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CRITERIA FOR USE OF RANKINE-MHD SYSTEMS IN SPACE

by Lester D. Nichols

Lewis Research Center
Cleveland, Ohio, U.S.A.

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Lester D. Nichols

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, U.S.A

REVISED ABSTRACT

A Rankine cycle that employs an MHD generator is being considered for use in space. These systems must be evaluated on the basis of minimum specific weight, which generally means minimum radiator area to reject the cycle waste heat. Limitations on materials, maximum available magnetic field strength, and the MHD power-generation process itself restrict the range of working fluid temperature and density.

The maximum temperature (set by materials limitations) and the minimum radiator area criterion can be used to specify the operating temperature extremes. The operating pressure and density extremes are then determined by the choice of working fluid from the vapor-pressure curve. Since the generator can be shown to be less efficient at high Mach numbers and to generate lower power density at low Mach numbers, a compromise Mach number of 1.0 is selected. The power density also depends on the electrical conductivity of the working fluid that must be of the 1 mho per meter for useful space application. Since the conductivity increases with electron number density and decreases with increasing fluid density, it may be necessary to seed the working fluid to provide adequate electron concentration at the relatively high densities needed to heat the fluid. These considerations, coupled with the temperature requirements of the cycle, place restrictions on the choice of the seeded working fluid.

Within the framework of these restrictions, certain freedom in the design of the generator system is possible. Nonequilibrium ionization of the seed gas can be used to enhance the conductivity of the working fluid at the prescribed conditions. In the case of magnetically induced ionization, this effect is limited, however, by the maximum magnetic field strength that can be attained. Also, the generator cross-sectional area can be determined to minimize the generator volume consistent with the condition that the inlet and exit pressures are specified.

Calculations based on these criteria indicate that cesium seeded lithium is a good choice for the working fluid. If lithium is boiled at

1365° K and superheated to 1645° K for a generator efficiency of 0.75, the radiator area is minimized at a radiator temperature of 1100° K. For these conditions in a Faraday segmented generator operating with an entrance Mach number of 1.0, a load parameter of 0.75, and a magnetic field strength of 15 Tesla, the magnetically induced nonequilibrium ionization can produce an electrical conductivity about of 0.22 mho per meter. This corresponds to a generator power density of about 10^7 watts per cubic meter. The MHD radiator area for this system is about the same as that for a turboalternator system for the same temperature limits.

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INTRODUCTION

The space environment places several restrictions on large, long-lifetime electrical power-generation systems. First, they must be light-weight; second, they probably will use a nuclear heat source as part of a closed cycle; and finally, they must reject the waste heat into space. It is the purpose of this paper to select criteria for the design of Rankine-MHD systems for space use. The criteria to be considered are (1) minimum radiator area, (2) generator efficiency, and (3) generator power density as influenced by generator design parameters and the electrical conductivity of the working fluid. The system will be subjected to the constraint of a maximum temperature compatible with material limitations and a maximum magnetic field strength that can be obtained with super-conducting magnets.

A thermodynamic cycle can be made more efficient by operating at a higher maximum temperature. However, the maximum temperature that can be obtained is limited by materials properties. For the temperature region in nuclear reactors with solid-fuel elements, the alkali metal vapors can be considered as Rankine cycle working fluids because of their low vapor pressure [1]. For space systems generating electrical power in the mega-watt range, the radiator required to reject the cycle waste heat will contribute a significant fraction of the total system weight. Therefore, the cycle operating conditions should be selected on the basis that this radiator weight be as small as possible [1]. In the analysis, it will be assumed that the radiator weight can be minimized by minimizing the radi-ating area. This may not always be justified [2], but for the relatively limited range of temperature considered in the analysis, the assumption appears to be valid.

It will be shown that the efficiency of the generator in the Rankine cycle (whether it is a turboalternator or MHD generator) is quite impor-tant in determining the area of the radiator. Because of the importance of this parameter, the effect of the entrance Mach number, generator load voltage, and other parameters on the efficiency of MHD generators is analyzed. To reduce the size and weight of the electromagnet, the channel area in the axial (or flow) direction is varied to minimize the generator volume. For the same reason, it is also important to design a generator with high power density. The power density depends on the entrance Mach number, the load voltage, the magnetic field strength, and the electrical conductivity of the working fluid. Reasonable values of the first two (Mach number and load voltage) can be determined from the generator analysis. The magnetic field strength is an independent parameter whose value is restricted to a maximum value, which, for the calculation in this report, is 20 Tesla. The electrical conductivity is the most critical of the variables. Even for seeded working fluids, at temperatures compatible with materials limitations, the conductivity for equilibrium conditions is too small to be utilized in MHD generators. A means of achieving a higher nonequilibrium conductivity for the working fluid is therefore needed. In this report, the method of magnetically induced ionization is considered [3-6] primarily because it does not require that part of the generator output be directed toward increasing the ionization.

RADIATOR AREA

Consider a thermodynamic cycle with efficiency η operating in space. The radiator area required to reject the waste heat can be expressed as

$$\frac{A^*_{rad}}{\dot{w}^* W^*} = \frac{1 - \eta}{\eta} \frac{1}{\epsilon \sigma^*_{SB} (T^*_{cond})^4} \tag{1}$$

(The symbols are defined in appendix A.) The radiator temperature will be determined as that value of T^*_{cond} which minimizes A^*_{rad}. For this calculation, it is necessary to determine the cycle efficiency which, in turn, depends on the generator efficiency.

The appropriate generator efficiency η_g is that fraction of the isentropic enthalpy change which is converted into electrical energy. To determine the effect of η_g on radiator area, two values of η_g will be assumed: 0.6 and 0.75. (The latter value is typical of those being considered for turboalternators for which the generator efficiency is the product of the turbine efficiency and the alternator efficiency.)

The effect of superheating the vapor on radiator area is also of interest. (It may be that the maximum temperature at which a working fluid can be superheated is greater than the maximum temperature at which it can be boiled, because of lower corrosion in the vapor state.) To illustrate this effect, the usual boiling point of 1365° K (2000° F) is used, with 0° and 280° K superheat.

The radiator area for the Rankine cycle will be compared to the radiator area for a Carnot cycle with maximum temperature $T^{*,C}_{max}$ equal to the working fluid boiling temperature (1365° K). For the Carnot cycle, the radiator temperature $T^{*,C}_{cond}$ is 3/4 $T^{*,C}_{max}$ the efficiency is 0.25, and the radiator area $A^{*,C}_{rad}$ from equation (1) is

$$\frac{A^{*,C}_{rad}}{\dot{w}^* W^*} = \frac{256}{27} \frac{1}{\epsilon \sigma^*_{SB} (T^{*,C}_{max})^4} \tag{2}$$

The ratio of radiator areas can be expressed as

$$\frac{A^*_{rad}}{A^{*,C}_{rad}} = \frac{27}{256}\left(\frac{1 - \eta}{\eta}\right)\left(\frac{T^{*,C}_{max}}{T^*_{cond}}\right)^4 \tag{3}$$

This ratio can be calculated for any Rankine cycle working fluid. For a turbine in space, potassium is a good choice, and it will be shown later that for an MHD generator, lithium is a good choice. The resulting area ratios are shown in Table I.

For potassium, superheating does not change the radiator area, and for lithium, only a few percent reduction is obtained. Thus, superheating is not needed to reduce radiator area, but it may have significant beneficial effects on the working-fluid conductivity. Superheating would tend to increase electron mobility, because of the reduced density, and to reduce the concentration of condensed droplets that may reduce the electrical conductivity [7].

GENERATOR ANALYSIS

Assumptions and Equations

Since the purpose of this paper is not a detailed performance analysis, but rather an illustration of the effect of certain parameters on performance, the following assumptions will be made: The working fluid is a perfect gas with (1) zero viscosity, (2) zero thermal conductivity, (3) constant specific heats, and (4) constant electrical conductivity. Further, consider this fluid flowing in a generator with variable cross-sectional area a^*, a constant magnetic field B^*, a constant Faraday electric field E^*, and with no spatial gradients in the plane of a^*.

The generator to be studied is a Faraday segmented generator identical to that studied in reference [3] with the exception that the cross-sectional area a^* is not constant. The area variation is necessary in order to achieve the necessary pressure change for a Rankine cycle. The equations governing this generator are

$$a\rho u = 1 \tag{4}$$

$$\rho u u' + p' + j(1 - K) = 0 \tag{5}$$

$$\rho u h' + \rho u^2 u' + jK(1 - K) = 0 \tag{6}$$

$$h = \frac{\gamma}{\gamma - 1}\,\frac{p}{\rho} \tag{7}$$

$$j = \frac{u - K}{1 - K} \tag{8}$$

where the prime denotes differentiation with respect to X, where

$$X \equiv \frac{\sigma^* B^{*2} X^*}{\rho^*_{ent} u^*_{ent}} = \frac{\sigma^* B^{*2} L^*}{\rho^*_{ent} u^*_{ent}}\;\frac{X^*}{L^*} \equiv \delta\,\frac{X^*}{L^*} \tag{9}$$

and X^* is the axial coordinate in the generator, and

$$p \equiv \frac{p^*}{\rho^*_{ent} u^{*2}_{ent}} \; ; \; h \equiv \frac{h^*}{u^{*2}_{ent}} \; ; \; K \equiv \frac{E^*}{u^*_{ent} B^*} \tag{10}$$

All other dependent variables are normalized with respect to their value at the entrance to the generator.

Generator Pressure Ratio

The generator pressure ratio can be determined from the saturation curve for the working fluid chosen and the operating temperature extremes. The working fluid expands through a nozzle from p^*_h to some entrance Mach number M_{ent} so that the static pressure at the entrance to the generator can be expressed as

$$p^*_{ent} = p^*_h \left(1 + \frac{\gamma - 1}{2} M^2_{ent}\right)^{-\gamma/(\gamma-1)} \tag{11}$$

At the exit of the generator the total pressure must equal the saturation pressure of the working fluid p^*_l at the radiator temperature; that is, the pressure p^*_{ex} and Mach number M^*_{ex} at the inlet to a diffuser at the generator exit must satisfy

$$p^*_l = p^*_{ex} \left(1 + \frac{\gamma - 1}{2} M^2_{ex}\right)^{\gamma/(\gamma-1)} \tag{12}$$

Since the generator is going to operate between these specified pressure limits p^*_l and p^*_h, the independent variable will be changed from X to p. To accomplish this, the system of equations (eqs. (4) to (8)) can be combined into

$$u' + ap' + a(u - K) = 0 \tag{13}$$

$$\frac{\gamma}{\gamma - 1} (aup)' + uu' + aK(u - K) = 0 \tag{14}$$

The derivative with respect to X can be eliminated, and equations (13) and (14) can be combined as follows:

$$K \left(\frac{du}{dp} + a\right) = \frac{\gamma}{\gamma - 1} \frac{d}{dp} aup + u \frac{du}{dp} \tag{15}$$

Equation (15) can be solved for u and a as functions of p, and these variables can be related to the length by using either equation (13) or (14). It is possible to integrate equation (15) in terms of a new variable A, where

$$A(p_{ex}) = \frac{1}{\gamma M^2_{ent}} + \int_{p_{ent}}^{p_{ex}} a \, dp \tag{16}$$

The integrated form of equation (14) is

$$K(u - 1) + K\left(A - \frac{1}{\gamma M_{ent}^2}\right) = \frac{\gamma}{\gamma - 1}\left(upA' - \frac{1}{\gamma M_{ent}^2}\right) + \frac{u^2 - 1}{2} \tag{17}$$

Equation (17) can be used to solve for u in terms of A and p_{ex}, where now the prime denotes differentiation with respect to p_{ex}

$$u = \frac{\gamma}{\gamma - 1}\left(K_1 + \sqrt{f} - A'p_{ex}\right) \tag{18}$$

(the sign of the square root in chosen to satisfy entrance conditions) and

$$f \equiv (A'p_{ex} - K_1)^2 + 2\frac{\gamma - 1}{\gamma}K_1 A - \frac{\gamma - 1}{\gamma}\left[2K_1 - \frac{\gamma - 1}{\gamma} - \frac{2(1 - K_1)}{\gamma M_{ent}^2}\right] \tag{19}$$

$$K_1 \equiv \frac{\gamma - 1}{\gamma}K \tag{20}$$

Specification of Generator Area Variation

The specification of the area variation can now be made. Since it is necessary to provide the magnetic field strength over the generator volume, it is desirable to keep this volume as small as possible. Therefore, the variation in the area will be such that the generator volume V is minimized. The volume is defined as

$$V = \frac{V^*}{a_{ent}^* L^*} = \frac{1}{L^*}\int_0^{L^*} a\, dx^* = \frac{1}{\delta}\int_0^{\delta} a\, dx \tag{21}$$

Equation (13) can be used to write a as

$$a = -\frac{u' + ap'}{u - K}$$

so that V becomes

$$V = \frac{1}{\delta}\left[\ln\frac{1 - K}{u - K} - \int_{p_{ent}}^{p_{ex}}\left(\frac{A'}{u - K}\right)dp\right] \tag{22}$$

This integral is minimized when the Euler equation

$$\frac{d}{dp}\left[\frac{\partial}{\partial A'}\left(\frac{A'}{u - K}\right)\right] - \frac{\partial}{\partial A}\left(\frac{A'}{u - K}\right) = 0 \tag{23}$$

is satisfied [8]. When the differentiation is performed, this equation may be rewritten as

$$\frac{d}{dp}\left(\frac{u(A'p)^2}{\sqrt{f}(u - K)^2}\right) = 0 \tag{24}$$

with initial conditions

$$\left.\begin{aligned} A(p_{ent}) &= \frac{1}{\gamma M_{ent}^2} \\ &\text{and} \\ A'(p_{ent}) &= 1.0 \end{aligned}\right\} \tag{25}$$

The solution of equation (24) can be used to describe the generator performance in terms of three parameters: Mach number, load parameter, and ratio of specific heats. This solution can then be used to calculate the Mach number $M = \sqrt{u/(\gamma p A')}$ and the total pressure ratio $p_l^*/p_h^* = p_l/p_h$ from equations (11) and (12). These variables can be related to the interacting length δ by integrating equation (13). The solutions to equation (24) have been by numerical integration for $\gamma = 5/3$ (since monatomic vapors are of interest for the closed-cycle space operation), $K = 1/4$, $1/2$, $3/4$ and $M_{ent} = 1/2$, 1, and 2.0. Shown in Figure 1 is one of the solutions (for $K = 3/4$ and $M_{ent} = 1.0$) that is typical of all the solutions. Also shown is the area variation for a constant velocity solution to equations (13) and (14) $(a^*/a_{ent}^*)_{vel}$ for the same entrance conditions. At a pressure ratio p_l^*/p_h^* of 0.1, the area ratio from equation (24) is about 40 percent of the area ratio for the constant velocity generator. Because of the steep increase in area (for either generator), it is apparent from Figure 1 that pressure ratios less than 0.1 would be difficult to achieve.

Generator Efficiency

In an idealized MHD generator, the electrical-energy output is equal to the actual change in total enthalpy of the working fluid. The appropriate generator efficiency is therefore the ratio of this actual enthalpy change to the isentropic change between the same total pressure limits [4]. (Note that this ratio is equivalent to the turbine efficiency alone in a turbo-alternator.) This efficiency η_g is given by[1]

$$\eta_g = \frac{N_{conv}}{1 - \left(\frac{p_l}{p_h}\right)^{1-(1/\gamma)}} = \frac{N_{conv}}{1 - \frac{N_{conv}}{ua\left(\gamma M_{ent}^2 p\right)^{1/\gamma}}} \tag{26}$$

where N_{conv} is the fraction of power in the working fluid that is converted to electrical power and can be written (from eqs. (8), (13), and (16)) as

$$N_{conv} = \frac{\int_0^{L^*} K u_{ent}^* B^* j^* a^* \, dx^*}{\rho_{ent}^* u^{*3} a_{ent}^* \left[\frac{1}{(\gamma - 1)M_{ent}^2} + \frac{1}{2}\right]} = \frac{K\left(1 + \frac{1}{\gamma M_{ent}^2} - u - A\right)}{\frac{1}{(\gamma - 1)M_{ent}^2} + \frac{1}{2}} \tag{27}$$

[1]Note the η_g is the same as η_s in reference [3] with the necessary modification to account for the area variation.

The values for u and A are determined from the solution to equation (24). The generator efficiency η_g and N_{conv} are also shown in Figure 1 for $M_{ent} = 1.0$ and $K = 3/4$.

Because of the strong effect of efficiency on radiator area, it will be assumed that if the Rankine-MHD system is to be of interest, the idealized generator efficiency must be at least equal to well-established turbo-alternator efficiencies. On this basis, η_g will arbitrarily be taken to be 0.75 or greater. This requirement imposes restrictions on M_{ent} and K. The effect of M_{ent} and K on the efficiency of an MHD generator operating with a total pressure ratio $p_l/p_h = 0.1$ is illustrated in Table II.

To achieve a value of $\eta_g = 0.75$, it is apparent that K must be about 3/4 and M_{ent} about 1.0 (or less). An additional restriction on the entrance Mach number results from the need for high generator power density (power generated divided by the generator volume).

Power Density

The generator power density is

$$\Pi^* = \frac{1}{V^*}\int_0^{L^*} K u_{ent}^* B^* j^* a^* \, dx^* = \frac{K u_{ent}^* B^* j_{ent}^*}{\delta V}\int_0^{\delta} ja \, dx \tag{28}$$

which can be rewritten by using equation (8) and (13) as

$$\Pi^* = \sigma^* B^{*2} \gamma \frac{R}{\mu} T_{max}^* \frac{M_{ent}^2 K\left(1 + \frac{1}{\gamma M_{ent}^2} - u - A\right)}{\delta V\left(1 + \frac{\gamma - 1}{2} M_{ent}^2\right)} \tag{29}$$

The effect of M_{ent} and K on the power density is given by the factor Π where

$$\Pi = \frac{M_{ent}^2 K\left(1 + \frac{1}{\gamma M_{ent}^2} - u - A\right)}{\delta V\left(1 + \frac{\gamma - 1}{2} M_{ent}^2\right)} = \frac{\eta_{conv}}{(\gamma - 1)\delta V} \tag{30}$$

and δV is defined in equation (21).

As shown in Table III, Π increases with increasing entrance Mach number, and for each Mach number there is a relative maximum for $K = 1/2$ (the load resistance is equal to the generator resistance). In order to achieve reasonable power densities it is therefore advisable to select as large a Mach number as possible consistent with maintaining a reasonable efficiency, and to select a load parameter between 1/2 (for maximum power density) and 1.0 (for maximum efficiency). For purposes of illustration, the Mach number will be selected as 1.0 and the load parameter 3/4, with the realization that these values may not necessarily be the most appropriate if other criteria are used to make the evaluation.

ELECTRICAL CONDUCTIVITY

Equilibrium Conductivity

For electrical power in the megawatt range it is reasonable to require the generator power density to be about 10 megawatts per cubic meter. As shown in equation (29) the power density depends on the electrical conductivity of the working fluid. The conductivity depends on the electron number density and the electron mobility (which, in turn, is inversely proportional to the working-fluid density). The working fluid can be seeded with an easily ionized vapor (usually cesium) in order to increase the electron number density at the gas temperature. However, for the alkali metal vapors, the mobility of the electrons is too low to yield sufficient conductivity below 2000° K. Since the electron-neutral collision cross sections for all the alkali metals are approximately the same, it is apparent that the electron mobility in the less dense vapors will be the highest. From this point of view, lithium is the most attractive of the alkali metals because it has, by far, the lowest vapor pressure. However, even for lithium boiling at 1365° K, superheated to 1920° K, seeded with 0.01 fraction of cesium at densities corresponding to $M_{ent} = 1.0$, the electrical conductivity is only 0.01 mho per meter. Therefore, even with a seeded working fluid with a low density, the temperatures required for equilibrium conductivity are too high to be compatible with materials.

Nonequilibrium Conductivity

It is apparent that some means of increasing the conductivity of the working fluid above its equilibrium value is necessary. The usual methods are to provide some means by which the electron number density (and the electron temperature) can be increased. Of the several methods that are available, two have been considered for working fluids appropriate for Rankine cycles. In reference [9] the method considered is photoionization, and it is shown that cesium seeded lithium can be made sufficiently conducting to attain reasonable power density. However, this method has the disadvantage that part of the generator output must be used to provide the ionization. Consequently, only magnetically induced ionization (Refs. [3] and [4]) is considered herein. Furthermore, only cesium-seeded lithium is considered for the working fluid. (In Ref. [5] it is condluded that cesium-seeded potassium can be utilized. However, for potassium as the working fluid, the magnetic field strength required for sufficient electrical power density is much greater than 20 Tesla for the conditions cited previously. In Ref. [6], zinc is analyzed, but the conductivity of saturated zinc vapor is less than saturated lithium vapor at the same electron temperature and number density.)

Magnetically Induced Ionization

As a result of magnetically induced ionization, the electron temperature at the entrance to a Faraday segmented generator can be calculated by (Ref. [4])

$$\frac{T_e^*}{T_{max}^*} = \frac{1 + \frac{\gamma}{3}\,\frac{(1 - K)^2}{\delta_{inel}}\,M_{ent}^2\left(\frac{\beta_e}{1 + \beta_e\beta_i}\right)^2}{1 + \frac{\gamma - 1}{2}\,M_{ent}^2} \tag{31}$$

and the electron concentration determined from the Saha equation at this electron temperature. The electron mobility can be determined when the working fluid and its density are specified. The parameters in this equation have been specified except for the "loss factor" δ_{inel} and the Hall parameter term. The loss factor will be assumed to be 1.0 (as in Refs. [3] and [5], and the Hall parameter term will be evaluated at the magnetic field strength that maximizes the power density. (This maximization occurs because of ion slip, as described in refs. [3] and [4] In Table IV, the magnetic field strength that maximizes the power density for $M_{ent} = 1.0$ and $K = 3/4$ is shown for lithium vapor boiling at 1365° K with 0.01 fraction of cesium seed. It is apparent that the magnetic field strength required for maximum power density decreases as the maximum temperature attainable is increased. The proper design point would depend upon the relative difficulty in attaining a large magnetic field strength as opposed to a high-temperature reactor. At 1645° K, the power density is greater than 10 megawatts per cubic meter and could be considered adequate. However, for purposes of illustration, the cycle with a maximum temperature of 1920° K is selected and shown in Figure 2 for the following conditions:

$$T^*_{boil} = 1365^\circ \text{ K} \qquad T^*_{cond} = 1100^\circ \text{ K}$$

$$T^*_{max} = 1920^\circ \text{ K} \qquad \eta_g = 0.774$$

For the conditions, the cycle efficiency η is 0.167, and the radiator area ratio $A^*_{rad}/A^{*,C}_{rad} = 1.25$. This radiator area ratio is lower than that for a potassium vapor turboalternator cycle with generator efficiency of 0.75 (Table I), primarily because of the effectiveness of superheat for the MHD case.

CONCLUSIONS

From this analysis of the Rankine-MHD cycle for space applications, several conclusions can be drawn:

1. Lithium is the best of the alkali metals as a working fluid for Rankine-MHD systems primarily because of its low vapor pressure and relatively large electron mobility.

2. Even though it may not be advantageous to superheat the working fluid for turboalternators (potassium), it is beneficial to superheat a typical MHD working fluid (lithium) for two reasons: the radiator area is smaller, and the generator power density is better.

3. It is possible to expand to lower pressures for the same area variation with a generator designed for minimum volume than with the constant velocity generator.

4. For efficient generator operation at low pressure ratio, the entrance Mach number should be low, whereas for large power density, the Mach number should be high.

5. The electrical conductivity of the working fluid must be enhanced by nonequilibrium means. The method of magnetic field ionization requires a large magnetic field strength if the ionization is to be accomplished with reasonable generator efficiency.

6. A Rankine-MHD system can be competitive with a Rankine-turbine system if the magnets (which must be superconducting in order to be light weight) have field strengths of the order of 10 to 20 Tesla.

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APPENDIX - SYMBOL LIST

A	cross-sectional-area parameter defined in equation (17)
A_{rad}	radiator area
a	channel cross sectional area
B	magnetic field strength
E	Faraday field in generator
f	function defined in equation (19)
h	enthalpy
j	current density
K	load parameter defined in equation (10)
K_1	load parameter defined in equation (20)
L	generator length
M	Mach number
N	fraction of stream power converted to electrical energy
p	pressure
R	universal gas constant
T	temperature
T_e	electron temperature
u	velocity
V	MHD generator volume
W	work done per unit mass of working fluid
$\dot{w}$	working fluid flow rate
X	interacting length defined in equation (9)
β_e, β_i	electron and ion Hall parameters
γ	ratio of specific heats
δ	interaction parameter defined in equation (9)
δ_{inel}	inelastic interaction parameter
ϵ	radiator emissivity
η	efficiency
μ	molecular weight
Π	generator power density
ρ	fluid density
σ	electrical conductivity
σ_{SB}	Stefan-Boltzmann constant

Superscripts:

C	Carnot cycle
*	dimensional quantities

Subscripts:

boil	boiling
cond	condensing
conv	conversion
ent	entrance
ex	exit
g	generator
h	high
l	low
max	maximum
rad	radiator
vel	constant velocity

TABLE I. - RADIATOR AREA RATIO (EQ. (2)) FOR

RANKINE CYCLES BOILING AT 1365° K

(a) Potassium

Degree of superheat	η_g	
	0.6	0.75
0	1.99	1.54
280	1.99	1.54

(b) Lithium

Degree of superheat	η_g	
	0.6	0.75
0	2.09	1.61
280	1.96	1.56

TABLE II. - GENERATOR

EFFICIENCIES FOR $\gamma = 5/3$

AND $p_l/p_h = 0.1$

K	M_{ent}		
	1/2	1.0	2.0
1/4	0.338	0.281	0.184
1/2	.610	.550	.393
3/4	.829	.774	.624

TABLE III. - Π FOR $\gamma = 5/3$

AND $p_l/p_h = 0.1$

K	M_{ent}		
	1/2	1.0	2.0
1/4	0.0411	0.133	0.309
1/2	.052	.169	.393
3/4	.037	.122	.249

TABLE IV. - MAGNETIC FIELD STRENGTH AND CONDUCTIVITY FOR MAXIMUM POWER DENSITY FOR CESIUM-SEEDED LITHIUM VAPOR BOILING AT 1365° K IN FARADAY SEGMENTED GENERATOR WITH $M_{ent} = 1.0$ AND $K = 0.75$

T^*_{max}, $^{\circ}K$	B^*, Tesla	σ^*, mho/M	Π, W/cu m
1365	18.0	0.03	3.11×10^{6}
1645	15.9	.22	2.02×10^{7}
1920	12.9	1.01	7.98×10^{7}
2200	11.3	3.37	2.61×10^{8}
2480	10.0	8.55	5.27×10^{8}

E-3417

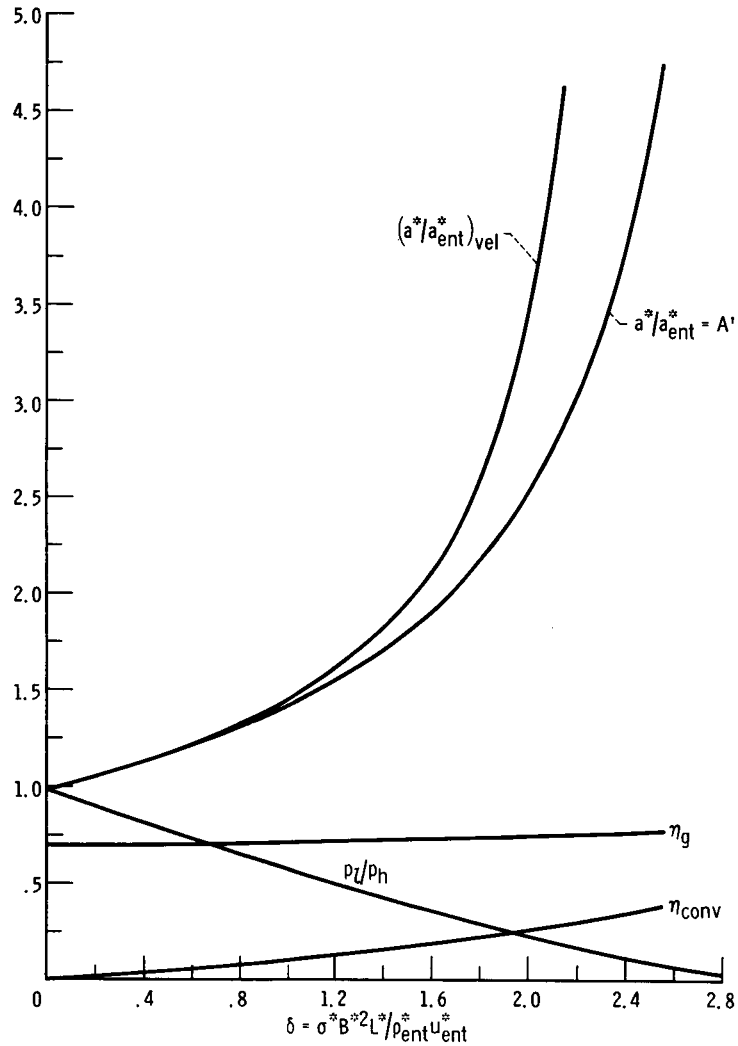

Figure 1. - Variation of areas, pressure ratio, η_g, and η_{conv} as function of interaction parameter for $M_{ent} = 1$, $K = 3/4$, and $\gamma = 5/3$.

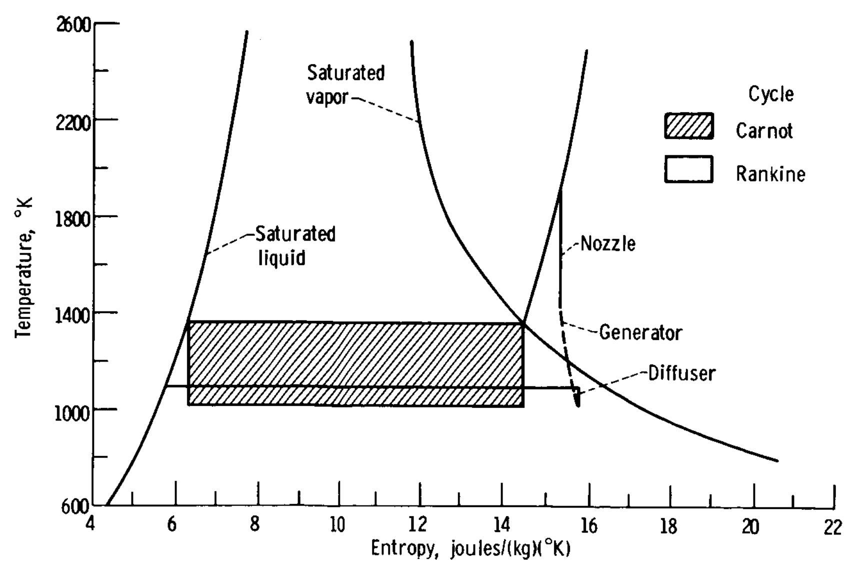

Figure 2. - Temperature-entropy diagram for lithium with MHD generator shown.