EXPLORING IN AEROSPACE ROCKETRY

2. PROPULSION FUNDAMENTALS

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Presented to Lewis Aerospace Explorers
Cleveland, Ohio
1966-67

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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2. PROPULSION FUNDAMENTALS

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Having considered the nature and the scope of the aerospace environment, let us direct our attention to the means by which man ventures out into space. Obviously, propulsion is the key which opens the door to all pioneering achievements in space. The "muscle" of the space program is the rocket engine. In it resides man's basic capability to hurl instrumented unmanned and manned payloads out beyond the restricting influences of the Earth's atmosphere and gravitational field.

Before we can intelligently examine any hardware details of the propulsion system and the vehicles, it is essential that we have some insight into what actually is a rocket engine. What are the fundamental design criteria? What factors influence performance? What is thrust? How is it derived from the hot jet issuing from the engine? The following discussion is centered on these questions.

Man's interest in exploring the "heavens" dates back to the earliest recorded civilizations. But a rational basis for modern rocketry was first established by Sir Isaac Newton in 1687; in that year, Newton published his book "Principia", in which he presented his principles of motion. Those principles, or laws, are examined in this chapter. (Symbols are defined in the appendix.)

THRUST

Newton's Second Law of Motion

Newton's Second Law of Motion (fig. 2-1) states simply that if a 1-pound force, or push, is applied to a body of unit mass (that is, 1 slug, defined as 32.2 lb), that body will accelerate 1 foot per second each second. In the illustration it is presumed that the body rides on frictionless wheels. Momentum is a term that describes a body in motion and may be defined as the product of mass and velocity. (Mass is related to weight by the equation $m = W/g$.) Newton's Second Law of Motion can be stated mathematically by the

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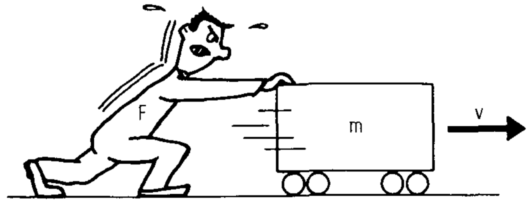


Momentum = mv

Figure 2-1. - Newton's Second Law of Motion: Acceleration - The change in a body's motion is proportional to the magnitude of any force acting upon it and in the exact direction of the applied force.

equation

$$F = ma = m\,\frac{\Delta v}{\Delta t} \tag{1}$$

or

$$F = \frac{\Delta mv}{\Delta t} = \frac{\text{Change in momentum}}{\text{Change in time}}$$

Simply, thrust is equal to the rate of change in momentum.

Newton's Third Law of Motion

Newton's Third Law of Motion (fig. 2-2) states that for every action there is an equal and opposite reaction. If we picture our little character here standing on frictionless

Figure 2-2. - Newton's Third Law of Motion. Reaction - For every acting force there is always an equal and opposite reacting force.

roller skates and holding a bowling ball, we will find that upon the action of throwing the ball there is a reaction force pushing him back in the opposite direction. The same situation exists with a high-pressure water hose. With a jet of water issuing from the nozzle, there is a force pushing back on the hose in the opposite direction. These are examples of the principle of action and reaction.

Probably, the simplest form of rocket with which we all are acquainted is a toy balloon. The balloon rocket (fig. 2-3) uses the principles of both laws of motion. When the

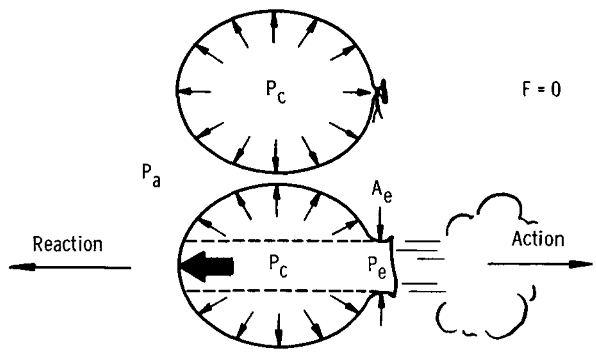


Figure 2-3. - Balloon rocket.

balloon is inflated and the outlet is tied off, the internal pressures acting in all directions against the wall of the balloon are in balance. Since no gas issues from the balloon, there is no thrust. When the outlet is opened, however, gas discharges through the opening (action), and the balloon moves in the opposite direction (reaction). The internal pressures are no longer balanced, and there is a reaction force equal to the open exit area times the difference between the internal and ambient pressures.

$$F = (P_c - P_a)A_e$$

This reaction is equal to the action force which is created by the exiting gas stream and which consists of a momentum term of mass flow rate $\dot{m}$ times the exit gas velocity v_e plus a pressure term $(P_e - P_a)A_e$.

$$F = \dot{m}v_e + (P_e - P_a)A_e \tag{2}$$

(Note: A dot over a symbol indicates a rate flow or means "per unit time.") In both the action and reaction forces, the effect of the surrounding environment has been taken into account by referring to P_a. Obviously, in a vacuum ($P_a = 0$) the thrust of a balloon rocket is larger than it is in the atmosphere (at sea level, for example, $P_a = 14.7$ psia).

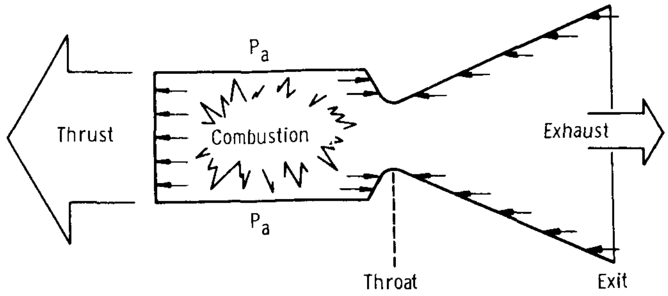


Figure 2-4. - Actual rocket engine.

In principle, there is no difference between a balloon rocket (fig. 2-3) and an actual rocket engine (fig. 2-4). In actuality, however, a rocket engine must have many practical refinements which a balloon rocket does not have. For example, a strong, high-temperature structure with intricate cooling provisions must be used to stably contain the high-pressure, hot gases. A combustion process (involving a fuel and an oxidizer at some particular mixture ratio, or o/f) is utilized to generate the hot, high-pressure gases. These gases are then expanded and exhausted finally through a nozzle at high velocity to the ambient environment. The purpose of the nozzle is to convert efficiently the random thermal energy of combustion into a high-velocity, directed kinetic energy in the jet. As with the balloon, thrust is determined by the momentum of the exit gas (eq. (2)). Thrust, of course, could also be obtained by integrating or summing up the incremental component pressures acting on all the internal surfaces of the combustion chamber and the nozzle. For an ideal nozzle, the gases are expanded such that the final exit pressure P_e is equal to the ambient pressure P_a. In this case, then, the pressure term is zero and

$$F = \dot{m}v_e = \left(\frac{\dot{W}}{g}\right)v_e \tag{3}$$

Equation (3) also generally holds in vacuum, or out in space, where the pressure term can be considered to be negligibly small. In space and in the absence of any external force fields, a spacecraft's motion can only be affected by thrust resulting from mass ejection.

Newton's Second Law of Motion can also be stated as

$$a = \frac{F}{m} = g\left(\frac{F}{W}\right)$$

In vertical flight, the net upward thrust equals the total thrust minus the vehicle weight, or F - W. Therefore,

$$a = g\,\frac{F - W}{W}$$

or

$$a = g\left(\frac{F}{W} - 1\right) \tag{4}$$

For lift-off, of course, F/W must be greater than 1; usually, it is approximately 1.3 to 1.5 for conventional rocket boosters.

ROCKET-ENGINE PARAMETERS

Specific Impulse

Specific impulse is a measure of rocket engine efficiency, just as "miles per gallon" is a measure of automobile economy performance, and is defined as follows:

$$I_{sp} = \frac{F}{\dot{m}g} = \frac{\text{Pounds force}}{\text{Pounds mass per second} \times g}$$

or

$$I_{sp} = \frac{F}{\dot{W}}$$ (usually specified in seconds; here, $\dot{W}$ is the propellant utilization rate in pounds per second.)

Rewriting equation (3) yields

$$\frac{F}{\dot{W}} = \frac{v_e}{g} = I_{sp} \tag{5}$$

Total Impulse

Total impulse is given by the following equation:

$$I_t = Ft \tag{6}$$

where t is the firing duration.

Some of the many factors which must be considered in the design of the rocket nozzle are chamber pressure P_c, ambient pressure P_a, ratio of specific heats for the particular gas γ, and nozzle area ratio ϵ. In thermodynamics, specific heat is a property of the gas that describes a work process involving changes in states (such as pressure, temperature, and volume). In practice, the effects of these factors are included in the ideal thrust equation:

$$F = C_F P_c A_t \tag{7}$$

where the thrust coefficient C_F varies from about 0.9 to approximately 1.8, depending on the nozzle pressure ratio. For an ideal nozzle (isentropic expansion to P_e, i.e., no energy losses), a rather complicated expression exists for C_F:

$$C_F = \sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{2}{\gamma - 1}\right)^{(\gamma+1)/(\gamma-1)}\left[1 - \left(\frac{P_e}{P_c}\right)^{(\gamma+1)/\gamma}\right]} + \frac{P_a - P_e}{P_c}\epsilon \tag{8}$$

Using equations (3) and (7) yields

$$F = C_F P_c A_t = \frac{\dot{W} v_e}{g}$$

$$v_e = \frac{g C_F P_c A_t}{\dot{W}} \tag{9}$$

Characteristic Exhaust Velocity

For the special case where C_F equals 1, v_e is designated as the characteristic exhaust velocity c^* (pronounced "see-star"). This quantity depends only on the combustion gases and is unaffected by what happens in the nozzle. As such, it is of value in comparing the potential of various propellants and is readily determined from experimentally measured parameters as follows:

$$c^* = \frac{g P_c A_t}{\dot{W}} \tag{10}$$

$$\text{Theoretical } c^* = \frac{\sqrt{\gamma g R T_c}}{\gamma\left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)}}$$

This theoretical value is determined from the properties of the hot combustion gas and thus is a function of the particular propellant combination. The ratio of experimental to theoretical c^* is generally used as an indicator of combustion efficiency.

From the preceding relation it can be shown that

$$c^* = (\text{a constant})\sqrt{\frac{T_c}{m}}$$

Stated another way, c^* is directly proportional to the square root of the combustion temperature and inversely proportional to the square root of the molecular weight of the propellant. From equations (9) and (10),

$$c^* = \frac{v_e}{C_F}$$

Then

$$F = \frac{\dot{W}}{g}\left(c^* C_F\right)$$

From equations (5) and (7),

$$I_{sp} = \frac{C_F P_c A_t}{\dot{W}} \tag{11}$$

Substituting with equation (10) yields

$$I_{sp} = \frac{c^* C_F}{g} \tag{12}$$

The interrelations of all the foregoing rocket-engine parameters are shown in detail in table 2-I. Review them for familiarity. All that is required are a few definitions and a little algebra.

TABLE 2-I. - PROPULSION FUNDAMENTALS

[Interrelation of rocket-engine parameters.]

Parameter	In terms of -							
	A_t	v_e	c^*	C_F	F	I_{sp}	P_c	$\dot{W}$
Nozzle throat area A_t		$\frac{v_e \dot{W}}{g P_c C_F}$	$\frac{c^* \dot{W}}{g P_c}$	$\frac{F}{C_F P_c}$	$\frac{F}{C_F P_c}$	$\frac{I_{sp} \dot{W}}{C_F P_c}$	$\frac{F}{C_F P_c}$	$\frac{\dot{W} c^*}{g P_c}$
Exhaust gas velocity v_e	$\frac{g A_t P_c C_F}{\dot{W}}$		$c^* C_F$	$c^* C_F$	$\frac{gF}{\dot{W}}$	$g I_{sp}$	$\frac{g P_c C_F A_t}{\dot{W}}$	$\frac{gF}{\dot{W}}$
Characteristic exhaust velocity c^*	$\frac{g A_t P_c}{\dot{W}}$	$\frac{v_e}{C_F}$		$\frac{v_e}{C_F}$	$\frac{gF}{C_F \dot{W}}$	$\frac{g I_{sp}}{C_F}$	$\frac{g P_c A_t}{\dot{W}}$	$\frac{gF}{\dot{W} C_F}$
Nozzle thrust coefficient C_F	$\frac{F}{P_c A_t}$	$\frac{v_e}{c^*}$	$\frac{v_e}{c^*}$		$\frac{F}{P_c A_t}$	$\frac{g I_{sp}}{c^*}$	$\frac{F}{P_c A_t}$	$\frac{gF}{\dot{W} c^*}$
Thrust F	$A_t P_c C_F$	$\frac{v_e \dot{W}}{g}$	$\frac{c^* C_F \dot{W}}{g}$	$C_F P_c A_t$		$I_{sp} \dot{W}$	$C_F P_c A_t$	$I_{sp} \dot{W}$
Specific impulse I_{sp}	$\frac{A_t C_F P_c}{\dot{W}}$	$\frac{v_e}{g}$	$\frac{c^* C_F}{g}$	$\frac{C_F c^*}{g}$	$\frac{F}{\dot{W}}$		$\frac{C_F P_c A_t}{\dot{W}}$	$\frac{F}{\dot{W}}$
Combustion-chamber pressure P_c	$\frac{F}{C_F A_t}$	$\frac{v_e \dot{W}}{g C_F A_t}$	$\frac{c^* \dot{W}}{g A_t}$	$\frac{F}{C_F A_t}$	$\frac{F}{C_F A_t}$	$\frac{I_{sp} \dot{W}}{C_F A_t}$		$\frac{c^* \dot{W}}{g A_t}$
Weight-flow rate $\dot{W}$	$\frac{g A_t P_c}{c^*}$	$\frac{gF}{v_e}$	$\frac{gF}{c^* C_F}$	$\frac{gF}{c^* C_F}$	$\frac{F}{I_{sp}}$	$\frac{F}{I_{sp}}$	$\frac{g P_c A_t}{c^*}$	

NOZZLE FLOW AND PERFORMANCE

Let us now turn our attention to the exhaust nozzle and examine some of the pertinent flow mechanisms. Figure 2-5 shows how a small disturbance propagates. Probably all of us have observed an action at some distance away and have noted the passage of a finite time before the sound reached our ears. A sharp noise generates a spherical sound wave which travels outward from the source at the speed of sound c. At sea level and room temperatures, c is about 1100 feet per second.

The effect of stream velocity on the propagation of small disturbances is shown in figure 2-6. In the subsequent discussion, Mach number M is simply the ratio of flow

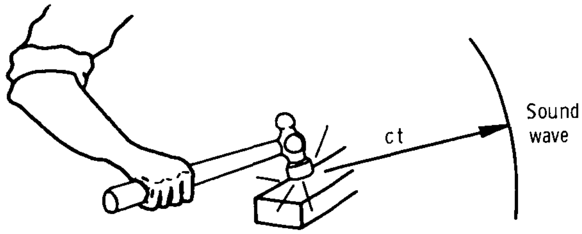


Figure 2-5. - Propagation of small disturbances in air. Instantaneous distance of sound wave from source = ct where c is speed of sound in air (at sea level, ≈ 1100 ft/sec) and t is time in seconds.

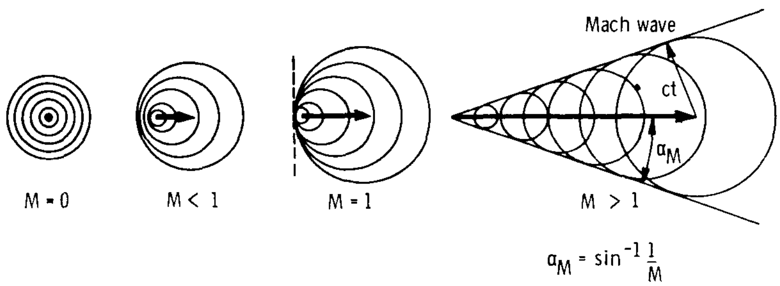


Figure 2-6. - Effect of velocity on sound-wave propagation.

velocity to the local speed of sound. If it is assumed that disturbances are generated in discrete increments of time, then for quiescent air conditions ($M = 0$) the waves are arrayed in concentric circles at any instant of time and are moving out uniformly in all directions. At subsonic speeds ($M < 1$), the waves are eccentric and are moving out in both directions, but they are moving faster in the downstream direction than in the upstream drection. At sonic flows ($M = 1$), there is no upstream propagation because the relative velocity is zero. At supersonic velocities, the envelope of small disturbances forms a Mach cone, the half-angle of which is equal to the Mach angle.

$$\alpha_M = \sin^{-1}\frac{1}{M} \tag{13}$$

At supersonic conditions, small disturbances can only propagate downstream within a volume defined by the Mach cone. A Mach wave thus defines the so-called region of influence.

Convergent-Divergent Nozzle

The flow conditions for a typical rocket nozzle are shown in figure 2-7. This figure shows the pressure distribution along the walls of a DeLaval convergent-divergent nozzle

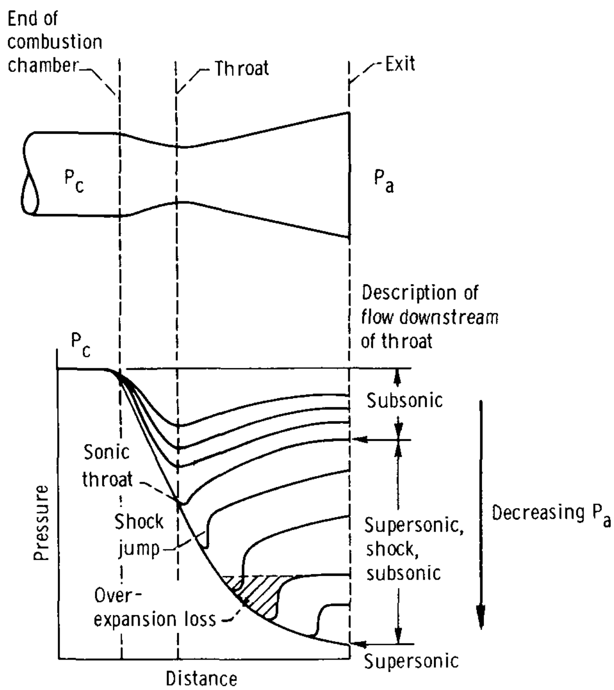


Figure 2-7. - Pressure distribution in DeLaval convergent-divergent nozzle.

for various pressure ratios. The pressure ratio is changed by varying the ambient pressure P_a and holding the chamber pressure P_c constant. At low pressure ratios, where P_a is relatively high, subsonic flow exists throughout the convergent and divergent portions of the nozzle. A small decrease in P_a causes the pressures to fall off all the way back to the combustion chamber. As was shown in figure 2-6, disturbances propagate upstream in the subsonic flow. This process continues with decreasing P_a until the pressure ratio in the throat corresponds to "choking" or sonic velocity. With further decreases in P_a, the flow upstream of the throat remains unaffected (remember: small disturbances cannot propagate upstream against a sonic or supersonic flow) while the flow downstream expands supersonically to a point where a normal shock is located. This normal shock is evidenced by a sharp rise in pressure (the so-called shock jump) and provides an abrupt transition of the flow back to the subsonic condition again. Further decreases in P_a cause the shock to move rearward in the nozzle and to occur at progressively higher Mach numbers. Eventually, the flow separates from the walls behind the shock wave. This is indicated here by the flatness in the distributions existing downstream of the shock jumps which are located well down in the nozzle. Note again that slight decreases in P_a can affect the location of the shock jump but cannot propagate any effects upstream thereof in the supersonic flow. When P_a is decreased to the point where supersonic flow is first established throughout the nozzle, and there is no shock jump, the nozzle is at design pressure ratio. Any further decreases in P_a will not affect the nozzle pressures, and the flow will continue to expand outside the nozzle.

At less than design pressure ratio, overexpansion losses occur. These are indicated for one such representative condition by the crosshatched area in figure 2-7. Overexpansion losses result from local pressures in the nozzle being less than ambient pressure. Pressures less than P_a constitute a loss in thrust or a drag on the propulsion system. For the particular P_a, the nozzle area ratio ϵ (equal to A_e/A_t) is simply too large and the flow overexpands with attendant energy losses.

At any point within the nozzle, the flow velocity, or Mach number, depends on the ratio of the local cross-sectional area to the throat area A/A_t as shown in figure 2-8. There is an evident need for a convergent channel to accelerate the gas subsonically and for a divergent channel to accelerate the gas supersonically. Theoretical considerations based on the conservation of mass, momentum, and energy across the nozzle yield the following expression to describe the variation in area:

$$\frac{A}{A_t} = \frac{1}{M}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M^2\right)\right]^{(\gamma+1)/2(\gamma-1)}$$

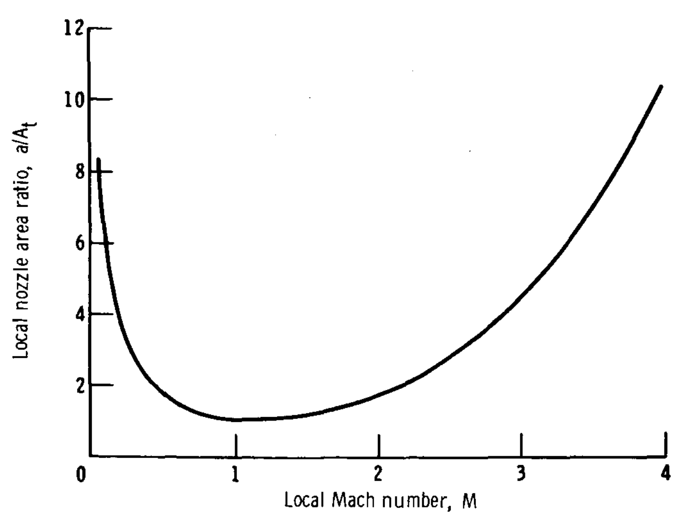


Figure 2-8. - Nozzle-area variation with flow Mach number.

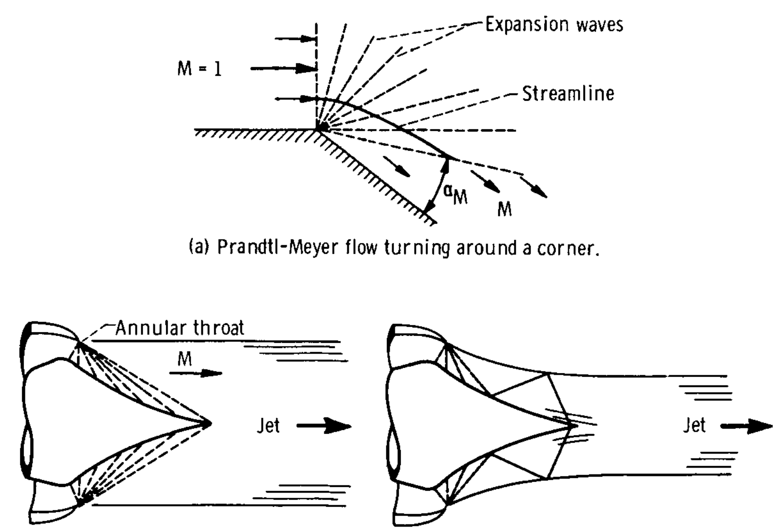
It should be understood that for larger nozzle pressure ratios P_c/P_a the gases can be expanded to higher exit Mach numbers M_e. Higher exit Mach numbers require larger nozzle area ratios A_e/A_t.

Variable-Area Nozzle

During an actual rocket boost phase through the atmosphere, the ambient pressure P_a decreases rapidly with altitude (as described in chapter 1). With a constant chamber pressure P_c, this changing ambient pressure causes a wide variation in the nozzle pressure ratio. For an ideal nozzle to match every point on the trajectory would require a variable-area nozzle and a concomitant variation in exit Mach number M_e to correspond to the variations in pressure ratio. Such a variable-area-ratio nozzle could possibly be accomplished mechanically. However, it would involve much structural complexity and added weight that would probably offset any gains in performance over a fixed-geometry nozzle. It would be especially difficult with a three-dimensional axisymmetric arrangement.

Free-Expansion Nozzle

Another possibility in achieving this adjustment in expansion ratio is to do it aerodynamically as suggested in figure 2-9. The Prandtl-Meyer flow-expansion theory permits



(a) Prandtl-Meyer flow turning around a corner.

(b) Plug nozzle (on design).

(c) Plug nozzle (off design).

Figure 2-9. - Altitude compensation by means of free-expansion nozzles.

calculation of supersonic flow turning about a point as illustrated in figure 2-9(a). Streamlines can be readily calculated through this expansion field at any specified radius. These theoretical streamlines can be used as surface contours for plug-type nozzles as shown in figures 2-9(b) and (c). Note the similarity in expansion fields. Operation of the plug nozzle is shown for both on- and off-design conditions. Also, note the variation in the size of the two exiting jets for on and off design. This aerodynamic adjustment of the flow is referred to as "altitude compensation." In this free-expansion process, no over-expansion losses are incurred as in the contrasting case for the convergent-divergent nozzles at off-design conditions. In some advanced engine concepts for the 20- to 30-million-pound-thrust category, modular combustors or clusters of small high-pressure engines are envisioned as feeding combustion gases onto a common plug nozzle for further expansion externally.

ROCKET-ENGINE PERFORMANCE

Let us now briefly consider the thrusting phase of a rocket flight. Because of the application of continuous thrust by the engine, the speed of the vehicle increases steadily and reaches a maximum when the propellant is finally consumed. At this point the velocity is referred to as the "burnout velocity" and is designated v_b. In determining this velocity increment, we will neglect for now the gravitational effects and aerodynamic drag. Now we will concern ourselves only with the thrust of the rocket engine. By using Newton's Second Law of Motion $(F = \Delta mv/\Delta t)$ and integrating over the burning period, a logarithmic expression can be derived for v_b:

$$v_b = v_e \ln \frac{m_i}{m_f} = 2.3\, v_e \log \frac{m_i}{m_f} \tag{14}$$

where m_i is the initial total mass of the vehicle (at lift off), and m_f is the final mass at burnout. By using equation (5) the expression for v_b can be rewritten as follows:

$$v_b = gI_{sp} \ln \frac{m_i}{m_f}$$

But, by definition

$$MR = \frac{m_f}{m_i} = \frac{m_f}{m_f + m_p}$$

Therefore, by substitution,

$$v_b = gI_{sp} \ln \frac{1}{MR} = 2.3\, gI_{sp} \log \frac{1}{MR} \tag{15}$$

This is the basic rocket equation. It shows the direct role of specific impulse in the attainment of high vehicle velocity. Burnout velocity is the parameter that best reflects rocket engine performance for either analyzing or accomplishing specific boost or space missions. It must be remembered, however, that in equation (15) we have neglected an additional gravity term. This gravity term will be taken into account in the next chapter, where we consider actual flight trajectories.

APPENDIX - SYMBOLS

A area, in.^{2}

A_e nozzle exit area, in.^{2}

A_t nozzle throat area, in.^{2}

a acceleration, (ft/sec)/sec

C_F nozzle thrust coefficient

c velocity of sound, ft/sec

c_p specific heat at constant pressure (for air, approx. 0.241), Btu/(lb)(^oF)

c_v specific heat at constant velocity (for air, approx. 0.17), Btu/(lb)(^oF)

c^* characteristic exhaust velocity, ft/sec

F thrust or force, lb

g acceleration due to gravity, 32.2 (ft/sec)/sec

I_{sp} specific impulse, sec

I_t total impulse, lb-sec

M Mach number

M_e Mach number at nozzle exit

MR mass ratio, m_f/m_i

m mass, slugs

m_f final mass of vehicle at burnout, slugs

m_i initial total mass of vehicle, slugs

m_p mass of propellant, slugs

$\dot{m}$ mass-flow rate, slugs/sec

$\mathcal{M}$ molecular weight (for hydrogen, 2; for oxygen, 32), lb/mole

o/f oxidizer to fuel (mixture) ratio

P_a ambient pressure, lb/in.^{2}

P_c combustion-chamber pressure, lb/in.^{2}

P_e nozzle exit pressure, lb/in.^{2}

R gas constant (universal gas constant, 1544 ft-lb/(^oR)(mole); specific gas constant for air, 53.3 ft-lb/(lb)(^oR))

T_c combustion temperature, ^oR

t time, sec

v velocity, ft/sec

v_b burnout velocity, ft/sec

v_e exhaust gas velocity

W weight, lb

$\dot{W}$ weight-flow rate, lb/sec

α_M Mach angle, deg

γ ratio of specific heats, c_p/c_v

Δ change in quantity or magnitude

ϵ nozzle area ratio, A_e/A_t

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