WORKING GAS SELECTION FOR THE CLOSED BRAYTON CYCLE

JOHN L. MASON

Chief Engineer, AiResearch Manufacturing Company, Los Angeles, California, U.S.A.

Systems based on the Rankine cycle have received primary attention in the last decade for nuclear-turboelectric space power production in the kilowatt range. However, recent developments in high-temperature energy sources, continued progress in component efficiency and reliability coupled with liquid metal handling problems in Rankine cycle systems have caused a refocusing of attention on the Brayton cycle. The properties of the cycle working gas influence the design of all major components except the alternator. Gas selection indices are developed here for a representative Brayton cycle turbocomponent, the compressor, and a representative heat transfer component, the recuperator. Especially at low power levels, these selection indices indicate substantial advantages associated with use of helium-xenon gas mixtures. Such binary inert-gas working fluids provide outstandingly low Prandtl numbers along with variable molecular weights to meet turbomachinery design requirements at various power levels.

INTRODUCTION

In the mid-1950s, the United States recognized the probable need for kilowatt quantities of power for space missions lasting months or years. Nuclear turboelectric power plants were studied. The Brayton and Rankine cycles...
were compared. The latter was preferred because of its apparent advantage in system weight and radiator area at peak cycle temperatures around 1600º to 1700ºR, the range projected for nuclear reactors of the SNAP II type. Study and development of Rankine cycle space power plants commenced, and, indeed, still goes on. For several years, the Brayton cycle was dropped from active consideration for space power. Even so, Brayton cycle technology continued to be developed through work on turbojet engines and small gas turbines. In the last 2 1/2 years, Brayton cycle applications to space power have been reexamined and reevaluated (Refs. 1 and 2). The Brayton cycle looks far better now than it did in 1955 for the following reasons:

1. Peak cycle temperatures well above 1600º to 1700ºR appear attainable, either from second-generation nuclear reactors or from solar or isotope energy sources. High heat-source temperatures dramatically improve Brayton cycle performance.

2. Continued progress in the design of Brayton cycle components—compressors, turbines, alternators, and heat exchangers—has led to estimates of improved component efficiency and, therefore, improved cycle efficiency.

3. Excellent reliability and service life have been achieved with commercial jet aircraft equipment similar to Brayton cycle components.

4. Rankine cycle system development difficulties, largely related to liquid metal handling problems, have led to renewed interest in the single-phase inert-gas working fluid used in the Brayton cycle.

5. Recently investigated binary inert-gas mixtures (Refs. 3 and 4) have led to reductions in predicted Brayton cycle system weight.

SYSTEM CONSIDERATIONS

One version of the recuperated Brayton cycle is shown schematically in Fig. 1a. Power output is electrical. Alternator, turbine, and compressor are mounted on a common shaft by means of gas bearings. Waste heat is recovered by means of a recuperator. In this single-loop system, the cycle working gas flows through both the radiator and the heat-source heat exchanger. All major components except the alternator are directly influenced by the properties of the cycle working gas. Selection of working gas is most critical at low power levels. Also, at least for the next few years, low power level is of primary interest from an applications standpoint. Therefore, this paper will consider only low power levels, for which single-stage centrifugal turbo-components (centrifugal compressors, radial inflow turbines) appear preferable.

Figure 1b shows a two-loop system with an auxiliary coolant linking the cycle heat-sink heat exchanger with the space radiator ultimate heat sink. A similar auxiliary thermal loop (not shown) can be used to link the heat source with the power cycle. An auxiliary loop, using liquid metal, is desirable when the heat source is a nuclear reactor. A liquid-cooled reactor is lighter, smaller, and easier to shield than a comparable gas-cooled reactor.
With solar and radioisotope heat sources, direct cooling by the cycle working gas is generally preferable to use of an auxiliary thermal loop.

![Brayton cycle schematic diagram, single-loop system.](image1)

**Fig. 1a.** Brayton cycle schematic diagram, single-loop system.

![Brayton cycle schematic diagram, with auxiliary coolant loop.](image2)

**Fig. 1b.** Brayton cycle schematic diagram, with auxiliary coolant loop.

The Brayton cycle as applied to space power will be designed to meet one of the following objectives:

1. Minimum system weight;
2. Minimum radiator area;

Usually these factors are mutually exclusive. In general, a system designed for minimum weight will not have minimum radiator area, nor will it have maximum conversion efficiency.

Minimum weight is a commonly used factor because of the ever-present penalty of boosting payload into orbit. Radiator area is limited by vehicle installation factors; as large launch vehicles become available, restrictions
on radiator area may become less stringent. Maximum conversion efficiency, seldom used as a direct figure of merit, is applicable whenever economy in heat-source provisioning is of overriding importance.

Gas selection is, of course, only one of the many choices to be made by the designer in putting together a Brayton cycle system to meet one of the above objectives. The purpose of this paper is not detailed system optimization, but rather development of gas selection indices, to be used to limit the number of gases requiring detailed investigation during an actual system design. These indices are necessarily based on simplifying assumptions.

Because Brayton cycle gas selection involves compromise between the turbomachinery and the heat transfer equipment in the system, gas selection indices are developed for a representative turbocomponent, the compressor, and a representative heat transfer component, the recuperator.

From the selection indices developed, previous recommendations (Refs. 3 and 4) of a Brayton cycle working gas are confirmed; the recommended gas is a binary mixture of xenon and helium, with gas composition (and therefore molecular weight) dependent on system power level.

The advantage of xenon-helium over pure monatomic gases is illustrated by comparison of a 28-per-cent-xenon, 72-per-cent-helium mixture with argon in a two-loop 3-kw system having an electrically simulated isotope heat source. The turbomachinery design and performance are essentially identical for the two gases, since they have the same molecular weight. The heat transfer equipment and associated structure are much lighter for the system using a gas mixture. Calculated system weight less radiator is 221 lb with xenon-helium, compared to 321 lb with argon. The latter is an actual weight for a system which has been built and successfully tested. Greater weight advantages can be expected in a system optimized for the xenon-helium gas.

CYCLE WORKING GAS SELECTION

Thermodynamic comparison

The first step in Brayton cycle working gas evaluation is to establish the thermodynamic indices by which different gases may be compared. The Appendix contains the derivation of an expression for cycle thermal efficiency, expressed in terms of the following dimensionless variables:
1. Compressor efficiency parameter, $B$;
2. Turbine efficiency parameter, $C$;
3. Recuperator effectiveness, $E$;
4. Cycle pressure drop parameter, $e$;
5. Compressor work function, $x_c$.

The expression for cycle thermal efficiency is:

$$\eta = \frac{BC \left( 1 - \frac{1}{x_c} \right) - x_c + 1}{(1 - E)(B - x_c + 1) + EBC \left( 1 - \frac{1}{x_c} \right)} \quad (1)$$

Symbols used herein are tabulated and defined in the list of Symbols on p. 249.
Equation (1) can be differentiated with respect to $x_c$, with $B$, $C$, $E$, and $\varepsilon$ being assumed constant, to give an approximate expression for $x_c^*$, the compressor work function for peak cycle efficiency:

$$x_c^* = \frac{1 + \sqrt{1 + YZ}}{Z}$$

where

$$Y = \frac{B}{2E - 1} (1 - E) - 1$$

and

$$Z = \varepsilon \left[ \frac{1}{C} \left( \frac{1 - E}{2E - 1} \right) + 1 \right]$$

Figure 2 presents $x_c^*$ as predicted by equation (2), in terms of the variables $B$, $C$, $E$, and $\varepsilon$.

The attainable compressor efficiency $\eta_c$ decreases with increasing $x_c^*$. Similarly, attainable turbine efficiency $\eta_t$ decreases with increasing $x_c$. Thus, both $\eta_c$ and $\eta_t$ decrease as $x_c$ increases, $\varepsilon$ being fixed. Therefore, $B$ and $C$ in equation (1) depend on $x_c$, and are not constant, as was assumed in the derivation of equation (2). The true $x_c^*$ is somewhat less than that predicted by equation (2). Nevertheless, equation (2) is a good basis for cycle analysis. The $x_c^*$ predicted by equation (2) is generally close to the $x_c$ for a minimum weight system for the cycle temperature levels ($T'_4$ and $T'_3$) being considered. In any event, $x_c$ is the function of compressor pressure ratio which directly influences cycle performance, independent of gas properties. Therefore, it is logical to compare different working gases in the Brayton cycle at equal values of $x_c$.

Typically, the Brayton cycle turbine and the compressor are of comparable configuration (Fig. 3) and rotate at the same shaft speed. Turbine and compressor are influenced by essentially the same variables, such as gas properties and cycle operating pressures and flows. The compressor design is usually the more critical from a thermodynamic standpoint; compressor efficiencies are generally several points below turbine efficiencies, and the compressor is less forgiving of unfavorable imposed conditions. For purposes of working gas selection and evaluation, compressor design criteria will be used as representative of both compressor and turbine. Gas properties, operating pressures, flows, and shaft speeds which are optimum for the compressor are likely to be optimum or near-optimum for the turbine. One precaution is necessary, that of conservatism in selection of compressor tip speeds to avoid overspeeding the turbine. The hot turbine wheel is usually 15 to 25 per cent larger in diameter than the compressor wheel, with a higher tip speed in proportion.

The Brayton cycle of Fig. 1 has three heat transfer components: the heat-source heat exchanger, the recuperator, and the radiator. All are large components; the size and weight of each are affected in the same general way by the cycle gas properties. The recuperator is the component selected.
to illustrate the effect of working gas on Brayton cycle heat transfer equipment size and weight.

Gas flow rate has a basic influence on both compressor and recuperator design. It is convenient to utilize the corrected molal flow rate $\hat{\dot{W}}/\theta$, which is

$$\dot{W} = \frac{HP \text{ lb mol}}{\Lambda \text{ sec}}$$

where

$$\Lambda = \frac{RT_1}{\eta_r} \left[ BC \left( 1 - \frac{1}{\beta x_c} \right) - x_c + 1 \right]$$

Fig. 2. $x_c$ as predicted by equation (2).
Compressor

The compressor adiabatic head is related to $x_e$ as follows:

$$H_{ad} = \frac{RT_1}{M_0} (x_e - 1)$$  \hspace{1cm} (7)

For a single-stage compressor, adiabatic head and tip speed are related as follows:

$$H_{ad} = \psi \frac{u_{x_e}^2}{g}$$  \hspace{1cm} (8)

The head coefficient $\psi$ is of the order of 0.5 for a practical single-stage centrifugal compressor design. Head coefficient is not significantly different for different gases, provided that compressor design is tailored to each gas.

Equations (7) and (8) combine to give:

$$u_{x_e}^2 = \frac{\varepsilon RT_1 (x_e - 1)}{\psi M_0}$$  \hspace{1cm} (9)

Fig. 3. Typical Brayton cycle turbomachinery.
or, in terms of compressor Mach number $N_M$:

$$N_M^2 = \frac{x_o - 1}{\psi(y - 1)}$$

(10)

Equations (9) and (10) can be used to compare a monatomic gas ($\gamma = 1.67$; $\theta = 0.400$) with a diatomic gas ($\gamma = 1.41$; $\theta = 0.286$) of equal molecular weight, in terms of the tip speed $u_T$ required to achieve a given $x_o$. At a given $T_1$ and $\psi$, the product $u_T^2\theta$ is fixed, and the diatomic gas needs the higher tip speed by a factor of $\sqrt{1.4}$. (A triatomic gas with $\theta = 0.22$ would require $\sqrt{1.8}$ times the tip speed of a monatomic gas to achieve equal $x_o$.) This tip speed advantage of monatomic gases is of considerable practical importance. It permits a single-stage compressor running at moderate tip speed to do a job that with a diatomic gas would require either two stages of compression or a high tip speed in a single-stage design.

Similarly, equation (10) shows that a monatomic gas can achieve a given $x_o$ at a lower compressor Mach number $N_M$ than a diatomic gas. This advantage is a manifestation of the low compressibility of monatomic gases.

The flow passages in a monatomic gas compressor show less area decrease in the direction of flow than do the passages of a diatomic gas compressor. As a result, the monatomic gas compressor is somewhat easier to design, and will usually show a higher efficiency, by two or three percentage points, than a diatomic gas compressor designed for the same $x_o$.

Two dimensionless variables have a major influence on compressor design. They are specific speed $N_S$ and Reynolds number $Re$:

$$N_S = \frac{N\sqrt{Q}}{(gH_{ad})^{\frac{1}{2}}}$$

(11)

$$Re = \frac{DP_{1}u_T}{\mu}$$

(12)

Figure 4 shows currently achievable compressor efficiency as a function of specific speed. The application regime of centrifugal compressors is approximately in the range from $N_S = 0.03$ to $N_S = 0.2$. Above $N_S = 0.2$, axial machines are preferred. The efficiency advantage of high specific speeds (above 0.1) is evident. Equation (11) is rewritten with the aid of equations (5) and (7), in order to express $N_S$ in terms of appropriate gas properties and cycle operating parameters:

$$N_S = \frac{\Lambda P_{1}^{\frac{1}{2}}H_{P}^{\frac{1}{2}}}{\Lambda P_{1}^{\frac{1}{2}}(RT_{1})^{\frac{1}{2}}(\gamma(x_o - 1))^{\frac{3}{2}}}$$

(13)

Figure 5 shows typical variation of compressor efficiency with Reynolds number. Below a critical Reynolds number $Re^*$, which is approximately $10^4$ for a centrifugal machine, efficiency decreases with decreasing $Re$, according to an equation of the form:

$$\frac{1 - \eta_e}{1 - \eta_e^*} = a + b \left(\frac{Re}{Re^*}\right)^{n}$$

(14)
Fig. 4. Compressor efficiency (with air) versus specific speed.

\[ N_s = \frac{N \sqrt{Q}}{(gH_{ad})^{3/4}} \]

- \( N \) = REV/SEC
- \( Q \) = FT\(^3\)/SEC
- \( H_{ad} \) = ADIABATIC HEAD, FT
- \( g \) = 32.2 FT/SEC\(^2\)
- \( Re \geq Re^b \geq 10^6 \)
Fig. 5. Typical variation in compressor efficiency with Reynolds number.
where \( a, b, \) and \( n \) are empirical constants. Reynolds numbers below critical should be avoided if at all possible. Equation (12) is rewritten with the aid of equation (9) in terms of \( x_c \) and other pertinent variables:

\[
Re = \frac{gP_1(x_c - 1)}{\pi \psi \theta \mu N}
\]

Table 1 lists the effect of the pertinent variables on \( N_S \) and \( Re \), as determined by equations (13) and (15). Table 1 also shows the effect of these same variables on heat transfer equipment weight and volume, as determined by equation (17), derived below. Table 1 is based on the assumption of fixed \( HP, \psi, x_c, T_1, T_4 \) and \( e \).

**Table 1. Influence of Selected Variables on Compressor \( N_S \), Compressor \( Re \), and Heat Transfer Equipment Weight and Volume**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Favoring high compressor ( N )</th>
<th>Favoring high compressor ( Re )</th>
<th>Favoring low heat transfer equipment weight and volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressor-turbine shaft speed, ( N )</td>
<td>high ( N )</td>
<td>low ( N )</td>
<td>---*</td>
</tr>
<tr>
<td>Compressor inlet pressure, ( P_1 )</td>
<td>low ( P_1 )</td>
<td>high ( P_1 )</td>
<td>high ( P_1 )</td>
</tr>
<tr>
<td>Compressor work function, ( x_c )</td>
<td>low ( x_c )</td>
<td>high ( x_c )</td>
<td>high ( x_c )</td>
</tr>
<tr>
<td>Gas specific heat function, ( \theta )</td>
<td>high ( \theta )</td>
<td>low ( \theta )</td>
<td>low ( \theta )</td>
</tr>
<tr>
<td>Gas molecular weight, ( M )</td>
<td>high ( M )</td>
<td>---</td>
<td>low ( M )</td>
</tr>
<tr>
<td>Gas viscosity, ( \mu )</td>
<td>---</td>
<td>low ( \mu )</td>
<td>---</td>
</tr>
<tr>
<td>Gas Prandtl number, ( Pr )</td>
<td>---</td>
<td>---</td>
<td>low ( Pr )</td>
</tr>
</tbody>
</table>

* Blank (---) = No effect.

Compromise is evidently necessary in selection of most of the variables of Table 1. If power level is above a few horsepower, selection of the variables to favor high \( Re \) becomes relatively unimportant, since in any event \( Re \) is likely to be well above \( Re^* \). Hence, selection of variables involves a compromise between high \( N_S \) and low heat transfer equipment weight and volume.

Increasing the shaft speed \( N \) to increase \( N_S \) causes a proportional decrease in \( Re \); that is, with respect to changes in shaft speed, \( Re N_S = \text{constant} \). Shaft speed increase is one of the better ways to increase \( N_S \), but there are limits to the tolerable increase in \( N \) over and above those imposed by Reynolds number decrease. Two of the most important factors limiting \( N \) are bearing design and critical speed problems. Also, increasing \( N \) at constant \( x_c \) (therefore, constant tip speed \( u_T \)) leads to a decrease in compressor wheel diameter \( D \), since \( u_T = \pi DN \). Small compressor wheels require close tolerances in their dimensions and surface finish. Internal leakage through
running clearances may become a sizable fraction of the throughflow of a small compressor, with consequent loss of efficiency. These factors combine to set maximum shaft speeds at about 80,000 to 100,000 rpm and minimum wheel diameters at about 2·5 to 3·5 in., for high efficiency at the current state of the art. These limits are not fixed. Figure 6 shows a 200,000-rpm, 10-w turboalternator now being developed for a reverse Brayton cycle cryogenic refrigeration system. It is questionable whether the efficiency of

![Image of turbomachinery](image.png)

*Fig. 6. Ultraminiature turbomachinery.*

this ultraminiature turbomachinery can be brought up to Brayton power cycle component requirements.

Decreasing the pressure to effect a given increase in $N_S$ has a strongly adverse effect on Reynolds number, since, with respect to pressure changes, $Re N_S^2$ = constant.

Of the gas properties $\theta$, $M$, $\mu$, and $Pr$ listed in Table 1, $\theta$ and $M$ involve compromise between the turbomachinery and the heat transfer equipment in the Brayton cycle. Low values of $\mu$ and $Pr$ are desirable without compromise. Table 2 shows room-temperature properties of a number of gases and gas mixtures (Ref. 5).
Table 2. Gas Properties at 18°C and 1 atm

<table>
<thead>
<tr>
<th>Gas</th>
<th>Molecular weight, M</th>
<th>Specific heat ratio, γ</th>
<th>Specific heat, ( c_p )</th>
<th>Viscosity, ( \mu )</th>
<th>Thermal conductivity, ( K )</th>
<th>Prandtl number, ( Pr = \frac{c_p}{K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tristonic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>44.01</td>
<td>1.28</td>
<td>0.216</td>
<td>0.21</td>
<td>0.0143</td>
<td>9.6 \times 10^{-6}</td>
</tr>
<tr>
<td>Steam ((P = 0))</td>
<td>18.02</td>
<td>1.33</td>
<td>0.248</td>
<td>0.45</td>
<td>0.0095</td>
<td>6.4 \times 10^{-6}</td>
</tr>
<tr>
<td><strong>Diatomic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oxygen</td>
<td>32.00</td>
<td>1.40</td>
<td>0.285</td>
<td>0.222</td>
<td>0.201</td>
<td>1.35 \times 10^{-3}</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>28.00</td>
<td>1.40</td>
<td>0.285</td>
<td>0.250</td>
<td>0.0165</td>
<td>1.11 \times 10^{-5}</td>
</tr>
<tr>
<td>Air</td>
<td>28.97</td>
<td>1.40</td>
<td>0.289</td>
<td>0.240</td>
<td>0.0173</td>
<td>1.16 \times 10^{-5}</td>
</tr>
<tr>
<td><strong>Monatomic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xenon</td>
<td>131.30</td>
<td>1.66</td>
<td>0.398</td>
<td>0.080</td>
<td>0.0224</td>
<td>1.51 \times 10^{-3}</td>
</tr>
<tr>
<td>Krypton</td>
<td>83.80</td>
<td>1.08</td>
<td>0.405</td>
<td>0.008</td>
<td>0.0240</td>
<td>1.67 \times 10^{-3}</td>
</tr>
<tr>
<td>Argon</td>
<td>39.94</td>
<td>1.668</td>
<td>0.400</td>
<td>0.124</td>
<td>0.0221</td>
<td>1.49 \times 10^{-3}</td>
</tr>
<tr>
<td>Neon</td>
<td>20.18</td>
<td>1.64</td>
<td>0.391</td>
<td>0.232</td>
<td>0.0510</td>
<td>2.08 \times 10^{-4}</td>
</tr>
<tr>
<td>Helium</td>
<td>4.00</td>
<td>1.66</td>
<td>0.398</td>
<td>1.25</td>
<td>0.0194</td>
<td>1.30 \times 10^{-3}</td>
</tr>
<tr>
<td>28.2% Xe-71.8% He</td>
<td>39.94</td>
<td>1.66</td>
<td>0.398</td>
<td>0.125</td>
<td>0.0252</td>
<td>1.69 \times 10^{-4}</td>
</tr>
<tr>
<td>30.2% Xe-69.8% He</td>
<td>42.4</td>
<td>1.66</td>
<td>0.398</td>
<td>0.177</td>
<td>0.0232</td>
<td>1.68 \times 10^{-4}</td>
</tr>
<tr>
<td>35.8% Xe-64.2% He</td>
<td>49.6</td>
<td>1.66</td>
<td>0.398</td>
<td>0.1007</td>
<td>0.0250</td>
<td>1.68 \times 10^{-4}</td>
</tr>
<tr>
<td>50.6% Xe-49.4% He</td>
<td>68.5</td>
<td>1.66</td>
<td>0.398</td>
<td>0.0728</td>
<td>0.0244</td>
<td>1.64 \times 10^{-4}</td>
</tr>
</tbody>
</table>
Equations (13) and (15) can be combined to give the following relationship
between compressor shaft speed \( N \) and system power level \( HP \):

\[
N \cdot HP = \frac{Re \, N^2 A [gRT(x_e - 1)]^{1/2} \psi u}{(MO)^{1/2}} \tag{16}
\]

The fluid properties group \( \mu/(M0)^{3/2} \) determines the product of shaft speed
and power at a given \( Re \) and \( N \), other cycle conditions being fixed.

The following sample set of system conditions will be used as a basis of
working gas comparison and evaluation:

1. \( E = 0.9 \);
2. \( e = 0.95 \);
3. \( T_1 = 500^\circ R \);
4. \( T_2 = 2000^\circ R \);
5. \( \eta_e = 0.77 \); \( \eta_t = 0.83 \). These are initial assumptions, needed to set
cycle throughflow;
6. \( x_e = 1.34 \), compared to \( x_e^* = 1.42 \) from equation (2).

Figure 7 is a plot of equation (16) for the sample problem written above,
with \( N \), \( Re \), argon, xenon, and three xenon-helium mixtures. Each pure gas or gas
mixture of definite molecular weight corresponds to a particular tip speed,
by equation (9). Tip speed and shaft speed combine to fix compressor
wheel diameter, selected values of which also are plotted in Fig. 7. Continuously variable gas molecular weight is obtainable by appropriate selection
of a binary mixture such as xenon-helium; any desired operating point in
Fig. 7 between the xenon and helium lines may be obtained.

Consider a specific power level such as 2 hp. To favor small heat-transfer
equipment weight and volume, it is desirable to operate at as low a molecular
weight as possible. Therefore, it is desirable to operate (on the 2-hp line)
at the highest allowable shaft speed, which results in a minimum allowable
molecular weight at the 2-hp level. This minimum molecular weight may
in fact be set by limits imposed on shaft speed for the mechanical design
reasons mentioned above. Instead, minimum molecular weight may be
set by maximum limits imposed on compressor tip speed or minimum limits
on compressor diameter. Which of these variables (\( N \), \( u_p \), or \( D \)) is actually
the effective limit on \( M \) depends on the numerical values assigned these
variables, as would be expected, and also on the power level. Let the following
limits be imposed: \( N = 100,000 \) rpm; \( u_p = 1000 \) ft per sec; \( D = 3 \) in.
At 2 hp, argon meets none of these limits, although it exceeds the tip speed
limit by only a small margin.

The three xenon-helium mixtures represented in Fig. 7 are selected to
meet these limits on \( N \), \( u_p \), and \( D \). The \( D = 3 \) in. limit, corresponding to
\( M = 685 \), is the most restrictive; that is, it determines the highest \( M \).

Table 3 presents detailed data associated with Fig. 7 for argon and the
three xenon-helium mixtures at the 2-hp level. Data also are presented for
one diatomic gas and one triatomic gas, both of which require excessive
shaft and tip speeds. Table 3 and Fig. 7 present essentially uncompromised
Fig. 7. Effect of gas properties on compressor N and D for constant N, and Re.
turbomachinery from the standpoint of \( N_g \) and \( \text{Re} \), since both of these quantities are fixed at values large enough to avoid serious compromise of \( \eta_r \). (An optimized design might deliberately compromise \( \eta_r \) for the sake of a larger gain in the heat transfer equipment, but the same gas selection logic would apply.) Any compromises of turbomachinery are clearly indicated by Table 3 and Fig. 7; they are represented by excessive \( N \), excessive \( u_p \), or insufficient \( D \). Reasonableness of \( N \), \( u_p \), and \( D \) at a desired power level (and at favorable \( N_g \) and \( \text{Re} \)) is offered as a gas selection criterion from the turbomachinery standpoint.

**Recuperator**

Appendix 1-C presents an idealized derivation of an expression for the surface area of a tubular counterflow heat exchanger as a function of cycle pressure level and gas properties, including the gas molecular weight \( M \), specific heat function \( \theta \), and Prandtl number \( Pr \):

\[
S = \frac{2P_1 \theta M^4}{P_1} \left( 1 + x_e \frac{1}{\theta} \right)^3
\]  

(17)

Weight and volume of recuperator matrix vary with \( S \) as follows:

\[
w = K_1 S t
\]  

(18)

\[
v = K_2 \frac{S}{r}
\]  

(19)

where \( t \) is the thickness of the matrix material and \( r \) is the matrix hydraulic radius. The quantity \( \lambda \) in equation (17) contains effectiveness, pressure drop, and other parameters, and is defined by equation (C-18) in the Appendix 1-C.

Table 4, based on equation (17), is a continuation of the gas comparison of Table 3 and Fig. 7, for the conditions tabulated in Fig. 7. Table 4 lists the quantities affecting the recuperator which are different for different working fluids, when the latter are compared at equal system power, equal effectiveness and percentage pressure drop, equal \( T_1 \), \( T_4 \), \( x_e \), equal compressor \( \text{Re} \), \( N_g \), and equal \( \eta_r \). The different gases are compared in required recuperator surface area \( S \) according to an arbitrary scale, with the value 100 assigned to argon.

The 30-2 per cent Xe–69-8 per cent He mixture, which (from Table 3) requires compressor shaft speed and tip speed comparable to argon, requires only 27 per cent of the recuperator surface area of argon. The other two argon mixtures show increased recuperator surface area requirements, but are still below argon. The 50-6-per-cent-Xe, 49-4-per-cent-He mixture, selected to give a compressor wheel diameter of 3-0 in., imposes a marked penalty on \( S \) relative to the other helium-xenon mixtures. These relative areas are influenced by built-in changes in pressure level reflective of the imposed conditions.

Nitrogen shows a favorably low recuperator surface area (50 per cent of that required for argon) but is unusable because of excessive compressor shaft speeds.
### Table 3. Working Gas Comparison 2-HP Sample Problem

<table>
<thead>
<tr>
<th>Gas</th>
<th>Remarks</th>
<th>M</th>
<th>Re</th>
<th>$N$</th>
<th>D</th>
<th>$P_1$</th>
<th>$\dot{W}_g$</th>
<th>$u_p$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>N too high</td>
<td>29.94</td>
<td>0-100</td>
<td>$10^4$</td>
<td>122,000</td>
<td>1-93</td>
<td>12-1</td>
<td>0-127</td>
<td>1030</td>
</tr>
<tr>
<td>30-7% Xe–69-8% He</td>
<td>N too high</td>
<td>42.4</td>
<td>0-100</td>
<td>$10^4$</td>
<td>127,000</td>
<td>1-81</td>
<td>14-2</td>
<td>0-127</td>
<td>1000</td>
</tr>
<tr>
<td>35-8% Xe–64-2% He</td>
<td>Good solution</td>
<td>49.6</td>
<td>0-100</td>
<td>$10^4$</td>
<td>100,000</td>
<td>2-12</td>
<td>11-1</td>
<td>0-127</td>
<td>926</td>
</tr>
<tr>
<td>50-6% Xe–49-4% He</td>
<td>Good solution</td>
<td>68.5</td>
<td>0-100</td>
<td>$10^4$</td>
<td>60,000</td>
<td>3-00</td>
<td>6-54</td>
<td>0-127</td>
<td>786</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>$N$, $u_p$ too high</td>
<td>28.0</td>
<td>0-100</td>
<td>$10^4$</td>
<td>254,000</td>
<td>1-31</td>
<td>13-5</td>
<td>0-090</td>
<td>1450</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>$N$, $u_p$ too high</td>
<td>44.0</td>
<td>0-100</td>
<td>$10^4$</td>
<td>172,000</td>
<td>1-77</td>
<td>5-94</td>
<td>0-068</td>
<td>1300</td>
</tr>
</tbody>
</table>
The superiority of the xenon-helium mixtures is a result of an extremely low Prandtl number $Pr$, far below that attainable with any known pure gas, monatomic or otherwise. Figure 8 shows $c_p$, $\mu$, $K$, and $Pr$ versus composition for xenon-helium mixture. $Pr$ is seen to be 0.3 or below for all xenon fractions between 0.11 and 0.78, corresponding to gas molecular weights between 18 and 103. Mixtures of helium and other monatomic gases also have low Prandtl numbers, but not as low as those for xenon-helium.

The gas properties $c_p$, $\gamma$, and $\theta$ are nearly independent of temperature for pure monatomic gases and for monatomic gas mixtures. If it is assumed that viscosity $\mu$ and thermal conductivity $K$ increase with temperature in such a way as to maintain nearly constant $Pr = c_p \mu / K$, the factors affecting $S$ in equation (17) do not vary appreciably with temperature, and it is legitimate to use room temperature properties in gas comparison, as is done in Table 4.

Viscosity has an effect on recuperator performance (apart from its influence on $Pr$) which is neglected by equation (17). Heat exchanger Reynolds number is inversely proportional to viscosity, and the friction and heat transfer factors $f$ and $j$ vary with Reynolds number according to

$$f = K_3 \Re^{-m} = K_3 \left( \frac{\mu S}{4WL} \right)^m$$

$$j = K_4 \Re^{-m} = K_4 \left( \frac{\mu S}{4WL} \right)^m$$

where the exponent $m$ usually lies between 0.2 and 0.5 for compact matrices. The different monatomic gases and gas mixtures do not differ enough in viscosity to justify the complication of accounting for the effect of viscosity (including its variation with temperature) on $Re$.

**System weight comparison**

To illustrate the advantage of xenon-helium mixtures, the effect of conversion of an existing Brayton cycle system from argon to xenon-helium was studied. If a 28-per-cent-Xe, 72-per-cent-He mixture is substituted for argon with no change in pressure, turbomachinery performance will remain...
unchanged, the effect of the 13-per cent viscosity increase being neglected. The heat transfer components, on the other hand, will all decrease in weight and volume.

A two-loop 3-kw system, built and tested by AiResearch in 1963, was selected for working gas conversion study. This system corresponds to that shown schematically in Fig. 1b. Figure 9 shows the combined rotating unit (turbine, compressor, and alternator). Figure 10 shows the heat transfer components. Left to right, they are heat-sink heat exchanger, recuperator, and heat-source heat exchanger. Figure 11 shows the complete system, less radiator loop. Heat source is electrical, simulating radioisotope capsules. The system was tested at 58,300 rpm and showed a measured overall efficiency (electrical power output/electrical heat input) of 18.1 per cent.

Table 5 presents the results of the system weight comparison. The weights shown in the argon column are actual equipment weights. The weights shown

Fig. 8. $\epsilon_{\infty}, \mu_{\infty}, K$, and Pr for xenon-helium at 18°C and 1 atm.
Fig. 9. Brayton cycle 3-kw combined rotating unit.

Fig. 10. Brayton cycle 3-kw heat transfer components: heat-sink heat exchanger, recuperator, and heat-source heat exchanger, with inter-connecting ducts.
Fig. 11. Brayton cycle 3-kw system.
in the 28-per-cent-Xe, 72-per-cent-He column are calculated weights, based on a detailed computer study for the recuperator and approximate calculations for the heat-source and heat-sink heat exchangers.

Table 5. System Weight Comparison

<table>
<thead>
<tr>
<th>Component</th>
<th>Actual weight with Argon lb</th>
<th>Estimated weight with 28% Xe-72% He lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined rotating unit</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Heat-source heat exchanger</td>
<td>36</td>
<td>28</td>
</tr>
<tr>
<td>Recuperator</td>
<td>93</td>
<td>38</td>
</tr>
<tr>
<td>Heat-sink heat exchanger</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>Ducts (7)</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>Structure</td>
<td>94</td>
<td>65</td>
</tr>
<tr>
<td>Total</td>
<td>321</td>
<td>221</td>
</tr>
</tbody>
</table>

Weights of the radiator, the radiator thermal loop, accessories and controls, and any required isotope shield are not included in Table 5. (The effect of low Prandtl number gases on single-loop system radiator design will be covered in a later paper.)

Table 5 includes weight of the system supporting structure, which was not designed for minimum weight. The reduction in structural weight shown for xenon-helium was arbitrary, by the same percentage as the reduction of total system weight.

CONCLUSIONS

Xenon-helium gas mixtures have been found to provide substantial advantages over other working gases for the closed Brayton cycle. Xenon-helium mixtures provide variable molecular weights to meet turbomachinery design requirements. Over a wide range of molecular weights, xenon-helium Prandtl number is outstandingly low. The overall result is markedly advantageous to the Brayton cycle, especially at low power levels. Selective leakage of helium is a potential problem, but one well worth taking on in view of the advantages to be gained.

The assistance of J. R. Frey, N. van Le, and K. O. Parker in the preparation of this paper is gratefully acknowledged.
APPENDIX

A. Cycle thermal efficiency

Cycle thermal efficiency is cycle power output divided by rate of heat input, with cycle power output defined as turbine output less compressor power input. Net power output could be obtained by subtracting bearing, windage, and alternator electrical losses from the cycle power output as defined above. An over-all efficiency could then be obtained from the net power output. Over-all efficiency is not used because it would require the introduction of additional variables not related to gas selection. Cycle power output per molal flow rate is:

\[
\frac{HP}{W} = \varepsilon T\eta (1 - \frac{1}{x_1}) - \frac{\varepsilon T_1}{\eta} (x_e - 1) \tag{A-1}
\]

Since \( R = c_p \theta \) and \( x_i = \frac{c_i}{c_p} \), equation (A-1) becomes:

\[
\frac{HP}{W} = \frac{RT_1}{\theta \eta} \left[ BC \left( 1 - \frac{1}{\frac{c_i}{c_p}} \right) - x_e + 1 \right] \tag{A-2}
\]

where \( BC = \eta \theta \tau \). With use of the definition of \( \Lambda \), equation (A-2) becomes:

\[
\frac{HP}{W} = \frac{\Lambda}{\theta} \tag{A-3}
\]

The rate of heat input to the cycle per molal flow rate is:

\[
\frac{q}{W} = \frac{R}{\theta} (T_4 - T_2) \tag{A-4}
\]

but

\[
T_4 - T_3 = T_4 - T_2 - (T_3 - T_2) = T_4 - T_2 - E(T_3 - T_2) \tag{A-5}
\]

\[
T_2 = T_1 + \frac{T_1(x_e - 1)}{\eta_e} \tag{A-6}
\]

and

\[
T_3 = T_4 - \eta_e T_4 \left( 1 - \frac{1}{x_e} \right) \tag{A-7}
\]

Equations (A-4) through (A-7) combine to give:

\[
\frac{q}{W} = \frac{RT_1}{\theta \eta_e} \left[ (1 - E)(B - x_e + 1) + EBC \left( 1 - \frac{1}{x_e} \right) \right] \tag{A-8}
\]
The cycle thermal efficiency $\eta$ is the quotient of equation (A-2) and (A-9):

$$\eta = \frac{BC \left(1 - \frac{1}{e_{c}}\right) - x_{c} + 1}{(1 - E)(B - x_{c} + 1) + EBC \left(1 - \frac{1}{e_{c}}\right)}$$ (A-9)

Equation (A-9) is differentiated with respect to $x_{c}$ ($B$, $C$, $E$, and $e$ being assumed constant); the result is set equal to zero and solved for $x_{c}$ to give an approximate expression for $x_{c}^*$. The procedure is straightforward and will not be detailed here. The result is:

$$x_{c}^* = \frac{1 + \sqrt{1 + \phi Z}}{Z}$$ (A-10)

B. Compressor Specific Speed and Reynolds Number

The definition of compressor specific speed is:

$$N_{S} = \frac{N\sqrt{Q}}{(gH_{ad})^{\frac{1}{2}}}$$ (B-1)

Volume flow at inlet stagnation conditions is:

$$Q = \frac{\dot{W}RT_{1}}{P_{1}}$$ (B-2)

Equations (A-3) and (B-2) combine to give:

$$Q = \frac{HP_{0}RT_{1}^{\frac{3}{2}}}{\sqrt{\lambda P_{1}}}$$ (B-3)

The compressor adiabatic head is:

$$H_{ad} = \frac{RT_{1}}{M\theta} (x_{c} - 1)$$ (B-4)

Equations (B-1), (B-3), and (B-4) combine to give:

$$N_{S} = \frac{N0^{\frac{3}{2}}HP_{0}(RT_{1})^{\frac{3}{4}}}{\lambda^{\frac{3}{4}}P_{1}^{\frac{3}{4}}(RT_{1})^{\frac{3}{4}}(x_{c} - 1)^{\frac{3}{4}}}$$ (B-5)

or

$$N_{S} = \frac{N0^{\frac{1}{2}}M^{\frac{3}{2}}HP_{1}^{\frac{3}{4}}}{\lambda^{\frac{3}{4}}P_{1}^{\frac{3}{4}}(RT_{1})^{\frac{3}{4}}(x_{c} - 1)^{\frac{3}{4}}}$$ (B-6)

The definition of compressor Reynolds number is:

$$Re = \frac{DP_{1}\omega_{r}}{\mu}$$ (B-7)
Since \( u_T = \pi DN \):
\[
\text{Re} = \frac{\rho u_T^2}{\mu N} = \frac{P_1 u_T^2}{RT_1 \mu N} \quad \text{(B-8)}
\]

Equation (9), previously derived for tip speed, is:
\[
u_T^2 = \frac{gRT_2(x_e - 1)}{\eta M_0} \quad \text{(B-9)}
\]

Equations (B-8) and (B-9) combine to give:
\[
\text{Re} = \frac{gP_1(x_e - 1)}{\pi \eta \mu N} \quad \text{(B-10)}
\]

C. Recuperator

Tubular counterflow recuperator design theory will be developed from seven basic equations. Axial heat conduction is neglected:

1. The Fanning equation for pressure drop:
\[
\Delta P = \frac{f W^2 L}{2g \rho A^2 r} \quad \text{(C-1)}
\]

2. The definition of the Colburn modulus \( j \):
\[
j = \frac{P_{1/2} hA}{Wc_p} \quad \text{(C-2)}
\]

3. The definition of the hydraulic radius \( r \):
\[
r = \frac{A L}{S} \quad \text{(C-3)}
\]

4. The relationship between required heat transfer coefficient times surface area \( US \) and the required effectiveness \( E \), for unit capacity rate ratio:
\[
US = \frac{Wc_p E}{1 - E} \quad \text{(C-4)}
\]

5. The relationship between hot-side percentage pressure drop \( \pi_H \), cold-side percentage pressure drop \( \pi_C \), and cycle pressure drop penalty \( \beta \):
\[
V - \beta = \beta \pi_H + \pi_C \quad \text{(C-5)}
\]

6. The summation of series thermal resistance, tube wall thermal resistance being neglected:
\[
\frac{1}{US} = \frac{1}{(hS)_H} + \frac{1}{(hS)_C} \quad \text{(C-6)}
\]

7. The empirical relationships between friction factor and Reynolds number, and Colburn modulus and Reynolds number:
\[
\begin{align*}
    f_H &= f_H(\text{Re}) \\
    f_C &= f_C(\text{Re}) \\
    j_H &= j_H(\text{Re}) \\
    j_C &= j_C(\text{Re})
\end{align*}
\]

247
Equation (C-1) and (C-3) are now combined; then the definition of \( \pi \) and the perfect-gas equation are introduced:

\[
\frac{\Delta P}{P} = \frac{jW^2S}{2gP_A^3P} \quad \text{(C-8)}
\]

\[
\pi = \frac{RTfW^2S}{2gPMA^2P} \quad \text{(C-9)}
\]

Equation (C-9) may be written separately for the hot and cold sides of the recuperator. For high effectiveness, \( \tilde{P}_H = \tilde{P}_C = \frac{P}{2} \). Also, \( W_H = W_C = W \) for unit flow rate ratio, and \( S_H = S_C = S \) for an all-prime surface heat exchanger. Also, it can be assumed with little error that \( \tilde{P}_u = P_2 \) and \( \tilde{P}_H = P_1 \)

\[
\pi_H = \frac{RTW^2f_{H}S}{2gP_2^3MA_1^2} \quad \text{(C-10a)}
\]

\[
\pi_C = \frac{RTW^2f_{C}S}{2gP_2^3MA_0^2} \quad \text{(C-10b)}
\]

Both sides of equation (C-2) are multiplied by \( S \), after which the equation is rearranged and applied separately to the hot and cold sides:

\[
S = \frac{Pr^4(hS)H}{A_H} = \frac{Pr^4(hS)C}{A_C} \quad \text{(C-11)}
\]

Equations (C-4), (C-6), and the definition of the thermal conductance ratio \( \phi = (hS)_H/(hS)_C \) are now combined:

\[
(hS)_H = \frac{Wc_PE(1 + \phi)}{1 - E} \quad \text{(C-12a)}
\]

\[
(hS)_C = \frac{Wc_PE\left(1 + \frac{1}{\phi}\right)}{1 - E} \quad \text{(C-12b)}
\]

Equation (C-11) is combined separately with equations (C-12a) and (C-12b):

\[
S = \frac{Pr^4A_HE(1 + \phi)}{j_H(1 - E)} \quad \text{(C-13a)}
\]

\[
S = \frac{Pr^4A_CE\left(1 + \frac{1}{\phi}\right)}{j_C(1 - E)} \quad \text{(C-13b)}
\]

The free flow area \( A \) is eliminated between equations (C-10a) and (C-10b) and (C-13a) and (C-13b):

\[
S^2 = \frac{Pr^4RTW^2f_{H}(1 + \phi)^3}{2gM(1 - E)j_H^3\pi_HP_1^3} \quad \text{(C-14a)}
\]

\[
S^2 = \frac{Pr^4RTW^2f_{C}\left(1 + \frac{1}{\phi}\right)^3}{2gM(1 - E)j_C^3\pi_CP_2^3} \quad \text{(C-14b)}
\]
Equation (C-14a) is divided by equation (C-14b) to give:
\[
\frac{\pi_H}{\pi_e} = \phi^2 \left( \frac{P_2}{P_1} \right)^2 \left( \frac{j_H}{j_C} \right) \left( \frac{j_C}{j_H} \right)^3
\]

To a good approximation, \( f_H = f_e = f \) and \( j_H = j_C = j \), since, for tubular counterflow, \( \text{Re}_H = \text{Re}_C = \text{Re} \). Equations (C-14a), (C-15), and (C-5) then combine to give:
\[
S^2 = \frac{P_2^2 R T W^2 \phi^2 f}{2 g M (1 - E)^2 \rho^2 (V - \beta)} \left[ \frac{(1 + \phi^2)(1 + r_e \phi^3)}{r_e \phi^3} \right]
\]

where \( r_e = P_2/P_1 \). It is desirable to select a reasonable value for \( \phi \). If \( \phi = r_e^{-1/2} \), the bracketed quantity in equation (C-16) is a minimum. The resulting \( S \) should be near-minimum. Equation (C-16) then combines with equation (A-2) and the identity \( W = \bar{W} M \) to give:
\[
S^2 = \frac{P_2^2 R T (H - \phi)^2 M^2 \rho^2 (1 + r_e^{-1/2})^4}{2 g A^2 P_2^2 (1 - E)^2 \rho^2 (V - \beta)}
\]

The terms of equation (C-17) that are independent of fluid properties are combined to define a new variable, \( \lambda \):
\[
\lambda = \frac{(A^2)^{1/2} (H - \phi)^2}{(2 g A^2 P_2^2 (1 - E)^2 \rho^2 (V - \beta))^{1/2}}
\]

Equation (C-17) can be rewritten in terms of \( \lambda \) and solved for \( S \), as follows:
\[
S = \frac{\lambda M P_2 (1 + r_e^{-1/2})^2}{P_1}
\]
or, since \( r_e = x_0^{1/2} \):
\[
S = \frac{\lambda M P_2 (1 + x_0^{-1/2})^2}{P_1}
\]

**SYMBOLS**

- **A** Recuperator free-flow area, ft²
- **a** Constant in equation (14)
- **B** Compressor efficiency function. \( B = \eta_B (\tau - 1) \)
- **b** Constant in equation (14)
- **C** Turbine efficiency function. \( C = \eta_C (\frac{\tau}{\tau - 1}) \)
- **c_p** Molal heat at constant pressure, Btu/lb-mol °R
- **c_v** Specific heat at constant pressure, Btu/lb °R
- **D** Compressor wheel diameter, ft
- **E** Recuperator effectiveness. \( E = \frac{T_3 - T_4}{T_3 - T_5} = \frac{T_6 - T_9}{T_6 - T_7} \)
- **f** Fanning friction factor, defined by equation (C-1)
- **g** 32.2 ft/sec²
JOHN L. MASON

HP  Turbine power output less compressor power input, ft-lb/sec
H\text{ad}  Compressor adiabatic head, ft
\( h \)  Individual heat transfer coefficient, Btu/hr-ft\(^2 \) \(^\circ\)R
\( j \)  Colburn modulus, defined by equation (C-2)
\( K \)  Thermal conductivity, \( \frac{\text{Btu}}{\text{sec-ft} \ \text{\(^\circ\)}\text{R}} \)
\( K_1 \)  Constant in equation (18)
\( K_2 \)  Constant in equation (19)
\( K_3 \)  Constant in equation (20)
\( K_4 \)  Constant in equation (21)
\( L \)  Recuperator length, ft
\( M \)  Gas molecular weight
\( m \)  Constant in equations (20) and (21)
\( N \)  Shaft speed, revolutions per minute or revolutions per second
\( N_{\text{M}} \)  Compressor Mach number, defined by equation (10)
\( N_{\text{S}} \)  Specific speed, defined by equation (11)
\( n \)  Constant in equation (14)
\( P \)  Pressure, psf\(^a\) or psia
\( \bar{P} \)  Average pressure in recuperator, psf\(^a\) or psia
\( \Delta P \)  Pressure drop, psf or psi
\( Pr \)  Prandtl number, \( Pr = \frac{\eta \mu}{K} \)
\( Q \)  Volume flow, cfs
\( q \)  Heat added to cycle, Btu/sec
\( R \)  Universal gas constant, \( R = 1544 \ \frac{\text{ft-lb}}{\text{mol} \ \text{\(^\circ\)}\text{R}} \)
\( Re \)  Reynolds number. For compressor, \( Re = \frac{D \eta \mu}{\mu} \). For recuperator, \( Re = \frac{4WL}{\mu S} \)
\( Re^* \)  Critical Reynolds number of compressor, \( \approx 10^4 \)
\( r \)  Heat transfer surface hydraulic radius, ft, defined by equation (C-3)
\( r_s \)  Compressor pressure ratio, \( \frac{P_2}{P_1} \)
\( S \)  Recuperator surface area, ft\(^2\)
\( T \)  Temperature, \( ^\circ\)R
\( T_\text{a} \)  Average temperature in recuperator, \( ^\circ\)R
\( t \)  Recuperator heat transfer surface thickness, in.
\( U \)  Overall heat transfer coefficient, Btu/hr-ft\(^2 \) \(^\circ\)R
\( \nu_T \)  Wheel tip speed, ft/sec
\( V \)  \( 1 - \frac{P_3 - P_4}{P_1} \) - \( \beta \left( \frac{P_3 - P_4}{P_1} \right) \). \( V = [1 - \text{fractional pressure drop in cycle other than in recuperator}] \)
\( v \)  Recuperator volume, ft\(^3\)
\( W \)  Weight flow rate, lb/sec
\( \bar{w} \)  Molal flow rate, lb-mol/sec
\( w \)  Recuperator weight, lb
WORKING GAS–BRAYTON CYCLE

\[ x_c = \left( \frac{P_2}{P_1} \right)^\theta \]
\[ x_c^* = \text{Compressor work function for peak cycle efficiency} \]
\[ x_t = \text{Turbine work function.} \]
\[ Y = \text{Function of } B \text{ and } E. \quad Y = B \left( \frac{1 - E}{2E - 1} \right) - 1 \]
\[ Z = \varepsilon \left( \frac{1 - E}{2E - 1} \right) + 1 \]
\[ \beta = \frac{\text{Turbine pressure ratio}}{\text{Compressor pressure ratio}} \]
\[ \gamma = \frac{c_p}{c_p - R} \]
\[ e = \frac{N_t}{N_e} = \beta^\theta \]
\[ \eta = \text{Cycle thermal efficiency.} \]
\[ \eta_c = \text{Compressor efficiency, isentropic work/actual work} \]
\[ \eta_t = \text{Turbine efficiency, actual work/isentropic work} \]
\[ \Theta = \frac{RT_1}{N_e} \left[ BC \left( 1 - \frac{1}{\Theta c} \right) - x_c + 1 \right] \]
\[ \lambda = \text{Function defined by equation (C-10)} \]
\[ \mu = \text{Viscosity, lb/ft-sec} \]
\[ \pi = 3.1416 \]
\[ \pi_h = \frac{\Delta P_{2,6}}{P_1} \]
\[ \pi_c = \frac{\Delta P_{2,3}}{P_2} \]
\[ \rho = \text{Specific weight, lb/ft}^3 \]
\[ \tau = \frac{T_4}{T_1} \]
\[ \phi = \frac{(hS)_{J}}{(hS)_{T}} \]
\[ \psi = \text{Head coefficient, defined in equation (8)} \]

**Subscripts**

Numbered subscripts refer to cycle points of Fig. 1a:

1. Compressor inlet
2. Compressor outlet
3. Recuperator outlet to heat source heat exchanger
4. Turbine inlet
REFERENCES


The following page(s) provide higher quality versions of graphics contained in the preceding article or section.
The compressor adiabatic head is related to \( \gamma \), as follows:

\[
H_{ad} = \frac{RT_1}{M_0} (\gamma - 1)
\]

(7)

For a single-stage compressor, adiabatic head and tip speed are related as follows:

\[
H_{ad} = \frac{\frac{1}{2} \nu_2^2}{g}
\]

(8)

The head coefficient \( \nu \) is of the order of 0.5 for a practical single-stage centrifugal compressor design. Head coefficient is not significantly different for different gases, provided that compressor design is tailored to each gas.

Equations (7) and (8) combine to give:

\[
\frac{\frac{1}{2} \nu_2^2}{g} = \frac{gRT_1(\gamma - 1)}{gM_0}
\]

(9)
running clearances may become a sizable fraction of the throughput of a small compressor, with consequent loss of efficiency. These factors combine to set maximum shaft speeds at about 80,000 to 100,000 rpm and minimum wheel diameters at about 2/5 to 3/5 m, for high efficiency at the current state of the art. These limits are not fixed. Figure 6 shows a 200,000-rpm, 10-kW turbomotor now being developed for a reverse Brayton cycle cryogenic refrigeration system. It is questionable whether the efficiency of

![Fig. 6. Ultraminature turbomachinery.](image)

this ultraminature turbomachinery can be brought up to Brayton power cycle component requirements.

Decreasing the pressure to effect a given increase in $N_s$ has a strongly adverse effect on Reynolds number, since, with respect to pressure changes, Re $N_s^3$ is constant.

Of the gas properties $g$, $M$, $\mu$, and $Pr$ listed in Table 1, $g$ and $M$ involve compromise between the turbomachinery and the heat transfer equipment in the Brayton cycle. Low values of $\mu$ and $Pr$ are desirable without compromise. Table 2 shows room-temperature properties of a number of gases and gas mixtures (Ref. 5).
Fig. 11. Brayton cycle 3-line system.