EXPERIMENTS ON AIM POINT ESTIMATION AT VARIOUS RATES OF CLOSURE

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SUMMARY AND ABSTRACT

As an aircraft approaches the earth all points on the ground appear to the pilot to expand radially out from a stationary point on the ground toward which the aircraft is aiming. If the pilot can detect this point of zero expansion, he knows precisely the point on the ground toward which his aircraft is aiming. Experimental evidence is presented as to how precisely a pilot can actually detect this aim point. Since any point in the pilot's field of view always moves radially out from the aim point, it is necessary to know only the magnitude of the velocity at any point to specify the pattern of motion seen by the pilot. This was done by plotting contours of equal angular rate. The hypothesis was then made that the expected error in detecting the aim point would be a linear function of the angular distance from the aim point to some threshold angular rate. Initial results of an experimental study in which subjects viewed a pattern of dots that simulated the apparent expansion pattern of the ground tend to confirm this hypothesis.

INTRODUCTION

A basic problem in psychology has been to determine the visual cues humans and other animals use to control their movement through their environment. This problem received new importance as men began to pilot airplanes and move about in three dimensions instead of the more usual two. Many of the cues used in two dimensions become inadequate and the pilot is forced to utilize new cues. That these cues are not always sufficient can be attested to by the large number of accidents attributed to "perceptual error."

As a specific example, consider the task of landing an aircraft. Analysis of the visual cues available shows that a "perfect" pilot, (i.e., a nonexistent being that could make errorless perceptual and control decisions) can extract from the visual field all the information necessary to control and land the aircraft. For example, if the intended touchdown point is kept 3° below the horizon, the aircraft will stay on a 3° glide slope. The position and orientation of the horizon indicates the pitch and roll attitude of the aircraft. The shape and apparent size of the runway give redundant glide slope information as well as further information on the position and attitude of the observer. The above cues all hold for a stationary observer. When motion is added the perfect pilot can extract the derivatives of these cues from the visual field. Also, when motion is added, a whole new set of cues becomes available. J. J. Gibson has pointed out that "so strict are the geometrical relationships between physical motion of the observer's body and retinal motion of the projected environment that the latter provides, in fact, the chief sensory guide for locomotion in space."

One of the cues that Gibson discusses is the one of concern in this paper. This cue is based on the fact that all points on the ground appear, to the pilot, to expand radially out from the point on the ground toward which the aircraft is aiming (i.e., the point where the aircraft velocity vector intersects the ground plane). This point of zero expansion is an exact indicator of the point on the ground toward which the perfect pilot is heading. If, for instance, this point begins to move down the runway away from the threshold, the perfect pilot knows that his rate of descent is too low and can take corrective action. The cue thus provides the pilot with valuable information on the rate of change of altitude.
Although all these cues and many others are available for use by the perfect pilot, it is not well known with what precision a real pilot can extract the same information from the visual field. In some cases his ability to perceive the information is clearly inadequate since most pilots would consider it an almost impossible task to land a large airliner with visual cues alone. Additional information on at least airspeed appears to be essential.

This paper is addressed to the problem of experimentally determining how accurately a pilot can detect the zero point of expansion in the field of view and thus his aim point. This accuracy will be related to quantitative measures of the visual stimulus.

SYMBOLS

\begin{align*}
AP & \quad \text{position of aim point on the ground} \\
D & \quad \text{distance from } O \text{ to } AP \\
h & \quad \text{altitude of observer} \\
O & \quad \text{position of observer} \\
P & \quad \text{any point on the ground} \\
V & \quad \frac{dD}{dt} \\
(a,r) & \quad \text{angular coordinates of } P \\
\beta & \quad \text{angle of approach of observer} \\
\delta & \quad \text{angle from } AP \text{ to } P \\
\dot{\delta} & \quad \frac{d\delta}{dt}
\end{align*}

MATHEMATICAL EXPRESSIONS AND HYPOTHESIS

The first step in determining how accurately a pilot can determine his aim point from visual cues in his expanding field of view is to quantify the visual stimulus. Since any point in the pilot's field of view always moves radially out from the aim point, it is only necessary to know the magnitude of the velocity at any point to completely specify the pattern of motion seen by the pilot. This was done by plotting contours of equal angular rate.
Figure 1.- Geometry for angular rate and contour radius formula.

From Figure 1 we can derive that

\[ \delta = \frac{V r (1 - \cos^2 \alpha \cos^2 \beta)^{1/2}}{D^2 + r^2 - 2D r \cos \alpha \cos \beta} \]  \hspace{2cm} (1)

If \( \beta = 90^\circ \) this simplifies to

\[ \delta = \frac{r V}{D^2 + r^2} \] \hspace{2cm} (2)

Solving equation (1) for \( r \) gives an expression for the contour of all points on the ground moving at a constant angular rate.

\[ r = \left[ D \cos \alpha \cos \beta + \frac{V}{2 \delta} (1 - \cos^2 \alpha \cos^2 \beta)^{1/2} \right] \]

\[ \pm \left[ D \cos \alpha \cos \beta + \frac{V}{2 \delta} (1 - \cos^2 \alpha \cos^2 \beta)^{1/2} \right]^2 - D^2 \] \hspace{2cm} (3)

If \( \beta = 90^\circ \) this simplifies to

\[ r = \frac{1}{2} \frac{V}{\delta} \left[ 1 \pm \left( 1 - \frac{4D^2 \delta^2}{V^2} \right)^{1/2} \right] \] \hspace{2cm} (4)

and the contours become circles independent of \( \alpha \). Equation (4) is expressed in terms of \( \delta \) in equation (5) to show that the ratio \( D/V \) completely specifies the pattern.
\[ \theta = \tan^{-1} \left( \frac{x}{D} \right) = \tan^{-1} \left\{ \frac{1}{2} \left( \frac{V}{D} \right) \left[ 1 \pm \left( 1 - \frac{4h^2}{V^2} \right)^{1/2} \right] \right\} \] (5)

These equations are similar to those derived by Gibson,\(^7\) Haveron,\(^8\), and General Electric.\(^9\)

Figures 2 and 3 represent the pattern of motion seen by the pilot at two phases in a landing approach. They were derived by perspective transforming the contours obtained from equation (3). For the vertical case which will be considered in this paper, the contours become circles with a radius equal to the angular distance from the aim point to the contour along a horizontal line through the aim point.

![Diagram](image1)

**Figure 2.** Pilot's view of constant angular velocity contours for \(B = 3\) deg. \([0.0522\text{ rad}], D = 6000\text{ ft }[1970\text{ m}], V = 200\text{ ft/sec }[65.6\text{ m/sec}]\) and \(D/V = 30\). The runway is \(150\text{ ft } \times 10,000\text{ ft }[49.2\text{ m } \times 3280\text{ m}]\) and the aim point is \(1000\text{ ft }[328\text{ m}]\) from the runway threshold. The contours are in min of arc/sec. The crosshatched area is described in the text.

![Diagram](image2)

**Figure 3.** Pilot's view of constant angular velocity contours for \(B = 3\) deg. \([0.0522\text{ rad}], D = 1500\text{ ft }[492\text{ m}], V = 200\text{ ft/sec }[65.6\text{ m/sec}]\) and \(D/V = 7.5\). The runway is \(150\text{ ft } \times 10,000\text{ ft }[49.2\text{ m } \times 3280\text{ m}]\) and the aim point is \(1000\text{ ft }[328\text{ m}]\) from the runway threshold. The contours are in min of arc/sec. The crosshatched area is described in the text.

The magnitude of the angular rates at the subject's eyes, and some simple assumptions about the subject's behavior and perceptual limitations lead to the following hypothesis: The angular error in locating the exact aim point will be linearly related to the angular distance from the aim point to some
particular contour on the isoangular velocity plots. In particular, the contour for some small angular rate should outline a region of confusion in which it would be expected that most of the subject's aim point judgements would fall. There are at least two reasonable assumptions about the subject's behavior that will lead to this linear hypothesis. First, the subject may look for an area of no perceptible motion and guess within this area. Second, the subject may look at various points located outside the region in which there is no perceptible movement and estimate the point away from which they are moving. If the standard deviation of the error in detecting the direction of movement of the points is a linear function of the velocity of the points, then this too will lead to the prediction that angular error in detecting the aim point should be a linear function of some contour radius. Dean Haveron made a similar hypothesis and by compiling psychophysical data on velocity thresholds he estimated that an upper angular rate threshold might be 10 minutes of arc per second (2.91 mrad/sec). The area outlined by this contour is shown by the crosshatched area on figures 2 and 3.

Haveron's result left the answer to the question of the value of the expanding gradient cue to aim point somewhat in doubt since, as can be seen in figures 2 and 3 the 10 min/sec (2.91 mrad/sec) contour outlines the entire runway during a large part of the approach, and it would not seem that the aim point could be detected with high enough accuracy to be of much use to the pilot. Haveron points out that one problem with his study is that it is based on threshold for velocity detection determined by having subjects observe one of two small stimuli which move for brief periods of time. Therefore, the possibility exists that pilots might make use of visual spatial and temporal summation when the whole field of view is moving. The two experiments reported in this paper are addressed to this question.

METHOD

The goal of the experimental program was to allow subjects to make actual aim point judgements, thus providing more direct information on human ability to detect an aim point than that provided by inferences based on psychophysical thresholds for velocity detection. Subjects were presented with an array of dots filling a square 30° (0.522 rad) field of view. These dots simulated the pattern of motion a pilot would observe during a vertical descent at various rates of closure. A vertical approach was chosen over a more typical aircraft approach of 30° because other cues the subject might use to estimate the location of the aim point could be eliminated. Equation (3) shows that for a vertical approach the ratio of the distance from the aim point to the velocity, D/V, completely characterizes the pattern of motion. Therefore, though in the actual experiment the final distance remained fixed and the velocity varied for the different conditions, the results apply to any combination of distance and velocity with the same ratio. By keeping the final distance constant it was possible to have the dots always spaced 20° (0.0348 rad) apart at the end of each trial.

In both experiments subjects sat in an enclosed booth with their eyes 18 inches from a rear projection screen. At the start of each trial a square array of dots appeared. The subject pushed a button and the dots appeared to start moving "toward" the subject, expanding about a randomly chosen aim point. After a certain length of time (3 sec in the first experiment), the
dots stopped and a tracking dot appeared. The subject moved this dot with an x-y controller to the point he estimated to be the aim point and then pushed a second button. His error was automatically recorded and a second dot appeared to show him the actual aim point. The trial was then repeated. After 23 such trials the velocity of approach was changed and a new condition began. Each session had five conditions and took about 20 minutes. The experiments were completely automatic with all data reduction and experimental control handled by a small digital computer and the display and perspective transformations performed by an analog computer.

In the first experiment the single experimental variable was the D/V ratio. The D/V ratios used (60, 30, 15, 7.5, 3.75) encompassed the range encountered by a pilot on a final approach. Patterns for a D/V ratio of 30 and 7.5 are shown in figures 2 and 3. Eight male subjects were used in the first experiment. They were told to imagine that they were moving toward the dot pattern and to estimate their impact point. Each subject had two practice sessions, plus additional practice before each daily session and before each condition within a session. The five D/V conditions and five data sessions were arranged in a balanced Latin Square. Thus, there were 100 data points per condition for each subject.

In the second experiment three subjects who participated in the first experiment were used and the variable of viewing time was added. These times were 1.5, 3.0, and 6.0 seconds. The 3-second condition served as a replication of the first experiment. Each subject had three sessions at each viewing time. There were five V/D conditions per session and 20 trials per session giving a total of 60 trials per time-D/V condition.

RESULTS

All statistics were based on the untested assumption that the horizontal and vertical errors were normally distributed with zero mean and equal variance. This implies that the magnitude of the error in detecting the aim point will have a radial normal or Rayleigh distribution. Figure 4 shows the results of the first experiment. The abscissa is the angular distance to the 10 min/sec (2.91 mrad/sec) contour from the aim point for the various D/V conditions. The ordinate is the maximum likelihood estimate for the mean of the radial normal distribution based on the data from the eight subjects. The broken line represents results averaged over all subjects.

![Figure 4. Angular error in detecting aim point vs. angular distance to 10 min/sec contour for 8 subjects.](image-url)
Statistical analysis showed that the differences between the D/V conditions were significant \((P = 0.001)\). The individual differences between subjects were also significant \((P = 0.01)\). In order to test for learning a plot of the average subject error for each condition and session was made. The two lowest D/V conditions showed a slight but significant \((P = 0.01)\) decrease in error of about 0.3° from session 1 to session 5. The other conditions showed no significant effect. Straight lines were fitted to the data to test the hypothesis that angular error in detecting the aim point is linearly related to some contour radius, in particular the 10 min/sec \((2.91\, \text{mrad/sec})\) contour. These results plus correlation coefficients are presented in Table I.

**TABLE I.- LEAST SQUARES STRAIGHT LINE FITS TO THE DATA FOR EXPERIMENT 1**

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>SLOPE</th>
<th>Y-INTERCEPT</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - ○</td>
<td>0.36</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>2 - □</td>
<td>0.32</td>
<td>1.38</td>
<td>0.96</td>
</tr>
<tr>
<td>3 - △</td>
<td>0.28</td>
<td>0.82</td>
<td>0.97</td>
</tr>
<tr>
<td>4 - △</td>
<td>0.24</td>
<td>1.22</td>
<td>0.97</td>
</tr>
<tr>
<td>5 - △</td>
<td>0.22</td>
<td>1.79</td>
<td>0.97</td>
</tr>
<tr>
<td>6 - ○</td>
<td>0.40</td>
<td>1.72</td>
<td>0.98</td>
</tr>
<tr>
<td>7 - ○</td>
<td>0.34</td>
<td>1.61</td>
<td>0.97</td>
</tr>
<tr>
<td>8 - ○</td>
<td>0.30</td>
<td>1.71</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Figure 5 shows the average performance of three subjects with time as a parameter. The broken line is the average performance for the same three subjects on the first experiment where the viewing time was also 3 seconds. The curves for the different viewing times show a slight decrease in error as viewing time is increased, but the improvement was only significant for the D/V = 3.75 and 7.5 conditions.

![Graph showing angular error in detecting aim point vs. angular distance to 10 min/sec contour for three different viewing times.](image)

Figure 5.- Angular error in detecting aim point vs. angular distance to 10 min/sec contour for three different viewing times.

After the first experiment the subjects were asked to comment on the strategy they found most effective in detecting the aim point. Although there were slight differences, a common strategy emerged. It can be characterized by the following steps: (1) the subject would fixate the center of the pattern and estimate the general location of the aim point, (2) after a
variable amount of time, inversely proportional to the speed of expansion of the pattern, the subject would shift his line of sight to his initial estimate of the aim point, and (3) in the time remaining he would refine his judgement by observing the local pattern of motion. The fraction of time spent in each of the two modes appeared to be a function of the expansion velocity. At the high rates, only a brief time was needed to detect the general region; whereas at the low rates, usually all of the time was spent looking for the general area of no movement. In the second experiment, the change in viewing time seemed to affect mainly the amount of time the subject was able to spend refining his initial judgement. In future experiments these observations will be quantified by measuring the subject's eye position during a trial.

DISCUSSION

The high linear correlation coefficients (0.96 to 0.99) in table I generally confirmed the hypothesis that the angular error in detecting the aim point is a linear function of the angular distance from the aim point to the contour for some low angular rate of expansion. The 10 min/sec (2.91 mrad/sec) contour was chosen as an independent variable in figures 4 and 5 and table I, but any other contour of this magnitude would have given the same linear result because for small angles the angular rate of the dots is a linear function of the angular distance from the aim point to the dots.

All the curves indicate the existence of a constant error of between 0.8° and 1.8°. This error might be due to carelessness in positioning the tracking dot, but more likely it is a result of the finite resolution of the dot pattern. It is perhaps remarkable that the average error on the high rate condition, D/V = 3.75, was less than the 2° spacing between dots on the pattern. Future experiments will include texture as a variable in order to determine how texture density affects the constant error term.

If the constant error is neglected, the slopes of the straight lines indicate that the 10 min/sec (2.91 mrad/sec) contour represents, approximately, a 99 percent confidence limit for the aim point judgements. This is the same conclusion Haveron reached by considering thresholds for detecting the angular motion of isolated dots. Therefore, it appears that the visual spatial summation provided by a 30° (0.522 rad) field of moving dots does not appreciably lower the threshold for detecting motion. The lack of any large difference between the different viewing times in the second experiment indicates that any visual temporal summation effects within the range of exposure times investigated are of little importance in estimating the aim point.

If we assume that the 10 min/sec contour does outline a 99 percent confidence region and that the data apply to a 3° approach, then the cross-hatched area of figures 2 and 3 gives an idea of how accurately a pilot can detect his aim point during a landing approach. This accuracy increases as the pilot nears the aim point, but it appears to be of marginal use until the last few thousand feet.
CONCLUSION

In this paper the experimental findings indicate that pilots can detect an aim point within an area bounded by the 10 min/sec contour. Further experiments are planned to test this prediction in a flight simulator.

REFERENCES


