of about $120^0 K$. These conditions, $p_o$ and $T_o$, would determine
the specific volume $v_o$ and internal energy $u_o$ of the reactant in the tank.

For this study, $p_o$ was assigned a value of $4.14 \times 10^5$ newtons per square meter (about
60 psia), which is the operating pressure of several present-day fuel cells. It was as-
sumed that the hydrogen would be stored at low temperatures and that it would be heated
up to $85^0 K$ during the lunar night. It was further assumed that the oxygen would be
stored near $120^0 K$ and that it would remain at that temperature. With these assumptions,
it was possible to express the tankage volume in terms of $m_o$:

$$V_T = \frac{4}{3} \pi r_T^3 = m_o v_o$$

Rearranged, the equation becomes

$$m_o = \frac{4\pi r_T^3}{3v_o}$$

At the end of the lunar daytime, during which the reactants would have been electro-
lytically regenerated, an amount of reactant $\dot{m}_T$ would have been added to the storage tank with an enthalpy of $h_L$. The tank would then contain the total amount of reactant $m_o + \dot{m}_T$ at pressure $p_L$, and temperature $T_L$. At these conditions, the reactant in the tank would have internal energy $u_L$, and specific volume $v_L$.

During the electrolysis period, heat from the lunar environment would have passed through the tank insulation at an average rate of $\overline{Q}_L$. An additional heat leak, through plumbing and supports, was assumed to occur at a rate of $0.5 \overline{Q}_L$ (ref. 7). Therefore, by the end of the electrolysis period a quantity of heat $1.5 \overline{Q}_L \tau$ would have entered the tank.

An energy balance on the storage tank yields

\[
\begin{align*}
\text{(Energy initially in tank)} & \quad \text{+ (Energy added to tank)} \quad = \quad \text{(Energy finally in tank)} \\
m_o u_o + (\dot{m}_T h_L - 1.5 \overline{Q}_L \tau) & \quad = \quad (m_o + \dot{m}_T) u_L,
\end{align*}
\]

(11)

($\overline{Q}_L$ was given a negative sense when entering the tankage.)

Equations (10a) and (11) were combined and rearranged to give

\[
u_L = \frac{4\pi r_T^3}{3 v_o} + \dot{m}_T h_L - 1.5 \overline{Q}_L \tau
\]

\[
\left(\frac{4\pi r_T^3}{3 v_o} + \dot{m}_T \right)
\]

\[
(12)
\]

For a given set of values for the parameters $p_E$, $T_L$, and $\overline{Q}_L$ ($p_E$ and $T_L$ being sufficient to define $h_L$), equation (12) expressed the relation between the unknown quantities $u_L$, and $r_T$. A value was assumed for $r_T$, and the equation was solved for $u_L$. This value of $u_L$, along with a given value for the parameter $p_L$, (the final tank pressure), defined the final tank temperature $T_L$ and final reactant specific volume $v_L$.

The correctness of the assumed value of $r_T$ was checked by calculating the dummy radius $r_T'$ of a tank which could contain the reactant mass $m_o + \dot{m}_T$ with the specific volume, $v_L$.

\[
r_T' = \left[\frac{3(m_o + \dot{m}_T)v_L}{4\pi}\right]^{1/3}
\]

(13)
This trial and error procedure was repeated until \( r_{T'} = r_T \). It was then possible to determine the tank mass.

The tank was assumed to be spherical and thin-walled. Therefore, the wall thickness was expressed as

\[
t_w = \frac{p_L r_T}{2S_T}
\]  

(14)

and the tank mass was written in terms of this wall thickness, the surface area of the tank, and the density of the tank material

\[
M_T = 4\pi r_T^2 t_w \rho_T
\]  

(15)

Finally, equations (13) (for \( r_{T'} = r_T \)) to (15) were combined, giving, for the tank mass,

\[
M_T = \frac{1.5 \rho_T p_L v_{L'} (m_0 + \dot{m} \tau)}{S_T}
\]  

(16)

By assigning to the tank material a strength-to-density ratio of 0.924×10^5 newton meters per kilogram, which would be typical of any strong, light weight material comparable to titanium (ref. 8), the tank mass became

\[
M_T = 1.623 \ p_L v_{L'} (m_0 + \dot{m} \tau) \times 10^{-5}
\]  

(16a)

**Tank insulation.** - The heat, which would be conducted through the tank insulation to the stored reactant, would first reach the insulation surface by radiation. As mentioned before, the radiation would come from a sink, the effective temperature of which would be less than that of the lunar daytime environment. This reduction of the effective sink temperature seen by the insulation surface would be brought about by the use of thin, multilayered solar shields, concentric to the tank.

During the electrolysis period, the reactant temperature in the tank would vary from \( T_0 \) to \( T_{L'} \), being affected by the introduction of reactant from the refrigerator and the heat leak. Over the same time, the lunar environment seen by the solar shields would vary with the position of the sun in the sky. Under these influences, the effective sink temperature seen by the insulation, the surface temperature of the insulation, and the heat-leak rate through the insulation would all vary with time.
It was therefore assumed that there was an average reactant temperature

\[ \overline{T_{L'}} = \frac{(T_{L'} + T_o)}{2} \]  

(At this point in the analysis, \( T_o \) had already been assigned a value, and \( T_{L'} \) had been determined by the energy balance on the tankage.) It was further assumed that, associated with this average reactant temperature, there would be an average effective insulation sink temperature \( \overline{T_{Si}} \), an average insulation surface temperature \( \overline{T_i} \), and an average heat-transfer rate through the insulation \( \overline{Q_L} \). With these assumptions, the simultaneous equations for the transfer of \( \overline{Q_L} \) by radiation and conduction would contain as unknowns the thickness \( t_i \) and average surface temperature \( \overline{T_i} \) of the insulation, in terms of the parameters \( \overline{T_{Si}} \) and \( \overline{Q_L} \).

For radiation between the insulation surface and the concentric solar shields, equation (7) was rewritten as

\[ \overline{Q_L} = \sigma F_e F_A A_i \left( \frac{T_i^4 - T_{Si}^4}{T_i - T_{Si}} \right) \]  

(18)

It can be shown that, for concentric surfaces,

\[ F_A = 1.0 \quad \text{and} \quad F_e = \frac{1}{1 + \frac{A_i}{A_{Si}} \left( \frac{1}{\epsilon_i} - 1 \right) \epsilon_{Si}} \]

\( \epsilon_i \) was chosen to be 0.90, and \( \epsilon_{Si} \) 0.03, for these surfaces. For close-spacing between the insulation surface and the innermost solar shield, the ratio \( A_i/A_{Si} \) is approximately equal to 1.0. These substitutions were made, giving \( F_e = 0.0298 \).

Next, the area of the insulation surface was expressed as

\[ A_i = 4\pi (r_T + t_i)^2 \]  

(19)

which assumes that the tank wall thickness is negligible relative to \( r_T + t_i \).

Finally, \( A_i \), \( F_e \), and \( F_A \) were substituted into equation (18), which was rearranged to yield the equation

12
For the conduction of $Q_L$ from the insulation surface to the stored reactant, it was assumed that the total resistance to heat flow was contributed by the insulation; therefore, the rate of heat transfer was expressed as

$$
\bar{Q}_L = \frac{kA_{lm}(\bar{T}_L - \bar{T}_i)}{t_i}
$$

(21)

The log-mean heat-transfer area $A_{lm}$ was rewritten in terms of the radii of the tank and the insulation surface.

$$
A_{lm} = \frac{2\pi [(r_T + t_i)^2 - r_T^2]}{\ln(r_T + t_i) - \ln r_T}
$$

(22)

Equations (21) and (22) were then combined and solved for the average insulation surface temperature $\bar{T}_i$:

$$
\bar{T}_i = \frac{\bar{Q}_L t_i}{2k\pi} \left[ \ln(r_T + t_i) - \ln r_T \right] + \bar{T}_L
$$

(23)

Finally, $T_i$ was eliminated from equations (20) and (23), giving

$$
\frac{\bar{Q}_L t_i}{2k\pi} \left[ \ln(r_T + t_i) - \ln r_T \right] + T_L = \left[ \frac{2.65 \bar{Q}_L}{\sigma(r_T + t_i)^2} + (T_{Si})^4 \right]^{1/4}
$$

(24)

For given values of the parameters $\bar{Q}_L$ and $T_{Si}$, and the previously-calculated values of $r_T$ and $T_L$, this equation was solved for the insulation thickness $t_i$ by trial and error. In this study, the insulation was assumed to have a thermal conductivity of $3.64 \times 10^{-5}$ watt per meter per °K, which is typical of superinsulations (ref. 7), and a density of 160.5 kilogram per cubic meter.

Knowing the tank radius $r_T$, the insulation thickness $t_i$ and the insulation density $\rho_i$, the mass of insulation is given by
Plumbing and supports. - It was assumed that the mass of the plumbing and the supports for the tank and insulation would amount to one-tenth the mass of the tank.

\[ M_p = 0.1 M_T \]  

Total mass. - For the reactant cooling-and-storage unit combining refrigeration and radiation, a final mass term \( M_{\text{tot}} \) was defined as

\[ M_{\text{tot}} = M_{\text{sc}} + M_R + m_o + M_T + M_i + M_p \]  

Cooling By Radiation

An equation for determining the prime area requirement of a radiator rejecting heat nonisothermally is presented in reference 9. For a reactant being cooled from \( T_E \) to \( T_L \) and for an effective sink temperature \( T_S \), the equation was written as

\[
A_R = \frac{\dot{m}C_P}{h_R} \left\{ \frac{1}{4} \ln \frac{T_{w,E}^4 - T_{w,L}^4}{T_{w,L}^4 - T_S^4} + \frac{1}{4\sigma c T_S^3} \ln \frac{(T_{w,E} - T_S)(T_{w,L} + T_S)}{(T_{w,L} - T_S)(T_{w,E} + T_S)} \right. \\
- 2 \left( \arctan \frac{T_{w,E}}{T_S} - \arctan \frac{T_{w,L}}{T_S} \right) \right\} 
\]  

In this expression, \( T_{w,E} \) and \( T_{w,L} \) are the radiator surface temperatures corresponding to the reactant temperatures \( T_E \) and \( T_L \), respectively. These two temperatures are defined according to the relations

\[
T_E = T_{w,E} + \frac{\sigma c}{h_R} \left( T_{w,E}^4 - T_S^4 \right) \]  

and
\[
T_L = T_{w, L} + \frac{\sigma \varepsilon}{h_R} \left( T_{w, L}^4 - T_s^4 \right)
\] (28b)

In equations (28) the term \( h_R \) is the reactant film heat-transfer coefficient related to radiating area. As representative values, this coefficient was assumed to be 85.2 watts per square meter per °K for hydrogen, and 17.2 watts per square meter per °K for oxygen.

Radiator. - As was the case with the radiator for the refrigerator, it was assumed that the effective sink temperature for the reactant radiator would be lower than the lunar daytime surface equilibrium temperature through the use of special surface geometries and/or the use of redundant working sections, a portion of which would always be directed away from the sun. The mass of the reactant radiator was determined by multiplying the prime area \( A_R \) by the assumed specific radiator mass \( S_R \):

\[
M_R = \dot{m} C_p S_R \left\{ \frac{1}{h_R} \ln \left( \frac{T_{w, E}^4 - T_s^4}{T_{w, L}^4 - T_s^4} \right) + \frac{1}{4\sigma \varepsilon T_s^3} \left[ \ln \left( \frac{T_{w, E} - T_s}{T_{w, L} + T_s} \right) \frac{\left( T_{w, L} + T_s \right)}{\left( T_{w, E} + T_s \right)} \right] - 2 \left( \arctan \frac{T_{w, E}}{T_s} - \arctan \frac{T_{w, L}}{T_s} \right) \right\} \] (29)

Storage tanks, insulation, plumbing and supports, and unused reactants. - The method for determining the masses of the remaining components of the purely radiative cooling and storage units was identical to that developed for combined radiative-refrigerative cooling. However, it was assumed that the reactants would be stored considerably above the lunar nighttime surface equilibrium temperature (120°K), and both would cool to 139°K (\( T_0 \)) during the night. Having made this adjustment prior to solving the equations, the masses of tankage, insulation, plumbing and supports, and unused reactants were obtained from equations (16a), (25), (26), and (10a), respectively.

Total mass. - For radiative reactant cooling,

\[
M_{tot} = M_R + m_o + M_T + M_i + M_p
\] (30)

Procedure

An output of ten kilowatts was chosen as the representative power level of a manned
lunar station. Using this power level, and assuming present fuel cell technology, reactant consumption rates of $1.305 \times 10^{-4}$ kilogram hydrogen per second, and $10.39 \times 10^{-4}$ kilogram oxygen per second were calculated. The respective rates of formation of these reactants by electrolysis $\dot{m}$ were assumed to be the same as for consumption.

The electrolysis unit was assumed to operate at $300^0 \text{K}$ $T_E^0$ and a maximum pressure $p_E$ of $2 \times 10^6$ newtons per square meter (about 300 psi). Operating temperatures above $300^0 \text{K}$ would make the water vapor pressure in the reactants excessively high, while operating pressures in excess of $2 \times 10^6$ newtons per square meter would require a heavy electrolysis unit.

The thermodynamic properties of hydrogen vary with its ortho-para composition, which is a function of temperature (ref. 10). When consideration was given to reactant cooling by radiation, the hydrogen was treated as being 75 percent ortho and 25 percent para. For combined refrigerative-radiative cooling, it was assumed to be 100 percent para.

The ranges of the parameters considered for the two types of reactant cooling are presented in tables I and II. These ranges represent estimates of conditions which might actually be met for lunar-based reactant cooling-and-storage units.

Equations (6) and (9) were added together, and a partial derivative was taken with respect to $T_H$. For a given value of $T_S$, the combined mass $M_R + M_{sc}$ was minimized by a particular value of $T_H$. This minimum mass was virtually unaffected by variations of the parameters $T_L$, $\eta$, and $\Delta$ over their respective ranges. When $M_R$ and $M_{sc}$ were

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Hydrogen</th>
<th>Oxygen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiator exit temperature, $T_{Lr}$ $^0\text{K}$</td>
<td>222 to 300</td>
<td>222 to 300</td>
</tr>
<tr>
<td>Radiator pressure, $p_{Er}$, N/m$^2$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
</tr>
<tr>
<td>Storage tank pressure, $p_{Lr}$, N/m$^2$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
</tr>
<tr>
<td>Average heat transfer through insulation, $Q_L$, W</td>
<td>-5 to -40</td>
<td>-1 to -20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hydrogen</th>
<th>Oxygen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refrigerator exit temperature, $T_{Lr}$ $^0\text{K}$</td>
<td>20 to 70</td>
<td>100 to 170</td>
</tr>
<tr>
<td>Refrigerator pressure, $p_{Er}$, N/m$^2$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
</tr>
<tr>
<td>Storage tank pressure, $p_{Lr}$, N/m$^2$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
<td>$1.4 \times 10^6$ to $2.0 \times 10^6$</td>
</tr>
<tr>
<td>Average heat transfer through insulation, $Q_L$, W</td>
<td>-1 to -20</td>
<td>-1 to -20</td>
</tr>
<tr>
<td>Refrigerator efficiency, $\eta$</td>
<td>0.05 to 0.20</td>
<td>0.05 to 0.20</td>
</tr>
<tr>
<td>Reactant-to-refrigerant temperature difference, $\Delta$, $^0\text{K}$</td>
<td>2.78 to 13.92</td>
<td>2.78 to 13.92</td>
</tr>
<tr>
<td>Radiator sink temperature, $T_{Sr}$ $^0\text{K}$</td>
<td>222 to 334</td>
<td>222 to 334</td>
</tr>
<tr>
<td>Average insulation surface sink temperature, $T_{Ss}$ $^0\text{K}$</td>
<td>222 to 278</td>
<td>222 to 278</td>
</tr>
</tbody>
</table>
TABLE III. - OPTIMUM REFRIGERATOR HEAT-REJECTION TEMPERATURES

<table>
<thead>
<tr>
<th>Effective sink temperature, $T_s$, K</th>
<th>Optimum refrigerator heat-rejection temperature, $T_H$, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>Oxygen</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>----------------------------------</td>
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<tr>
<td>222</td>
<td>361</td>
</tr>
<tr>
<td>278</td>
<td>395</td>
</tr>
<tr>
<td>334</td>
<td>434</td>
</tr>
</tbody>
</table>

evaluated for various combinations of the parameters, the optimum value of $T_H$ was used for each $T_s$. These optimum values are presented in table III.

Using these conditions and assumptions, the preceding equations were evaluated over the ranges of the parameters.

RESULTS AND DISCUSSION

The results of these computations are shown graphically in this section. Each graph presents the combined mass $M_{tot}$ of the components of a particular cooling-and-storage unit as a function of the temperature $T_L$ to which the reactant was cooled. On each graph is a family of curves, each representing one of a series of values for a particular parameter.

It is important to recall that $M_{tot}$ for the reactant cooling-and-storage system utilizing combined refrigeration and radiation does not include a mass term for the refrigerator.

Hydrogen Refrigeration and Storage Unit

In general, refrigeration of the hydrogen to low values of $T_L$ caused an increase in the mass of those components involved in the refrigeration, such as the solar cells and the refrigerator radiator. At the same time, those components related to hydrogen storage decreased in mass. The mass exchange between the two sets of components for decreasing values of $T_L$ resulted in a minimum value of $M_{tot}$.

These considerations explain the general form of the curves presented in figure 3. Each of these curves shows the variation of $M_{tot}$ with $T_L$ for a range of one particular parameter, while other parameters are held constant.
Variation of refrigeration pressure. - The effect of the hydrogen pressure in the refrigerator \( p_E \) on \( M_{\text{tot}} \) is seen in figure 3(a). When the hydrogen was refrigerated to temperatures above 300 K, a mass advantage was realized by performing the refrigeration at the higher pressure. The reason is that, for the ranges of pressure and temperature considered, the enthalpy of hydrogen decreases with increasing pressure. Therefore, when \( p_E \) was high, less energy was transferred to the storage tank. This being the case, the final storage temperature \( T_L' \) was lower for greater \( p_E \). Since the final storage pressure \( p_L' \) was the same for both values of \( p_E \), the lower \( T_L' \) results in a lower specific volume \( v_L' \). Thus, the tank was smaller, the amount of unusable reactant retained in the tank was less, the insulation requirement for a given heat leak was less, and the tank mass was less (see eq. (16a)).

Of course, in order to cool the hydrogen to the lower enthalpy \( h_L' \), more refrigerator power was required. This led to higher values of \( M_{\text{sc}} \) and \( M_R \) (eqs. (7) and (10)), but this penalty was greatly outweighed by the decreases in \( M_T, M_i, \) and \( M_o \).
For values of $T_L$ below $30^\circ$ K the hydrogen was, for both refrigeration pressures, quite dense. Its enthalpy was therefore nearly unaffected by $p_E$, and $M_{tot}$ became independent of $p_E$.

It will be noted that both curves of figure 3(a) are nearly vertical between $30^\circ$ and $40^\circ$ K because over this range the hydrogen was being cooled to and below its critical temperature. Both values of $p_E$ were supercritical, as was $p_{L'}$, so that the hydrogen was refrigerated and stored as a supercritical fluid; however, the enthalpy of the hydrogen still decreased sharply with temperature over this range, and therefore so did $M_{tot}$.

Variation of final storage pressure. - For values of $T_L$ above $35^\circ$ K, the final pressure in the storage tank had a considerable effect on $M_{tot}$ (fig. 3(b)). When the hydrogen was stored at the lower pressure, it had a greater specific volume, and required a correspondingly larger tank. This tank, at the end of the lunar night, therefore contained a greater amount of unused hydrogen; and it also required more insulation for a given heat leak.

The effect of $p_{L'}$ on the mass of the tank, itself, can be understood by referring to equations (13) to (16a). In these equations, the term $p_{L'}$ determines the wall thickness required to contain that pressure, while $[v_{L'}(m_o + m_r)]$ determines the tank radius. Even though the lower value of $p_{L'}$ allowed thinner tank walls, this advantage was lost to the increased specific volume and total mass of the stored hydrogen. However, for values of $T_L$ below $28^\circ$ K there was little difference in the hydrogen specific volume at the two values of $p_{L'}$; therefore, tank size, insulation requirement, and $m_o$ became nearly independent of $p_{L'}$. Under these conditions $M_{tot}$ was greater for the higher $p_{L'}$ because the tank walls were thicker.

Variation of the average heat leak into the tank. - For a given value of $T_L$, the final storage temperature $T_L'$, was lowered by decreasing $Q_L$. This resulted in a lower hydrogen specific volume, which, in turn, led to lower values of $M_T$, $M_P$, and $m_o$. However, lowering $Q_L$ also increased the insulation requirement $M_i$. Because of this exchange, there was an optimum value of $Q_L$ which minimized $M_{tot}$. For $T_L$ between $40^\circ$ and $70^\circ$ K this optimum $Q_L$ was about -10 watts. Between $20^\circ$ and $40^\circ$ K the optimum was about -5 watts (fig. 3(c)).

Variation of refrigerator efficiency. - The efficiency of the hydrogen refrigerator $\eta$ had a large effect on $M_{tot}$ (fig. 3(d)). The effect of $\eta$ was to alter $M_{sc}$ and $M_R$ according to equations (6) and (9), respectively. The lower the efficiency, the greater the mass of solar cells to power the refrigerator and the greater the mass of radiator to reject the waste heat.

Increasing $\eta$ from 0.05 to 0.20 caused the optimum value of $T_L$ to fall from $32^\circ$ to $26^\circ$ K, and the corresponding minimum $M_{tot}$ to decrease from 1860 to 610 kilograms. Even at an efficiency of 0.05, the mass of the cooling-and-storage unit was decreased by 1380 kilograms by lowering $T_L$ from $70^\circ$ to $32^\circ$ K.
Variation of refrigerant-to-reactant temperature difference. - Reexamination of equations (6) and (9) leads to an understanding of the effect of the temperature difference $\Delta$ between the refrigerant and the hydrogen on $M_{\text{tot}}$. In the term

$$\ln\left(\frac{T_E - \Delta}{T_L - \Delta}\right)$$

the numerator of the fraction is insensitive to $\Delta$ because of the magnitude of $T_E$ ($300^\circ$ K). This is not true of the denominator, since $T_L$ is relatively small. Therefore, for a given value of $T_L$, increasing $\Delta$ led to an increase in the magnitude of the term. Thus, $M_R$ and $M_{sc}$ (and, therefore, $M_{\text{tot}}$) both increased with increasing $\Delta$ (fig. 3(e)). As $T_L$ became lower, the effect of variations in $\Delta$ became greater, leading to the divergence of the curves between $20^\circ$ and $35^\circ$ K.

Variation of the radiator sink temperature. - It was previously pointed out that, for a given $T_s$, there was an optimum heat-rejection temperature $T_H$ which would minimize
For constant values of refrigerant-to-reactant temperature difference. Refrigerator efficiency, 0.20; storage tank pressure and refrigeration pressure, $2.0 \times 10^6$ newtons per square meter; radiator sink temperature and insulation surface sink temperature, $222^\circ$ K; average heat leak into tank, -10 watts.

For constant values of radiator sink temperature. Refrigerator efficiency, 0.20; refrigerant-to-reactant temperature difference, $2.78^\circ$ K; storage tank pressure and refrigeration pressure, $2.0 \times 10^6$ newtons per square meter; radiator sink temperature, $222^\circ$ K; average heat leak into tank, -10 watts.

For constant values of insulation surface sink temperature. Refrigerator efficiency, 0.20; refrigerant-to-reactant temperature difference, $2.78^\circ$ K; storage tank pressure and refrigeration pressure, $2.0 \times 10^6$ newtons per square meter; radiator sink temperature, $222^\circ$ K; average heat leak into tank, -10 watts.

Figure 3. - Concluded.
\( M_R + M_{sc} \). It is shown in figure 3(f) that increasing \( T_s \) from 222\(^\circ\) to 334\(^\circ\) K caused only a small increase in \( M_{tot} \), when the corresponding optimum \( T_H \) values were used in the computations. To show the importance of using the optimum \( T_H \), an additional point was calculated for \( T_L = 30^\circ \) K, \( T_s = 334^\circ \) K, and \( T_H = 361^\circ \) K (instead of the optimum value of 435\(^\circ\) K). This caused an increase of 248 kilograms in \( M_R + M_{sc} \).

The midrange \( T_s \) value of 278\(^\circ\) K, is the equilibrium temperature that would be attained during the lunar daytime by a typical radiator facing the sun, insulated from the lunar surface, and rejecting no net quantity of heat.

**Variation of insulation surface sink temperature.** - The last parameter considered was \( T_{si} \), the effective sink temperature of the hydrogen storage tank insulation surface. In order to maintain a constant heat leak from the sink to the insulation surface it was necessary that, as \( T_{si} \) increased, the insulation surface temperature also increased. To maintain the same heat leak through the insulation to the stored hydrogen, it was then necessary that the insulation thickness increase, too. The mass effects of these increases in \( T_{si} \) and the insulation thickness are seen in figure 3(g). The effect was greater at higher values of \( T_L \), where the tank size was large. At low \( T_L \), the increase in \( M_1 \) (and \( M_{tot} \)), due to increased \( T_{si} \) was negligible.

**Summary of parametric effects.** - The most significant fact was that, for all the parameters, there was a subcritical value of \( T_L \) which minimized \( M_{tot} \). However, even cooling which occurred above this optimum \( T_L \) caused considerable decreases in \( M_{tot} \).

The minimum \( M_{tot} \) was strongly affected by the refrigerator efficiency and, to a lesser extent, by the heat leak into the storage tank. Furthermore, there was an optimum value of \( Q_L \) determined by the exchange of \( M_l \) against \( M_T + M_P + m_o \). For the values of the parameters considered, the optimum \( Q_L \) was between -5 and -10 watts.

The minimum \( M_{tot} \) was slightly affected by \( \Delta \) and \( T_s \). Increasing \( \Delta \) from 2.78\(^\circ\) to 13.92\(^\circ\) K increased the minimum \( M_{tot} \) by 130 kilograms, and increasing \( T_s \) from 222\(^\circ\) to 334\(^\circ\) K raised it 70 kilograms. The second is true only if the optimum value of \( T_H \) which minimizes \( M_R + M_{sc} \) is used for each value of \( T_s \). If not, \( M_{tot} \) can be hundreds of kilograms greater than shown in figure 3(b).

The final two parameters, \( P_E \) and \( P_{L'} \), had almost no effect on the minimum values of \( M_{tot} \). However, for values of \( T_L \) greater than 40\(^\circ\) K, \( M_{tot} \) was decreased by hundreds of kilograms by raising the final storage pressure (\( P_{L'} \)) from \( 1.4 \times 10^6 \) to \( 2.0 \times 10^6 \) newtons per square meter (about 200 to 300 psia).

**Hydrogen Radiative Cooling-and-Storage Unit**

All the curves in figure 4 show \( M_{tot} \) decreasing linearly as \( T_L \) was decreased from 280\(^\circ\) to 225\(^\circ\) K. In all cases, the radiator sink temperature was assumed to be
Figure 4. Effect of radiative cooling of hydrogen on combined mass of tankage, insulation, plumbing and supports, and unusable reactants. Radiator sink temperature and insulation surface sink temperature, 222°K.

(a) For constant values of radiator pressure. Storage-tank pressure, 1.4x10^6 newtons per square meter; average heat leak into tank, -10 watts.

(b) For constant values of final storage-tank pressure. Radiator pressure, 2.0x10^6 newtons per square meter, average leak into tank, -10 watts.

(c) For constant values of average heat leak into tank. Storage-tank pressure and refrigeration pressure, 2.0x10^6 newtons per square meter.
222° K. It would have been necessary to cool the hydrogen to 222.5° K before the rate of increase of $M_R$ exceeded the rate of decrease ($M_T + M_i + m_o$). At that temperature $M_{tot}$ would pass through its minimum value. Even then, $M_R$ would be less than 2 percent of $M_{tot}$.

Variation of radiator pressure. - The pressure of the hydrogen in the radiator had no effect on $M_{tot}$ (fig. 4(a)). The absence of effect was because, over the ranges of pressure and temperature considered, the enthalpy of hydrogen is independent of its pressure. Therefore, for a given value of $T_L$, the energy content of the hydrogen leaving the radiator $h_L$ was independent of $p_E$. This being the case, the final storage temperature and specific volume of the hydrogen were also independent of $p_E$, and, consequently, so were $M_T$, $M_i$, and $m_o$.

Variation of final storage pressure. - The effect of the final storage pressure of the hydrogen on $M_{tot}$ (fig. 4(b)) was considerable. For a given $T_L$, the lower storage pressure resulted in a much greater unit mass because the lower $p_L$, called for greater tank volume, and, consequently, left more unusable hydrogen and required more insulation. Also, the tank mass was greater (referring again to eq. (16a)) because a given decrease in $p_L$, caused an even greater increase in the term $v_L(m_o + m_I)$. Therefore, at a $T_L$ of 225° K, an increase of $p_L$, from $1.4 \times 10^6$ to $2.0 \times 10^6$ newtons per square meter lowered $M_{tot}$ by 1130 kilograms.

Extrapolating the lower curve of figure 4(b) to 300° K (no cooling) shows that $M_{tot}$ can be decreased by about 1400 kilograms by cooling the hydrogen from 300° to 225° K.

Variation of average heat leak into the tank. - As $Q_L$ was decreased, an exchange occurred between the increasing mass of insulation and the decreasing masses of tank, plumbing, and unusable reactants. Over the range of $T_L$ considered (fig. 4(c)) this exchange resulted in an optimum value for $Q_L$ between -5 and -10 watts.

Summary of parametric effects. - The mass of the purely radiative cooling-and-storage unit for hydrogen had a minimum value at 222.5° K, when the radiator sink temperature was 222.0° K. By cooling the hydrogen from 300° to 225° K, $M_{tot}$ was decreased from 6730 to 5340 kilograms.

For values of $T_L$ between 225° and 280° K, $M_{tot}$ was decreased by about 1280 kilograms when the final storage pressure was raised from $1.4 \times 10^6$ to $2.0 \times 10^6$ newtons per square meter. Furthermore, for each $T_L$ there was an optimum heat leak of between -5 and -10 watts which resulted in a minimum value for $M_{tot}$. The pressure at which hydrogen was cooled had no effect on $M_{tot}$.

Comparison of Radiative Cooling and Combined Refrigeration-Radiation for Hydrogen

When the hydrogen was stored at the electrolyzer temperature (300° K), the mass of
the storage unit was 6730 kilograms. By using a radiator to cool the hydrogen from 300°
to 225° K, the mass of the cooling-and-storage unit was decreased to 5340 kilograms.
When a refrigeration of 5 percent efficiency replaced the radiator, M\text{tot} was minimized at 1860 kilograms. Since this value of M\text{tot} does not include the mass of the refrigerator itself, the difference (5340 to 1860 kg) represents the mass of the heaviest such refrigerator which could be considered for hydrogen cooling. A refrigerator of 20 percent efficiency minimized M\text{tot} at 610 kilograms, greatly increasing the allowable refrigerator mass. Of course, a combined refrigerative-radiative cooling-and-storage unit which offered little or no mass advantage over a purely radiative cooling-and-storage unit would have to be disregarded because of its greater complexity.

Oxygen Refrigeration and Storage Unit

The general shape of the curves for oxygen refrigeration (fig. 5) is quite similar to that of those for hydrogen refrigeration (fig. 3). The difference is that the ranges of p\text{E} and p\text{L}, considered were, for oxygen, subcritical, and for hydrogen, supercritical. Therefore, the oxygen was liquefied upon refrigeration, giving rise to the vertical portion of the curves.

By the same token, the explanations of the curves describing the effect of the various parameters on M\text{tot}, are quite similar for both reactants. Therefore, the discussion here will not be as detailed as before.

Variation of refrigeration pressure. - The enthalpy of the oxygen leaving the refrigerator at supercritical values of T\text{L} was lower for higher refrigerator pressures. Therefore, when p\text{E} was high, less energy was transferred to the storage tanks. This caused the final storage temperature to be low, and, consequently, the final storage specific volume of the oxygen was also low. The result was that the tank was smaller and lighter, less insulation was required, and less unusable oxygen inventoried. Therefore, M\text{tot} was lower for higher p\text{E} as shown in figure 5(a).

For subcritical values of T\text{L}, the refrigerator pressure had no appreciable effect on the oxygen enthalpy. The M\text{tot} then became independent of p\text{E}, and minimized at approximately 175 kilograms for T\text{L} \sim 110° K.

Variation of the final storage pressure. - Increasing the final storage pressure of the oxygen, p\text{L}, caused a decrease in tank size. The effect of this change is shown in figure 5(b). Being smaller, the tank contained less unusable reactant at the end of the lunar night. This meant that the total capacity of the tank m\text{o} + m\tau was less. The tank was therefore lighter and required less insulation, making M\text{tot} less.

Variation of the average heat leak into the tank. - As was the case for the hydrogen refrigeration and storage unit, each value of T\text{L} had associated with it an optimum Q\text{L}
Figure 5. Effect of oxygen refrigeration on combined mass of radiator, solar cells, tank, plumbing and supports, insulation, and unusable reactant.
Figure 5. - Continued.

(c) For constant values of average heat leak into tank, Refrigerator efficiency, 0.20; refrigerant-to-reactant temperature difference, 2.78° K; refrigeration pressure and storage tank pressure, 2.0x10^6 newtons per square meter; radiator sink temperature and insulation surface sink temperature, 222° K.

(d) For constant values of refrigerator efficiency. Refrigerant-to-reactant temperature difference, 2.78° K; storage tank pressure and refrigeration pressure 2.0x10^6 newtons per square meter; radiator sink temperature and insulation surface sink temperature, 222° K; average heat leak into tank, -10 watts.
Refrigerant-to-reactant temperature difference, 

storage tank pressure and refrigeration pressure, 2.0 x 10^5 newtons per square meter; radiator sink temperature and insulation surface sink temperature, 222° K; average heat leak into tank, -10 watts.

(f) For constant values of radiator sink temperature. Refrigerator efficiency, 0.20, refrigerant-to-reactant temperature difference, 2.78° K; storage tank pressure and refrigeration pressure, 2.0 x 10^5 newtons per square meter; insulation surface sink temperature, 222° K; average heat leak into tank, -10 watts.

Figure 5. Continued.
For constant values of insulation surface sink temperature. Refrigerator efficiency, 0.20; refrigerant-to-reactant temperature difference, 2.78°K; storage tank pressure and refrigeration pressure, 2.0x10^6 newtons per square meter; radiator sink temperature, 222°K; average heat leak into tank, -10 watts.

Figure 5. - Concluded.
which minimized $M_{tot}$ (fig. 5(c)). For supercritical $T_L$, this optimum $\bar{Q}_L$ was about -10 watts. For subcritical $T_L$, it was between -5 and -1 watts.

Variation of refrigerator efficiency. - Increasing $\eta$ from 0.05 to 0.20 caused the minimum value of $M_{tot}$ to fall from 500 to 175 kilograms (fig. 5(d)). The percentage change in the minimum $M_{tot}$ was very nearly the same as for the hydrogen refrigeration unit, which decreased from 1860 to 610 kilograms for the same increase in efficiency.

Variation of refrigerant-to-reactant temperature difference. - The temperature difference between the refrigerant and the oxygen, $\Delta$, had virtually no effect on $M_{tot}$ (fig. 5(e)). The absence of effect is understood by considering the term $\ln(T_E - \Delta/T_L - \Delta)$ in equations (6) and (9), for $M_{sc}$ and $M_{R'}$ respectively. In this term, $T_E$ and $T_L$ are both large relative to $\Delta$. Therefore, the magnitude of the term is quite insensitive to small changes in $\Delta$.

Variation of the radiator sink temperature. - Increasing $T_S$ from 222° to 334° K has virtually no effect on $M_{tot}$ (fig. 5(f)). It was necessary that the optimum value of $T_H$ be used for each value of $T_S$, as discussed previously. To illustrate this necessity, $M_R + M_{sc}$ was calculated for $T_L = 120°$ K, $T_S = 334°$ K, and $T_H = 361°$ K, producing a $M_R + M_{sc}$ value of 198 kilograms. The same calculation using the optimum $T_H$ (424° K, instead of 361° K) gave a $M_R + M_{sc}$ value of 137 kilograms.

Variation of insulation surface sink temperature. - When the sink temperature seen by the insulation surface was increased, the thickness of insulation necessary to maintain a given heat leak also increased. For supercritical values of $T_L$ (when the storage tanks were large), this increased insulation thickness resulted in a moderate increase in $M_i$ and, therefore, $M_{tot}$ (fig. 5(g)).

Summary of parametric effects. - There was a subcritical value of $T_L$ which minimized $M_{tot}$ for all parameters considered. All cooling of the oxygen which occurred above this optimum $T_L$ caused $M_{tot}$ to decrease.

For supercritical values of $T_L$, $M_{tot}$ was decreased somewhat (<250 kg) by raising the pressure of the oxygen in the refrigerator from $1.4 \times 10^6$ to $2.0 \times 10^6$ newtons per square meter (about 200 to 300 psia). For subcritical $T_L$, $p_E$ had no effect on $M_{tot}$.

The final storage pressure had considerable effect on $M_{tot}$ for supercritical $T_L$. By raising $p_{L'}$ from $1.4 \times 10^6$ to $2.0 \times 10^6$ newtons per square meter, $M_{tot}$ was decreased by more than 550 kilograms. The effect of $p_{L'}$ for subcritical $T_L$ was much less (<150 kg).

For each value of $T_L$, there was an optimum $\bar{Q}_L$ which minimized $M_{tot}$. For supercritical $T_L$, this optimum was about -10 watts, and for subcritical $T_L$, it was between -5 and -1 watts.

The oxygen refrigerator efficiency had a small effect on $M_{tot}$ for supercritical $T_L$. The minimum $M_{tot}$, however, was decreased from 500 to 175 kilograms when $\eta$ was increased from 0.05 to 0.20.
(a) For constant values of radiator pressure. Storage tank pressure, 1.4x10^6 newtons per square meter; average heat leak into tank, -5 watts.

(b) For constant values of storage tank pressure. Radiator pressure, 2.0x10^6 newtons per square meter; average heat leak into tank, -5 watts.

(c) For constant values of average heat leak into tank. Radiator pressure and storage tank pressure, 2.0x10^6 newtons per square meter.

Figure 6. - Effect of radiator cooling of oxygen on combined mass of tankage, insulation, plumbing and supports, and unusable reactants. Radiator sink temperature and insulation sink temperature, 222°K.
The remaining three parameters $A$, $T_{Si}$, and $T_s$ had little or no effect on $M_{tot}$ over the ranges of variation considered. For this to be true for $T_s$, it was necessary that the optimum $T_H$ be used for each value of $T_s$.

**Oxygen Radiative Cooling-and-Storage Unit**

The effect of the parameters $p_E$, $p_L$, $Q_L$ and $T_L$ on the mass of the oxygen radiative cooling-and-storage unit was virtually the same as for the corresponding hydrogen unit (fig. 6). When the pressure of the oxygen in the radiator was raised from $1.4 \times 10^6$ to $2.0 \times 10^6$ newtons per square meter, $M_{tot}$ decreased by about 50 kilograms. The same variation in $p_L$, decreased $M_{tot}$ by about 1350 kilograms, depending on $T_L$. Over the range of $T_L$ considered, the optimum heat leak was between -1 and -5 watts.

For a radiator sink temperature of 222.0°K, it was necessary to cool the oxygen to 222.5°K in order to minimize $M_{tot}$. Extrapolating the lower curve of figure 6(b) to 300°K (no cooling) shows that $M_{tot}$ decreased by about 1000 kilograms when $T_L$ was lowered from 300°K to 225°K.

**Comparison of Radiative Cooling and Combined Refrigeration-Radiation for Oxygen**

For the radiative cooling of oxygen over the ranges of parameters considered, the lowest minimum $M_{tot}$ was about 3360 kilograms. The minimum $M_{tot}$ for combined refrigerative-radiative cooling with 5-percent efficient refrigerator was 500 kilograms. Since this figure does not include the mass of the refrigerator, the difference (3360 to 500 kg) represents the heaviest such refrigerator which could be considered for oxygen cooling. Similarly, a 20-percent efficient refrigerator could have a mass of 3185 kilograms. However, as mentioned before, the combined cooling-and-storage unit would be much more complex than the unit with purely radiative cooling. It would therefore have to offer a considerable mass advantage in order to be seriously considered.

**CONCLUSIONS**

The effect of prestorage reactant cooling on the mass of a lunar-based regenerative fuel cell system was studied. The bases of this analysis were a 10-kilowatt hydrogen-oxygen fuel cell and an electrolyzer which operated at 300°K with a maximum pressure of $2.0 \times 10^6$ newtons per square meter (about 300 psia). A parametric mass analysis was made of units using radiators either alone or in conjunction with refrigerators. The dif-
ference in mass between both units established the maximum allowable mass of refrigerators, which were not included as part of the total mass of the refrigerative-radiative cooling-and-storage units.

For both hydrogen and oxygen and for both cooling methods, it was possible to minimize the total mass of the cooling-and-storage unit by cooling the reactant to a specific temperature. In the case of reactant refrigeration, this optimum temperature was subcritical. For radiative cooling, it was about $0.5^\circ K$ above the radiator sink temperature. Over the ranges of the parameters considered, even cooling to a temperature higher than the optimum temperature decreased the total mass of the unit.

In all cases, the pressure of the reactant while in the cooling device had little, if any, effect on the total mass. The final storage pressure also had very little effect when the reactant was cooled to subcritical temperatures. However, when the reactant was cooled to supercritical temperatures, a decrease in the storage pressure caused a considerable increase in total mass.

For each temperature to which the reactant was cooled, there existed an optimum allowable heat leak into the tankage. This optimum heat leak resulted in a minimum total unit mass.

The mass of the combined refrigerative-radiative cooling-and-storage unit for hydrogen was affected only slightly by the reactant-to-refrigerant temperature difference in the refrigerator. This parameter had no effect on the mass of the corresponding oxygen unit.

The refrigerator efficiency, which determined the power and heat-rejection requirement of the refrigerator, had considerable influence on the total mass. This was particularly true for the subcritical cooling of hydrogen.

The average sink temperature for the surface of the tankage insulation had only a minor effect on the mass of the refrigerative cooling-and-storage unit. This was also the case with the sink temperature for the refrigerator radiator, provided the optimum heat-rejection temperature of the radiator was maintained. This optimum temperature varied with the sink temperature and minimized the sum of the masses of the solar cells and the radiator required by the refrigerator.

Based on the ranges of the parameters considered, the minimum total mass for the hydrogen radiative cooling-and-storage unit was 5340 kilograms. The corresponding mass for cooling with a 5-percent efficient refrigerator was 1860 kilograms. This figure does not include the mass of the refrigerator itself. Therefore, the difference between these two masses (3480 kg) represents the heaviest 5-percent efficient refrigerator which could be considered for hydrogen cooling. Similarly, a 20-percent efficient hydrogen refrigerator could have a maximum mass of 4730 kilograms.

The lightest radiative cooling-and-storage unit for oxygen had a mass of 3360 kilograms. Therefore, the heaviest permissible oxygen refrigerators of 5-percent and 20-percent efficiency would have masses of 2860 and 3185 kilograms, respectively.
Of course, a refrigerative cooling-and-storage unit that did not have a considerable mass advantage would have to be disregarded in favor of the less complex radiative unit.

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REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute ... to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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