

TECHNICAL REPORT NUMBER 2

THE ANALYSIS AND DESIGN OF AN ELLIPTICALLY  
POLARIZED, CAVITY BACKED, CROSSED-SLOT ANTENNA

Prepared by

ANTENNA RESEARCH LABORATORY

E. R. GRAF, PROJECT LEADER

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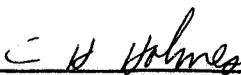
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## FOREWORD

This is a technical report of a study conducted by the Electrical Engineering Department of Auburn University under the auspices of Auburn Research Foundation toward the fulfillment of the requirements prescribed by NASA Contract NAS8-20557.

## ABSTRACT

The increased use of electronically scanned antenna arrays has created a need for array elements which have a hemispherical beamwidth and elliptical polarization. The physical size and construction of the elements must also be suited for use in closely spaced arrays for reduction of the sidelobes. A cavity backed, crossed-slot element which is suitable for this application has been constructed and analyzed, and the results are presented in this report.

A theoretical discussion of the electromagnetic field and polarization characteristics of a crossed-slot antenna is presented through the application of Babinet's Principle to the dipole antenna.

The feed system that was used to obtain the necessary slot excitation to produce an elliptically polarized wave is unique. It requires a minimum number of components and is well suited for use in large arrays. The crossed slots are excited by two orthogonal modes in a resonant cavity of square cross-section. The two modes are excited by two loops magnetically coupled to the cavity at a magnetic field maximum, with a conducting stub located between the loops for coupling and tuning purposes. One of the loops is excited by a coaxial feed, and the second loop is excited parasitically by coupling between the loops and through the conducting stub. The polarization ellipticity and input impedance of the element is dependent upon the penetration

of the conducting stub, and may be adjusted to obtain low voltage standing-wave ratios and polarizations of small ellipticity.

Design considerations and physical characteristics of the antenna are presented together with calculated and measured polarization and azimuth patterns.

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THE ANALYSIS AND DESIGN OF AN ELLIPTICALLY  
POLARIZED, CAVITY BACKED, CROSSED-SLOT ANTENNA

E. R. Graf and R. A. Heaton

I. INTRODUCTION

Advances in space exploration and high-speed vehicles have created a need for high-grain, narrow-beam, steerable, tracking antennas. One of the primary antenna systems used for this purpose is the electronically scanned antenna array. This system employs an array of antenna elements to obtain high gains and a narrow beam width, with the beam steering being accomplished by controlling the phase and/or the amplitude of the individual array elements.

The individual elements in the electronically scanned antenna array are normally identical, and, ideally, must have two primary characteristics:

- (1) The beam of the element should be hemispherical.
- (2) The radiation field should be circularly polarized.

The criteria of a hemispherical beam enables the antenna array to have a hemispherical coverage, and circular polarization allows operation independent of the vehicle orientation. The crossed-slot antenna may ideally have these characteristics, and the physical structure of the antenna is very well suited to array applications.

The major problem in the design of the crossed-slot antenna is the method of exciting the slots to obtain the required polarization

characteristics. A unique cavity-type feed system, which has been constructed and tested, is analyzed in this study. The feed system requires a minimum number of components and has characteristics suitable for use in electronically scanned tracking antennas.

## II. THEORETICAL DISCUSSION

### Radiation Fields of a Crossed-Slot Antenna

The radiation fields of a slot antenna are determined by the electromagnetic field that exists in the slot. The electromagnetic field in the slot depends primarily on the boundary conditions imposed by the conducting material surrounding the slot. The boundary conditions are that the electric field must be everywhere normal to the conducting surface, and that the magnetic field must be everywhere parallel to the conducting surface; or

$$\bar{n} \times \bar{E} = 0 \quad (1)$$

and

$$\bar{n} \cdot \bar{B} = 0 \quad , \quad (2)$$

where  $\bar{n}$  is a unit vector everywhere normal to the conductor,  $\bar{E}$  is the electric field vector, and  $\bar{B}$  is the magnetic vector.

For the case of a slot of length  $\lambda/2$ , the  $\bar{E}$  field will have an approximate distribution of [1]

$$\bar{E} = \bar{E}_0 \cos kz \quad , \quad (3)$$

where  $z$  is measured from the center to the ends of the slot and  $k = 2\pi/\lambda$ . This electric field distribution is the complement of the magnetic field configuration on a  $\lambda/2$  dipole. This observation leads to the duality principle which is stated as follows:

If an e.m.f. of frequency  $f$  is applied to an ideal slot antenna from an arbitrary source, the electromagnetic field vectors,  $E$  and  $H$ , in the slot will have the same directions and will be the same functions of the space coordinates as the directions and the functions of the vectors  $H$  and  $E$  respectively of the field of a dipole consisting of an ideally conducting infinitely thin plate, located in free space and having the same shape and dimensions as the slot, when an e.m.f. of the same frequency  $f$  is applied to the plate at corresponding points[2].

Thus, the radiation fields of a crossed-slot antenna in a flat sheet may be derived from the fields of a complementary dipole antenna by applying the duality principle.

Consider the coordinate system of Figure 1 with  $\lambda/2$  slot antennas located along the  $y$  and  $z$  axes. The  $\bar{E}$  field of the slot located on the  $z$  axis is, by the duality principle, the same as the  $\bar{H}$  field of a  $\lambda/2$  dipole located along the  $z$  axis. It has only a  $\phi$  component, and the normalized expression is [3].

$$E_{\phi} = \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} \quad (4)$$

The electric field of the slot antenna located along the  $y$  axis may be obtained by using equation (4) and rotating coordinates. Equation (4) may be written as

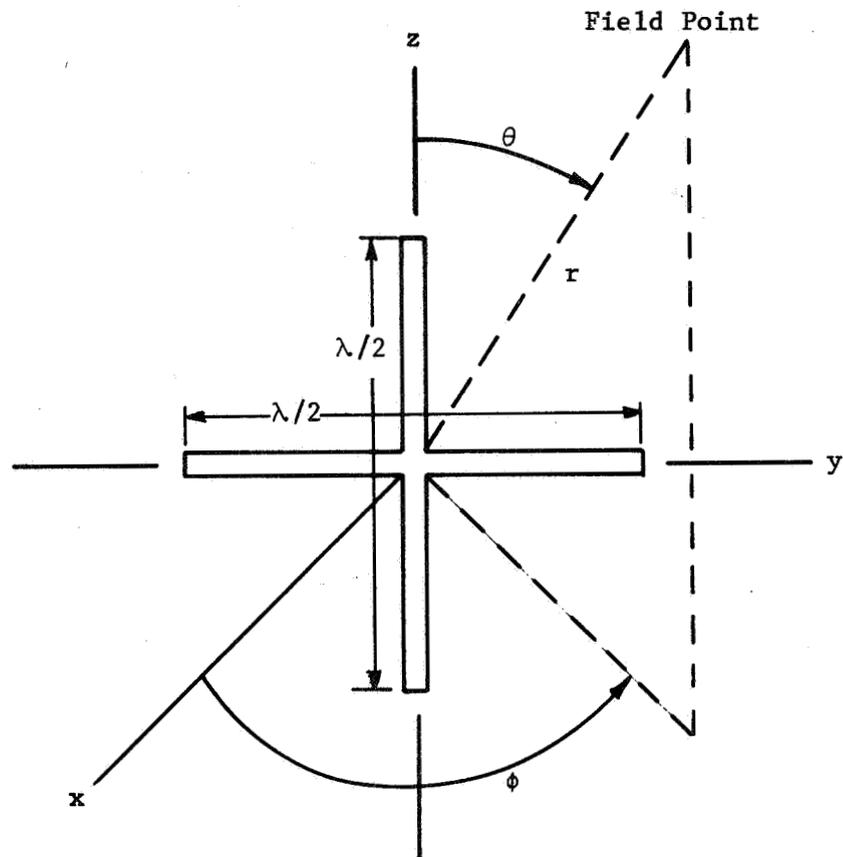


Fig. 1--The Coordinate System Showing the Orientation of the Crossed-Slot Antenna.

$$\bar{E} = \frac{\cos(\pi/2 \cos \theta)}{\sin \theta} [-\bar{a}_x \sin \phi + \bar{a}_y \cos \phi], \quad (5)$$

where  $\bar{a}_x$  and  $\bar{a}_y$  are unit vectors along the x and y axes, respectively.

In this coordinate system,  $\cos \theta = z/r$ ,  $\sin \theta = \frac{\sqrt{x^2 + y^2}}{r}$ ,

$$\cos \phi = \frac{x}{\sqrt{x^2 + y^2}} \text{ and } \sin \phi = \frac{y}{\sqrt{x^2 + y^2}}.$$

Substitution of these relationships in equation (5) results in the following equation for the electric field:

$$\bar{E} = \frac{\cos(\pi/2 z/r)}{\frac{\sqrt{x^2 + y^2}}{r}} \left[ \frac{-y \bar{a}_x}{\sqrt{x^2 + y^2}} + \frac{x \bar{a}_y}{\sqrt{x^2 + y^2}} \right]. \quad (6)$$

If the coordinate system is rotated  $90^\circ$  in a counterclockwise direction about the x axis, the  $\bar{E}$  field of the y axis slot may be obtained. Let the new coordinates be  $x'$ ,  $y'$ , and  $z'$ , where  $z = -y'$ ,  $y = z'$ ,  $x = x'$ .

In the new coordinate system, the  $\bar{E}$  field becomes

$$\bar{E} = \frac{\cos(\pi/2 y'/r)}{\frac{x'^2 + z'^2}{r^2}} \left[ \frac{-z' \bar{a}_{x'}}{r} + \frac{x' \bar{a}_{z'}}{r} \right]. \quad (7)$$

The following equations hold for the new coordinate system:

$$\frac{y'}{r} = \sin \theta \sin \phi, \quad (8)$$

$$\frac{x'}{r} = \sin \theta \cos \phi, \quad (9)$$

$$\frac{z'}{r} = \cos \theta \quad , \quad (10)$$

and

$$\frac{x'^2 + z'^2}{r^2} = 1 - \sin^2 \theta \sin^2 \phi \quad . \quad (11)$$

One may substitute equations (8), (9), (10), and (11) into equation (7) to obtain

$$\bar{E} = \frac{\cos (\pi/2 \sin \theta \sin \phi)}{1 - \sin^2 \theta \sin^2 \phi} \left[ -\bar{a}_x' \cos \theta + \bar{a}_z' \sin \theta \cos \phi \right] . \quad (12)$$

The components of electric field in the  $\theta$  and  $\phi$  directions may be obtained by computing  $\bar{E} \cdot \bar{a}_\theta$  and  $\bar{E} \cdot \bar{a}_\phi$ , respectively. This calculation results in the following expressions for the electric field components of a slot antenna aligned along the y axis:

$$E_{\theta y} = - \frac{\cos \phi \cos (\pi/2 \sin \theta \sin \phi)}{1 - \sin^2 \theta \sin^2 \phi} \quad (13)$$

and

$$E_{\phi y} = \frac{\sin \phi \cos \theta \cos (\pi/2 \sin \theta \sin \phi)}{1 - \sin^2 \theta \sin^2 \phi} \quad . \quad (14)$$

The radiation field of the crossed-slot antenna may be obtained by adding the components from the individual slots. The expressions for the electric field of the crossed-slot antenna shown in Figure 1 are

$$E_{\theta} = - \frac{\cos \phi \cos (\pi/2 \sin \theta \sin \phi)}{1 - \sin^2 \theta \sin^2 \phi} \quad (15)$$

and

$$E_{\phi} = \frac{\cos (\pi/2 \cos \theta)}{\sin \theta} + \frac{\sin \phi \cos \theta \cos (\pi/2 \sin \theta \sin \phi)}{1 - \sin^2 \theta \sin^2 \phi} \quad (16)$$

#### Polarization Characteristics of a Crossed-Slot Antenna

The polarization of an antenna is described by the orientation of the electric vector, as a function of time, in a plane normal to the direction of propagation [4]. The direction of propagation of the electric vector from the crossed-slot antenna shown in Figure 1 is along the x axis where  $\phi = 0^{\circ}$  and  $\theta = 90^{\circ}$ . Along this axis, the normalized electric fields for the y and z axis slots are

$$E_{\phi y} = 0 \quad (17)$$

$$E_{\phi z} = 1 \quad (18)$$

$$E_{\theta y} = -1 \quad (19)$$

Since the electric vectors produced by the individual slots are always in the same direction, each slot is said to be linearly polarized. The total electric field is the sum of two linearly polarized fields in space quadrature, with the actual magnitude and phase of the two fields dependent upon the magnitude and phase of the excitation in the respective slots. If the slots are excited with unequal amplitudes and phase, the fields may be expressed as [3]

$$E_y = E_1 \sin \omega t \quad (20)$$

$$E_z = E_2 \sin (\omega t + \alpha) , \quad (21)$$

where  $E_y$  and  $E_z$  are linearly polarized fields in the y and z directions, respectively, as shown in Figure 2. Expansion of equation (21) yields

$$E_z = E_2 (\sin \omega t \cos \alpha + \cos \omega t \sin \alpha) . \quad (22)$$

Equation (20) may be used to obtain the following relationships:

$$\sin \omega t = \frac{E_y}{E_1} \quad (23)$$

$$\cos \omega t = \left[ 1 - \left( \frac{E_y}{E_1} \right)^2 \right]^{\frac{1}{2}} . \quad (24)$$

Substitution of equations (23) and (24) in equation (22) and rearrangement yields

$$\frac{E_y^2}{E_1^2 \sin^2 \alpha} - \frac{2E_y E_z \cos \alpha}{E_1 E_2 \sin^2 \alpha} + \frac{E_z^2}{E_2^2 \sin^2 \alpha} = 1 \quad (25)$$

Equation (25) is a general equation for an ellipse and the most general equation for an elliptically polarized wave, since the electric vector describes an ellipse in the plane normal to the direction of propagation, as shown in Figure 3. Therefore, it has been shown that by properly exciting the slots of a crossed-slot antenna, an elliptically polarized wave will result.

The ellipticity of an elliptically polarized wave is defined as the minor to major axis ratio expressed in db. The case of zero ellipticity, or circular polarization, occurs when the two component fields are equal in amplitude and differ in time phase by  $90^\circ$ . For this case, equation (25) reduces to

$$\left(\frac{E_y}{E_1}\right)^2 + \left(\frac{E_z}{E_1}\right)^2 = 1 \quad (26)$$

indicating that the electric vector describes a circle in the plane normal to the direction of propagation. This is the ideal polarization for an antenna which is to receive a signal of arbitrary polarization.

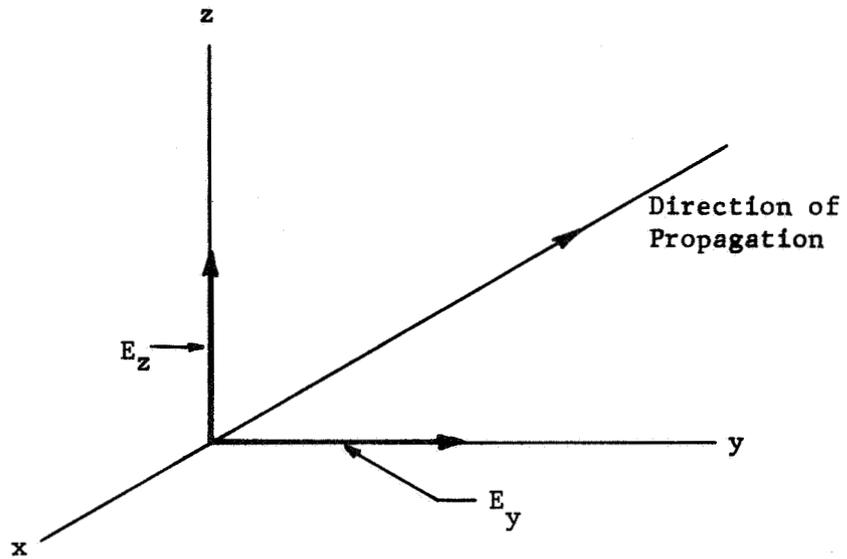


Fig. 2--The Electric Fields Produced by the Crossed-Slot Antenna

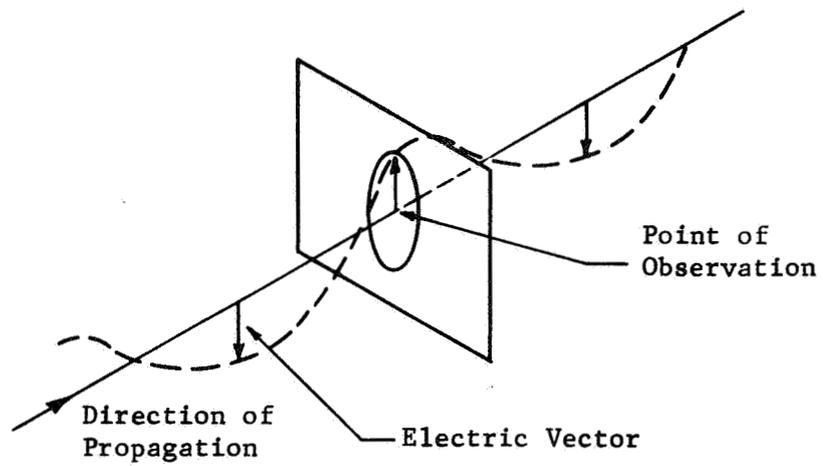


Fig. 3--An Elliptically Polarized Wave.

### III. FEED SYSTEM

#### Design of the Feed System

A slot antenna may be excited by an energized cavity placed behind it, by a transmission line connected across the slot, or through a waveguide [4]. However, if the antenna is to radiate from only one side and at the same time be suitable for use in arrays, cavity excitation is the most desirable of these methods.

Since the antenna is to be elliptically polarized, which requires that each slot be excited with different amplitudes and/or phase, there must be two modes in space quadrature excited in the cavity with these characteristics. The most fundamental approach to obtain the two modes in the cavity is to make the cavity square in cross section. The dimensions of the cross section is determined by the placement and length of the slots in the end of the cavity. The resonant crossed slots may be oriented normal to the side walls of the cavity, or they may be oriented along the diagonals of the cross section. Orientation along the diagonals, however, allows a reduction in the cross-sectional area of the cavity and would necessarily permit the use of the antenna in arrays with more closely spaced elements.

The two dominant space quadrature modes,  $TE_{011}$  and  $TE_{101}$ , can be excited by probe coupling through the sides of the cavity, or by loop coupling in the end of the cavity. Loop coupling is the most desirable of the two methods since entrance through the sides of the cavity

negates the possibility of using the elements in a closely spaced array. To achieve maximum coupling and the desired excitation of the slots, the loops must be in space quadrature and should be located as closely as possible to the maximum magnetic field. If the cavity is oriented as shown in Figure 4, the  $\bar{H}$  field for the  $TE_{101}$  mode is [5]

$$\bar{H} = \bar{a}_x K_1 \sin\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi z}{b}\right) . \quad (27)$$

Therefore, for maximum coupling, the loop used to excite the  $TE_{101}$  mode should be located in the end of the cavity at  $x = a/2$ . The  $\bar{H}$  field for the  $TE_{011}$  mode is [5]

$$\bar{H} = \bar{a}_y K_2 \sin\left(\frac{\pi y}{a}\right) \cos\left(\frac{\pi z}{b}\right) . \quad (28)$$

Again, for maximum coupling, the loop used to excite the  $TE_{011}$  mode should be located in the end of the cavity at  $y = a/2$ .

The requirement of different amplitudes and/or phases of the two modes in the cavity requires that the current in each of the loops be different in amplitude and/or phase. As noted previously, the case of minimum ellipticity occurs when the loops are excited with equal amplitude and differ in time phase by  $90^\circ$ .

The most fundamental approach to obtain the proper amplitude and phase is to excite the loops from a two-way power divider with transmission lines which differ in length by  $\lambda/4$ . However, this method has two major disadvantages:

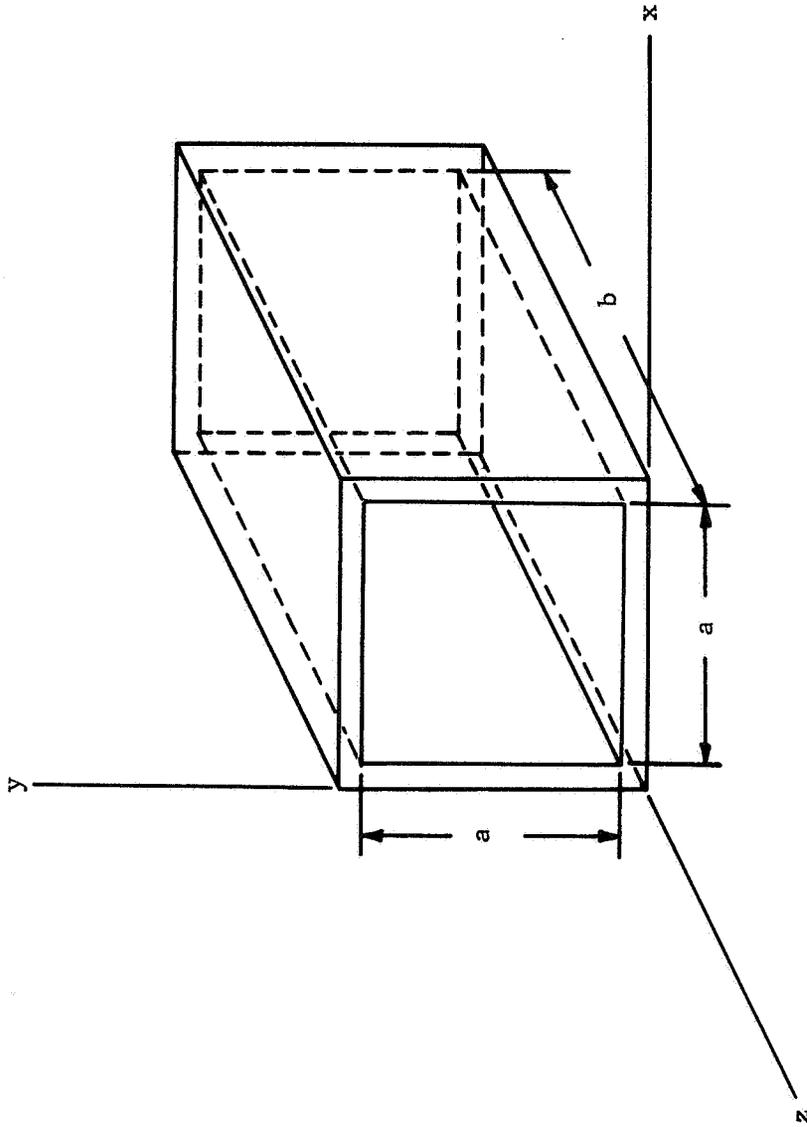


Fig. 4--The Coordinate System Showing the Orientation of the Resonant Cavity.

- (1) It is very difficult to obtain a perfect impedance match at the input to the cavity. Therefore, the use of transmission lines which differ in length by  $\lambda/4$  results in an impedance mismatch at the power divider, causing the loops to be excited with different amplitudes, thereby increasing the polarization ellipticity of the antenna.
- (2) The use of a power divider and critical length lines increases the number of components necessary, especially if the elements are used in a large array.

A simplified feed system was experimentally developed which eliminates the use of a power divider and critical length lines and results in a polarization ellipticity comparable with that obtained using the feed system described previously. This feed system, shown in Figure 5, incorporates a conducting stub placed in the center of the cavity wall containing the two loops. One of the loops is excited by a transmission line, while the other is terminated in a matched load and is excited parasitically by coupling between the loops and through the conducting, or coupling, stub. The relative magnitude and phase of the loop currents is dependent upon the penetration of the coupling stub.

#### Analysis of the Feed System

An exact analysis of the radiation characteristics of the feed system and antenna is rendered very difficult. However, an approximate analysis may be made by using lumped parameter circuits, the most general form being that shown in Figure 6. Specific identification of the circuit elements is unnecessary since they will not be used in the analysis. The

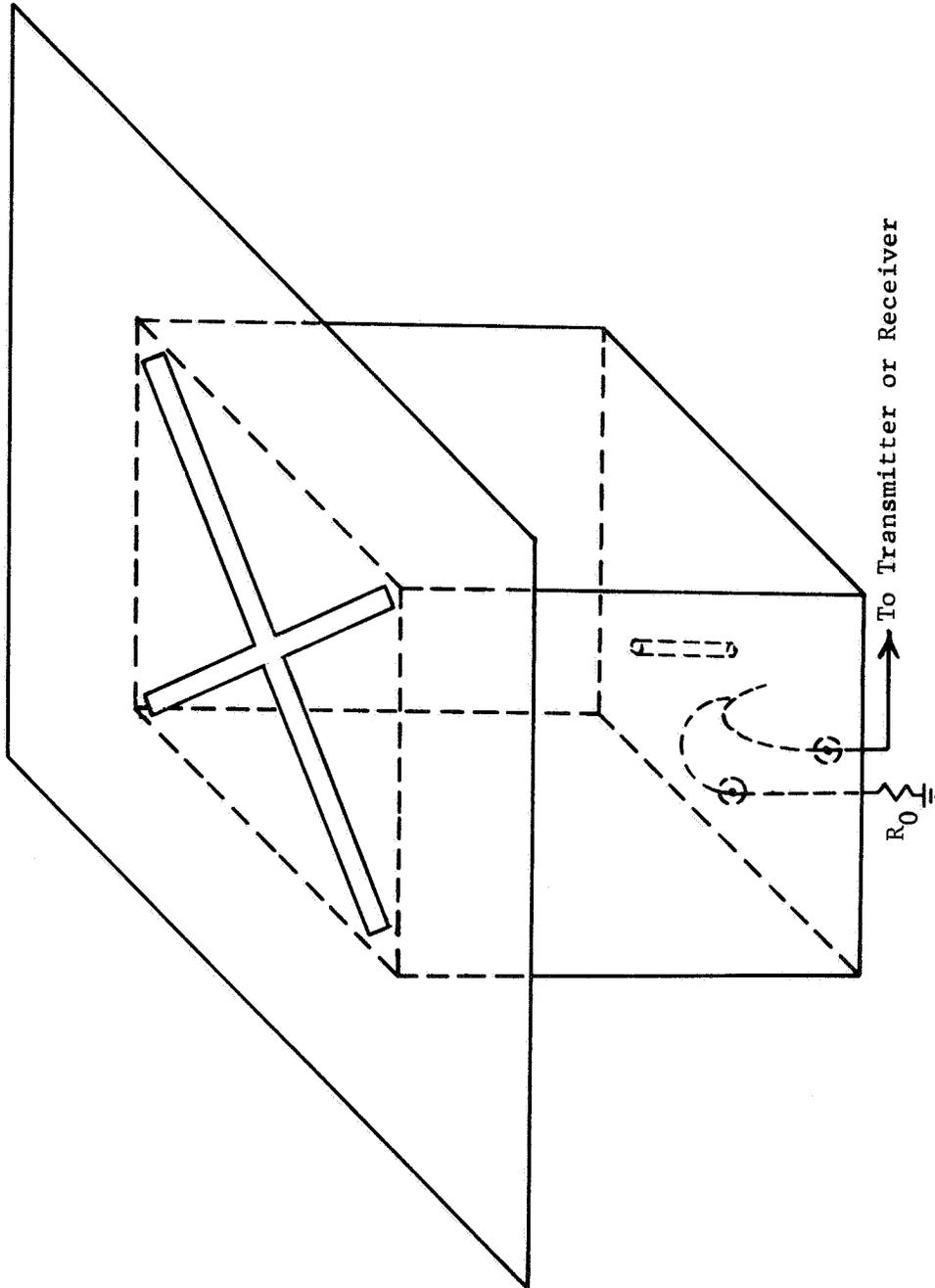


Fig. 5--The Feed System and Crossed-Slot Antenna.

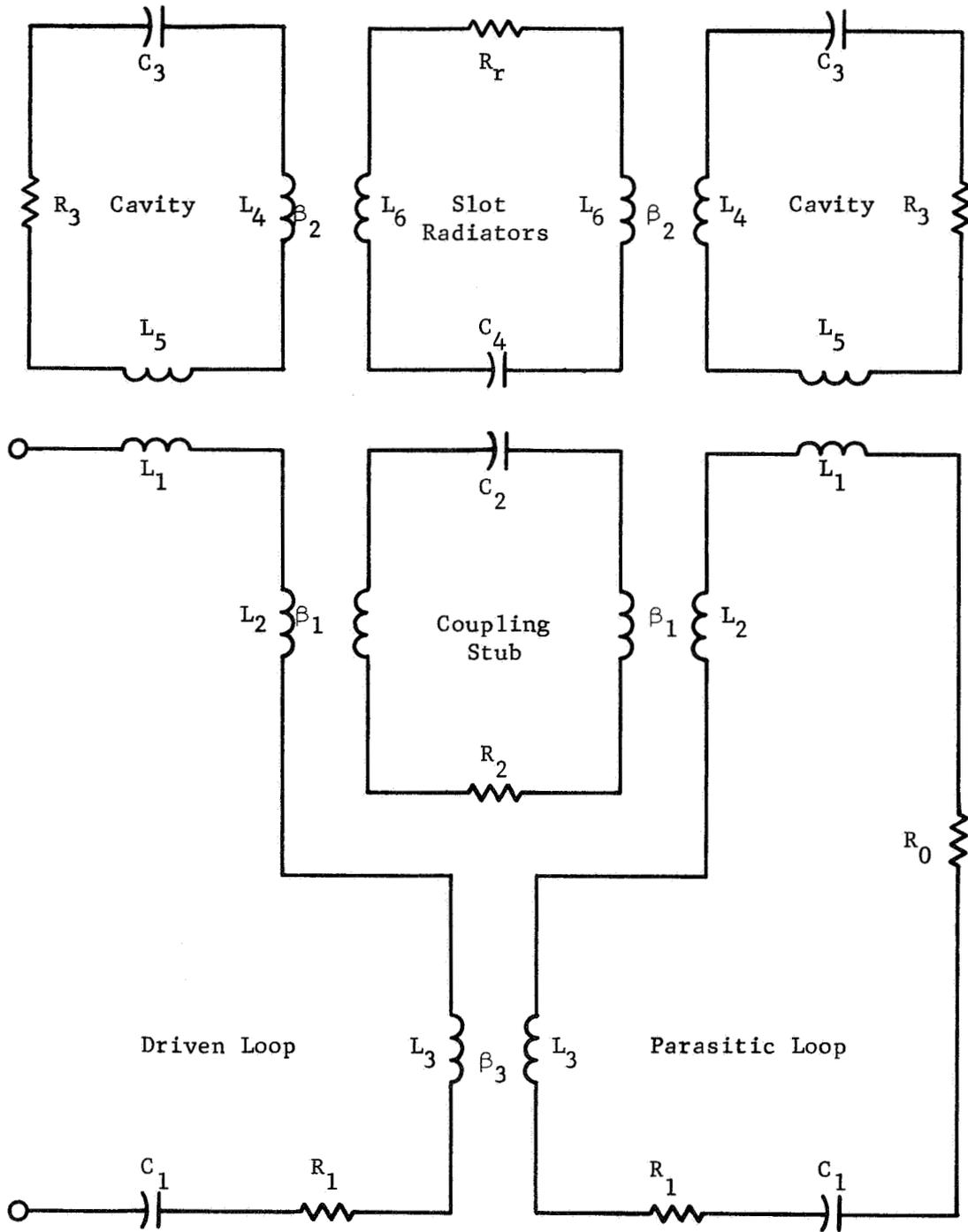


Fig. 6--The Approximate Equivalent Circuit.

capacitor  $C_2$ , the resistor  $R_2$  and the coupling coefficient  $\beta_1$  are proportional to the penetration of the conducting stub into the cavity. The variation of these parameters with stub penetration also changes the relative phase and magnitude of the loop currents which, in turn, changes the polarization characteristics of the crossed-slot antenna.

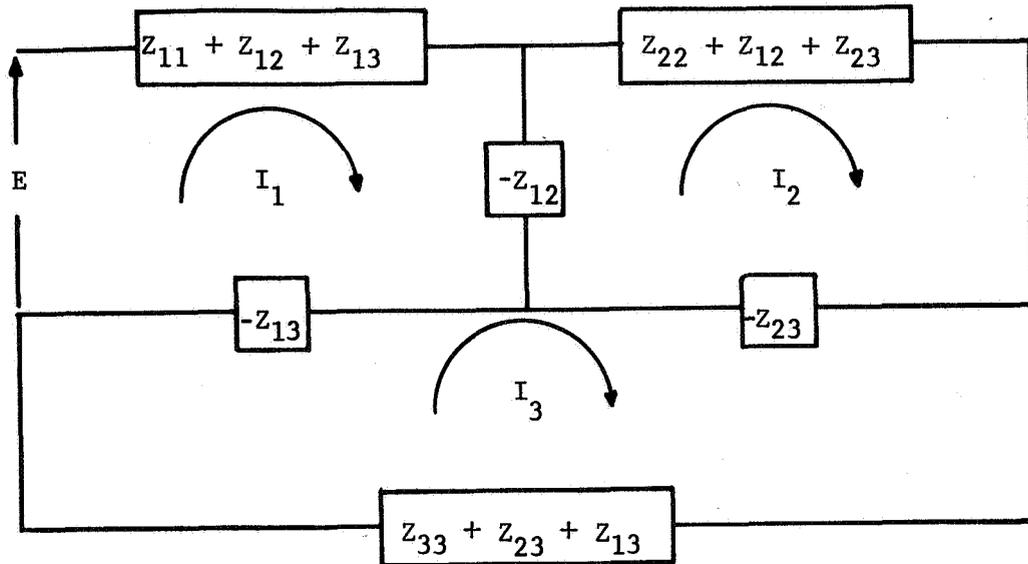
The approximate equivalent circuit shown in Figure 6 is very complex and contains a number of elements which cannot be determined. Therefore, it is necessary to simplify the circuit to be able to analyze the feed system. The complexity of the circuit may be reduced by combining several of the current loops into one complex impedance. A simplified approximate equivalent circuit is shown in Figure 7 with the circuit elements identified as they pertain to the feed system. The phasor equations which describe the simplified circuit are:

$$E = I_1 Z_{11} + I_2 Z_{12} + I_3 Z_{13} \quad (29)$$

$$0 = I_1 Z_{12} + I_2 Z_{22} + I_3 Z_{23} \quad (30)$$

$$0 = I_1 Z_{13} + I_2 Z_{12} + I_3 Z_{33} \quad (31)$$

It is necessary to determine from these equations the ratio of feed loop currents,  $I_3$  and  $I_1$ , to be able to determine the polarization characteristics of the antenna. This ratio may be determined by eliminating certain variables in equations (29), (30), and (31), and retaining only those parameters which can be measured or calculated.



$Z_{11}$  = the self impedance of the driven loop plus the reflected impedance of the cavity and slots.

$Z_{22}$  = the self impedance of the conducting stub.

$Z_{33}$  = the self impedance of the parasitic loop plus the reflected impedance of the cavity and slots.

$Z_{12} = Z_{23}$  = the mutual impedance between the conducting stub and each loop.

$Z_{13}$  = the effective mutual impedance between the loops.

Fig. 7--The Reduced Equivalent Circuit.

Division of equation (29) by  $I_1$  yields

$$Z_i^{(1)} = \frac{E}{I_1} = Z_{11} + \frac{I_2 Z_{12}}{I_1} + \frac{I_3 Z_{13}}{I_1}, \quad (32)$$

where  $Z_i^{(1)}$  is the input impedance to the feed system. The ratio of  $I_2$  to  $I_1$  may be obtained from equation (30) with  $Z_{23} = Z_{12}$ , and is

$$\frac{I_2}{I_1} = \frac{-Z_{12} \left(1 + \frac{I_3}{I_1}\right)}{Z_{22}} \quad (33)$$

Solution of equation (31) for  $Z_{13}$  yields

$$Z_{13} = -\frac{I_2}{I_1} Z_{12} - \frac{I_3}{I_1} Z_{33} \quad (34)$$

Substituting equations (33) and (34) into equation (32) and solving for the ratio of  $I_3$  to  $I_1$ , one may obtain the following expression:

$$\frac{I_3}{I_1} = \left[ \frac{Z_i^{(1)} Z_{22} + Z_{12} - Z_{11} Z_{22}}{Z_{12} - Z_{33} Z_{22}} \right]^{\frac{1}{2}} \quad (35)$$

The impedance  $Z_{12}$ , which varies with the penetration of the coupling stub, is the mutual impedance between the stub and each loop. The value of this impedance may be determined by removing one feed loop from the cavity and making input impedance measurements for different stub penetrations. The equivalent circuit for the feed with one loop removed is shown in Figure 8, and the phasor equations which describe this circuit are

$$E' = I_1' Z_{11} + I_2' Z_{12} \quad (36)$$

$$0 = I_1' Z_{12} + I_2' Z_{22} \quad (37)$$

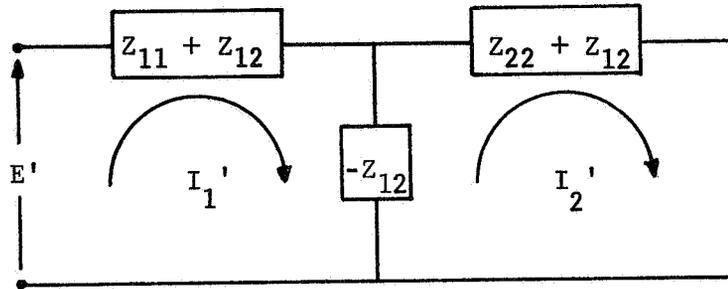


Fig. 8--Equivalent Circuit for Feed System with One Loop Removed.

Division of equation (36) by  $I_1'$  yields

$$Z_i^{(2)} = \frac{E'}{I_1'} = Z_{11} + \frac{I_2'}{I_1'} Z_{12} \quad (38)$$

where  $Z_i^{(2)}$  is the input impedance of the feed system with one feed loop removed. Solving equation (37) for the ratio of  $I_2'$  to  $I_1'$ , substituting in equation (38), and solving for  $Z_{12}$ , one obtains the following expression for  $Z_{12}$ :

$$Z_{12} = [Z_{22}(Z_{11} - Z_i^{(2)})]^{1/2} \quad (39)$$

Equation (39) may be substituted for  $Z_{12}$  in equation (35) which yields

$$\frac{I_3}{I_1} = \left[ \frac{Z_i^{(1)} - Z_i^{(2)}}{Z_{11} - Z_i^{(2)} - Z_{33}} \right]^{\frac{1}{2}} \quad (40)$$

The impedances involved in equation (40) may all be determined by impedance measurements. As mentioned previously,  $Z_i^{(1)}$  and  $Z_i^{(2)}$  may be determined by measuring the input impedance of the feed system with both feed loops and then with one feed loop, respectively. Both of these measurements are dependent upon the penetration of the conducting stub in the cavity.  $Z_{11}$  may be determined by removing both the conducting stub and the parasitic loop from the cavity and measuring the input impedance. This measurement is also sufficient to determine  $Z_{33}$ , since the feed loops are identical, except for the additional 50 ohms in  $Z_{33}$  because of the matched termination on the parasitic loop. Once these impedance measurements have been made, the relative magnitude and phase of the loop currents,  $I_3$  and  $I_1$ , may be determined for any penetration of the conducting stub, which is sufficient to determine the polarization characteristics of the antenna.

Consider the coordinate system of Figure 9 with the crossed slots and feed system oriented as shown. The relative magnitude and phase of the two orthogonal modes in the cavity is the same as the relative magnitude and phase of the loop currents which excite them. If  $E_d$  and  $E_p$  represent the magnitude and phase of the electric vectors excited by the driven loop and parasitic loops, respectively, and the ratio of the currents  $I_3$  and  $I_1$  is equal to a magnitude  $A$  with phase angle  $\beta$ , then,

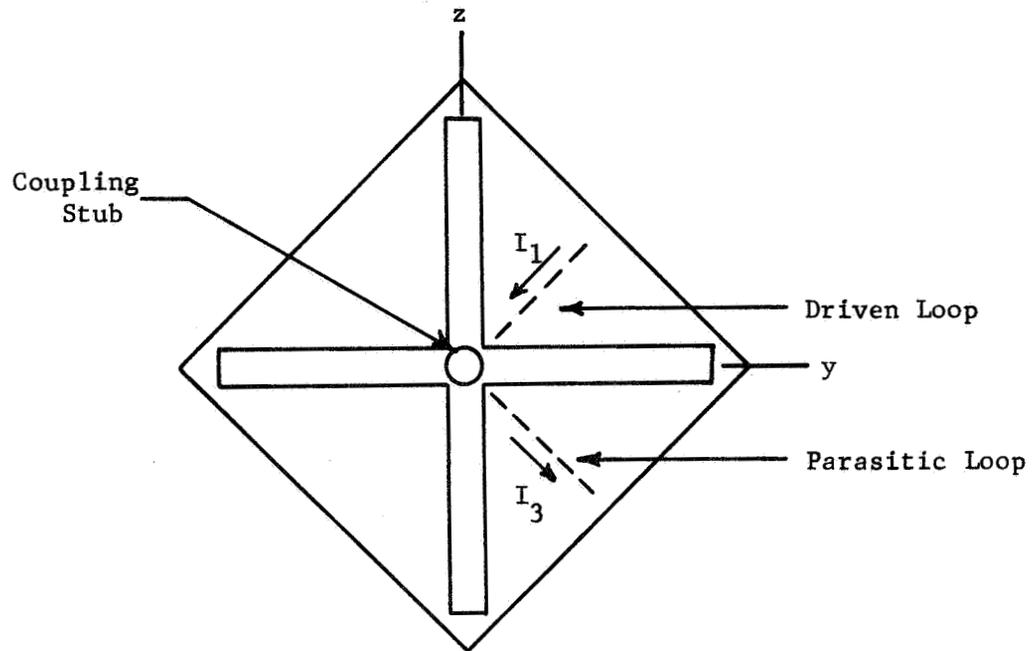


Fig. 9--Coordinate System Showing the Location of the Crossed Slots and Feed System.

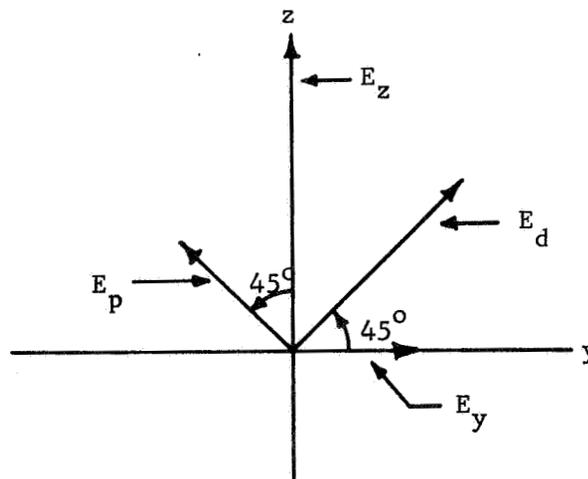


Fig. 10--The Electric Fields Existing in the Resonant Cavity and in the Slot Radiators.

$$\frac{E_p}{E_d} = \frac{I_3}{I_1} = A \angle \beta \quad . \quad (41)$$

Since the relative magnitude and phase is desired, then  $I_1$  and  $E_d$  may be assumed to have a magnitude unity and phase angle of  $0^\circ$ ; then,

$$E_d = kI_1 = 1 \angle 0 \quad (42)$$

$$E_p = kI_3 = A \angle \beta \quad , \quad (43)$$

where  $k$  is a proportionality constant.

The direction of the electric fields in the cavity will be parallel to the plane of the loops which excite them, as shown in Figure 10. The directions shown reflect that  $I_1$  was assumed into the driven loop and  $I_3$  was assumed out of the parasitic loop.

The boundary conditions on the slot radiators require the electric field in the slot to be everywhere normal to the long dimension of the slot. Therefore, for the orientation shown in Figure 10, the  $z$  component of the electric field in the cavity will excite the  $y$  axis slot, and the  $y$  component of the electric field will excite the  $z$  axis slot. The fields of the two orthogonal cavity modes may be converted to  $y$  and  $z$  components, as shown in Figure 10, where

$$E_y = \frac{E_d - E_p}{\sqrt{2}} \quad (44)$$

$$E_z = \frac{E_d + E_p}{\sqrt{2}} \quad (45)$$

One may substitute equations (42) and (43) in equations (44) and (45) to obtain the following relationships:

$$E_y = \frac{1 \angle 0 - A \angle \beta}{\sqrt{2}} \quad (46)$$

$$E_z = \frac{1 \angle 0 + A \angle \beta}{\sqrt{2}} \quad (47)$$

The polarization characteristics of the crossed-slot antenna were previously shown to be dependent upon the relative magnitude and phase of the excitation in each of the crossed slots. Therefore, if  $E_y$  is taken as the phase reference, then equations (46) and (47) may be represented as

$$E_y = E_1 \sin \omega t \quad (48)$$

$$E_z = E_2 \sin (\omega t + \alpha) \quad (49)$$

where  $E_1$ ,  $E_2$ , and  $\alpha$  are dependent on  $A$  and  $\beta$ . These equations represent the exciting fields in the  $z$  and  $y$  axis slots, respectively.

The radiation fields that determine the polarization characteristics of the crossed-slot antenna have been shown previously to be the  $y$  component of the  $z$  axis slot, and the  $z$  component of the  $y$  axis slot.

Therefore, since the radiated fields are linearly proportional to the excitation, equations (48) and (49) may be used to determine the polarization ellipse of the antenna.

#### IV. EXPERIMENTAL EVALUATION

The experimental antenna was designed to operate at a frequency of 2.2 GHz, with the resonant cavity loaded with a dielectric material to reduce its physical size. The pertinent information regarding the construction of the antenna is listed in Table 1, and a photograph of the antenna and feed system components is shown in Figure 11. One end of each loop feed was brought out of the cavity to a coaxial connector with the other end terminated in a variable capacitor to ground. The capacitor was adjusted to obtain minimum VSWR, and it was determined that this value of capacitance was not affected by the penetration of the coupling stub.

The polarization pattern of the antenna was measured for several different penetrations of the coupling stub by transmitting a linearly polarized wave and by recording the power received by the crossed-slot antenna as it was rotated in the plane of the crossed slots. It was observed that the polarization changed considerably as the penetration of the stub was changed, with a minimum polarization ellipticity of 4.2 db occurring when the length of the stub was .625 in.

Several impedance measurements were needed in order to compare the analysis of the antenna with the measured polarization characteristics. These measurements were made using slotted line techniques [6].

TABLE 1  
THE PHYSICAL CHARACTERISTICS OF THE  
ANTENNA AND FEED SYSTEM

Material	Aluminum
Slot Width	.15 cm
Slot Length	6.3 cm
Cavity Width	5.08 cm
Cavity Height	5.08 cm
Cavity Length	8.1 cm
Dielectric Constant	2.74
Operating Frequency	2.2 GHz
Stub diameter	.48 cm
Loop diameter	1 cm
Capacitance	1-10 pf.

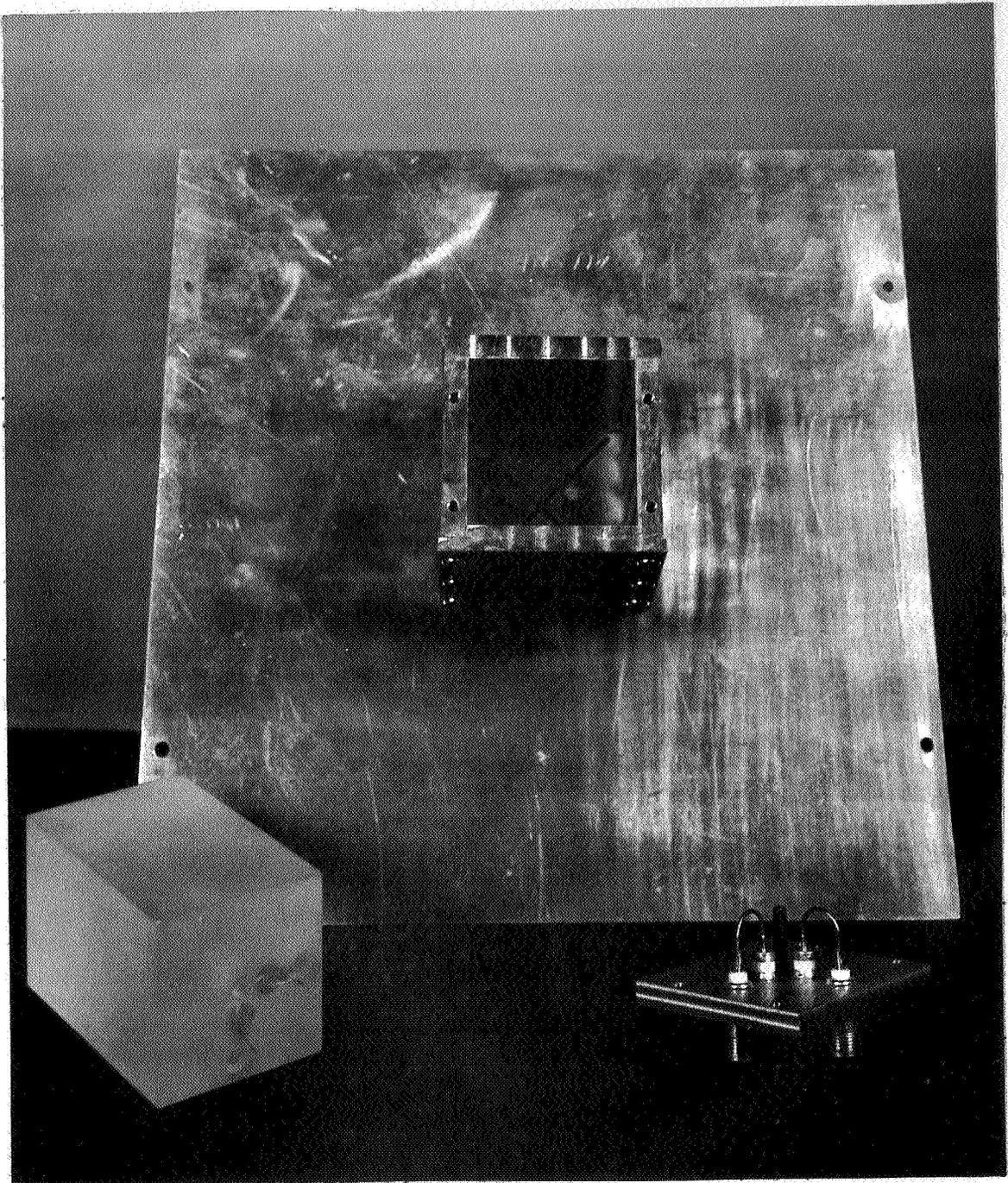


Fig. 11--A Photograph of the Crossed-Slot Antenna and Feed System.

The results of these measurements are shown in Figures 12 and 13. The impedance measurements were then used in previously derived equations to calculate polarization ellipses for several different penetrations of the coupling stub.

In order to compare the calculated and measured polarization characteristics, the calculated polarization ellipses must be converted to equivalent polarization patterns [3]. Consider an antenna which radiates the elliptical polarization shown in Figure 14. If the polarization pattern of this antenna is measured by means of a linearly polarized transmitter, the field intensity observed at each position of the antenna is proportional to the maximum component of  $\vec{E}$  in the direction of the antenna. Therefore, for a given orientation OP of the antenna, the response is proportional to the greatest ellipse dimension measured normally to OP, as shown in Figure 14. This procedure may be repeated for any orientation which results in the polarization pattern.

Polar plots of the calculated and measured polarization patterns are contained in Figures 15, 16, and 17, for several values of L, the coupling stub length. Very good agreement was obtained for the case of minimum ellipticity. Differences in the measured and calculated patterns can be attributed to two fundamental causes:

- (1) the approximate nature of the feed system analysis; and,
- (2) the difficulty in making accurate impedance measurements at microwave frequencies.

The latter of the two causes was a predominant cause of error for the patterns with large ellipticities, since the voltage standing-wave

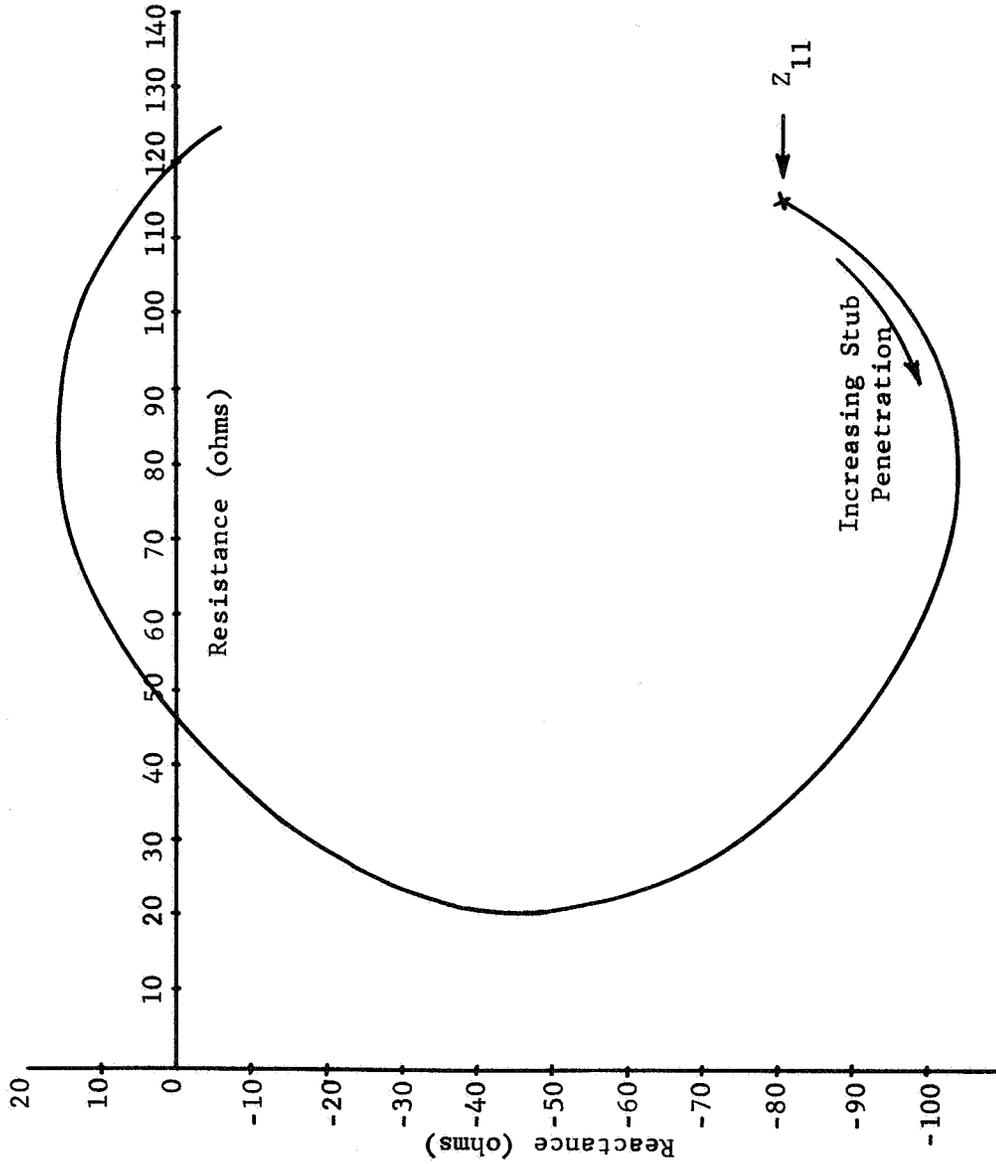


Fig. 12--The Input Impedance of the Feed System with One Loop as a Function of Stub Penetration.

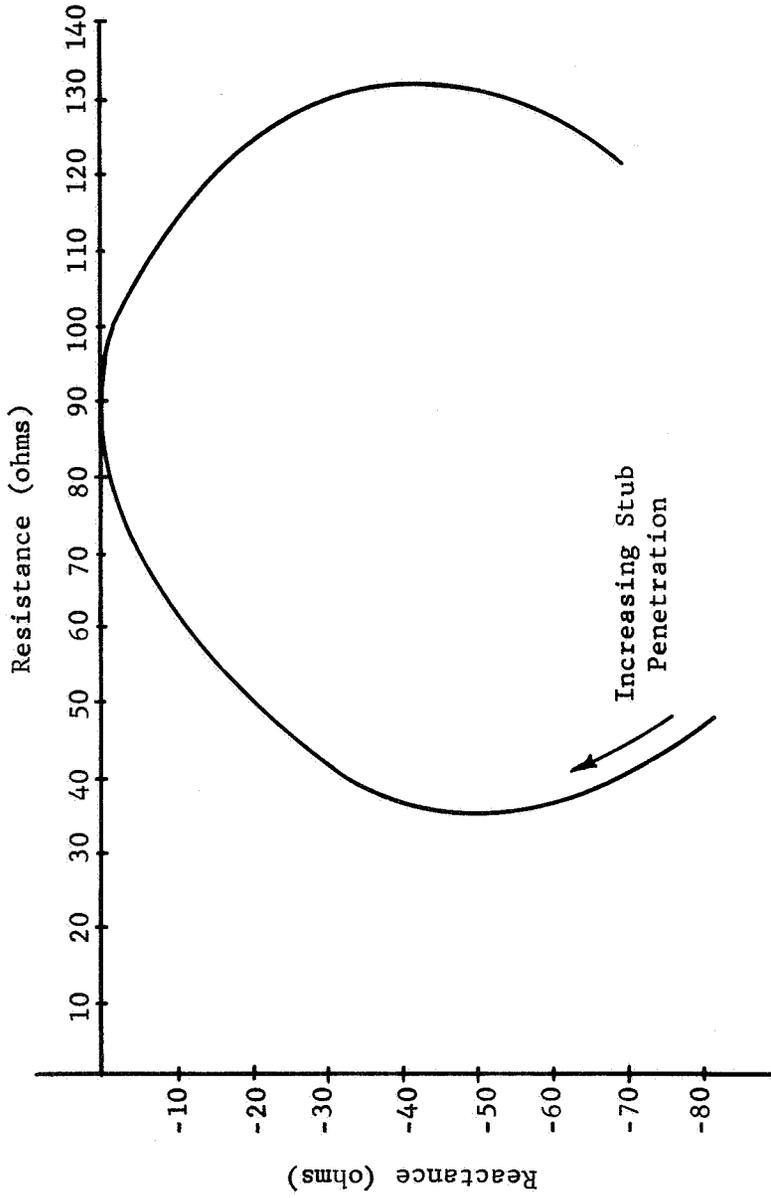


Fig. 13--The Input Impedance of the Feed System with Two Loops as a Function of Stub Penetration.

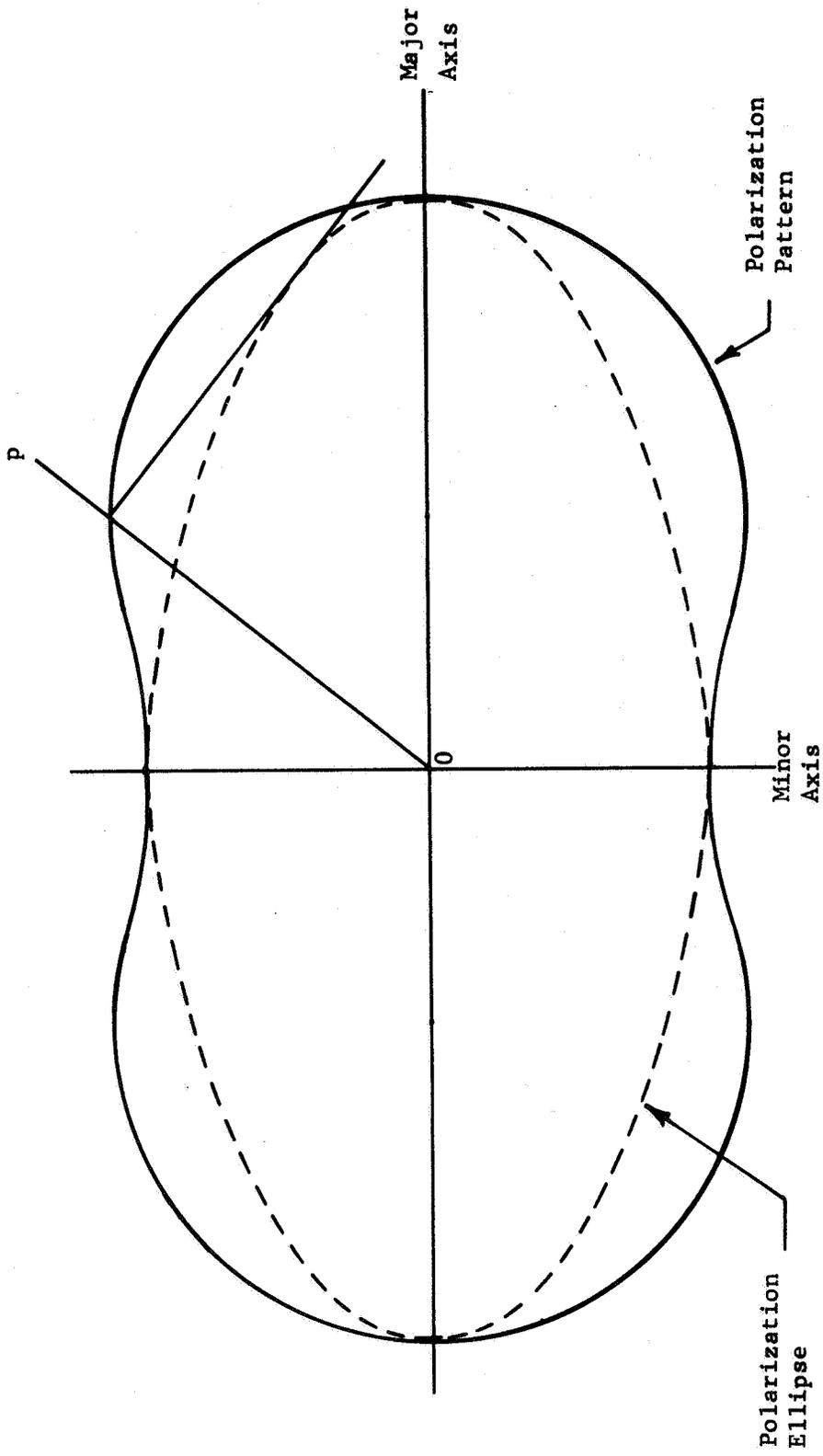


Fig. 14--An Illustration of the Construction of the Polarization Pattern from the Polarization Ellipse.

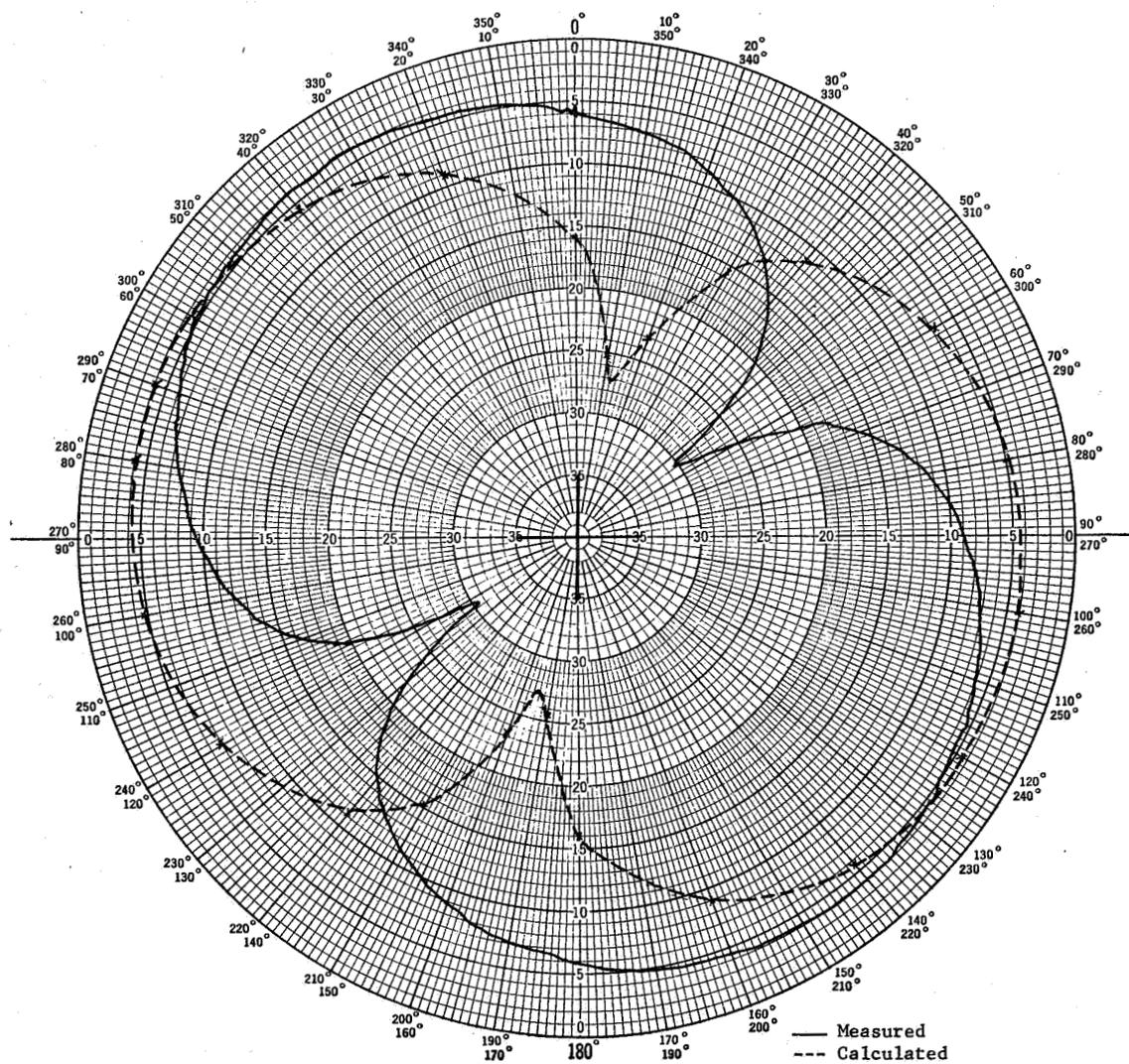


Fig. 15--The Measured and Calculated Polarization Patterns,  $L = 1.2$  cm.

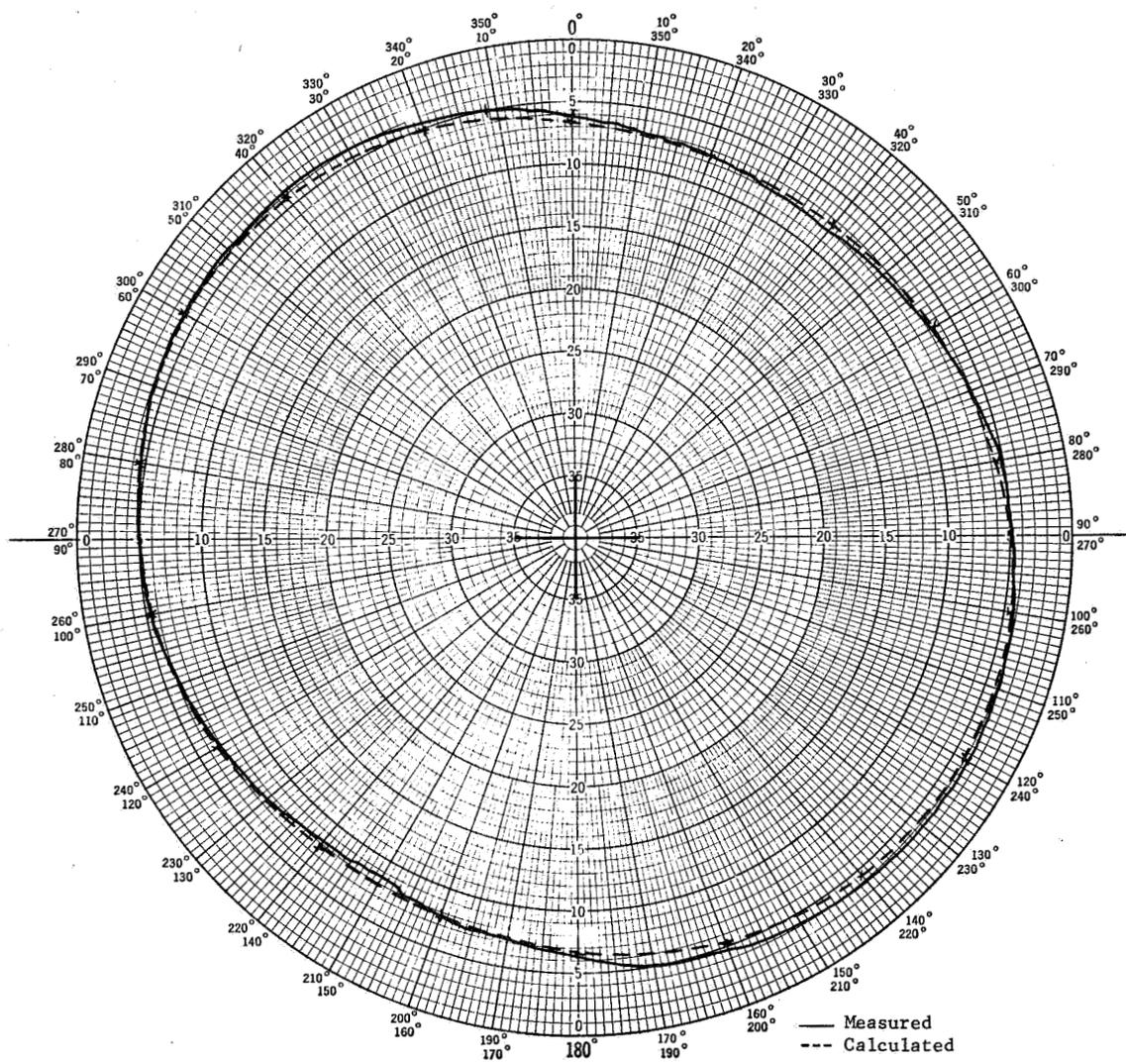


Fig. 16--The Measured and Calculated Polarization Patterns,  $L = 1.6$  cm.

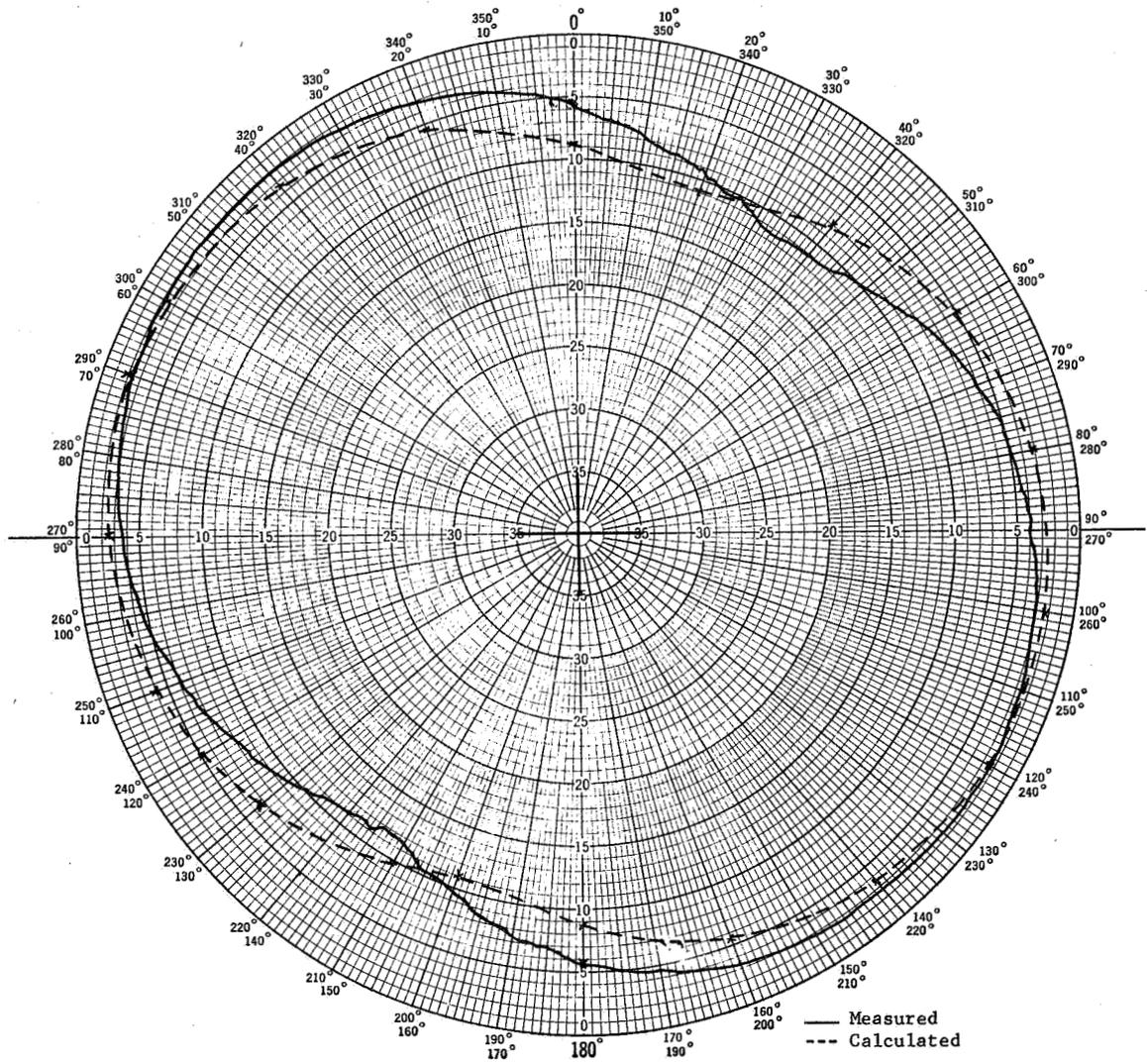


Fig. 17--The Measured and Calculated Polarization Patterns,  $L = 2.0$  cm.

ratios were large for these positions of the coupling stub. The voltage standing wave ratio was a minimum value of 1.4 for the polarization pattern of minimum ellipticity, and the one which reflected the closest agreement.

The beamwidth of the crossed-slot antenna may be determined theoretically from equations (15) and (16), for the  $E_\theta$  and  $E_\phi$  components, respectively. The variations of the  $E_\theta$  and  $E_\phi$  components in the  $\theta = 90^\circ$  plane were measured, and the theoretical and measured variations are shown in Figures 18 and 19. The discrepancies of the calculated and measured patterns are to be expected since the calculation was based on an infinite ground plane even though a 4 x 4 foot ground plane was used to obtain the measured patterns.

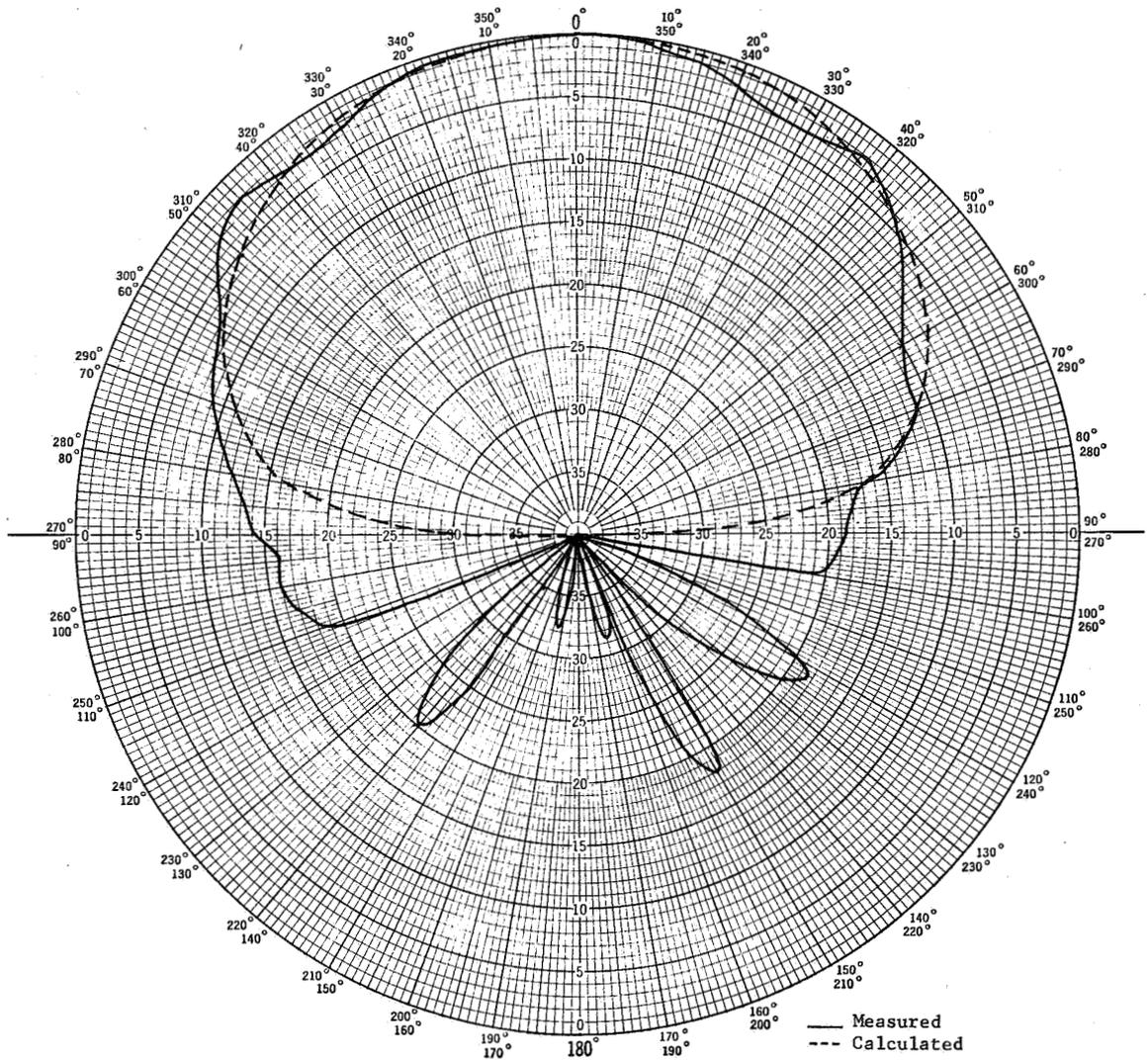


Fig. 18--Variation of the  $E_{\theta}$  Component in the  $\theta = 90^{\circ}$  Plane.

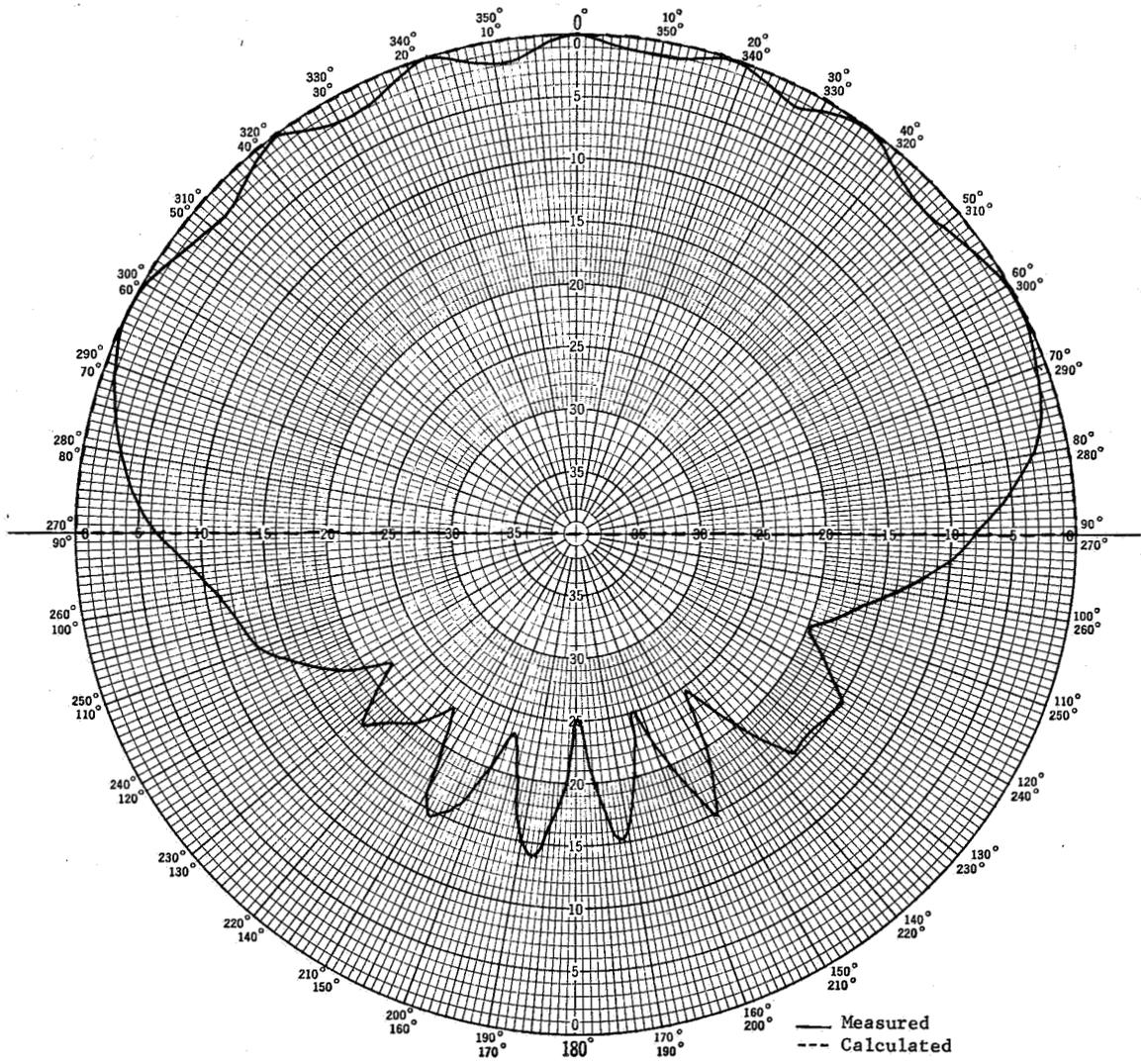


Fig. 19--Variation of the  $E_\phi$  Component in the  $\theta = 90^\circ$  Plane.

## V. CONCLUSIONS

The crossed-slot antenna and unique feed system that were constructed and analyzed are well suited for use in electronically scanned arrays. The feed system utilizes a minimum number of components which is very desirable for arrays with a large number of elements. Although the radiation fields obtained did not have the ideal characteristics of a hemispherical beam and circular polarization, the beam width and polarization ellipticities obtained are reasonable for array applications.

The feed system analysis produced results which were comparable with measured results, and discrepancies were primarily those resulting from measurement inaccuracies and the approximate nature of the analysis.

#### REFERENCES

- [1] Samuel Silver (ed.), Microwave Antenna Theory and Design (New York: Dover Publications, Inc., 1965), p. 293.
- [2] A. Z. Fradin, Microwave Antennas, trans. Morton Nadler (London: Peramon Press, 1961), p. 456.
- [3] John D. Kraus, Antennas (New York: McGraw-Hill Book Company, 1950), p. 358, pp. 466-480.
- [4] Henry Jasik (ed.), Antenna Engineering Handbook (New York: McGraw-Hill Book Company, 1961), pp. 1-8.
- [5] Simon Ramo and John Whinnery, Fields and Waves in Modern Radio (New York: John Wiley and Sons, 1953), pp. 426-427.
- [6] Walter C. Johnson, Transmission Lines and Networks (New York: McGraw-Hill Book Company, 1950), pp. 185-186.