THE EFFECT OF A RANDOM SAMPLING INTERVAL ON A
SAMPLED-DATA MODEL OF THE HUMAN OPERATOR

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INTRODUCTION

There are a number of proposed mathematical models of the human operator in a compensatory tracking function. Some features of a human operator, such as time delay, have been relatively easy to incorporate in most of these models. However, the remnant, or those frequencies in the human operator's output which are not linearly correlated with the input, has never been easy to include in a satisfying, organic fashion in many of the models. In recent years, sampled-data models have been advanced for a number of reasons one of which is that such models will produce naturally frequencies not present in the input due to the presence of the sampler.

Historically, sampled-data models of the human operator have assumed a periodic sampler. The behavior of such samplers has been studied extensively and certain characteristic features have been well described. For instance, if the model contains such a sampler then the output spectra will have "humps" containing direct and reversed images of the input spectra centered about integral values of the sampling frequency. In addition, there is a null point in the output spectra at each integral value of sampling frequency rather like a deep valley between two hills.

In AFDDL-TR-65-15, (Reference 1) it is stated that carefully analyzed records of actual human operator outputs failed to show the humped spectra characteristic of a periodic sampler. In that report, the authors theorized that the remnant is due to time varying parameters, such as gains, time constants and time delays within the human operator transfer function. If these parameters may vary, how would the behavior of a human operator model be affected if, assuming it contained a sampler, the sampling interval were allowed to vary in some fashion? Our experiment is an investigation of a sampled-data human operator model in which the sampling interval varies randomly about some mean value.

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EXPERIMENTAL PLAN

We designed the experiment based on a simple analog computer simulation of a sampled-data model of the human operator in a one dimensional compensatory tracking loop with a constant gain. Two types of data hold, a zero order and a first order were investigated, and different amounts of randomness were used for a parametric study. Starting with the zero order hold and a purely periodic sampler, the sampling interval was made to vary in a Gaussian fashion about the mean interval. The amount of randomness was increased in three steps to a maximum variation of 1 $\sigma = 17\%$ of a mean .33 sec. sampling interval. The experiment was then repeated with a first order hold, and with two successive amounts of randomness with a maximum 1 $\sigma$ variation of 12.5% of a mean .3 sec sampling interval. All other model parameters were held fixed for a given case. The model input consisted of the sum of five essentially non-harmonic sine waves. A linear, continuous human operator model was also constructed and run simultaneously with the sampled-data model. This model was an approximate one, including a gain and lag terms, but without a time delay. This simplification is justifiable in view of the use to which the model was put, as explained below.

Output data was recorded and used as follows. The continuous model output was recorded and power spectral analyses were performed on the two purely periodic runs. Although useful for comparison, this signal was not of major interest. The sampled model output also was recorded and power spectral analyses performed mainly for comparison purposes. Although the behavior of this model as the randomness is increased was the primary objective, another signal was used for the reasons given below. This signal was the difference of continuous and sampled model outputs, as shown in Figure 1. The time constant and gain ($K''$ and $K'$) of the continuous model were adjusted so that its frequency response matched that of the sampled model as closely as possible in the low frequency or input region. This difference signal, when compared to the sampled model output, enhances the higher frequency remnant area and allows scaling in the data reduction to decrease the effect of data channel noise. Due to this fact, our results are based on spectral analysis of this signal. Finally, the sampling interval statistics were collected to confirm the experimental distribution used and were checked by a chi-squared test to show that they were closely Gaussian.
EXPERIMENTAL SETUP

The block diagram (Figure 1) shows the general arrangement of the analog simulation. The models and the input sine-wave generators are self explanatory. The hold on the sampled model, however, must be supplied with a random pulse train, and the remaining blocks in the diagram represent the hardware that generated it. The interval between pulses was required to vary in a Gaussian fashion about a fixed mean. Our method for accomplishing this afforded us the capability of conveniently and independently adjusting both the mean and the variation of the interval between samples.

A triangle wave oscillator was used to provide a stable and symmetrical wave form with a linear slope and equal positive and negative voltage excursions. At each positive and negative peak of the triangle wave a sample of the signal from the Gaussian noise generator was taken and held. The running slope of the triangle wave was then compared with this sample of the noise in a comparator. The output of this comparator was operated on by a pulse forming network to give a train of clean rectangular samples with a fixed "on" time and a random variation about a mean sampling interval of one half the period of the triangle wave. The statistics of the interval variation were Gaussian and their variance was controlled by the relative amplitude of the rms. noise voltage to the peak-to-peak voltage of the triangle wave. The noise voltage was limited to a value no greater than the peak-to-peak voltage to prevent any embarrassing extraneous pulses. The ratio of the limit voltage to rms. noise voltage was chosen sufficiently large as to have little effect on the interval statistics.

RESULTS AND DISCUSSION

Figure 2 shows the power spectrum of the output of the sampled-data model using a zero order hold. Note the input frequencies and the "humped"spectra on either side of the 3 cps sampling frequency. This graph, with its strong "valley" trend at 3 cps clearly displays the expected characteristics of a sampled data model. In Figure 3 the spectra of the difference signal is shown for the same set of experimental conditions. There is approximately a 10 db enhancement of the sampling frequency area over Figure 2, the sampled model output. Again the null at 3 cps is evident. One can easily pick out the peaks which correspond to the direct and reversed images of the input frequencies. Note also the relatively quiet region between the input and sampling frequency regions.

The next three figures (Figures 4 through 6) show the same output, zero order hold and difference signal, for increasing amounts of sampling interval randomness. Histograms of the sampling interval (ΔT) statistics are plotted for each case on
the corresponding spectrograph. Chi-squared tests on these substantiated the claim that the distributions are Gaussian. These figures show the diminution of the peaks in the sampling region as the randomness increases. The average sampling frequency and the points where spectral peaks would be in the periodic case are marked on each figure. With a $\Delta T (1 \sigma)$ of only 7% we can see a definite decrease. More significant however, is the "filling in" of the valley at the sampling frequency and also of the normally quiet region between the input frequencies and the sampling area. For the $\Delta T (1 \sigma) = 12\%$ case (Figure 5) we note a 5 - 10 db decrease in the peaks, approximately a 7 db increase in average power level at the valley and a considerable increase in the noise in the formerly quiet region. This trend continues with increasing randomness and in Figure 6 ($\Delta T, (1 \sigma) = 17\%$) it is hard to find any resemblance to the spectrum of a sampler, even with the general area marked. Similar results were obtained for the case of model containing a first order hold (Figures 7 through 9). In these cases the sampling frequency is 3.3 cps and two degrees of sampling interval randomness, $\Delta T (1 \sigma) = 6.5\%$ and 12.5%, are presented in terms of the difference signal.

CONCLUSIONS

Our experimental results show that:

(1) It might be impossible to determine the presence of an internal sampler by examining the human operator's output spectra if a random sampler is assumed. Noteworthy is the fact that this assumption does not require an unreasonable amount of randomness in the sampling interval.

(2) Assuming a randomly varying sampler our results are compatible with both those presented in Reference 1 and those obtained from earlier sampled-data models.

(3) This then demonstrates the feasibility of using sampled-data models for the human operator, provided only that the sampling interval include a reasonable amount of randomness.

(4) Finally, our results suggest that attempts at enhancing the remnant portion of the human operator's output and improvement of the signal processing and power spectral analysis will be important in further determination of the validity of sampled-data models for the human operator.

AREAS OF POSSIBLE EXTENSION

We have worked with a random variation in the sampling interval. Furthermore we used noise with a Gaussian probability distribution to control the randomness.
It is not particularly evident that this would be the "best" distribution to use if one were specifically trying to match the model output spectrum to actual human operator output spectra. In fact, it does not seem unlikely that some sort of deterministic "criterion function" could be used to determine the sampling intervals (in conjunction with some degree of randomness). All these possibilities are in the realm of model matching and form the basis for possible future work.

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REFERENCE

FIG 7

FIRST ORDER HOLD
PERIODIC