CAPILLARY RISE IN THE ANNULAR REGION OF CONCENTRIC CYLINDERS DURING COAST PERIODS OF ATLAS-CENTAUR FLIGHTS

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SUMMARY

The Centaur space vehicle liquid oxygen and liquid hydrogen tanks each contain a mass sensing system which utilizes flow through a helical inlet tube into a probe consisting of two concentric cylinders. During the coast periods of the Atlas-Centaur flights, each mass sensing system acted as a capillary system under low gravity conditions. Data for the capillary rise height in the annular region of concentric cylinders as a function of time were obtained from the flights of three Atlas-Centaurs at acceleration levels of approximately $7.0 \times 10^{-3}$ and $4.2 \times 10^{-4}$ g with durations of up to 100 and 1341 seconds, respectively. The capillary rise data were correlated with an analytical expression which accurately described the fluid flow through the sensing system under low gravity conditions.

INTRODUCTION

In a satellite or space vehicle under a very low gravitational field, the capillary effect can be used to position liquids. Drop tower investigations by Petrash (ref. 1) and Siegel (ref. 2) have described the capillary effect and transient capillary rise for liquids into vertical tubes during a zero gravity period. These investigations, however, were necessarily limited to drop tower time durations ($<3$ sec) and to noncryogenic liquids.

During the low gravity coast periods of the Atlas-Centaur flights, the mass sensing probes for the liquid oxygen and liquid hydrogen tanks acted as capillary systems and provided an opportunity to extend the data of references 1 and 2 to considerably longer periods of time and to the use of cryogenic fluids. The geometry of the Centaur mass sensing probes, the acceleration levels of the Centaur vehicle, and the duration of the coast period were such that a complete transient capillary rise rate profile from start to
steady-state conditions was nearly provided. An analytical investigation was conducted at the Lewis Research Center in order to correlate the capillary rise data of the concentric tube configuration and the environment of the mass sensing probes with known fluid flow principles. The results of the analysis, which accounts for the pressure losses that occur throughout the fluid, are presented herein.

**SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area, m²</td>
</tr>
<tr>
<td>$F_s$</td>
<td>surface tension force</td>
</tr>
<tr>
<td>$F_T$</td>
<td>total force</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration, m/sec²</td>
</tr>
<tr>
<td>$K_1$</td>
<td>inlet tube entrance loss coefficient</td>
</tr>
<tr>
<td>$K_2$</td>
<td>inlet tube helical shape correction term coefficient</td>
</tr>
<tr>
<td>$K_3$</td>
<td>inlet tube exit loss coefficient</td>
</tr>
<tr>
<td>$L$</td>
<td>annulus liquid level, m</td>
</tr>
<tr>
<td>$l$</td>
<td>tube length, m</td>
</tr>
<tr>
<td>$\Delta P_a$</td>
<td>acceleration pressure loss, N/m²</td>
</tr>
<tr>
<td>$\Delta P_{EA}$</td>
<td>inlet tube exit pressure loss, N/m²</td>
</tr>
<tr>
<td>$\Delta P_{EI}$</td>
<td>inlet tube entrance pressure loss, N/m²</td>
</tr>
<tr>
<td>$\Delta P_{FA}$</td>
<td>frictional pressure loss in annulus, N/m²</td>
</tr>
<tr>
<td>$\Delta P_{FI}$</td>
<td>frictional pressure loss in inlet tube, N/m²</td>
</tr>
<tr>
<td>$P_s$</td>
<td>surface tension pressure, N/m²</td>
</tr>
<tr>
<td>$P_u$</td>
<td>ullage pressure, N/m²</td>
</tr>
<tr>
<td>$r$</td>
<td>radius, m</td>
</tr>
<tr>
<td>$Z$</td>
<td>capillary rise height, m</td>
</tr>
<tr>
<td>$\dot{Z}$</td>
<td>capillary rise liquid velocity, m/sec</td>
</tr>
<tr>
<td>$\ddot{Z}$</td>
<td>capillary rise liquid acceleration, m/sec²</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$r_{ic}/r_{oc}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>contact angle between liquid and solid, rad</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity, (N)(sec)/m²</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
\[ \rho \] liquid density, kg/m\(^2\)

\[ \sigma \] surface tension, N/m

Subscripts:

A annulus
C Centaur tank
I inlet tube
ic inner cylinder
oc outer cylinder

**EXPERIMENTAL DESCRIPTION**

The Centaur space vehicle utilized two acceleration levels during the coast period of the Atlas-Centaur 8, 9, and 12 flights (designated AC-8, AC-9, and AC-12, respectively). The acceleration levels and their durations are given in table I.

<table>
<thead>
<tr>
<th>Flight</th>
<th>Acceleration level 1, g</th>
<th>Duration of acceleration level 1, sec</th>
<th>Acceleration level 2, g</th>
<th>Duration of acceleration level 2, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC-8</td>
<td>7.15 \times 10^{-3}</td>
<td>100</td>
<td>4.28 \times 10^{-4}</td>
<td>820</td>
</tr>
<tr>
<td>AC-9</td>
<td>7.15</td>
<td>76</td>
<td>4.28</td>
<td>1341</td>
</tr>
<tr>
<td>AC-12</td>
<td>6.90</td>
<td>76</td>
<td>4.15</td>
<td>1214</td>
</tr>
</tbody>
</table>

The first acceleration level is applied after Centaur first main engine cutoff in order to reduce the propellant disturbances associated with main engine thrust termination. The second, lower, acceleration level is employed to maintain the propellants at the rear of the tank (for engine restart) throughout the coast period. A complete description and analysis of the use of these acceleration levels for the coast period of AC-8 is given in reference 3.

The Centaur space vehicle utilizes a mass sensing probe in the liquid hydrogen and liquid oxygen tanks to regulate the mixture ratio of the propellants during main engine
firing. Each mass sensing probe consists, in part, of concentric cylinders with a helical inlet tube.

A schematic of the mass sensing probes, together with a listing of the probe dimensions, is shown in figure 1. As shown in the figure, saturated liquid enters the helical inlet tube and passes into the annular region. As the liquid rises in the annular region, the capacitance changes between the two plates. The change in capacitance is telemetered to a ground station providing a continuous reading of the varying mass in the probe. Since the probe calibration in terms of mass and capacitance and the annular cross-sectional area of the probe are known, the liquid level in the annular region is easily determined. Hence, the transient liquid level rise in the annular region of concent-

![Figure 1. Schematic of Centaur liquid hydrogen and liquid oxygen mass sensing probes.](image)

<table>
<thead>
<tr>
<th>Sensing probe dimensions(a)</th>
<th>Liquid hydrogen</th>
<th>Liquid oxygen</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_{oc})</td>
<td>0.0372</td>
<td>0.0595</td>
</tr>
<tr>
<td>(r_{ic})</td>
<td>0.0228</td>
<td>0.0473</td>
</tr>
<tr>
<td>(r_{l})</td>
<td>0.0051</td>
<td>0.0046</td>
</tr>
<tr>
<td>(L_{T})</td>
<td>1.13</td>
<td>0.49</td>
</tr>
<tr>
<td>(b_{T})</td>
<td>0.36</td>
<td>0.51</td>
</tr>
</tbody>
</table>

\(a\) All dimensions in meters. 
\(b_{T}\) = length of inlet tube.

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4
tric cylinders was obtained for the two Centaur coast period acceleration levels.

ANALYSIS

It is suggested in references 1 and 2 that in order to predict analytically the transient movement of liquids into vertical capillaries the pressure losses that occur anywhere in the fluid must be taken into account. For the particular geometry of the Centaur liquid oxygen and liquid hydrogen mass sensing probes, the various pressure losses are indicated in figure 2. The summation of these pressure losses for the capillary rise in the annular region can be expressed as follows:

\[ g \text{, Acceleration field} \]
\[ h_I \text{, Height of liquid in inlet tube} \]
\[ L \text{, Height of liquid in annulus at } 1 \text{ g} \]
\[ \Delta P_g \text{, Pressure loss from fluid acceleration} \]
\[ \Delta P_{EA} \text{, Pressure loss in annulus entrance} \]
\[ \Delta P_{EI} \text{, Pressure loss in inlet tube entrance} \]
\[ \Delta P_{FA} \text{, Friction loss in annulus} \]
\[ \Delta P_{FI} \text{, Friction loss in inlet tube} \]
\[ P_s \text{, Surface tension pressure in annulus} \]
\[ P_u \text{, Ullage pressure} \]
\[ Z \text{, Capillary rise height} \]
\[ \rho \text{, Fluid density} \]

Figure 2. - Summation of pressures for capillary rise in annular region of concentric cylinder mass sensing probe.
\[ P_u + \rho g h + \rho g L = P_u + \rho g Z + \rho g L + \rho g h_1 + \Delta P_{FA} + \Delta P_{EA} + \Delta P_{FI} + \Delta P_{EI} + \Delta P_a - P_s \]

Then,

\[ P_s - \Delta P_{EI} - \Delta P_{FI} - \Delta P_{EA} - \Delta P_{FA} - \Delta P_a - \rho g Z = 0 \quad (1) \]

Each pressure term will now be defined.

**Surface Tension Pressure \( P_s \)**

The effects of surface tension on liquid rising in an annulus has been investigated by Seebold (ref. 4). He has shown that, in a low gravity environment, the interface advances faster on one wall than the other and that the distortion of the interface is a function of Bond number and annular radii ratio \( r_{ic}/r_{oc} \). For the case at hand, both the experimental data and that of reference 4 indicate this distortion was negligible so that a radially symmetric curved interface may be assumed with little error.

The summation of the surface tension forces around the annulus perimeter is

\[ F_s = 2\pi (r_{ic} + r_{oc})\sigma \cos \theta \]

Dividing each side by the annulus area \( A_A \) results in the pressure due to surface tension

\[ \frac{F_s}{A_A} = P_s = \frac{2\pi (r_{ic} + r_{oc})\sigma \cos \theta}{\pi (r_{oc}^2 - r_{ic}^2)} = \frac{2\sigma \cos \theta}{r_{oc} - r_{ic}} \quad (2) \]

**Inlet Tube Entrance Loss \( \Delta P_{EI} \)**

At the entrance to the inlet tube a pressure loss results from the momentum changes necessary to form the velocity profile. The entrance loss is expressed as (ref. 5)

\[ \Delta P_{EI} = \frac{K_1}{2} \rho (\dot{Z}_I)^2 = \frac{K_1}{2} \rho (\dot{Z}_A)^2 \left( \frac{A_A}{A_I} \right)^2 \quad (3a) \]

where (ref. 5)
When the entrance loss is expressed in terms of the annulus fluid velocity, equation (3a) can be expressed as

\[ K_1 = 0.4 \left( 1.25 - \frac{A_I}{A_C} \right) \]  

(3b)

For the Centaur vehicle, \( A_I / A_C \approx 0 \). Then, from equation (3b), \( K_1 = 0.5 \).

\[ \Delta P_{EI} = \frac{K_1}{2} \rho Z \left( \frac{r_{oc}^2 - r_{ic}^2}{r_i^2} \right)^2 \]  

(4)

Inlet Tube Drag Loss \( \Delta P_{FI} \)

As the fluid passes through the inlet tube frictional losses will occur. These losses can be expressed from the Hagan-Poiseville relation as (ref. 6)

\[ \Delta P_{FI} = \frac{8 \mu \dot{I}_I K_2}{r_i^2} \]  

(5)

where \( K_2 \) is a correction term for the helical inlet tube shape. When the inlet tube drag loss is expressed in terms of the annular fluid velocity, equation (5) can be expressed as

\[ \Delta P_{FI} = 8 \frac{\mu \dot{I}_I Z A K_2}{r_i^2} \left( \frac{r_{oc}^2 - r_{ic}^2}{r_i^2} \right) \]  

(6)

where \( K_2 \) for the helical inlet tubes using cryogenic fluids is approximately 1 (ref. 5).

Inlet Tube Exit Loss \( \Delta P_{EA} \)

As the fluid enters the annular region from the inlet tube, a pressure loss due to expansion will occur. This pressure loss can be expressed as (ref. 5)
\[ \Delta P_{EA} = \frac{1}{2} Z_I^2 \left( 1 - \frac{A_I}{A_A} \right)^2 \rho \]  

or in terms of the annular fluid velocity

\[ \Delta P_{EA} = \frac{1}{2} Z_A^2 K_3 \left( \frac{r_{oc}^2 - r_{ic}^2}{r_I^2} \right)^2 \rho \]  

where

\[ K_3 = \left( 1 - \frac{A_I}{A_A} \right)^2 \]

### Annulus Drag Loss \( \Delta P_{FA} \)

As the fluid passes through the annulus, a frictional loss with the cylinder walls will occur. The frictional loss in the annulus can be expressed as a modified form of the Hagan-Poiseville equation (ref. 6)

\[ P_{FA} = 8 \frac{\mu (L + Z)}{r_{oc}^2} \frac{Z_A}{r_I^2} \left( \frac{1 - \beta^4}{\ln \beta} - \frac{1 - \beta^2}{1 - \beta^2} \right) \]

where \( \beta = r_{ic}/r_{oc} \).

### Acceleration Losses \( \Delta P_a \)

As the fluid is accelerated through the inlet tube and the annular region, a pressure loss due to fluid acceleration occurs. This pressure loss can be derived from Newton's force-mass relation as follows:

\[ F_T = m_I \ddot{Z}_I + m_A \ddot{Z}_A \]
\[ F_T = l_I \rho A_I \ddot{Z}_I + (L + Z) \rho A_A \ddot{Z}_A \]

\[ \Delta P_a = \frac{l_I \rho A_I}{A_I} \ddot{Z}_I + (L + Z) \rho \frac{A_A}{A_A} \ddot{Z}_A \]

or, in terms of the annular fluid acceleration,

\[ \Delta P_a = l_I \rho \ddot{Z}_A \frac{A_A}{A_I} + (L + Z) \rho \ddot{Z}_A \]

\[ \Delta P_a = \rho \ddot{Z}_A \left( L + Z + l_I \frac{A_A}{A_I} \right) \]

\[ \Delta P_a = \rho \ddot{Z}_A \left[ L + Z + l_I \left( \frac{2}{r_{oc}^2 - r_{ic}^2} \right) \right] \]

Equation (11) was solved for the acceleration \( \ddot{Z}_A \), the velocity \( \dot{Z}_A \), and the displacement \( Z \) in the annular region as a function of time by successive integrations on a high speed IBM 7094 computer. The Runge-Kutta method of forward integration was used.

\[ \frac{2 \sigma \cos \theta}{r_{oc} - r_{ic}} - \frac{K_1}{2} \rho Z_A^2 \left( \frac{r_{oc}^2 - r_{ic}^2}{r_I^2} \right) - 8 \frac{\mu l_I}{r_I^2} K_2 \left( \frac{r_{oc}^2 - r_{ic}^2}{r_I^2} \right) - \frac{\rho Z_A^2}{2} K_3 \left( \frac{r_{oc}^2 - r_{ic}^2}{r_I^2} \right) \]

\[ - 8 \frac{\mu (L + Z)}{r_{oc}^2} \dot{Z}_A \left( \frac{1 - \beta^4}{1 - \beta^2} \ln \frac{1}{\beta} \right)^{-1} - \rho \ddot{Z}_A \left[ L + Z + l_I \left( \frac{r_{oc}^2 - r_{ic}^2}{r_I^2} \right) \right] - \rho g Z = 0 \]  

At steady-state conditions, equation (11) reduces to

\[ \frac{2 \sigma \cos \theta}{r_{oc} - r_{ic}} - \rho g Z = 0 \]  

Equation (11) was solved for the acceleration \( \ddot{Z}_A \), the velocity \( \dot{Z}_A \), and the displacement \( Z \) in the annular region as a function of time by successive integrations on a high speed IBM 7094 computer. The Runge-Kutta method of forward integration was used.
Figure 3. Liquid oxygen capillary rise.
Figure 4. - Liquid hydrogen capillary rise.
RESULTS AND DISCUSSION

The capillary liquid level rise in the annular region of the concentric cylinder liquid oxygen and liquid hydrogen mass sensing probes obtained from the coast phase data of AC-8, AC-9, and AC-12 flights is presented in figures 3 and 4. The data are estimated to be accurate to within ±15 percent for the high acceleration level.

The transition in acceleration level that occurred at 100 seconds for AC-8 and at 76 seconds for AC-9 and AC-12 after main engine cutoff is indicated in each figure by the break in slope of the capillary liquid level rise. The effect of the acceleration level on the steady-state liquid levels is clearly shown in each figure. The greater acceleration level results in a lower steady-state liquid level. A comparison of figures 3 and 4 also indicates the effect of fluid properties on the capillary rise and steady-state liquid levels. The lower density liquid hydrogen (fig. 4) rises at a greater rate to a steady-state liquid level; the rate is nearly twice that observed with those obtained from the derived analytical expression (eq. (11)). Generally, the theoretical curve agrees very favorably with the experimental liquid oxygen liquid levels. The liquid hydrogen liquid level rise appears to be slightly delayed in initial response to the acceleration change, compared to the theoretical curve. The steady-state liquid levels predicted from equation (12) at the two acceleration levels are compared to the actual steady-state values in table II. Examination of table II reveals that the predicted steady-state liquid levels agree favorably with the actual steady-state liquid levels.

<table>
<thead>
<tr>
<th>Flight</th>
<th>Acceleration level, g</th>
<th>Actual steady-state liquid level, m</th>
<th>Predicted steady-state liquid level, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LOX tank</td>
<td>LH₂ tank (a)</td>
</tr>
<tr>
<td>AC-8</td>
<td>7.15×10⁻³</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>AC-9</td>
<td>7.15</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>AC-12</td>
<td>6.90</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>AC-8</td>
<td>4.28</td>
<td>0.48</td>
<td>0.97</td>
</tr>
<tr>
<td>AC-9</td>
<td>4.28</td>
<td>0.48</td>
<td>0.98</td>
</tr>
<tr>
<td>AC-12</td>
<td>4.15</td>
<td>0.50</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*(a) Based on extrapolated data.*
SUMMARY OF RESULTS

During the coast period of the Atlas-Centaur flights, the liquid hydrogen and liquid oxygen concentric cylinder mass sensing systems acted as capillary systems under low gravity conditions. Data for the capillary rise height in the annular region of a concentric cylinder as a function of time was obtained from the flights for acceleration levels of approximately $7.0 \times 10^{-3}$ and $4.2 \times 10^{-4}$ g. The duration of these acceleration periods were up to 100 and 1341 seconds, respectively, and were of sufficient duration to nearly provide a transient capillary rise profile from start to steady-state conditions.

The capillary liquid level rise data were correlated with an analytical expression derived from a balance of fluid pressure losses. The analytical expression correlated well with the actual flight data and accurately described the fluid flow through the sensing system under low gravity conditions.

Lewis Research Center,
National Aeronautics and Space Administration, Cleveland, Ohio, January 10, 1968, 891-01-00-06-22.

REFERENCES


