STUDY OF A NAVIGATION AND TRAFFIC
CONTROL TECHNIQUE EMPLOYING SATELLITES

(Interim Report)

VOLUME II
SYSTEM ANALYSIS
By David A. Conrad

DECEMBER 1967

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Prepared under Contract No. NAS 12-539 by

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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SECTION

2.1: None

2.2: E elevation angle of line of sight from horizontal
E-ground elevation angle at ground
E-ionosphere elevation angle at ionosphere
RSS root-sum-square
σ standard deviation of measurement error

2.3.1 and 2.3.2: C95 radius of circle containing the user with probability 0.95
RSS root-sum-square
σ standard deviation
T time after T_0
T_0 epoch at which system constellation is defined
T_{45}, T_{90}, etc times 45 min, 90 min, etc after T_0
u radial position of satellite
v in-track position of satellite
w cross-track position of satellite

2.3.3: \( \delta x_1 \) \( \hat{x}_1-x_1 \)
\( \delta x_2 \) \( \hat{x}_2-x_2 \)
E ( ) expectation operator
RSS root-sum-square
σ standard deviation
\( \Sigma_{ij} \) \( E[\delta x_i \delta x_j^T] \) \( i, j = 1, 2 \)
\( \Sigma_{in} \) portion of \( \Sigma_{ii} \) due to random errors
\( \Sigma_{is} \) portion of \( \Sigma_{ii} \) due to satellite errors
\[ \Sigma_R = E \left[ (\delta x_1 - \delta x_2) (\delta x_1 - \delta x_2)^T \right] \]

Time after \( T_0 \)

\( x_1 \) deviation of position of user 1 from nominal

\( x_2 \) deviation of position of user 2 from nominal

\( \hat{x}_1 \) estimate of \( x_1 \)

\( \hat{x}_2 \) estimate of \( x_2 \)

\( J_2, J_{22} \text{ etc} \) coefficients of earth gravitational potential harmonics

\( \mu \) earth gravitational coefficient

\( \sigma_u \) standard deviations of \( u, v, w \)

\( u \) radial satellite position error

\( v \) in-track satellite position error

\( w \) cross-track satellite position error

\( A_i \) sensitivity matrix \( \frac{\partial y_i}{\partial x_1} \)

\( B_i \) sensitivity matrix \( \frac{\partial y_i}{\partial z} \)

\( C_{95} \) radius of circle containing user with probability 0.95

\( Q \) giant normal matrix defined in Eq (8)

\( Q_i \) normal matrix defined in Eq (16)

\( W_i \) weighting matrix defined in Eq (17)

\( x \) giant state vector defined in Eq (9)

\( x_i \) deviation of \( i^{th} \) satellite position vector from nominal

\( y_i \) vector of measurement deviations to \( i^{th} \) satellite

\( z \) vector of deviations of satellite-independent parameters
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y$</td>
<td>standard deviation of velocity error</td>
</tr>
<tr>
<td>$\psi (\text{psij})$</td>
<td>standard deviation of heading error</td>
</tr>
<tr>
<td>$A$</td>
<td>sensitivity matrix $\frac{\partial y}{\partial x}$</td>
</tr>
<tr>
<td>$b$</td>
<td>vector of measurement bias deviations</td>
</tr>
<tr>
<td>$\hat{b}$</td>
<td>minimum variance estimate of $b$</td>
</tr>
<tr>
<td>$C$</td>
<td>column vector with all elements unity</td>
</tr>
<tr>
<td>$C_m$</td>
<td>$m$-dimensional column vector with unity elements</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>vector of random measurement errors</td>
</tr>
<tr>
<td>$E$</td>
<td>expectation operator</td>
</tr>
<tr>
<td>$Im$</td>
<td>$m$-dimensional identity matrix</td>
</tr>
<tr>
<td>$m$</td>
<td>dimension of $z$</td>
</tr>
<tr>
<td>$n$</td>
<td>dimension of $y$, $C$ and $\epsilon$</td>
</tr>
<tr>
<td>$p$</td>
<td>dimension of $x$</td>
</tr>
<tr>
<td>$P$</td>
<td>$A^T T^T T^T TA$ Eq (34)</td>
</tr>
<tr>
<td>$Q$</td>
<td>$A^T T^T TA$ Eq (33)</td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>standard deviation of components of $\epsilon$</td>
</tr>
<tr>
<td>$T$</td>
<td>transformation matrix from range to range-difference measurements Eq. (12)</td>
</tr>
<tr>
<td>$T^{(\text{superscript})}$</td>
<td>matrix transpose</td>
</tr>
<tr>
<td>$T_k$</td>
<td>$k^{th}$ row of $T$</td>
</tr>
<tr>
<td>$W$</td>
<td>equivalent weighting matrix defined in Eq (21)</td>
</tr>
<tr>
<td>$\bar{W}$</td>
<td>equivalent weighting matrix defined in Eq (18)</td>
</tr>
<tr>
<td>$x$</td>
<td>user position vector (deviation from nominal)</td>
</tr>
<tr>
<td>$\hat{x}$</td>
<td>minimum variance estimate of $x$</td>
</tr>
</tbody>
</table>
\( \hat{x}_2 \) suboptimal estimate of \( x \) according to Eq. (28)

\( y \) pseudorange measurement vector (deviation from nominal)

\( z \) range-difference measurement deviation vector

\( z_k \) \( k \)th element of \( z \)

\( \Lambda_{\delta} \) covariance matrix of range-difference measurement errors.

\( \Lambda_{\hat{x}_2} \) covariance matrix of \( \hat{x}_2 \)

3: See page 89

4.1: \( \Delta V \) characteristic velocity

e orbital eccentricity

\( \lambda_N \) longitude of ascending node

\( \lambda_{N_0} \) initial longitude of ascending node

i orbital inclination to equator

\( J_2, J_{22} \) etc coefficients of earth gravitational potential harmonics

\( \Omega \) right ascension of ascending node

\( \Omega_M \) heliocentric longitude of ascending node of moon.

\( T_0 \) epoch at which system constellation is defined.

\( T_{45}, T_{90} \) etc \( T_0 + 45 \) min, \( T_0 + 90 \) min. etc
STUDY OF A NAVIGATION AND TRAFFIC
CONTROL TECHNIQUE EMPLOYING SATELLITES

Volume II. System Analysis
By David A. Conrad

1. INTRODUCTION

This volume documents the analysis made of the satellite constellation and ground-station network and presents the results of tracking accuracy and error analysis studies. User equations are also derived and presented.

Sec. 2 contains the satellite constellation analysis. The navigation accuracy obtained is discussed in subsec. 2.1; measurement errors and orbit determination errors are discussed in subsecs. 2.2 and 2.3, respectively.

Sec. 3 presents the navigation equations used by four classes of user.

Sec. 4 describes certain supporting studies, including orbital perturbations and stationkeeping requirements, eclipse histories, and the selection of injection nodes. The appendixes contain descriptions of the computer programs used in the analysis and derivations of some of the equations used in these studies.

The findings being submitted to NASA-ERC were the result of a strong team effort. While numerous technical personnel made contributions to the study results contained in the various volumes of this interim report, the following TRW Systems people made significant contributions to the analyses presented in this volume:

Coverage: H. T. Ekstrand, E. B. Mielak, P. D. Burgess

Error Sources: A. J. Mallinckrodt, A. Garabedian

Navigation Accuracy: S. Y. Itoga, D. J. Johnson, J. E. Land, D. A. Conrad
Orbit Determination: D. J. Johnson


Orbit Perturbations: G. S. Gedeon

Eclipse Periods and Injection Nodes: H. T. Ekstrand, A. J. Mallinckrodt

SPIT Program and Applications: A. J. Mallinckrodt, T. P. Nosek, C. L. Whitman

NAVSAP Program: S. Y. Itoga, D. Kuhn, D. J. Johnson

2. ANALYSIS OF SELECTED CONSTELLATION

2.1 SYSTEM DEFINITION

The analysis of various possible satellite system configurations was based on the following criteria:

- The selected system must be compatible with an interim system that provides near-continuous coverage for the North Atlantic; that is, the interim system must be a portion of the final system. This requirement is most easily satisfied by satellites with 24-hr orbital periods.
- The final system must provide global coverage, with the possible exception of the polar regions.
- There should be sufficient redundancy so that at least three satellites are visible in the ±60° latitude band after one satellite has failed.
- The four satellites covering a given area should be positioned in such a way that there is minimum geometric degradation of accuracy.
- The number of orbit planes should be as small as possible to minimize establishment and maintenance costs.

The constellation selected on the basis of these criteria consists of two orbit planes with eight satellites in each plane. Both planes are inclined 18.5° to the equatorial plane with their ascending nodes spaced 157.5° apart. The satellites are positioned within their orbit planes to yield the configuration shown in Figure 1.

This constellation was selected from a variety of possible constellations on the basis of coverage and accuracy considerations. The portion of the earth between 60° north and south latitudes was to be emphasized. One of the assumptions made for the comparative analysis was that the minimum elevation angle for user antennas would be 5° above the horizon. During the study (see Vol. III) it developed that a more suitable compromise

*Constellations evaluated and discarded are described in Ref. 1 and include 1 x 12, (1 orbit plane x 12 satellites per orbit plane) 2 x 12, and 4 x 3 configurations, all at 30° inclination. Ascending node spacing was 180° for the 2 x 12 system and both 90° and 75° were considered for the 4 x 3 system. Orbital period was 24 hr in all cases.
TWO ORBIT PLANES, INCLINED 18.5° TO EQUATOR, LINES OF NODES 157.5° APART

INTERIM SYSTEM CONSISTS OF SATELLITES 1, 2, 3, 4

Figure 1. Satellite Geometry
for aircraft-mounted antennas would be to limit the elevation angle to a minimum of 10°.

It is clear that selection of an optimum constellation requires exact definition of the coverage requirements for each region of the earth under consideration. Following this, detailed accuracy, coverage, and booster analyses can be performed. Such a study was beyond the scope of this contract, nevertheless, the results presented for the selected constellation are indicative of the performance that may be expected from the proposed NAVSTAR system. Performance would, of course, be slightly improved for an optimized constellation.

2.2 MEASUREMENT ERROR SOURCES

Although it is a relatively straightforward matter to identify the error sources associated with user measurements and to assign a number to each source, a difficulty arises in properly qualifying these numbers with respect to their important correlation properties. In general, each measurement may be associated with a particular time, a particular location, and a particular satellite by either a ground station or a user. The related types of measurement correlation are:

- **Time Serial Correlation**, affecting error sources which are neither pure (constant) bias nor independent for each measurement sample. This intermediate class of error sources may be highly correlated over many measurements but not over all available data; proper treatment of this case requires an estimate of the effective correlation time so that the effect of serial smoothing can be suitably represented. An example is ionospheric error, which is a slowly varying quantity.

- **Inter-station Correlation**, where a phenomenon is physically common to some or all measurements associated with the same satellite. An example is the error due to satellite oscillator drift.

- **Intersatellite Correlation**, where a phenomenon is physically common to some or all measurements associated with the same station, such as ground survey error.

In general, information is not available on which to base a detailed functional correlation model for partial correlations when they exist. In

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*Some of this work has been subsequently performed by TRW for the Navy (Ref. 4).*
many cases, it would not be possible to incorporate such partial correlations within the framework of present programs even if they were known. As a feasible approximation, we have chosen to represent such correlations as an "on-off" phenomenon. That is, the related measurements are represented as either fully correlated or completely independent, as a function of the estimated time constant, distance separation, etc. Present programs will generally permit such gross representations in terms of appropriately constrained biases or measurement errors in the appropriate domain.

The sources of measurement error are as follows:

- Tropospheric retardation
- Ionospheric refraction
- Receiver noise
- Receiver drift
- Quantization
- Multipath effects
- Oscillator error
- Speed of light

The characteristics of these error sources are discussed separately in the following paragraphs and are summarized in Table I.

2.2.1 Tropospheric Error (Refs. 2, 3, 9, 10, and 11)

The total tropospheric retardation is rather accurately estimated by

$$\sigma = 8 \text{ ft } \csc E \text{ (class b user)}$$

where $E$ is the elevation angle of the line of sight from the horizontal.

This will be taken as the total error $\sigma$, for a low-accuracy user who does not make a refraction correction (class b user). For a high-accuracy (class a) user or ground station the residual from a standard correction of this type is about 5 percent of the correction itself or

$$\sigma = 0.4 \text{ ft } \csc E \text{ (class a user or ground station)}$$

This error is considered correlated for time differences less than 1 hr, for ground position differences less than 20 mi, and for all satellites viewed by a given station.
TABLE I  
ERROR SUMMARY

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma$ (ft)</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ground Station Range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.4 Csc $E \sqrt{10^0 + E^2}$</td>
<td>All ranges for which $\tau &lt; 1$ hr, $\Delta &lt; 20$ mi</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>6.9 Csc $\sqrt{10^0 + E^2}$</td>
<td>All ranges for which $\tau &lt; 1$ hr, $\Delta &lt; 600$ mi</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>7.8</td>
<td>none</td>
</tr>
<tr>
<td>Receiver Quantization</td>
<td>5.1</td>
<td>none</td>
</tr>
<tr>
<td>Multipath</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Receiver Drift</td>
<td>12</td>
<td>All ranges from a given ground station for $\tau &lt; 1$ hr</td>
</tr>
<tr>
<td><strong>User Range Measurement</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.4 Csc $E$</td>
<td>All ranges for which $\tau &lt; 1$ hr, $\Delta &lt; 20$ mi</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>6.9 Csc $\sqrt{(10^0)^2 + E^2}$</td>
<td>All ranges for which $\tau &lt; 1$ hr, $\Delta &lt; 20$ mi</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>14</td>
<td>none</td>
</tr>
<tr>
<td>Receiver Quantization</td>
<td>10.2</td>
<td>none</td>
</tr>
<tr>
<td>Multipath</td>
<td>45</td>
<td>none</td>
</tr>
<tr>
<td>Receiver Drift</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Osc Error</td>
<td>9.2</td>
<td>All user range measurements to a given satellite for $\tau &lt; 2$ hr</td>
</tr>
<tr>
<td>Speed of Light</td>
<td>$&lt;4$</td>
<td></td>
</tr>
</tbody>
</table>
2.2.2 Ionospheric Error (Refs. 2 and 3)

Accounting for the elevation angle effect in ionospheric error is a little more complicated because the significant variable is the elevation angle at the ionosphere. Approximating this reasonably well by

\[ E_{\text{ionosphere}} = \sqrt{(10^o)^2 + (E_{\text{ground}})^2} \]

we can write for the average daytime (worst-case) ionospheric retardation at 1500 MHz

\[ \sigma = 13.8 \text{ ft} \text{ Csc} \sqrt{(10^o)^2 + E^2} \text{ (class b user)} \]

this will be taken as the total error for the low-accuracy (class b) user. For the high-accuracy user or ground station who makes a correction based on a precomputed table as a function of local apparent time, geomagnetic latitude, and elevation angle, the anticipated residual is reduced as much as 50 percent:

\[ \sigma = 6.9 \text{ ft} \text{ Csc} \sqrt{(10^o)^2 + E^2} \text{ (class a user and ground station)} \]

This error is considered correlated for distances less than 600 mi, for time less than 1 hr and for all satellites seen from a given ground station.

2.2.3 Thermal Noise

The error due to thermal noise is a function of the received SNR. For the user and ground station this error will be 32 and 18 ft, respectively, in a 26-Hz bandwidth. It is now further planned that after acquisition, the class a user and the ground station will provide a further bandwidth narrowing by averaging over 8 frames of the 78-Hz component or \( T = 8/78 = 0.102 \) sec. The effective bandwidth of this averaging process is \( 1/2T = 4.9 \) Hz resulting in a further improvement factor of \( \sqrt{4.9/26} = 1/2.3 \). This leads to net errors of

\[
\sigma = \begin{cases} 
32 \text{ ft (class b user)} \\
14 \text{ ft (class a user)} \\
7.8 \text{ ft (ground station)}
\end{cases}
\]
These errors are fully correlated during any one observation, uncorrelated between successive (16 sec) frames, and uncorrelated between all independent ranges (not range differences).

2.2.4 Quantizing Noise

The user has a 10-MHz clock for range count, whereas a ground station will have a 20-MHz clock. These result in 29 and 14.5 ft 1-σ quantization errors, respectively, for the user and ground station. These are independent at 1/78 sec basic sample intervals and it is presently planned that for a class a user or ground station a complete measurement will consist of an accumulation or average of 8 such measurements for a further advantage of \( \sqrt{8} \) resulting in:

\[
\sigma = \begin{cases} 
29 \text{ ft} & \text{(class b user)} \\
10.2 \text{ ft} & \text{(class a user)} \\
5.1 \text{ ft} & \text{(ground station)} 
\end{cases}
\]

This error is completely uncorrelated between all range measurements and serially between frames.

2.2.5 Oscillator Error (Ref. 5)

From Ref. 5 (Fig. 4), for a quartz oscillator and a 2-hr typical extrapolation period

\[
\sigma = 9.2 \text{ ft}
\]

This is linearly proportional to T for times other than 2 hr. As such, the error is to be considered correlated for all times less than 2 hr and for all ranges from all stations to a given satellite.

2.2.6 Multipath

This factor is assumed negligible for the ground station or for a surface ship due to ground antenna directivity and short multipath lengths (Ref. 6). For the aircraft, present planning is to utilize modulation characteristics to ensure worst-case (elevation angle 10°, worst altitude) multipath error less than 45 ft. This error may be considered essentially uncorrelated from frame to frame (10 sec) and between all range measurements from all stations to all satellites.
2.2.7 Receiver Drift Errors

The drifts in the IF, carrier phase-locked loop, and the range signal phase-locked loop have been estimated to RSS to

\[
\sigma = \begin{cases} 
17 \text{ ft} & \text{(class a or b user)} \\
12 \text{ ft} & \text{(ground station)} 
\end{cases}
\]

This should be considered correlated for times less than about 1 hr and for all ranges measured by a given ground station.

2.2.8 Speed of Light

The present fractional uncertainty in the velocity of light is estimated at \(0.3 \times 10^{-6}\). However, it is of course completely correlated between all range measurements. Ideally, it should be modelled as an unrecovered systematic error source common to all ground and user measurements. Short of this, it is suggested that the error can be bounded by a representation as an additional user position error (not range-measurement error) of \(0.3 \times 10^{-6}\) of the distance to the "average," (or in the case of relative navigation, to the reference) ground station. Taking that distance conservatively as 2000 mi the effective position error is 4 ft or less, which can safely be ignored.

2.2.9 Summary

It is difficult to RSS these diverse error sources since they are, in many cases, not directly comparable because of different correlation effects and have to be treated as separate error sources. Nevertheless, to give an idea of the resulting orders of magnitude, ignoring all correlations and RSSing all measurement errors for an assumed elevation angle of \(10^\circ\), the tabulation below yields the following results.

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<tr>
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<th>Ground Station</th>
<th>Class a User</th>
<th>Class b User</th>
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<tr>
<td>RSS</td>
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</table>
It is to be re-emphasized, however, that these RSS numbers are not for direct program input. For such purposes, these numbers must be appropriately modified taking into account the restrictions of the program in which the data are to be used and the serial and intermeasurement correlations.

2.3 NAVIGATION ACCURACY AND COVERAGE

The overall navigation accuracy provided by the proposed system was analyzed for the complete worldwide system and for an interim system covering the North Atlantic. (The interim system consists of the four satellites labeled numerically in Figure 1.) Special analyses were made of the accuracy of velocity estimates from doppler measurements and of the accuracy of relative navigation. These analyses are discussed separately in this section.

2.3.1 Worldwide System Accuracy and Coverage

The navigation accuracies obtained by a user of the system depend on three elements:

- Measurement noise and bias (discussed in detail in Subsection 2.2)
- Satellite position uncertainties (discussed in subsec. 2.4)
- Relative geometry between user and satellites, which varies with user location and time of day.

For purposes of analysis, the (range) measurement noise standard deviation is taken as 50 ft for either the interim or worldwide system. This value is derived by taking the RSS of the receiver noise, quantization error, multipath error, satellite oscillator error, and a portion of the tropospheric and ionospheric errors.*

* Tropospheric and ionospheric errors are correlated among all satellites, but have different values depending on elevation angle. Half of the 28-ft ionospheric error (see subsec. 2.2) is treated as a bias and the other half as random; the 14-ft random half is included in the RSS calculation. The remaining errors behave like biases in the user equipment and are dominated by the uncertainty in user oscillator calibration.
Since no attempt is made to calibrate the user oscillator, a large a priori bias is assumed in the user equipment. This parameter is then solved for along with the user position, as indicated in the discussion of the navigation equations (subsec. 3.2). The equivalence of this procedure to range difference is discussed in par. 2.5.3.

Navigation accuracies were determined first in the absence of orbit determination uncertainties (i.e., assuming perfect knowledge of satellite positions). This analysis illustrates the effect of measurement errors alone and thus serves to establish an upper bound on usable accuracies of an ideal satellite tracking network. It will be seen later that the effect of tracking (i.e., satellite position) errors is to cause only a 5 to 10 percent decrease in accuracy.

The results of the navigation accuracy analysis assuming no satellite position errors are shown in Figure 2 for the worldwide system and for an assumed uncertainty of 75 ft (1σ) in a priori knowledge of user altitude. The accuracy figures and those of the following subsection were obtained from the MSAT computer program described in app. B. The coverage boundaries were computed using the program described in apps. C and D. The following information is presented in the figure:

- The subsatellite points for those satellites in the northern hemisphere
- The absolute navigation accuracy obtainable within each contour, defined as the C95 value, or the uncertainty corresponding to 95 percent confidence that the actual location is within a circle of the given radius. It will be noted that near the equator the accuracy contours do not always coincide with the coverage regions.

In interpreting these results, it should be kept in mind that the figures are absolute accuracies, that is, accuracies of position determination relative to an earth-centered coordinate system. If the user accuracy is desired relative to another point on the earth, then it is necessary to add (RSS) the uncertainty in the location of that point. As will be seen later, in par. 2.3.4 on relative navigation, some of the errors may cancel when the two points are in the same vicinity and both estimate position using navigation satellites.
Figure 2 and the succeeding polar plots through Figure 14 are to be used with the clear polar overlay found in the pocket inside the back cover of this report. At time $T_0$ the Greenwich meridian of the overlay (found in pocket inside back cover) aligned with the indexing axis on the map.

Some of the characteristics to be seen from the figure irrespective of the overlay orientation are: 1) for latitudes up to $55^\circ$, navigation accuracy is within 250 ft (C95) at all longitudes; 2) the highest accuracies are associated with large numbers of visible satellites, but the number required for a given accuracy decreases toward the poles because of more favorable geometry; 3) there are two small regions of indeterminacy near the pole.

Navigation accuracies for particular regions of the northern hemisphere can be determined for 3-hr intervals after time $T_0$ by rotating the overlay $45^\circ$ counterclockwise for each 3 hr. The system configuration is such that at the end of 3 hr each satellite is at the position occupied 3 hr earlier by the one leading it. The system is thus identical, but the earth has rotated $45^\circ$ eastward during this period.

Accuracy contours for times within this 3-hr interval are given in Figure 3, 4, and 5 for 45, 90, and 135 min after time $T_0$, respectively. These maps show that the accuracy contours change in size and shape through the interval, with a general movement to the west. Navigation accuracy remains high up to $55^\circ$ latitude and is generally equivalent to that obtainable at $T_0$.

These plots provide a good general idea of the coverage and accuracy provided by the system and of the variations of coverage with time. More detailed data on overall navigation accuracy as a function of geographical location are presented in the computer-generated tables, Tables II through VI. ** Table II presents results for the system at time $T_0$, with zero satellite position uncertainties. Table III presents the same data computed with satellite position uncertainties of the expected magnitudes (see subsec. 2.4 for a discussion of these uncertainties). Although the altitude

---

* $T_0$ denotes an arbitrary epoch at which the system is defined. $T_{45}$, $T_{90}$, etc. denote times 45 min, 90 min, etc. after $T_0$.
** A one (1) in these and the following tables denotes that insufficient satellites are visible to provide a fix.
uncertainty for Table III is 150 ft rather than the 75 ft of Table II, the
prime contributor to the C95 increase is the satellite error. * Comparison
of the two tables shows that the increase in C95 due to the inclusion of
satellite errors ranges from less than 10 to 50 ft, occasionally reaching
values between 50 and 100 ft. The effect tends to increase at higher
latitudes.

Tables IV and V present the same data for the system at T_{45}, indic-
ating that the change in geometry over this period results in very minor
changes in C95 position uncertainty of a few feet either more or less.
Some large changes can be seen at high latitudes because of the rotation
of the regions of indeterminacy near the poles.

Table VI shows the same information for T_{90}, with the satellite posi-
tion errors included. Again the changes from the comparable T_0 values
are small, on the order of 20 ft or less, except in certain high-latitude
regions.

It will be recalled that these results are based on an a priori user
altitude uncertainty (except as noted) of 75 ft. This assumption will be
valid for surface vessels and may hold for aircraft with recently cali-
brated altimeters. In general, however, altimeter readings using pres-
sure equivalents may not have this accuracy after long flight intervals.
The sensitivity of navigation uncertainty to a priori altitude accuracy is
indicated in Figure 6, which is based on an a priori altitude sigma of
2500 ft. This is equivalent to essentially no a priori information. In the
regions where only three satellites are visible, there is a loss of accuracy.
In regions with more redundancy, however, the variation is much less.

*Figure 4-24 of Reference 4 shows the variation of C95 with altitude for a
4-satellite interim system. In that case, with 75 ft altitude error, the
C95 is 240 ft, and with 150 ft altitude error, the C95 is 320 ft. With more
satellites visible, this variation will be sharply reduced.
Figure 2. Worldwide Accuracy at $T_0$

A PRIORI ALTITUDE SIGMA = 75 FEET

NOTE:
• INDICATES SUBSATELLITE POINT (ABOVE EQUATOR)
Figure 3. Worldwide Accuracy at T_{45}

A PRIORI ALTITUDE SIGMA = 75 FEET

NOTE:
- INDICATES SUBSATELLITE POINT (ABOVE EQUATOR)

KEY
- C95 < 150 FT
- 150 - 250 FT
- 250 - 400 FT
- 400 - 600 FT
- > 600 - 5000 FT
- INDETERMINATE
Figure 4. Worldwide Accuracy at $T_{90}$
Figure 5. Worldwide Accuracy at T₁₃₅
Figure 6. Worldwide Accuracy at $T_0$ With Poor Altitude Data

KEY

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A PRIORI ALTITUDE SIGMA = 2500 FEET

NOTE:
- INDICATES SUBSATELLITE POINT (ABOVE EQUATOR)
### Table II

**WORLDWIDE NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT \( T = 0 \) WITHOUT SATELLITE ERRORS**

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**SYSTEM: 2x8**

**ORBITAL PERIOD: 24 hours**

**TIME FROM EPOCH: 0 hours**

**ORBITAL INCLINATION: 18.5°**

**SPACING OF ASCENDING NODES: 157.5°**

**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH: 0°**

**SATELLITE SPACING WITHIN EACH PLANE: 45°**

**MEASUREMENT NOISE (1σ): 50 feet**

**USER ALTITUDE UNCERTAINTY (1σ): 75 feet**

**SATELLITE POSITION UNCERTAINTY (1σ)**

- **RADIAL (v): 0**
- **IN-TRACK (v): 0**
- **CROSS-TRACK (w): 0**

**MINIMUM USER ELEVATION ANGLE: 5°**

(1) Denotes indeterminate point
### TABLE III

WORLDWIDE NAVIGATION ACCURACY (C95) AS A FUNCTION OF
USER LONGITUDE AND LATITUDE AT T = 0 WITH
SATELLITE ERRORS

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**SYSTEM:** 2x8

**ORBITAL PERIOD:** 24 hours

**TIME FROM EPOCH:** 0 hours

**ORBITAL INCLINATION:** 18.5°

**SPACING OF ASCENDING NODES:** 157.5°

**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 0°

**SATELLITE SPACING WITHIN EACH PLANE:** 45°

**MEASUREMENT NOISE (σ):** 50 feet

**USER ALTITUDE UNCERTAINTY (σ):** 150 feet

**SATELLITE POSITION UNCERTAINTY (σ):**

- **RADIAL (u):** 15 feet
- **IN-TRACK (v):** 117 feet
- **CROSS-TRACK (w):** 38 feet

**MINIMUM USER ELEVATION ANGLE:** 5°

(1) Denotes indeterminate point

21
## TABLE IV

**WORLDWIDE NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 45 MIN. WITHOUT SATELLITE ERRORS**

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**SYSTEM:** 2x8

**ORBITAL PERIOD:** 24 hours

**TIME FROM EPOCH:** .75 hours

**ORBITAL INCLINATION:** 18.5°

**SPACING OF ASCENDING NODES:** 157.5°

**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 0°

**SATELLITE SPACING WITHIN EACH PLANE:** 45°

**MEASUREMENT NOISE (1σ):** 50 feet

**USER ALTITUDE UNCERTAINTY (1σ):** 75 feet

**SATELLITE POSITION UNCERTAINTY (1σ):**

- **RADIAL (u):** 0
- **IN-TRACK (w):** 0
- **CROSS-TRACK (v):** 0

**MINIMUM USER ELEVATION ANGLE:** 5°

(1) Denotes indeterminate point
### TABLE V

WORLDWIDE NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT \( T = 45 \) MIN.

WITH SATELLITE ERRORS

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**SYSTEM:** 2x8  
**ORBITAL PERIOD:** 24 hours  
**TIME FROM EPOCH:** .75 hours  
**ORBITAL INCLINATION:** 18.5°  
**SPACING OF ASCENDING NODES:** 157.5°  
**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 0  
**SATELLITE SPACING WITHIN EACH PLANE:** 45°  
**MEASUREMENT NOISE (10):** 50 feet  
**USER ALTITUDE UNCERTAINTY (10):** 150 feet  
**SATELLITE POSITION UNCERTAINTY (10):**  
**RADIAL (u):** 15 feet  
**IN-TRACK (v):** 117 feet  
**CROSS-TRACK (w):** 38 feet  
**MINIMUM USER ELEVATION ANGLE:** 5°  

(1) Denotes indeterminate point
### TABLE VI
WORLDWIDE NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 90 MIN.
WITH SATELLITE ERRORS

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<td>90  177  176  175  174  173  172  171  170  169</td>
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</table>

**SYSTEM:** 2x8

**ORBITAL PERIOD:** 24 hours

**TIME FROM EPOCH:** 1.5 hours

**ORBITAL INCLINATION:** 18.5°

**SPACING OF ASCENDING NODES:** 157.5°

**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 0°

**SATELLITE SPACING WITHIN EACH PLANE:** 45°

---

**MEASUREMENT NOISE (1σ):** 50 feet

**USER ALTITUDE UNCERTAINTY (1σ):** 150 feet

**SATELLITE POSITION UNCERTAINTY (1σ):**

- **RADIAL (u):** 15 feet
- **IN-TRACK (v):** 117 feet
- **CROSS-TRACK (w):** 38 feet

**MINIMUM USER ELEVATION ANGLE:** 5°

(1) Denotes indeterminate point
2.3.2 Interim System Accuracy and Coverage

The interim system analyzed consists of two satellites in each of the same two orbit planes used for the worldwide system. The satellites are positioned to provide the best coverage over the North Atlantic.

With only two satellites in each plane, the system configuration at time $T_0$ does not repeat every 3 hr as in the $2 \times 8$ worldwide system, but only after 24 hr. Figures 7 through 14 show coverage and accuracy contours for this system every 3 hr of the 24-hr cycle. Detailed numerical data for the same time periods are presented in Tables VII through XIV for the case of no satellite errors and in Tables XV through XXII for the case of the assumed nominal satellite position uncertainties. It can be seen that for the interim system also the effect of including satellite errors is relatively minor.

In general, the above data indicate that the four satellite system yields C95 uncertainties of less than 400 ft at latitudes below $50^\circ$. An aircraft flying from New York to London would have a navigation uncertainty varying from about 400 ft at the beginning of the trip to about 600 ft at the end. This is two orders of magnitude better than the navigation accuracy available today.

As shown by the maps for the various times of day, the accuracy contours change during the day because of changing user-to-satellite geometry, but the average C95 accuracies are comparable to the $T_0$ values except for $T = 18$ and $T = 21$ hr, when the geometry is unfavorable for users in the northern hemisphere.

The variation in C95 navigation accuracy with time of day is summarized in Figure 15, where it is shown for three typical user locations. The locations selected are near New York, near the midpoint of the flight corridor, and a point south of the latter location. It can be seen that the accuracy is approximately constant except for the period between $T = 16$ and $T = 22$ hr when the uncertainty rises rapidly. For a short period, the user's position cannot be determined because only three satellites are visible and the measurements from the three are redundant due to adverse geometry. At one point, the three satellites and the user are in the same plane.
The position would still be determinable at this time except for the large bias in the user measurement. The coplanarity of the user and three satellites causes the spherical surfaces of constraint from satellite to user to be tangent, so that the user's location on a line in the northwest/southeast direction cannot be determined.

In general, it is clear that the interim system provides high accuracy during most of the day, but is degraded for a period of about an hour. This difficulty can be alleviated by increasing the orbit plane inclination at the cost of decreased average accuracy, or by adding one or two satellites to the interim system.
Figure 7. Interim System Accuracy at T₀

**NOTE:**

1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE

2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)

○ SUB-SATELLITE POINTS BELOW EQUATOR

3) A PRIORI ALTITUDE SIGMA = 75 FEET
NOTE:
1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE
2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)
   ○ SUB-SATELLITE POINTS BELOW EQUATOR
3) A PRIORI ALTITUDE SIGMA = 75 FEET

Figure 8. Interim System Accuracy at $T_0 + 3$ Hours
Figure 9. Interim System Accuracy at T_0 + 6 Hours

NOTE:

1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE

2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)
  ○ SUB-SATELLITE POINTS BELOW EQUATOR

3) A PRIORI ALTITUDE SIGMA = 75 FEET
NOTE:

1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE

2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)

○ SUB-SATELLITE POINTS BELOW EQUATOR

3) A PRIORI ALTITUDE SIGMA = 75 FEET

Figure 10. Interim System Accuracy at $T_0 + 9$ Hours
Figure 11. Interim System Accuracy at $T_0 + 12$ Hours

**NOTE:**

1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE

2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)
   ○ SUB-SATELLITE POINTS BELOW EQUATOR

3) A PRIORI ALTITUDE SIGMA = 75 FEET
Figure 12. Interim System Accuracy at $T_0 + 15$ Hours

NOTE:
1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE
2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)
   ○ SUB-SATELLITE POINTS BELOW EQUATOR
3) A PRIORI ALTITUDE SIGMA = 75 FEET
NOTE:

1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE

2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)
   ○ SUB-SATELLITE POINTS BELOW EQUATOR

3) A PRIORI ALTITUDE SIGMA = 75 FEET

Figure 13. Interim System Accuracy at $T_0 + 18$ Hours
NOTE:

1) NUMBERS IN CIRCLES INDICATE NUMBER OF SATELLITES VISIBLE

2) ● INDICATES SUB-SATELLITE POINT VISIBLE ON MAP (ABOVE EQUATOR)
   ○ SUB-SATELLITE POINTS BELOW EQUATOR

3) A PRIORI ALTITUDE SIGMA = 75 FEET

Figure 14. Interim System Accuracy at T₀ + 21 Hours
TABLE VII
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION
OF USER LONGITUDE AND LATITUDE AT T = 0 HR
WITHOUT SATELLITE ERRORS

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SYSTEM: 2x8 Interim (2x2) MEASUREMENT NOISE (1σ): 50 feet
ORBITAL PERIOD: 24 hours USER ALTITUDE UNCERTAINTY (1σ): 75 feet
TIME FROM EPOCH: 0 hours SATELLITE POSITION UNCERTAINTY (1σ)
ORBITAL INCLINATION: 18.5° RADIAL (u): 0
SPACING OF ASCENDING NODES: 157.5° IN-TRACK (v): 0
ARGUMENT OF LATITUDE OF THE FIRST SATELLITE CROSS-TRACK (w): 0
IN EACH PLANE AT EPOCH: 45°, 225° MINIMUM USER ELEVATION ANGLE: 5°
SATellite spacing WITHIN EACH PLANE: 45°

(1) Denotes indeterminate point
TABLE VIII
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 3 HR WITHOUT SATELLITE ERRORS

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SYSTEM: 2x8 Interim (2x2)

MEASUREMENT NOISE (1σ): 50 feet
USER ALTITUDE UNCERTAINTY (1σ): 75 feet
SATELLITE POSITION UNCERTAINTY (1σ)
RADIAL (u): 0
IN-TRACK (v): 0
CROSS-TRACK (w): 0

MINIMUM USER ELEVATION ANGLE: 5°

(1) Denotes indeterminate point
### TABLE IX

**INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 6 HR WITHOUT SATELLITE ERRORS**

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**SYSTEM**: 2x8 Interim (2x2)

**ORBITAL PERIOD**: 24 hours

**TIME FROM EPOCH**: 6 hours

**ORBITAL INCLINATION**: 18.5°

**SPACING OF ASCENDING NODES**: 157.5°

**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH**: 45°, 225°

**SATELLITE SPACING WITHIN EACH PLANE**: 45°

**MEASUREMENT NOISE (1σ)**: 50 feet

**USER ALTITUDE UNCERTAINTY (1σ)**: 75 feet

**SATELLITE POSITION UNCERTAINTY (1σ)**

|RADIAL (u)| 0 |
|IN-TRACK (v)| 0 |
|CROSS-TRACK (w)| 0 |

**MINIMUM USER ELEVATION ANGLE**: 5°

(1) Denotes indeterminate point
TABLE X
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 9 HR WITHOUT SATELLITE ERRORS

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SYSTEM: 2x8 Interim (2x2)
ORBITAL PERIOD: 24 hours
TIME FROM EPOCH: 9 hours
ORBITAL INCLINATION: 18.5°
SPACING OF ASCENDING NODES: 157.5°
ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH: 45°, 225°
SATellite SPACING WITHIN EACH PLANE: 45°

MEASUREMENT NOISE (1σ): 50 feet
USER ALTITUDE UNCERTAINTY (1σ): 75 feet
SATELLITE POSITION UNCERTAINTY (1σ)
RADIAL (u): 0
IN-TRACK (v): 0
CROSS-TRACK (w): 0
MINIMUM USER ELEVATION ANGLE: 5°

(1) Denotes indeterminate point
### TABLE XI
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 12 HR, WITHOUT SATELLITE ERRORS

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**SYSTEM:** 2x8 Interim (2x2)
**MEASUREMENT NOISE (1σ):** 50 feet
**USER ALTITUDE UNCERTAINTY (1σ):** 75 feet
**SATELLITE POSITION UNCERTAINTY (1σ):**
  - RADIAL (u): 0
  - IN-TRACK (v): 0
  - CROSS-TRACK (w): 0
**MINIMUM USER ELEVATION ANGLE:** 5°

**ORBITAL PERIOD:** 24 hours
**TIME FROM EPOCH:** 12 hours
**ORBITAL INCLINATION:** 18.5°
**SPACING OF ASCENDING NODES:** 157.5°
**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 45°, 225°
**SATELLITE SPACING WITHIN EACH PLANE:** 45°

(1) Denotes indeterminate point
### TABLE XII

**INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 15 HR WITHOUT SATELLITE ERRORS**

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**SYSTEM:** 2x8 Interim (2x2)

**MEASUREMENT NOISE (1σ):** 50 feet

**USER ALTITUDE UNCERTAINTY (1σ):** 75 feet

**SATELLITE POSITION UNCERTAINTY (1σ):**
- Radial (u): 0
- In-Track (v): 0
- Cross-Track (w): 0

**ORBITAL PERIOD:** 24 hours

**TIME FROM EPOCH:** 15 hours

**ORBITAL INCLINATION:** 18.5°

**MINIMUM USER ELEVATION ANGLE:** 5°

**SPACING OF ASCENDING NODES:** 157.5°

**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 45°, 225°

**SATELLITE SPACING WITHIN EACH PLANE:** 45°

(1) Denotes indeterminate point
### TABLE XIII
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 18 HR WITHOUT SATELLITE ERRORS

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#### SYSTEM: 2x8 Interim (2x2)

- **Orbital Period:** 24 hours
- **Time from Epoch:** 18 hours
- **Measurement Noise (1σ):** 50 feet
- **User Altitude Uncertainty (1σ):** 75 feet
- **Satellite Position Uncertainty (1σ):**
  - *Radial (u):* 0
  - *In-Track (v):* 0
  - *Cross-Track (w):* 0
- **Minimum User Elevation Angle:** 5°
- **Orbital Inclination:** 18.5°
- **Spacing of Ascending Nodes:** 157.5°
- **Argument of Latitude of the First Satellite in Each Plane at Epoch:** 45°, 225°
- **Satellite Spacing Within Each Plane:** 45°

(1) Denotes indeterminate point
### TABLE XIV

**INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 21 HR WITHOUT SATELLITE ERRORS**

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#### OUTPUTS:

**SYSTEM:** 2x8 Interim (2x2)
**ORBITAL PERIOD:** 24 hours
**TIME FROM EPOCH:** 21 hours
**ORBITAL INCLINATION:** 18.5°
**SPACING OF ASCENDING NODES:** 157.5°
**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 45°, 225°
**SATELLITE SPACING WITHIN EACH PLANE:** 45°

**MEASUREMENT NOISE (lσ):** 50 feet
**USER ALTITUDE UNCERTAINTY (l):** 75 feet
**SATELLITE POSITION UNCERTAINTY (lσ):**
  - RADIAL (u): 0
  - IN-TRACK (v): 0
  - CROSS-TRACK (w): 0
**MINIMUM USER ELEVATION ANGLE:** 5°

(1) Denotes indeterminate point
TABLE XV
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION
OF USER LONGITUDE AND LATITUDE AT T = 0 HR
WITH SATELLITE ERRORS

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SYSTEM: 2x8 Interim (2x2)
ORBITAL PERIOD: 24 hours
TIME FROM EPOCH: 0 hours
ORBITAL INCLINATION: 18.5°
SPACING OF ASCENDING NODES: 157.5°
ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH: 45°, 225°
SATELLITE SPACING WITHIN EACH PLANE: 45°
MEASUREMENT NOISE (\(\sigma\)): 50 feet
USER ALTITUDE UNCERTAINTY (\(\sigma_{h}\)): 150 feet
SATELLITE POSITION UNCERTAINTY (\(\sigma_{r}\))
RADIAL (\(u\)): 15 feet
IN-TRACK (\(v\)): 117 feet
CROSS-TRACK (\(w\)): 38 feet
MINIMUM USER ELEVATION ANGLE: 5°

(1) Denotes indeterminate point
TABLE XVI
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION
OF USER LONGITUDE AND LATITUDE AT T = 3 HR
WITH SATELLITE ERRORS

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SYSTEM: 2x8 Interim (2x2)
ORBITAL PERIOD: 24 hours
TIME FROM EPOCH: 23 hours

MEASUREMENT NOISE (1σ): 50 feet
USER ALTITUDE UNCERTAINTY (1σ): 150 feet
SATELLITE POSITION UNCERTAINTY (1σ):
   RADIAL (u): 15 feet
   IN-TRACK (v): 117 feet
   CROSS-TRACK (w): 38 feet
MINIMUM USER ELEVATION ANGLE: 5°

ORBITAL INCLINATION: 18.5°
SPACING OF ASCENDING NODES: 157.5°
ARGUMENT OF LATITUDE OF THE FIRST SATELLITE
   IN EACH PLANE AT EPOCH: 45°, 225°
SATELLITE SPACING WITHIN EACH PLANE: 45°

(1) Denotes indeterminate point
TABLE XVII
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 6 HR WITH SATELLITE ERRORS

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SYSTEM: 2x8 Interim (2x2)
ORBITAL PERIOD: 24 hours
TIME FROM EPOCH: 6 hours

MEASUREMENT NOISE (σ): 50 feet
USER ALTITUDE UNCERTAINTY (σ): 150 feet
SATELLITE POSITION UNCERTAINTY (σ):
RADIAL (u): 15 feet
IN-TRACK (v): 117 feet
CROSS-TRACK (w): 38 feet
MINIMUM USER ELEVATION ANGLE: 5°

ORBITAL INCLINATION: 18.5°
SPACING OF ASCENDING NODES: 157.5°
ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH: 45°, 225°
SATELLITE SPACING WITHIN EACH PLANE: 45°

(1) Denotes indeterminate point
TABLE XVIII
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 9 HR WITH SATELLITE ERRORS

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**SYSTEM**: 2x8 Interim (2x2)
**MEASUREMENT NOISE (1σ)**: 50 feet
**ORBITAL PERIOD**: 24 hours
**USER ALTITUDE UNCERTAINTY (1σ)**: 150 feet
**TIME FROM EPOCH**: 9 hours
**SATELLITE POSITION UNCERTAINTY (1σ)**
**ARGUMENT OF LATITUDE**: 18.5°
**RADIAL (u)**: 15 feet
**SPACE OF ASCENDING NODES**: 157.5°
**IN-TRACK (v)**: 117 feet
**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE**
**CROSS-TRACK (w)**: 38 feet
**IN EACH PLANE AT EPOCH**: 45°, 225°
**MINIMUM USER ELEVATION ANGLE**: 5°
**SATellite SPACING WITHIN EACH PLANE**: 45°

(1) Denotes indeterminate point
TABLE XIX
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 12 HR
WITH SATELLITE ERRORS

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**SYSTEM:** 2x8 Interim (2x2)
**ORBITAL PERIOD:** 24 hours
**TIME FROM EPOCH:** 12 hours
**ORBITAL INCLINATION:** 18.5°
**SPACING OF ASCENDING NODES:** 157.5°
**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 45°, 225°
**SATELLITE SPACING WITHIN EACH PLANE:** 45°

**MEASUREMENT NOISE (1σ):** 50 feet
**USER ALTITUDE UNCERTAINTY (1σ):** 150 feet
**SATELLITE POSITION UNCERTAINTY (1σ):**
  - **RADIAL (u):** 15 feet
  - **IN-TRACK (v):** 117 feet
  - **CROSS-TRACK (w):** 38 feet
**MINIMUM USER ELEVATION ANGLE:** 5°

(1) Denotes indeterminate point
### TABLE XX
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 15 HR WITH SATELLITE ERRORS

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**SYSTEM:** 2x8 Interim (2x2)
**MEASUREMENT NOISE (1σ):** 50 feet

**ORBITAL PERIOD:** 24 hours
**USER ALTITUDE UNCERTAINTY (1σ):** 150 feet

**TIME FROM EPOCH:** 15 hours
**SATELLITE POSITION UNCERTAINTY (1σ):**
- **RADIAL (u):** 15 feet
- **IN-TRACK (v):** 117 feet
- **CROSS-TRACK (w):** 38 feet

**ORBITAL INCLINATION:** 18.5°
**MINIMUM USER ELEVATION ANGLE:** 5°

**SPACING OF ASCENDING NODES:** 157.5°
**(1) Denotes indeterminate point**
## TABLE XXI

INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 18 HR

WITH SATELLITE ERRORS

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<td>1193</td>
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<td>890</td>
<td>934</td>
<td>999</td>
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</table>

**SYSTEM:** 2x8 Interim (2x2)  
**ORBITAL PERIOD:** 24 hours  
**TIME FROM EPOCH:** 18 hours  
**ORBITAL INCLINATION:** 18.5°  
**SPACING OF ASCENDING NODES:** 157.5°  
**ARGUMENT OF LATITUDE OF THE FIRST SATELLITE IN EACH PLANE AT EPOCH:** 45°, 225°  
**SATELLITE SPACING WITHIN EACH PLANE:** 45°  

**MEASUREMENT NOISE (1σ):** 50 feet  
**USER ALTITUDE UNCERTAINTY (1σ):** 150 feet  
**SATELLITE POSITION UNCERTAINTY (1σ):**  
- **RADIAL (u):** 15 feet  
- **IN-TRACK (v):** 117 feet  
- **CROSS-TRACK (w):** 38 feet  
**MINIMUM USER ELEVATION ANGLE:** 5°  

(1) Denotes indeterminate point
TABLE XXII
INTERIM SYSTEM NAVIGATION ACCURACY (C95) AS A FUNCTION OF USER LONGITUDE AND LATITUDE AT T = 21 HR WITH SATELLITE ERRORS

NORTH LATITUDE

<table>
<thead>
<tr>
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<th>30</th>
<th>40</th>
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<td>1</td>
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<td>90</td>
<td>930</td>
<td>573</td>
<td>885</td>
<td>931</td>
<td>1000</td>
<td>1090</td>
<td>1210</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>-80</td>
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<td>1008</td>
<td>1127</td>
<td>1298</td>
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<td>1</td>
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<tr>
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<td>455</td>
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<td>830</td>
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<td>1242</td>
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<td>1</td>
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<tr>
<td>-60</td>
<td>354</td>
<td>403</td>
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<td>602</td>
<td>642</td>
<td>912</td>
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<td>1202</td>
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<tr>
<td>-50</td>
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<td>381</td>
<td>455</td>
<td>559</td>
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<td>986</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

SYSTEM: 2x8 Interim (2x2)  
MEASUREMENT NOISE (1σ): 50 feet  
ORBITAL PERIOD: 24 hours  
USER ALTITUDE UNCERTAINTY (1σ): 150 feet  
TIME FROM EPOCH: 21 hours  
SATELLITE POSITION UNCERTAINTY (1σ)  
ORBITAL INCLINATION: 18.5°  
RADIAL (u): 15 feet  
SPACING OF ASCENDING NODES: 157.5°  
IN-TRACK (v): 117 feet  
ARGUMENT OF LATITUDE OF THE FIRST SATELLITE  
IN EACH PLANE AT EPOCH: 45°, 225°  
IN EACH PLANE: 45°  
GROSS-TRACK (w): 38 feet  
MINIMUM USER ELEVATION ANGLE: 5°
(1) Denotes indeterminate point
Figure 15. Variation of Interim System Navigation Accuracy with Time of Day
2.3.3 Velocity Estimation from Doppler Data

By using the Navigation Satellite Accuracy Program (NAVSAP), it was determined that a user can accurately estimate his velocity from doppler measurements at one instant of time.

Required modifications to the usual mode of program operation are described in app. E. Six user locations in the North Atlantic were chosen for the analysis, and a $5^\circ$ (min) elevation angle was assumed for visibility. Two runs were made with two different a priori error covariance matrices for satellite velocity, as determined from orbit determination runs, but the difference between the two cases proved insignificant (0.1 ft/sec).

The measurement noise was pessimistically assumed to be 0.707 ft/sec (1σ), and the user velocity error covariance matrix was diagonal with a standard deviation of 100 ft/sec in each direction. Results are given in Table XXIII, which shows the RSS user velocity error in ft/sec at each of the six locations. The range is from 1.22 to 1.92 ft/sec. This error can easily be reduced, and will be determined by the cost of the user hardware.

**TABLE XXIII**

<table>
<thead>
<tr>
<th>West Longitude</th>
<th>North Latitude</th>
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<tr>
<td></td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>1.92</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>1.80</td>
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<tr>
<td>$0^\circ$</td>
<td>1.40</td>
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</table>
2.3.4 Relative Navigation Accuracy Analysis

Relative navigation accuracy is the accuracy with which a user can determine his position relative to (1) another user, (2) a ground station, or (3) his own home base. All three modes have the characteristic that certain common error sources can be expected to cancel, providing the possibility of increased accuracy over the absolute navigation case.

In mode 1, each user estimates his position independently with common error sources, especially satellite errors, causing common position estimation errors. The users then communicate their estimates to each other and compute their relative positions. It can be expected that the common errors will tend to cancel in the subtraction, leaving only the effects of the independent random errors made by the two users. However, the random effects add (RSS).

In mode 2, a ground station replaces the second user. The station takes measurements like those of the user, but its advantage is in being stationary and capable of taking many measurements, thereby reducing the effect of noise. Hence, not only do the common error sources cancel, but the doubling of the noise effect occurring in mode 1 is eliminated in mode 2. Furthermore, if the absolute position of the ground station is known, the computed position can be used for calibration purposes, enabling the user to obtain a more accurate absolute position determination.

In mode 3, the user is assumed to make a preliminary fix while at his home base and then, during his subsequent flight, to navigate with respect to this base. This is similar to mode 2, except that user receiver bias is calibrated at the home base. Also, due to the elapsed time between calibration and navigation fix, there may be drifts in some of the calibrated errors, such as receiver drift and tropospheric and ionospheric errors. In the case of the last two errors, distance, as well as time, determines the degree of correlation and, hence, cancellation (see subsec. 2.2 on Measurement Error Source).

The ideas expressed in the preceding paragraphs can be investigated in terms of the covariance matrices of the two participants, whether they are two users, mode 1; a user and a ground station, mode 2; or the same
user at two different locations, mode 3. The covariance matrix of relative position error is

\[
E\left[(\delta x_1 - \delta x_2)(\delta x_1 - \delta x_2)^T\right] = E(\delta x_1 \delta x_1^T)
+ E(\delta x_2 \delta x_2^T) - E(\delta x_1 \delta x_2^T) - E(\delta x_2 \delta x_1^T)
\]

where \(\delta x_1\) and \(\delta x_2\) are the estimation errors \(\hat{x}_1 - x_1\) and \(\hat{x}_2 - x_2\), and \(E\) denotes the expected value. The difference \(\delta x_1 - \delta x_2\) is the error in estimating the position of user 1 relative to user 2. \(x_1, x_2\) are the actual position vectors and \(\hat{x}_1, \hat{x}_2\) are their estimates.

This equation contains the essential elements of the relative navigation problem. The first two terms are the individual estimation errors of the two users, including the effects of common error sources (in this case, satellite position and satellite clock errors). The last two terms are the correlations between the errors of the two users. These correlations are due to the common satellite errors and can be expected to reduce the portions of the first two terms that are attributable to satellite errors. The effect is brought out by rewriting Eq. (1) in the form

\[
\Sigma_R = \frac{\Sigma_{1n} + \Sigma_{1s}}{\Sigma_{11}} + \frac{\Sigma_{2n} + \Sigma_{2s}}{\Sigma_{22}} - \Sigma_{12} - \Sigma_{21}
\]

Here the \(\Sigma\)'s denote the covariance matrices (e.g., \(\Sigma_{12} = E(\delta x_1 \delta x_2^T)\)), and \(\Sigma_{11}\) and \(\Sigma_{22}\) have been divided into two parts, the first part due to random errors (e.g., \(\Sigma_{1n}\)) and the second due to satellite errors (e.g., \(\Sigma_{1s}\)).

Equation (2) assumes its minimum value either when the satellite errors are zero (in which case \(\Sigma_{12} = \Sigma_{21} = \Sigma_{1s} = \Sigma_{2s} = 0\)) or when the correlations directly cancel the satellite errors of the two users. This minimum is given by

\[
\Sigma_{R_{\text{min}}} = \Sigma_{1n} + \Sigma_{2n}
\]
These equations apply to all three modes of relative navigation. In modes 2 and 3, however, the component $\Sigma_{11}$ will be significantly smaller than in mode 1, where it is of the same order of magnitude as $\Sigma_{22}$.

The individual terms in Eq. (2) were evaluated for the interim system at $T = 0$ by using the NAVSAP program (details of the analysis are included in app. F). Some typical results are shown in Table XXIV, where user 2 moves to several positions north of a base station (user 1), who is at latitude $0^\circ$ and longitude $-30^\circ$. The eight columns of the table are headed by the appropriate covariance matrices in Eq. (2); the numerical values given are the C95 values corresponding to these covariance matrices.

The first two columns show the uncertainties in the positions of user 1 and user 2 as determined in earth-centered coordinates. User 1's uncertainty is, of course, independent of user 2's position; therefore all figures in the first column are the same, with his latitude as shown. User 2's uncertainty changes with his latitude as shown.

Columns 3 and 4 are the same as columns 1 and 2, except that no satellite errors are included ($\Sigma_{1s}$ and $\Sigma_{2s}$ are zero)*. Since mode 2 assumes that the effect of noise in user 1's measurement ($\Sigma_{1n}$) is zero, column 4 can be interpreted as mode 2 relative navigation without satellite errors.

Columns 5 and 6 are the RSS of columns 1 and 2, and 3 and 4, respectively (covariance matrices add; C95's RSS). They can be interpreted to represent the uncertainty in user 2's position in relation to someone like user 1 (who sees similar satellite configurations), but located far from user 1 and, therefore, seeing different satellites. Column 5 includes satellite errors, while column 6 excludes them (or assumes that they are correlated and therefore produce a negligible effect). In this case, the user correlation terms $\Sigma_{12}$ and $\Sigma_{21}$ do not

*Par. 2.5.1 on satellite error correlations shows that correlation effects may reduce the effect of satellite errors to negligible values. Thus, cases excluding satellite errors may alternatively be considered as cases in which intersatellite correlation is taken into account.
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude of User 2</td>
<td>$\Sigma_{21}$</td>
<td>$\Sigma_{22}$</td>
<td>$\Sigma_{1n}$</td>
<td>$\Sigma_{2n}$</td>
<td>$\Sigma_{11} + \Sigma_{22}$</td>
<td>$\Sigma_{1n} + \Sigma_{2n}$</td>
<td>$\Sigma_{R}$</td>
</tr>
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<td>269</td>
<td>269</td>
<td>187</td>
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<td>269</td>
<td>277</td>
<td>187</td>
<td>386</td>
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<td>193</td>
<td>202</td>
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<tr>
<td>15°</td>
<td>269</td>
<td>285</td>
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<td>212</td>
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<td>20°</td>
<td>269</td>
<td>298</td>
<td>187</td>
<td>402</td>
<td>283</td>
<td>212</td>
<td>212</td>
</tr>
</tbody>
</table>

Notes: Measurement noise = 50 ft (15 m), satellite error = 75 ft (23 m), radial error = 360 ft (110 m), cross-track error = 99 ft (30 m).
subtract as they do in local (using the same satellites) relative navigation. This situation will be designated worldwide relative navigation. It is representative of the intuitive notion of navigation as being relative to some distant point of the earth's surface rather than relative to an arbitrary earth-centered coordinate system.

Column 7 is the C95 corresponding to local relative navigation in mode 1, including all the terms of Eq. (2). The significant point to be noted is that the values are nearly identical to those of column 6. That is, local relative navigation causes cancellation of nearly all the effects of satellite errors*. The difference between columns 7 and 5 should also be noted; it is due to the effects of user correlation (Σ₁₂ and Σ₂₁) and illustrates the intuitive idea of error reduction in relative navigation.

Column 8 is column 7 less the effect of noise in determining the base station (user 1) position. Thus, column 8 gives the uncertainties for mode 2, local relative navigation, where the base station can take many measurements and reduce the effects of noise to a negligible value. Since the relative navigation effect causes the nearly complete cancellation of satellite errors, column 8 is nearly equal to column 4.

This brief analysis of relative navigation accuracy leads to the following conclusions:

- Local relative navigation results in cancellation of satellite errors
- Satellite errors are not the major source of navigation uncertainties, so that the improvement of local relative navigation accuracy (column 7) with respect to worldwide relative navigation (column 5) is not as pronounced as might be expected.
- If intersatellite correlations are taken into account, it is expected (although not yet proved) that column 6 is a better representation of worldwide relative navigation than is column 5. In that case, worldwide and local relative navigation uncertainties are nearly identical and both approach the minimum value of Eq. (3).

*In this case, both users see identical satellites. If only some of the satellites were common, the cancellation would be less complete.
2.4 ORBIT DETERMINATION ERRORS

The accuracy with which a user can determine his position depends, in part, on the accuracy of his knowledge of the satellite ephemerides. The effect of tracking errors on satellite position determination accuracy was analyzed. A series of preliminary analyses were made to obtain answers to the following subsidiary questions:

- Which uncertainties make the largest contributions to the total position uncertainty?
- Which parameters should be solved for in processing the tracking data?
- Are range measurements alone sufficient, or will angle and range-rate measurements increase tracking accuracy?
- Which geopotential harmonics should be estimated?
- Should measurement biases be assumed constant or changing?
- How many tracking stations are required for satisfactory position determination?
- How long a tracking period is required?

In order to find answers to these questions, several tracking configurations were analyzed in addition to the finally selected configuration. Details of these preliminary analyses and results are given in app. K, and the minimum-variance estimation methods of the TRW System's ESPOD computer program used to derive the results are briefly described in app. G.

The conclusions reached can be summarized as follows:

- The predominant error source is the uncertainty in the earth's gravitational constant $\mu$. This uncertainty leads to large period errors, which appear as large $v$ (in-track) errors.

- Solving for the parameters (measurement bias errors, survey errors, and uncertainties in $\mu$ and $J_2$) results in a considerable improvement in accuracy, particularly in the $v$ (downrange) direction.
With the angle (AE) and range-rate ($\dot{R}$) measurements to the range (R) measurements does not affect the system accuracy; therefore, only range measurements are required.

The $J_{22}$ geopotential harmonic should be solved for. With this coefficient solved for, the position uncertainties are unaffected, but if it is not solved for, it leads to large increases in total error, especially in the along-track and cross-track directions. A run was also made to determine the effects of $J_{33}$; this term had no effect on the position uncertainties.

The use of three stations rather than two substantially reduces the in-track error, an effect that can be expected to be even more pronounced for shorter (less than 72 hr) tracking intervals, in which case it would also affect cross-track errors significantly.

Reducing the tracking period from 72 to 36 hr has little significant effect on the results. Hence, 36 hr is sufficient.

With this background, it was possible to analyze a realistic tracking configuration, corresponding to the proposed system, which uses essentially the same equipment as a user, taking measurements from a particular satellite at a rate of one every 16 sec. Three stations were chosen, collocated with present tracking facilities. The station locations and the satellite ground track used in this analysis are shown in Figure 16. The error sources considered and their values are shown in Table XXV.

The results of the proposed tracking configuration analysis for the single satellite and set of ground stations selected are shown in Figure 17, where the epoch is at the end of a 36-hr tracking interval.
Figure 16. Recommended Tracking Configuration
TABLE XXV
ERROR SOURCES FOR ORBIT DETERMINATION ANALYSIS

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<th>Measurement errors</th>
<th></th>
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<td>Noise</td>
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<td>Bias</td>
<td>50 ft</td>
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<table>
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<th>Station location errors</th>
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<tr>
<td>Longitude</td>
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<tr>
<td>Altitude</td>
<td>100 ft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gravitational potential uncertainties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( 1.06 \times 10^{-11} ) ft(^3)/sec(^2)</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>( 2.0 \times 10^{-7} )</td>
</tr>
<tr>
<td>( J_{22} )</td>
<td>( 2.0 \times 10^{-7} )</td>
</tr>
<tr>
<td>( J_{33} )</td>
<td>( 2.6 \times 10^{-7} )</td>
</tr>
</tbody>
</table>

At the end of 36-hr tracking, the figure shows the following tracking errors:

\[
\begin{align*}
\sigma_u \text{ (radial)} & \quad 10.8 \text{ ft} \\
\sigma_v \text{ (in-track)} & \quad 140.0 \text{ ft} \\
\sigma_w \text{ (cross-track)} & \quad 5.0 \text{ ft}
\end{align*}
\]

These results assume a constant bias in the measurements. In a real tracking situation, however, the biases are slowly varying. To represent this condition in the program approximately, piecewise constant biases (all uncorrelated) were assumed over 3-hr tracking intervals. The first 15 hr of the tracking period were then re-run, which produced the results shown in Figure 18. The results shown in Figure 17 are included for easier comparison.

It can be seen that the effect is a significant degradation in accuracy over extended tracking intervals.

Additional study of the tracking of several satellites simultaneously is indicated. Measurements from additional satellites provide more information for determining geopotential terms and biases, and the satellite position errors become correlated because of common error sources, particularly the uncertainties in the geopotential model. Covariance matrices containing these correlations result in smaller
Figure 17. Results of Tracking for 36 Hr at 16-sec Data Interval
user position errors, as discussed in par. 2.5.1. The magnitude of the
correlations should be determined for the tracking procedures used in
the actual system.

Another important source of error that must be investigated in detail
is the effect of satellite oscillator drift on range measurements and, con-
sequently, on navigation accuracy. Present error models assume that the
satellite clocks can be calibrated so that the drift error after 3 hr is less
than 10 ft. Actually, the oscillator drift characteristics must be estimated
from the tracking data along with the orbital parameters.

To determine the error in the estimate of satellite oscillator drift
and the consequent effect on orbit determination accuracy and navigation
accuracy will require 1) a suitable mathematical model of the oscillator
drift mechanism and 2) the incorporation of this model into the present
series of error analysis programs (NAVSAP and ESPOD).
2.5 ANALYSIS OF OTHER FACTORS AFFECTING NAVIGATION ACCURACY AND COVERAGE

Results of analyses of a number of factors affecting the overall navigation accuracy provided by the system are presented in this subsection. Par. 2.5.1 discusses the effects of satellite error correlations and illustrates these effects. The remaining paragraphs discuss the effects of:
1) user motion, 2) using range-difference measurements and 3) increasing the minimum elevation angle of the antennas.

2.5.1 Effect of Correlated Satellite Position Errors

The majority of navigation-satellite user position accuracy studies assumed that position errors of the several satellites are uncorrelated, with equal covariance matrices resulting from tracking studies using the TRW orbit-determination program. It is apparent, however, that correlations do exist, arising from numerous common error sources. In particular, station measurement biases and location errors will equally influence all satellites seen from that station, and earth potential model uncertainties cause correlations between errors in all satellites.

It has been postulated that the correlations arising from these common errors may have a significant effect on the resulting user position determination errors. This claim has been corroborated by the results of the Single Point in Time (SPIT) accuracy program presented in app. H. In that analysis, tracking stations and users make simultaneous observations of a group of satellites. Correlations in satellite position errors arise from common ground-station bias errors, and these correlated satellite errors are used directly to compute user position errors. Appendix I contains a more complete description of the SPIT program and its assumptions. The results show a high degree of correlation and a significant reduction in position errors over the uncorrelated case. In fact, the position errors closely approach those obtained with the assumption of perfectly known satellite positions.

To confirm the effect, using more realistic satellite errors, additional runs have been made using the Navigation Satellite Accuracy
Program (NAVSAP), in which perfectly correlated satellite errors were assumed; that is

\[ E(x_1 x_2^T) = E(x_1^T x_1) = E(x_2^T x_2) \]  

(4)

where \( x_1 \) and \( x_2 \) are satellite position vectors. This implies \( x_1 = x_2 \) with probability one. These results are shown in Table XXVI, along with comparable results for no satellite errors and uncorrelated errors. Again, the position errors are reduced to values near to those obtained with zero satellite errors.

A more complete analysis of this effect is proposed for future work. In particular, the correlations that arise in a realistic tracking situation must be determined. Since the present TRW orbit determination program is limited to a single satellite, multiple runs must be made and the resulting normal matrices assembled to determine a joint normal matrix, which can then be inverted to yield the overall satellite error.

**TABLE XXVI**

**NAVIGATION ACCURACY WITH CORRELATED SATELLITE ERRORS**

<table>
<thead>
<tr>
<th>Latitude</th>
<th>0°</th>
<th>30°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Satellite Errors</td>
<td>327</td>
<td>341</td>
<td>364</td>
</tr>
<tr>
<td>Correlated Errors</td>
<td>337</td>
<td>349</td>
<td>369</td>
</tr>
<tr>
<td>Uncorrelated Errors</td>
<td>442</td>
<td>455</td>
<td>474</td>
</tr>
</tbody>
</table>

**NOTES:**

1. Range measurements from all visible satellites - User oscillator uncalibrated
2. Measurement Noise = 100 ft. (1σ)
3. User \textit{a priori} altitude error = 150 ft. (1σ)
4. Satellite \textit{a priori} position error covariance matrix from app. K (450 ft downrange error)
5. User longitude = 60° west
covariance matrix. More specifically, consider a linearized tracking model associated with tracking the $i^{th}$ satellite

$$y_i = A_i x_i + B_i z + \epsilon_i$$

(5)

$x_i$ is the satellite state vector, consisting of positions, velocities, and possibly satellite oscillator bias. $z$ is a vector of satellite-independent parameters, including station locations, biases, and earth potential parameters. $y_i$ is the measurement vector, $A_i$ and $B_i$ are the appropriate partial-derivative matrices, and $\epsilon_i$ is the measurement error. $x_i'$, $y_i'$, and $z$ are to be interpreted as small deviations from reference values.

For each satellite, the tracking program will generate a normal matrix of the form

$$Q_i = \begin{bmatrix} A_i^T W_i A_i & A_i^T W_i B_i \\ B_i^T W_i A_i & B_i^T W_i B_i \end{bmatrix}$$

(6)

where

$$W_i = \left[ E(\epsilon_i \epsilon_i^T) \right]^{-1}$$

(7)

These individual matrices are then assembled into the giant normal matrix $Q$ as follows:

$$Q = \begin{bmatrix} A_1^T W_1 A_1 & 0 & 0 & \cdots & 0 & A_1^T W_1 B_1 \\ 0 & A_2^T W_2 A_2 & 0 & \cdots & 0 & A_2^T W_2 B_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & A_n^T W_n A_n & A_n^T W_n B_n \\ B_1^T W_1 A_1 & B_2^T W_2 A_2 & \cdots & B_n^T W_n A_n & \sum_{i=1}^n B_i^T W_i B_i \end{bmatrix}$$

(8)
The inverse of this matrix is the covariance matrix of the giant state vector

\[
x = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n \\
z
\end{bmatrix}
\]

assuming all of the components of \( z \) are estimated. If some are not, their errors can be taken into account, using a well known formula involving submatrices of Eq. (8). These computations are readily performed by the TRW Matrix Abstraction Program (MAP).

2.5.2 Sequential Estimation of Position of a Rapidly Moving User (SST)

The navigation equations presented in Section 3 provide for continual updating of user position as measurements are processed. Every 16 sec, the system recycles through the visible satellites, and the new measurements are processed to refine the previous estimate. If the user were stationary, this recursive estimation procedure would result in a continual reduction of the errors due to measurement noise. The same is true if the user were flying along a perfectly predictable flight path. In that case, the estimate is propagated to the new measurement time, and the new fix is used to refine the propagated estimate. Unfortunately, the flight path of an SST is not perfectly predictable; hence, errors are introduced in propagating the previous estimate forward. If large enough, these errors can effectively nullify the previous estimate and force reliance only on current data to produce a current fix. In that case, there is no beneficial effect of noise reduction from multiple measurements.

This subsection presents an analysis allowing an approximate assessment of the effect of the uncertainty in the user flight path on navigation accuracy. The NAVSAP program (app. J) considers a user
moving along a nominal great circle flight path, taking an instantaneous fix from all visible satellites every 16 sec.\* The program does not provide for the estimation of user velocity, but does permit uncertainties in user velocity and heading to be introduced, using the state noise feature of the Kalman filter. Although this is a simplified model of the actual system operation, the results do point up some important aspects of the sequential estimation problem.

Three runs were made with varying magnitudes of the flight path uncertainties as tabulated below:

<table>
<thead>
<tr>
<th>Case</th>
<th>Heading Error (rad-1σ)</th>
<th>Velocity Error (ft/sec-1σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>0.10</td>
<td>100</td>
</tr>
</tbody>
</table>

The user was assumed to be flying just outside of London on a great circle route to New York at a speed of 2000 ft/sec and an altitude of 50,000 ft. Satellite estimation errors were 98 ft radial, 720 ft in-track, and 198 ft cross-track (Table K-II - app. K), and the measurement noise was 50 ft (1σ). The exact values used are relatively unimportant; the purpose is to illustrate the effect. The satellites considered are the five shown in Figure 19.

The results of the three cases are presented in Table XXVII and plotted as the top two curves in Figure 20. The results for cases 2 and 3 are nearly identical, despite the wide variation in flight-path errors. These errors are only 12 to 15 ft greater than in the case of zero errors, which demonstrates the relatively minor influence of the velocity and heading errors. The curves show an initial rapid drop while the user oscillator is being calibrated.\** Thereafter, the curves exhibit a slower decrease toward an asymptotic value determined by the satellite errors. The lower curves illustrate these effects, in both cases, with no heading or velocity errors. The bottom curve shows the results for no satellite.

\* This is, of course, an approximation, as the data for the fix are taken throughout the 16-sec frame, not instantaneously.

\** Uncertainties in velocity and heading do not degrade the estimate of oscillator bias.
TABLE XXVII

C95 VERSUS TIME FOR A MOVING USER WITH VARIOUS VELOCITY AND HEADING ERRORS

<table>
<thead>
<tr>
<th>Time (Sec)</th>
<th>$\sigma_v = 0$</th>
<th>$\sigma_v = 20$ fps</th>
<th>$\sigma_v = 100$ fps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_\psi = 0$</td>
<td>$\sigma_\psi = 0.01$</td>
<td>$\sigma_\psi = 0.01$</td>
</tr>
<tr>
<td>Before</td>
<td>After Observation</td>
<td>Before After Observation</td>
<td>Before After Observation</td>
</tr>
<tr>
<td>0</td>
<td>302</td>
<td>302</td>
<td>302</td>
</tr>
<tr>
<td>16</td>
<td>302</td>
<td>259</td>
<td>505</td>
</tr>
<tr>
<td>32</td>
<td>259</td>
<td>243</td>
<td>484</td>
</tr>
<tr>
<td>48</td>
<td>243</td>
<td>235</td>
<td>477</td>
</tr>
<tr>
<td>64</td>
<td>235</td>
<td>230</td>
<td>474</td>
</tr>
<tr>
<td>80</td>
<td>230</td>
<td>226</td>
<td>471</td>
</tr>
<tr>
<td>96</td>
<td>226</td>
<td>223</td>
<td>470</td>
</tr>
<tr>
<td>112</td>
<td>224</td>
<td>222</td>
<td>469</td>
</tr>
<tr>
<td>128</td>
<td>222</td>
<td>220</td>
<td>468</td>
</tr>
<tr>
<td>144</td>
<td>220</td>
<td>219</td>
<td>467</td>
</tr>
<tr>
<td>160</td>
<td>219</td>
<td>218</td>
<td>467</td>
</tr>
<tr>
<td>176</td>
<td>218</td>
<td>217</td>
<td>466</td>
</tr>
<tr>
<td>192</td>
<td>217</td>
<td>216</td>
<td>466</td>
</tr>
<tr>
<td>208</td>
<td>216</td>
<td>215</td>
<td>465</td>
</tr>
<tr>
<td>224</td>
<td>215</td>
<td>214</td>
<td>465</td>
</tr>
<tr>
<td>240</td>
<td>214</td>
<td>214</td>
<td>465</td>
</tr>
<tr>
<td>256</td>
<td>214</td>
<td>213</td>
<td>464</td>
</tr>
<tr>
<td>272</td>
<td>213</td>
<td>212</td>
<td>464</td>
</tr>
<tr>
<td>288</td>
<td>212</td>
<td>211</td>
<td>464</td>
</tr>
<tr>
<td>304</td>
<td>211</td>
<td>210</td>
<td>464</td>
</tr>
<tr>
<td>320</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
errors, in which case the error continues to decrease as the effect of noise is reduced. In the second curve from the bottom, satellite errors are included, but measurement noise is zero. In that case, the estimation error rapidly approaches the minimum value established by the satellite position errors.

From these results it can be concluded that a sequence of fixes results in an initial increase in accuracy due to improved estimation of the bias (user oscillator calibration). Continual estimation, as provided by the proposed system, will provide accuracies near this reduced value. The minimum attainable is determined by satellite errors, and may approach the case of zero satellite errors due to the correlation effects discussed in par. 2.5.1.

The effect of heading and velocity errors during the interval (≥2 sec) separating observations from different satellites has not yet
been studied. This study would require several modifications to the
NAVSAP program as follows:

1) Logic for sequential measurements from one satellite
   at a time

2) Capability for estimation of user velocity

3) Improved model of aircraft motion.

2.5.3 Effect of Correlations in Range-Difference Measurements

The proposed NAVSTAR measurement system is based on measuring
the transmission time of a signal from the satellite to a ground station.
If satellite and user clocks are synchronized, the absolute transit time
is obtained directly and is proportional to range. However, if a less
stable clock is available to the user, the equipment will be more econo-
mical. With a less stable clock, the user time reference is accurate for
only relatively short time measurements as it is not synchronized with
the highly precise satellite clocks. In this case, it is impossible to
measure the absolute range from user to satellite directly; however,
the arrival times of signals from various satellites can be compared,
and will yield, in effect, a range-difference measurement. Thus, a
pseudo-range is produced from the difference between the arrival time
of the signal and the local clock time. In effect, this measurement is
the actual range from satellite to user, plus a large unknown bias caused
by lack of synchronization of the satellite and user clocks. Therefore,
this same bias appears in simultaneous or near-simultaneous measure-
ments to all visible satellites. These raw measurements, including the
common bias, can then be differenced in the computer to produce range-
difference measurements, which can then be processed further to produce
a position estimate.

An alternate approach is to process the pseudo-range measurements
directly, producing a simultaneous estimate of user position plus the
unknown bias. It will be shown, subsequently, that the two approaches
give the same result under suitable conditions. Since sequential data
processing is more straightforward in the second method, it is recom-
mended wherever maximum accuracy is required.
The linearized measurement model relates the pseudo-range measurement vector $y$ to the user position $x$ and bias $b$ by

$$y = Ax + Cb + \epsilon$$  \hspace{1cm} (10)$$

where $y$, $x$, and $b$ are small deviations from reference values, $A$ is the sensitivity matrix of the observations with respect to position, $\epsilon$ is the measurement error, and $C$ is a column vector with all elements unity.

$$C^T = (1, 1, 1, \ldots, 1)$$  \hspace{1cm} (11)$$

Hence, the bias $b$ is a scalar, and the vector $Cb$ adds the bias to each observation.\(^*\)

2.5.3.1 Optimum Weighting of Range-Difference Measurements

An $m$-vector $z$ of range-difference measurements can be interpreted as a linear transformation on $y$

$$z = Ty$$  \hspace{1cm} (12)$$

where $T$ is an $m \times n$ matrix, each row of which contains one $+1$, one $-1$, and all other elements zero, i.e.,

$$z_k = T_k y = y_i - y_j$$  \hspace{1cm} (13)$$

The $i^{th}$ element of $T_k$ is $+1$ and the $j^{th}$ is $-1$. Multiplying Eq. (10) by $T$ and noting that $TC = 0$ results in the bias-free observation equation for $z$

$$z = TAx + T\epsilon$$  \hspace{1cm} (14)$$

\(^*\)Let $y$, $C$, and $\epsilon$ be of dimension $n$, and $x$ be of dimension $p$.\)
The range-measurement errors are uncorrelated \( \mathbb{E}(\epsilon_i \epsilon_j) = 0 \) and are assumed to be of equal variance \( \sigma^2 \). Therefore, the range-difference errors are correlated, with covariance matrix

\[
\Lambda_\delta = \sigma^2 \mathbf{Z} \mathbf{Z}^T
\]

The minimum variance estimate \( \hat{x} \) of \( x \), based on the range-difference measurements \( z \), appropriately weighted by \( \Lambda_\delta^{-1} \) is readily found to be

\[
\hat{x} = \left[ \Lambda_\delta \right]^{-1} \mathbf{Z} \mathbf{Z}^T \left( \mathbf{Z} \mathbf{Z}^T \right)^{-1} \mathbf{z}
\]

\[
\hat{x} = \left( \Lambda_\delta \right)^{-1} \mathbf{Z} \mathbf{Z}^T \left( \mathbf{Z} \mathbf{Z}^T \right)^{-1} \mathbf{z}
\]

where

\[
\mathbf{W} = \frac{1}{\sigma^2} \mathbf{Z} \mathbf{Z}^T \left( \mathbf{Z} \mathbf{Z}^T \right)^{-1} \mathbf{z}
\]

An interesting result, demonstrated by Soule at the Aerospace Corporation, is that \( \mathbf{W} \) is independent of \( T \), i.e., independent of the choice of range-difference measurements. It is required only that \( \mathbf{Z} \mathbf{Z}^T \) be nonsingular, which in turn requires that \( m \) be \( n-1 \). The invariance of \( \mathbf{W} \), along with Eq. (17) shows that the minimum variance estimate is independent of the choice of range differences.

2.5.3.2 Estimation Using Pseudo-Range Measurements

The alternative approach to estimating \( x \) is to process the pseudo-range data \( y \) directly and attempt to solve for the bias. The resulting estimate, for equally weighted pseudo-range measurements, is

\[
\begin{bmatrix}
\hat{x} \\
\hat{b}
\end{bmatrix} = \begin{bmatrix}
\mathbf{A}^T \mathbf{A} & \mathbf{A}^T \mathbf{C} \\
\mathbf{C}^T \mathbf{A} & \mathbf{C}^T \mathbf{C}
\end{bmatrix}^{-1} \begin{bmatrix}
\mathbf{A}^T \\
\mathbf{C}^T
\end{bmatrix} \mathbf{y}
\]

(19)
Partitioning the inverse leads to the \( \hat{x} \) component

\[
\hat{x} = (A^T W A)^{-1} A^T W y
\]  \hspace{1cm} (20)

with

\[
W = \frac{1}{\sigma^2} (I - \frac{1}{n} CC^T)
\]  \hspace{1cm} (21)

where \( CC^T \) is an \( n \times n \) square matrix with all elements unity.

That \( W = \overline{W} \) can be shown by assuming the invariance demonstrated heuristically by Soule and computing \( \overline{W} \) for a particular, \( T \), namely,

\[
T = \begin{bmatrix}
1 & -1 & 0 & 0 & \ldots & 0 \\
1 & 0 & -1 & 0 & \ldots & 0 \\
1 & 0 & 0 & -1 & \ldots & 0 \\
odots & & & & & \\
1 & 0 & 0 & 0 & \ldots & -1
\end{bmatrix}
= \begin{bmatrix} C_m & -I_m \end{bmatrix}
\]  \hspace{1cm} (22)

Then

\[
T T^T = \begin{bmatrix}
2 & 1 & 1 & \ldots & 1 \\
1 & 2 & 1 & \ldots & 1 \\
1 & 1 & 2 & \ldots & 1 \\
odots & & & & \\
1 & 1 & 1 & \ldots & 2
\end{bmatrix}
= I_m + C_m C_m^T
\]  \hspace{1cm} (23)
where $C_m$ is an $m$ vector, distinct from $C = C_n$, which is an $n$ vector.
The inverse of Eq. (23) is
\[
(TT^T)^{-1} = I_m - \frac{1}{1+m} C_mC_m^T
\]  
(24)

Then, by using the partitioned form of $T$, from Eq. (22), it follows after some computation that
\[
T^T(TT^T)T = \begin{bmatrix} 1 & 0 \\ 0 & I_m \end{bmatrix} - \frac{1}{1+m} \begin{bmatrix} 1 & C_m^T \\ C_m & C_mC_m^T \end{bmatrix}
\]
\[
= I_n - \frac{1}{n} C_n C_n^T
\]  
(25)

where the last step follows from the previously noted equality, $m = n-1$.
Hence, $\bar{W} = W$ and the estimates of Eqs. (20) and (16) are identical, with error-covariance matrices
\[
\Lambda_A = \sigma_e^2 \left[ A^T(I - \frac{1}{n} C C^T)A \right]^{-1}
\]
\[
= \sigma_e^2 \left[ \sum_{i=1}^{n} A_i^T A_i - \frac{1}{n} \left( \sum_{i=1}^{n} A_i^T \right) \left( \sum_{i=1}^{n} A_i \right) \right]^{-1}
\]  
(26)

where $A_i$ is the $i^{th}$ row of $A$, corresponding to the $i^{th}$ measurement $y_i$. 

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Of further interest is the second term of the estimates Eq. (20) or (17)

\[
\hat{A}^T W_y = \frac{1}{\sigma^2} A^T \left( I - \frac{1}{n} CC^T \right) y
\]

\[
= \frac{1}{\sigma^2} \left[ \left( \sum_{i=1}^{n} A_i^T y_i \right) - \frac{1}{n} \left( \sum_{i=1}^{n} A_i^T \right) \left( \sum_{i=1}^{n} y_i \right) \right]
\]

\[
= \frac{1}{\sigma^2} \sum_{i=1}^{n} A_i^T \left( y_i - \frac{1}{n} \sum_{k=1}^{n} y_k \right)
\]

This shows that either scheme uses a kind of weighted difference, where the average of all of the measurements is subtracted from each measurement \(y_i\).

2.5.3.3 Suboptimal Weighting of Range-Difference Measurements

A third possible estimate of \(x\) offers advantages from a data-processing point of view, namely,

\[
\hat{x}_2 = \left( A^T T^T A \right)^{-1} A^T T^T z
\]

(28)

This estimate would be minimum variance if the range-difference errors were uncorrelated. Taking the correlations into account, however, the covariance matrix of this estimate becomes

\[
\Lambda_{\hat{x}_2} = \sigma^2 \left( A^T T^T A \right)^{-1} T^T T^T \left( A^T T^T A \right)^{-1}
\]

(29)
In general, different choices of \( T \) can be expected to give different accuracies. The case considered in Eq. (22) can, however, be computed explicitly. In that case

\[
A_T T T = (A_1^T - A_2, A_1^T - A_3, \ldots, A_1^T - A_n)
\]

and

\[
A_T T T C_m = \sum_{i=2}^{n} (A_1 - A_i)^T
\]

So, by using Eq. (23) in the middle of Eq. (29), and applying the partitions of \( A \) and \( T \), the covariance matrix Eq. (29) can be brought into the form

\[
A = (Q^{-1} + Q^{-1} P Q^{-1}) \sigma^2
\]

where

\[
Q = A_T T T A = \sum_{i=2}^{n} (A_1 - A_i)^T (A_1 - A_i)
\]

\[
= n A_1 A_1 + \sum_{i=1}^{n} A_i^T A_i - A_1^T \sum_{i=1}^{n} A_i - \sum_{i=1}^{n} A_i^T A_i
\]

\[
P = A_T T T T T A = \left[ \sum_{i=2}^{n} (A_1 - A_i)^T \right] \left[ \sum_{i=2}^{n} (A_1 - A_i) \right]
\]

\[
= n^2 A_1 A_1 + \left( \sum_{i=1}^{n} A_i^T \right) \left( \sum_{i=1}^{n} A_i \right) - n A_1^T \sum_{i=1}^{n} A_i - n \sum_{i=1}^{n} A_i^T A_i
\]
These formulas can be evaluated and the result compared with Eq. (26) to determine the penalty associated with the suboptimal estimate.

2.5.3.4 Summary

It has been shown that the use of range-difference measurements with optimum weighting to account for correlation is equivalent to the direct application of the heavily biased pseudo-range measurement to solve for the position and the bias. The latter method is recommended, since it is more convenient to implement with the Kalman filter proposed for the user computer. When less accuracy is required, the correlations can be ignored and range differences processed according to Eq. (29). In that case, the accuracy of the estimate can be assessed by means of Eqs. (32) through (34).

2.5.4 Effect of Increasing Minimum Elevation Angle

The navigation accuracies given in pars. 2.1.1 and 2.1.2 are based on a minimum user-to-satellite angle of 5° above the horizon, as noted earlier. Since hardware considerations indicated that a value of 10° or more might be more realistic, the effect of increasing the minimum elevation angle to 10°, 20°, and 30° was investigated.

Results are shown in Figures 21, 22, and 23 for 10°, 20°, and 30°, respectively. A comparison of the 10° case (Figure 21) with the corresponding map of Figure 2 shows that system coverage is degraded. The areas of indeterminacy (0 or 2 satellites visible) extend over some areas of interest, and a single failed satellite would make the system of questionable value above 50° latitude at certain times of the day.
Figure 21. Worldwide System Coverage at $T_0$ With $10^\circ$ Minimum Incidence Angle (Numbers Denote Satellites Visible)
Figure 22. Worldwide System Coverage at $T_0$ With $20^\circ$ Minimum Incidence Angle (Numbers Denote Satellites Visible)
Figure 23. Worldwide System Coverage at $T_0$ With $30^\circ$ Minimum Incidence Angle (Numbers Denote Satellites Visible)
3. NAVIGATION EQUATIONS

The computations described in the following sections must be performed for a user to determine his position, and possibly his velocity, from satellite observations. The four classes of user considered (three with standard ephemeris data, one with extra data) are the following: 1) a relatively sophisticated user, such as a supersonic transport, desiring maximum accuracy; 2) a user with somewhat more limited computational facilities than the SST, but who nevertheless requires a reasonably high degree of accuracy; 3) a simplest class of user, who will use charts and make hand calculations to compute his position to within nominal accuracy requirements, and 4) user who is provided additional data to make his computations near-trivial.

Two kinds of constraints must be satisfied by an effective set of user equations: computational requirements must be such that 1) the computations can be performed by a reasonably small computer; 2) the estimates produced by them can achieve the desired degree of accuracy. The equations presented here consequently serve as inputs to two separate studies: 1) determination of the computer size necessary for actual implementation, and 2) analysis of the estimation accuracy of the filter equations.

3.1 THE KALMAN FILTER

The Kalman filter permits sequential computation of a minimum-variance estimate of the state of a linear, discrete-time system excited by a Gaussian white-noise random sequence. This filter has the added advantage that during the process of computing the estimate, it generates the covariance matrix of the estimation error.*

If it is desired to estimate the state of a system described by nonlinear difference equations, the Kalman filter may still be used if sufficiently good linear approximations to the nonlinear equations can be found.

*Several programs used in the error analysis portion of this study use the better known batch processing techniques known as weighted-least-squares. The prime advantage of batch processing is that it permits analysis of the effect of errors in parameters which are not estimated. For example, a user will not estimate the satellite position, but errors in these positions are important. While these effects can also be treated in the Kalman filter program, the method is cumbersome and somewhat inefficient with respect to computer time. The results are, of course, independent of whether a sequential or batch processing algorithm is used.
This is usually done by expanding the system equations at some sampling instant about the state estimate at the previous sampling instant and neglecting second and higher order terms. The Kalman filter equations presented here have been linearized in this manner. Since the derivation of these equations appears in many places in the technical literature, it will not be reproduced here.

The system whose state it is desired to estimate is described by the difference equations:

\[
\begin{align*}
    x(n+1) &= f(x(n)) + z(n) \\
    y(n) &= g(x(n)) + w(n)
\end{align*}
\]

where \(x(n)\) and \(y(n)\) are the dynamic state and measurement vectors with \(n_x\) and \(n_y\) components, respectively, \(f(\cdot)\) and \(g(\cdot)\) are \(n_x\) and \(n_y\) vector functions of \(x\), and \(z(n)\) and \(w(n)\) are zero-mean random vectors with covariance matrices:

\[
E \left[ z(n) z^T(m) \right] = Z(n) \delta_{mn}
\]

\[
E \left[ w(n) w^T(m) \right] = W(n) \delta_{mn}
\]

where \(\delta_{mn}\) is the Kronecker delta.

Define the \(n_x \times n_x\) matrix \(U\) by:

\[
U = \begin{bmatrix} \frac{\partial f_i}{\partial x_j} \end{bmatrix}
\]

and the \(n_y \times n_x\) matrix \(M\) by:

\[
M = \begin{bmatrix} \frac{\partial g_i}{\partial x_j} \end{bmatrix}
\]

where \(f_i\), \(x_i\) and \(g_i\) are the \(i\)th components of \(f\), \(x\) and \(g\), respectively.
We denote the estimate of the state at the $n^{th}$ sampling instant before the measurement is processed by $\hat{x}(n)$, and its error-covariance matrix by $J(n)$. The estimate after the $n^{th}$ measurement is processed is denoted by $\hat{x}(n^+)$, and the corresponding error-covariance matrix by $J(n^+)$. The estimate error-covariance matrix is first propagated from the $n^{th}$ to the $(n+1)^{th}$ sampling instant by:

$$J(n+1) = U(\hat{x}(n^+)) J(n^+) U^T(\hat{x}(n^+)) + Z(n)$$

The estimate is propagated by:

$$\hat{x}(n+1) = f(\hat{x}(n^+))$$

The predicted observation is:

$$\hat{y}(n+1) = g(\hat{x}(n+1))$$

The residual between actual and predicted observations is:

$$\Delta(n+1) = y(n+1) - \hat{y}(n+1)$$

The filter gain (weighting matrix of the residual) is:

$$K(n+1) = J(n+1) M^T(\hat{x}(n+1)) [M(\hat{x}(n+1)) J(n+1) M^T(\hat{x}(n+1)) + W(n+1)]^{-1}$$

The estimate is then updated by the $(n+1)^{th}$ measurement as:

$$\hat{x}(n+1^+) = \hat{x}(n+1) + K(n+1) \Delta(n+1)$$

Finally, the error-covariance matrix of the new estimate is obtained as:

$$J(n+1^+) = \left[I - K(n+1) M(\hat{x}(n+1))\right] J(n+1)$$

Estimates are computed sequentially in this manner; the filter is initialized with an a priori guess and an a priori error-covariance matrix.
We will now proceed to a description of the application of the equations written above to the determination of a NAVSTAR user's position and velocity.

3.2 EQUATIONS FOR HIGH ACCURACY

3.2.1 Data Received

The user receives three types of data:

1) At each 2-sec interval, a number \( R_i^* \) from which the range to the \( i \)th satellite is to be determined, and a measurement \( \dot{R}_i^* \) of the range rate between the user and the \( i \)th satellite.

2) At intervals greater than 2 sec, numbers \( b_1 \) and \( b_{11}' \) which are to be used to correct the range measurement for oscillator drift in the \( i \)th satellite.

3) At intervals greater than those for which oscillator drift corrections are sent, numbers \( \Delta \rho_i, \Delta \lambda_i \) and \( \Delta i_i \) which are to be used to correct the ephemeris of the \( i \)th satellite for drift from a circular orbit.

3.2.2 Sequence of Calculations

The measurements are to be processed in a simplified Kalman filter with peripheral logic. In general terms, the sequence of calculations shown in the flow chart of Figure 24 is the following:

1) Calculate the coordinates of the satellites in an earth-fixed rectangular system and the time derivatives of these coordinates from stored ephemeris data; the stored-ephemeris data are to be periodically corrected by the transmitted perturbations mentioned in (3) above.

2) Correct the measurement \( R_i^* \) for satellite-oscillator drift, and convert this corrected measurement into a range measurement.

3) Process the range measurement and the range-rate measurement in a Kalman filter, and obtain an estimate of the user position and velocity in an earth-fixed rectangular system.

4) Convert the estimate of (3) into an estimate of user latitude, longitude, altitude, position, and heading.
Figure 24. Filter Flow Chart for High Accuracy Navigation Equations
It should be emphasized at this point that we are processing a range measurement rather than a range-difference measurement. This introduces the necessity of solving for the bias $B_o$ (caused by the difference in turn-on time between the satellite and user oscillators) but eliminates the following problems which are present when range differences are used.

1) The noise on the range-difference measurements is correlated between measurements; optimally processing noise of this nature makes the filter equations very cumbersome.

2) The bias $B_o$ is actually not constant between measurements, but will change due to user oscillator drift; this introduces an error in the range differences. The nonconstancy of $B_o$ can easily be handled when processing range measurements by the addition of state noise. (See par. 3.2.3).

3) Processing ranges eliminates the necessity of having to decide which satellite ranges should be differenced to produce the range differences.

As indicated above, the range (suitably modified) and range-rate measurements are to be processed in a filter of the type described in subsec. 3.2 in which the (nonlinear) system equations have been linearized. Of course, the equations are linearized first about the a priori estimate and for succeeding calculations about the current state estimate. The possibility thus arises that if the a priori state estimate is not sufficiently close to the actual state the linearization of the equations will not be valid and the equations should be relinearized two or more times. Alternatively, the estimate could simply be propagated to the next measurement interval and a new measurement processed; after one or two measurements, the estimate should be sufficiently close to the actual value so that the linearization will be valid. This is the procedure used to avoid the necessity of several filter iterations with the first measurement.

We will now proceed to an expanded description of the filter calculations, and a detailed explanation of each. The computations performed in each block are stated and explained in the order in which the blocks are numbered in the flow chart, Figure 24.
Symbols used in the equations are as follows:

\[
\begin{align*}
\hat{x} & \text{ estimate of user’s coordinates in earth-fixed Cartesian system (computed)} \\
\hat{y} & \text{ estimate of Cartesian components of user’s velocity (computed)} \\
\hat{z} & \\
\hat{x'} & \\
\hat{y'} & \\
\hat{z'} & \\
X_i & \text{ Cartesian coordinates of } i^{th} \text{ satellite (computed)} \\
Y_i & \\
Z_i & \\
\dot{X}_i & \text{ velocity components of } i^{th} \text{ satellite (computed)} \\
\dot{Y}_i & \\
\dot{Z}_i & \\
\lambda & \text{ user’s longitude (input or computed)} \\
\phi & \text{ user’s latitude (input or computed)} \\
v & \text{ user’s velocity (input or computed)} \\
h & \text{ user’s altitude above sea level (input or computed)} \\
\psi & \text{ user’s heading east of north (input or computed)} \\
J_u & \text{ user’s a priori error-covariance matrix (input), or error-covariance matrix of current estimate (computed)}
\end{align*}
\]

\[J_u = \begin{bmatrix} J_p & J_{pv} \\ J_T & J_v \end{bmatrix} ; \quad J_p \text{ is } 4 \times 4, \quad J_v \text{ is } 3 \times 3\]
\( R_e \) radius of earth (input)

\( R_i^* \) range measurement from user to \( i^{th} \) satellite (transmitted)

\( r_i^* \) range-rate measurement between user and \( i^{th} \) satellite (transmitted)

\( b_{i0} \) bias on \( i^{th} \) satellite clock (transmitted)

\( b_{i1} \) drift rate on \( i^{th} \) satellite clock (transmitted)

\( \rho_{oi} \) nominal ephemeris of \( i^{th} \) satellite (input)

\( \lambda_{oi} \) \( i_{oi} \) \( t_i \) perturbations to nominal ephemeris (transmitted)

\( \Delta\rho_{oi} \) \( \Delta\lambda_{oi} \) \( \Delta i_{oi} \) current time (transmitted)

\( R_i^{*'} \) range measurement corrected for satellite clock drift (computed)

\( \hat{R}_i \) range computed from estimate of user position

\( \hat{\rho}_o \) bias in range measurement due to difference in initial phase between satellite and user clock (not known)

\( \hat{\rho}_{o} \) estimate of \( \rho_o \) from user filter (computed)

\( \hat{\rho}_{o} \) estimate of \( \rho_o \) used to correct range measurement for 2,000-mile ambiguity (computed)

\( R_i^{*''} \) range measurement corrected for satellite clock drift and 2,000-mile ambiguity (computed)

\( \Delta_{R_i} \) range measurement residual (computed)

\( \hat{R}_i \) range rate computed from estimate of satellite position and velocity
\[\Delta_D\] 
range rate residual (computed)

M 
measurement matrix (computed)

M is partitioned as:

\[
M = \begin{bmatrix}
M_1 & 0 \\
0 & M_2
\end{bmatrix}; \ M_1 \ is \ 1 \times 4, \ M_2 \ is \ 1 \times 3
\]

W 
covariance matrix of observation noise (input)
W is 2 x 2, and written:

\[
W = \begin{bmatrix}
w_{11} & w_{12} \\
w_{12} & w_{22}
\end{bmatrix}
\]

\[K_{p1}\] 
weight of range residual in position estimate
(4 x 1 matrix) (computed)

\[K_{p2}\] 
weight of range-rate residual in position estimate
(3 x 1 matrix) (computed)

\[K_{v1}\] 
weight of range residual in velocity estimate
(4 x 1 matrix) (computed)

\[K_{v2}\] 
weight of range-rate residual in velocity estimate
(3 x 1 matrix) (computed)

\[\hat{x} \text{ or } \hat{x}^+\] 
the 4 x 1 array:

\[
\begin{bmatrix}
\hat{x} \\
\hat{y} \\
\hat{z} \\
\hat{B}_o
\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}
\hat{x}^+ \\
\hat{y}^+ \\
\hat{z}^+ \\
\hat{B}_o^+
\end{bmatrix} \quad \text{(computed)}
\]
\( \hat{x} \) or \( \hat{x}^+ \) are quantities related to earth flattening (input)

\( a \) is the major axis of earth ellipsoid (input)

\( \omega \) is the earth rotation rate (input)

\( \hat{x} \) is an estimate of \( x \) before a measurement is processed

\( \hat{x}^+ \) is an estimate of \( x \) after a measurement is processed

I. INITIALIZE

\begin{align*}
\text{Input:} & \quad \lambda, \phi, v, h, \psi \\
\text{Input:} & \quad J_u = \begin{bmatrix} J_p & J_{pv} \\ J_{Tpv} & J_v \end{bmatrix} \\
\text{Calculate A Priori Estimates:} & \\
\hat{x} &= (R_e + h) \cos \phi \cos \lambda \\
\hat{y} &= (R_e + h) \cos \phi \sin \lambda \\
\hat{z} &= (R_e + h) \sin \phi \\
\hat{x} &= -v(\sin \phi \cos \lambda \cos \psi + \sin \lambda \sin \psi) \\
\hat{y} &= -v(\sin \phi \sin \lambda \cos \psi - \cos \lambda \sin \psi) \\
\hat{z} &= v(\cos \phi \cos \psi) \\
B_0 &= 0 \text{ nmi}
\end{align*}
In this box, the user calculates a priori estimates of his position and velocity in an earth-fixed rectangular system from an estimate of his latitude, longitude, altitude, mean-earth radius, velocity, and heading east of north. The quantities $\hat{x}, \hat{y},$ and $\hat{z}$ will be used in Block VI to aid in the resolution of range ambiguity.

The user also inputs an initial error-covariance matrix $J_u$.

Letting $\hat{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ \hat{\theta_0} \end{bmatrix}$ and $\hat{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix}$

then

$$J_p = E \left| \hat{X} \hat{X}^T \right|$$

$$J_{pv} = E \left| \hat{X} \hat{X}^T \right|$$

$$J_v = E \left| \hat{X} \hat{X}^T \right|$$

This matrix can be stored permanently in the user's computer and need not be input each time the filter is initialized.
RECEIVE MEASUREMENT

Each Two Second Interval:

\[ R_i^* \]
\[ \dot{R}_i^* \]
\[ t \]

Less Frequently:

\[ b_{i0} \]
\[ b_{i1} \]

Still Less Frequently:

\[ \Delta p_i \]
\[ \Delta \lambda_i \]
\[ \Delta i_i \]
\[ t_i \]

Every 2 sec the user receives a range measurement \( R_i^* \). (The subscript denotes the range and range rate to the \( i^{th} \) satellite).

The measurement \( R_i^* \) is related to the range \( R_i \) as follows:

\[ 0 \leq R_i^* \leq 2,000 \]

\[ R_i^* = R_i + B_o + \Delta_i + w_i - 2K_i \times 1,000. \]
Here,

\[ R_i = \text{actual range from user to } i^{\text{th}} \text{ satellite} \]
\[ B_o = \text{bias due to difference in turn-on time} \]
\[ \Delta_i = \text{error due to satellite oscillator drift} \]
\[ w_i = \text{white noise on range measurement} \]
\[ K_i = \text{a positive integer} \]

The modified measurement we wish to process in the filter is:

\[ R_i^{*'} = R_i + B_o + w_i \]

To obtain \( R_i^{*'} \), we must correct \( R_i^* \) for the error due to satellite drift (Block VIII), and determine the integer \( K_i \) (Block IX).

At intervals greater than 2 sec, numbers \( b_{10} \) and \( b_{11} \) are received which represent a bias and drift rate on the \( i^{\text{th}} \) satellite oscillator respectively and are to be used to correct for the error \( \Delta_i \).

At still greater intervals, numbers \( \Delta_{\rho_i}, \Delta_{\lambda_i}, \Delta_{i_1} \) are received which are perturbations to be applied to the nominal ephemeris data of the \( i^{\text{th}} \) satellite.
III and IV. CORRECT EPHEMERIS AND CALCULATE SATELLITE POSITION

**CALCULATE NEW EPHEMERIS**

- \( \rho_i = \rho_{oi} + \Delta \rho_i \)
- \( \lambda_i = \lambda_{oi} + \Delta \lambda_i \)
- \( i_i = i_{oi} + \Delta i_i \)

**CALCULATE SATELLITE POSITION AND VELOCITY**

\[
X_i = \frac{1}{2} \rho_i \left[ (1 + \cos i_i) \cos \lambda_i + (1 - \cos i_i) \cos (2\omega \tau_i - \lambda_i) \right]
\]

\[
Y_i = \frac{1}{2} \rho_i \left[ (1 + \cos i_i) \sin \lambda_i + (1 - \cos i_i) \sin (2\omega \tau_i - \lambda_i) \right]
\]

\[
Z_i = \rho_i \sin i_i \sin \omega \tau_i
\]

\[
X_i = -\omega \rho_i (1 - \cos i_i) \sin (2\omega \tau_i - \lambda_i)
\]

\[
Y_i = \omega \rho_i (1 - \cos i_i) \cos (2\omega \tau_i - \lambda_i)
\]

\[
Z_i = \omega \rho_i \sin i_i \cos \omega \tau_i
\]

(\( \tau_i = t - t_i \))
The perturbations received and their use to correct the ephemeris data are shown in the "Calculate New Ephemeris" block.

The satellite position and velocity are then calculated in an earth-fixed coordinate system (with $X$, $Y$ axes in the equatorial plane with $X$ axis at zero longitude and positive $Z$ axis pointing north). The present time $t$ is transmitted with the range and range-rate measurements.

These calculations assume a nominal circular orbit. The effects of this approximation on accuracy have not yet been assessed. However, the necessary analysis has been developed and is presented in app. L. Numerical results should be obtained in a future study.

V and VI. CHANGE BIASES AND CALCULATE RANGE CORRECTION

![Diagram of the process]

The measurement is next corrected for the error caused by drift in the $i^{th}$ satellite oscillator. The most recently transmitted drift bias $b_{io}$ and drift rate $b_{il}$ are used for this purpose.
VII. CALCULATE RESIDUALS

**CALCULATE RESIDUALS**

\[
\hat{R}_i = \left\{ (X_i - \hat{x})^2 + (Y_i - \hat{y})^2 + (Z_i - \hat{z})^2 \right\}^{1/2}
\]

\[
\frac{\hat{R}_i}{2,000} = K_i + \frac{\Delta \hat{R}_i}{2,000} \quad \{ K_i \text{ is an integer} \}
\]

\[
\Delta \hat{R}_i < 2,000
\]

\[
\hat{B}_o' = R_i'^* - \Delta \hat{R}_i
\]

**RANGE RESIDENTIAL**

\[
\hat{R}_i = \left\{ (X_i - \hat{x})^2 + (Y_i - \hat{y})^2 + (Z_i - \hat{z})^2 \right\}^{1/2}
\]

Round \( \{ \hat{R}_i + \hat{B}_o' - \hat{R}_i'^* \} \) to nearest integer multiple of 2,000.

Denote this by \( K_i \times 2,000 \).

\[
R_i'^** = R_i'^* + K_i \times 2,000
\]

\[
\Delta R = R_i'^** - \hat{R}_i - \hat{B}_o
\]

**RANGE-RATE RESIDUAL**

\[
\hat{R}_i = \frac{1}{\hat{R}_i} \left\{ (\hat{x} - X_i) \hat{x} + (\hat{y} - Y_i) \hat{y} + (\hat{z} - Z_i) \hat{z} + (X_i - \hat{x}) \hat{x}_i \
+ (Y_i - \hat{y}) \hat{y}_i + (Z_i - \hat{z}) \hat{z}_i \right\}
\]

\[
\Delta D = \hat{R}_i'^* - \hat{R}_i
\]
VIII and IX. MEASUREMENT MATRIX AND FILTER GAINS

CALCULATION OF MEASUREMENT MATRIX

Let: \[ M_1 = \begin{bmatrix} \frac{\hat{x} - x_I}{R_i} & \frac{\hat{y} - y_I}{R_i} & \frac{\hat{z} - z_I}{R_i} \\ \end{bmatrix} \]

Let: \[ M_2 = \begin{bmatrix} \frac{\hat{x} - x_I}{R_i} & \frac{\hat{y} - y_I}{R_i} & \frac{\hat{z} - z_I}{R_i} \\ \end{bmatrix} \]

CALCULATION OF FILTER GAINS

Let: \[ b_{11} = M_1 J_p M_1^T + w_{11} \]

Let: \[ b_{12} = M_2 J_{pv} T M_1^T + w_{12} \]

Let: \[ b_{22} = M_2 J_v M_2^T + w_{22} \]

Let \[ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \]

Invert \( B \): \[ B^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{12} & B_{22} \end{bmatrix} \]

Let: \[ K_{p1} = J_p M_1^T B_{11} + J_{pv} M_2^T B_{12} \]

Let: \[ K_{p2} = J_{pv} M_1^T B_{12} + J_{pv} M_2^T B_{22} \]

Let: \[ K_{v1} = J_{pv} M_1^T B_{11} + J_v M_2^T B_{12} \]

Let: \[ K_{v2} = J_{pv} M_1^T B_{12} + J_v M_2^T B_{22} \]
The calculations for the filter equations are performed in blocks VII through XI, beginning with the calculation of the range and range-rate residual. The first operation, shown in the "Calculate Residuals" box, is to correct the measurement $R_i^*$ for range ambiguity (justification for this correction is given in app. M). It is assumed that the bias $B_o$ will be approximately constant over the 16-sec measuring interval. At the beginning of the interval, an estimate $B_o^A$ is calculated from the first-range measurement (say, from the $i^{th}$ satellite) by the three steps shown. The estimate is used throughout the 16-sec interval to correct the range measurements for the 2000-mi ambiguity. The range and range-rate residuals are then calculated as shown.

The range and Doppler measurements are processed simultaneously, since the observation noise on each (from a given satellite) will usually be correlated. The measurement matrix $M$ has the form:

$$M = \begin{bmatrix}
\frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} & \frac{\partial R_i^*}{\partial B_o} & \frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} \\
\frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} & \frac{\partial R_i^*}{\partial B_o} & \frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} \\
\frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} & \frac{\partial R_i^*}{\partial B_o} & \frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} \\
\frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z} & \frac{\partial R_i^*}{\partial B_o} & \frac{\partial R_i^*}{\partial x} & \frac{\partial R_i^*}{\partial y} & \frac{\partial R_i^*}{\partial z}
\end{bmatrix}$$

If we assume

$$\frac{\partial R_i^*}{\partial x} = \frac{\partial R_i^*}{\partial y} = \frac{\partial R_i^*}{\partial z} = 0,$$

then $M$ has the block diagonal form

$$M = \begin{bmatrix}
M_1 & 0 \\
0 & M_2
\end{bmatrix}$$

with $M_1$ and $M_2$ as shown in the upper box.
X and XI. UPDATE ESTIMATE AND ERROR-COVARIANCE MATRIX, PROPAGATE ESTIMATE AND MATRIX

**ESTIMATE AND ERROR COVARIANCE MATRIX UPDATE**

Estimate Update:

\[
\hat{x}^+ = \hat{x}^0 + K_{p1} \Delta R + K_{p2} \Delta D
\]

\[
\hat{x} = \hat{x}^0 + K_{v1} \Delta R + K_{v2} \Delta D
\]

Error-Covariance Matrix Update:

\[
J_p^+ = [I - K_{p1} M_1] J_p - K_{p2} M_2 J_{pv}^T
\]

\[
J_{pv}^+ = [I - K_{p1} M_1] J_{pv} - K_{p2} M_2 J_v
\]

\[
J_v^+ = [I - K_{v2} M_2] J_v - K_{v1} M_1 J_{pv}
\]

**ESTIMATE AND ERROR COVARIANCE MATRIX PROPAGATION**

Estimate Propagation:

\[
\hat{x} = \hat{x}^+ + \hat{x} \Delta t
\]

\[
\hat{y} = \hat{y}^+ + \hat{y} \Delta t
\]

\[
\hat{z} = \hat{z}^+ + \hat{z} \Delta t
\]

Error-Covariance Matrix Propagation:

Let \( K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \)

\[
J_p = J_p^+ (K J_{pv}^+ T + J_{pv}^+ T + K J_v^+ K^T \Delta t^2
\]

\[
J_{pv} = J_{pv}^+ + K J_v^+ \Delta t
\]

\[
J_v = J_v^+
\]
The block diagonal form of $M$ makes partitioned computations of the filter gain particularly easy. Recall that the optimal filter gain (weight to be applied to the residuals) is:

$$K = J_u M^T \left[ M J_u M^T + W \right]^{-1}$$

If we let

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = M J_u M^T + W,$$

then in terms of partitions of $M$ and $J_u$, we have the results shown in the lower box.

The position and velocity estimates, before the current measurement is processed are combined with the range and range-rate residuals to produce the estimate update.

The error-covariance matrix of this updated estimate is

$$J_u^+ = \left[ I - K M \right] J_u = \begin{bmatrix} J^+_p & J^+_pv \\ J^+pv & J_v \end{bmatrix}$$

In terms of the partitions of $J_u$, $K$ and $M$, this has the form shown for the error-covariance matrix update.

Recalling that

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \\ B_0 \end{bmatrix}, \quad \mathbf{\dot{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$
and that the user is assumed to move in a straight line with constant velocity, the manner in which the position estimate propagates over the time interval between measurements is determined as follows:

Let

\[ H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \]

Then

\[ \hat{x} = \hat{x}^+ + H^T \hat{x}^+ \Delta t \]

where \( \Delta t \) = time interval between measurements

Of course,

\[ \hat{x} = \hat{x}^+. \]

The error-covariance matrix propagates as:

\[ J_u = \begin{bmatrix} J_p & J_{pv} \\ J_{pv}^T & J_v \end{bmatrix} = \begin{bmatrix} I & H\Delta t \end{bmatrix} J_u^+ \begin{bmatrix} I & O \\ O & I \end{bmatrix} \]

Formulas for the partitions are shown in the block.
LONGITUDE:
\[ \lambda = \tan^{-1} \left( \frac{\frac{\Delta}{x}}{\frac{\Delta}{y}} \right) \]

LATITUDE:
\[ \phi_o = \tan^{-1} \left( \frac{\frac{\Delta}{x}}{\frac{\Delta}{y}} \right) \]

Let \( v(\phi_o) = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi_o}} \)

Let \( f(\phi_o) = \frac{\cos \phi_o}{\sqrt{\Delta^2 + \frac{\Delta^2}{y^2}}} - \frac{\sin \phi_o}{\frac{\Delta}{z} + e^2 v(\phi_o) \sin \phi_o} \)

Let \( \phi_1' = \phi_o + \Delta \phi_o \)

COMPUTE \( v(\phi_1'), f(\phi_1') \)

\[ \phi = \phi_o - \frac{f(\phi_1') - f(\phi_o)}{f(\phi_o) \Delta \phi_o} \]

ALTITUDE:
\[ h = \frac{\Delta}{\cos \phi \sin \lambda} - v(\phi) \]

VELOCITY:
\[ v = \sqrt{\frac{\Delta^2}{x^2} + \frac{\Delta^2}{y^2} + \frac{\Delta^2}{z^2}} \]

HEADING:
\[ \psi = \cos^{-1} \left( \frac{\frac{\Delta}{x}}{\frac{\Delta}{y} \cos \phi} \right) \]
When the user calculates his latitude, longitude, and altitude, significant errors can result if earth aspheroidicity is neglected.

The equations for the user's coordinates in an earth-fixed rectangular system with \( x \) and \( y \) axes in the equatorial plane and \( x \) axis at zero longitude, and with positive \( z \) axis coincident with the polar axis and in the direction of the north pole, are (Ref. 7)

\[
x = (\nu(\phi) + h) \cos \phi \cos \lambda
\]
\[
y = (\nu(\phi) + h) \cos \phi \sin \lambda
\]
\[
z = \left[ (1 - e^2) \nu(\phi) + h \right] \sin \phi
\]

\[
\nu(\phi) = \frac{a}{\left[ 1 - e^2 \sin^2 \phi \right]^{1/2}}
\]

Here,

\( a \) = major axis of earth ellipsoid

\( e \) = eccentricity of earth ellipsoid

\( h \) = altitude perpendicular to earth ellipsoid

\( \phi \) = ellipsoidal latitude

\( \lambda \) = ellipsoidal (or geocentric) longitude

Eqs (35) and (36) are straightforwardly solved for \( \lambda \) as shown.

Eliminating \( \lambda \) from Eqs. (35) and (36), we get:

\[
\sqrt{x^2 + y^2} = (\nu(\phi) + h) \cos \phi
\]

Eliminating \( h \) from this and Eq. (37) gives:

\[
\frac{\cos \phi}{\sqrt{x^2 + y^2}} - \frac{\sin \phi}{z + e^2 \nu(\phi) \sin \phi} = 0.
\]
This transcendental equation is solved recursively for $\phi$. A first approximation is:

$$
\phi_o = \tan^{-1}\left(\frac{z}{\sqrt{x^2 + y^2}}\right)
$$

Letting

$$
f(\phi) = \frac{\cos \phi}{\sqrt{x^2 + y^2}} - \frac{\sin \phi}{z + e^2 \nu(\phi) \sin \phi}
$$

and

$$
\phi_1' = \phi_o + \Delta \phi_o \text{ (where } \Delta \phi_o \text{ is an input constant)},
$$

a second approximation is:

$$
\phi_1 = \phi_o - \frac{f(\phi_1') - f(\phi_o)}{f(\phi_o) \Delta \phi_o}
$$

This should be sufficiently close to the true value. The user altitude is then obtained straightforwardly as indicated.
3.2.3 Remarks

The major simplifying assumptions that have been made in the development of these equations are:

1) Satellite coordinates and velocities computed from ephemeris data are correct

2) The state equations assume a particularly simple form: the bias $B$ is constant, and the user moves with constant velocity during the 2 sec between measurements.

In addition, the measurement matrix has been simplified by neglecting the partial derivatives of the doppler measurement with respect to the user position coordinates.

Since fairly extensive satellite tracking facilities are available, assumption (1) is reasonably good. This will not necessarily be the case with assumption (2), since the user will invariably perform maneuvers of varying degrees of severity in the course of the flight. Also, the user clock will drift over long periods of time, so the bias $B_o$ will not be constant. Each of these effects can be approximately accounted for by the addition of state noise on the user velocities and on the bias $B_o$. This prevents the filter from putting too much weight on a priori estimate which is erroneous because of incorrect modeling.

It should be pointed out that the equations presented here are intended for a user with considerable computational facilities and reasonably high accuracy demands. Substantial refinements would be necessary for a very high accuracy user (such as a tactical bomber), and simplifications can be made for a user with less stringent accuracy demands (such as an ocean liner). The following section presents equations for the latter case.
3. 3 EQUATIONS FOR INTERMEDIATE ACCURACY

3. 3. 1 Discussion

The calculations for a set of filter equations which should satisfy the demands of a "simple user" are described here. A simple user is defined to be one who is moving fairly slowly (less than 30 mi/hr), has limited computational facilities, desires fixes relatively infrequently (no more often than every 15 min), requires no velocity estimate, but who, nevertheless, requires a reasonably high degree of accuracy. This simple user is divided into two classes:

1) Three measurements are available
2) More than three measurements are available.

User 1) is further divided as:

1-a) The measurements consist of two range differences and altitude above mean sea level
1-b) The measurements consist of three range differences.

User 2) may or may not have altitude information. The simple user processes range differences rather than ranges. The consequences of this were discussed in par. 2. 5. 3.

The simple user makes the following basic assumptions:

1) The satellite positions, as computed from transmitted ephemeris data, are correct.
2) A suboptimal filter which considers the measurement noise negligible is sufficiently accurate.

In addition, an assumption is made regarding satellite motion over the time interval when the two (or three) range-difference measurements are obtained. Range measurements are received at separate time instants and differenced to determine the range-difference measurement to be processed; consequently, this difference corresponds to the range difference between satellites at two distinct time instants. It will be assumed that the motion of the satellites and user over the measurement interval is small, so that these measurements can be considered to have occurred
simultaneously. The range rate of a satellite, relative to a stationary user, is of the order of 200 ft/sec, hence, the above assumption will be reasonably good if measurements occurring 2 sec apart are differenced.

An additional motivation for keeping the time interval small over which measurements are differenced is the following: the user oscillator will invariably drift, and this drift will be significant over long periods of time. If the time interval over which range measurements are differenced is too long, the user oscillator bias will not be completely cancelled in the differencing process and an erroneous range-difference measurement will result.

The measurements which the respective users thus process are the following:

User 1-a:

\[
\begin{align*}
\Delta_{ij} &= \sqrt{(x-X_i)^2 + (y-Y_i)^2 + (z-Z_i)^2} - \sqrt{(x-X_j)^2 + (y-Y_j)^2 + (z-Z_j)^2} \\
\Delta_{jk} &= \sqrt{(x-X_j)^2 + (y-Y_j)^2 + (z-Z_j)^2} - \sqrt{(x-X_k)^2 + (y-Y_k)^2 + (z-Z_k)^2} \\
h &= \sqrt{x^2 + y^2 + z^2} - R_e
\end{align*}
\]

User 1-b:

\[
\begin{align*}
\Delta_{ij} &= \sqrt{(x-X_i)^2 + (y-Y_i)^2 + (z-Z_i)^2} - \sqrt{(x-X_j)^2 + (y-Y_j)^2 + (z-Z_j)^2} \\
\Delta_{jk} &= \sqrt{(x-X_j)^2 + (y-Y_j)^2 + (z-Z_j)^2} - \sqrt{(x-X_k)^2 + (y-Y_k)^2 + (z-Z_k)^2} \\
\Delta_{kl} &= \sqrt{(x-X_k)^2 + (y-Y_k)^2 + (z-Z_k)^2} - \sqrt{(x-X_l)^2 + (y-Y_l)^2 + (z-Z_l)^2}
\end{align*}
\]

User 2:

- more than three range-difference measurements (of Type 1-b).
Each user will linearize these equations about some nominal value of \( x, y, z \), and solve. The solution will be performed iteratively using the Kalman filter equations, since this both permits relinearization after each measurement is processed, and affords a very convenient way of processing the redundant data of User 2.

3. 3. 2 Sequence of Calculations

The sequence of calculations is shown in the flow chart of Figure 25. The following pages show the computations performed in each block, accompanied by discussion where appropriate.

I. INITIALIZE

```
INITIALIZE

Input \( \lambda, \phi, h \)

(for User 1, \( h = 0 \); for User 2,
\( h = \) Altimeter Reading)

Calculate A Priori Estimates

\[
\hat{x} = (R_e + h) \cos \phi \cos \lambda \\
\hat{y} = (R_e + h) \cos \phi \sin \lambda \\
\hat{z} = (R_e + h) \sin \phi
\]

The user inputs an a priori estimate of his latitude \( \phi \), longitude \( \lambda \), and altitude \( h \). User 1 - a inputs \( h = 0 \), while Users 1 - b and 2 input an altimeter reading if one is available. The a priori estimates of user position are then calculated as shown.
```
Figure 25. Flowchart for Intermediate Accuracy Navigation Equations
II. RECEIVE MEASUREMENT

<table>
<thead>
<tr>
<th>RECEIVE MEASUREMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranges: $R_i^<em>$, $R_j^</em>$</td>
</tr>
<tr>
<td>Corrections for Satellite</td>
</tr>
<tr>
<td>Clock Drift: $b_{i0}$, $b_{i1}$, $b_{j0}$, $b_{j1}$</td>
</tr>
<tr>
<td>Time $t$</td>
</tr>
<tr>
<td>Ephemeris Perturbations: $\Delta \alpha_i$, $\Delta \lambda_i$, $\Delta \tau_i$</td>
</tr>
<tr>
<td>User I-(a): Altitude $h$</td>
</tr>
</tbody>
</table>

All users will receive range measurements and difference them; measurements received over consecutive time intervals should be differenced. User 1-a considers his altimeter reading as a measurement and processes it accordingly. In addition, users have stored biases and drift rates on all satellite clocks involved in the range-difference measurement. These are periodically retransmitted. At intervals, the user also receives corrections to the nominal stored ephemeris data.
III. CALCULATE SATELLITE POSITION

**CALCULATE SATELLITE POSITION**

Correct Ephemeris:

\[
\rho_i = \rho_{0i} + \Delta \rho_i \\
\lambda_i = \lambda_{0i} + \Delta \lambda_i \\
\iota_i = \iota_{0i} + \Delta \iota_i
\]

Calculate Satellite Position:

\[
x_i = \frac{1}{2} \rho_i \left[ (1 + \cos \iota_i) \cos \lambda_i + (1 - \cos \iota_i) \cos(2\omega \tau_i - \lambda_i) \right]
\]

\[
y_i = \frac{1}{2} \rho_i \left[ (1 + \cos \iota_i) \sin \lambda_i + (1 - \cos \iota_i) \sin(2\omega \tau_i - \lambda_i) \right]
\]

\[
z_i = \rho_i \sin \iota_i \sin \omega \tau_i \quad (\tau_i = t - t_i)
\]

The first step in the estimate determination is the calculation of the positions of the satellites involved in the difference measurements, using the most recently corrected ephemeris data. The orbit parameters \(\rho_i\), \(\lambda_i\), and \(\iota_i\) are corrected if the corresponding perturbations have been received. For a range difference between satellite \(i\) and \(j\), for example, the coordinates of the \(i^{th}\) satellite are calculated as shown, and those of the \(j^{th}\) satellite are determined in the same way.
IV. RANGE-DIFFERENCE COMPUTATION

**COMPENSATE RANGE DIFFERENCE FOR SATellite OSCILLATOR DRIFT**

\[ \Delta = R_i^* + b_{10} + b_{11} \tau_i - R_j^* - b_{j0} - b_{j1} \tau_j \]

The range-difference measurement shown above is computed and corrected for the oscillator drift of satellites \( i \) and \( j \).

V. RANGE-DIFFERENCE AMBIGUITY RESOLUTION AND RESIDUAL CALCULATION

**RESIDUAL**

\[ \hat{R}_i - \hat{R}_j = \sqrt{(x_i - \hat{x})^2 + (y_i - \hat{y})^2 + (z_i - \hat{z})^2} \]

\[ - \sqrt{(x_j - \hat{x})^2 + (y_j - \hat{y})^2 + (z_j - \hat{z})^2} \]

Round \( \hat{R}_i - \hat{R}_j - \Delta \) To Nearest Multiple of 2,000.

Call this \( K \times 2,000 \):

\[ \Delta_R = K \times 2,000 + \Delta - (\hat{R}_i - \hat{R}_j) \]

The corrected measurement \( \Delta \) calculated in the previous box must still be adjusted for the range-difference ambiguity. This is done by first calculating a range difference \( \hat{R}_i \) based on the a priori (or current) estimate of the user's position. The difference is then rounded and corrected for the 2000-mi ambiguity and the range-difference residual \( \Delta_R \) is computed as shown. (See app. M for justification of this procedure.)
VI. MEASUREMENT MATRIX CALCULATION

**MEASUREMENT MATRIX**

\[
M_i = \frac{1}{\hat{R}_i} [\delta - x_i, \gamma - y_i, z - z_i]
\]

\[
M_j = \frac{1}{\hat{R}_j} [\delta - x_j, \gamma - y_j, z - z_j]
\]

\[
M_{ij} = M_i - M_j
\]

User 1 Processing Altitude

\[
M_h = \frac{1}{\sqrt{\delta^2 + \gamma^2 + z^2}} [\delta, \gamma, z]
\]

For a range-difference measurement between the \(i^{th}\) and \(j^{th}\) satellites, the measurement matrix has the form \(M_{ij} = M_i - M_j\). User 1-a processes his altimeter reading as a measurement, using the measurement matrix \(M_h\) shown in the box.

VII. FILTER GAIN CALCULATION

**FILTER GAINS**

\[
K_{ij} = J M_{ij}^T \left[ M_{ij} J M_{ij}^T \right]^{-1}
\]

User 1-a Processing Altitude:

\[
K_h = J M_h^T \left[ M_h J M_h^T \right]^{-1}
\]
The filter gains are the same as those which would be used for an optimal filter with no observation noise and have the form shown. When user 1-a precesses his altimeter reading, he uses a gain of $K_h$ of the form shown.

VIII. ESTIMATE AND COVARIANCE MATRIX UPDATE

<table>
<thead>
<tr>
<th>ESTIMATE AND COVARIANCE MATRIX UPDATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x}^+ = \hat{x} + K_{ij} \Delta R$</td>
</tr>
</tbody>
</table>

For User 1-a: $\hat{x}^+ = \hat{x} + K_h h$

$J^+ = [I - K_{ij} M_{ij}]J$

User 1-a Processing Altitude:

$J^+ = [I - K_h M_h]J$

The updated estimates are calculated using the residuals and the gains as $\hat{x}^+$ in the box for each range measurement and for user 1-a altitude measurement. The error-covariance matrices are then updated as shown for the range-difference and altitude measurements.
LONGITUDE:

\[ \lambda = \tan^{-1} \left( \frac{y}{x} \right) \]

LATITUDE:

Let \( \varphi_o = \tan^{-1} \left( \frac{y}{\sqrt{x^2 + y^2}} \right) \)

Let \( v(\varphi_o) = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi_o}} \)

Let \( f(\varphi_o) = \frac{\cos \varphi_o}{\sqrt{x^2 + y^2}} - \frac{\sin \varphi_o}{\sqrt{1 - e^2 v(\varphi_o) \sin \varphi_o}} \)

Let \( \varphi'_1 = \varphi_o + \Delta \varphi_o \)

Compute \( v(\varphi'_1), f(\varphi'_1) \)

\[ \varphi = \varphi_o - \frac{f(\varphi'_1) - f(\varphi_o)}{f(\varphi_o) \Delta \varphi_o} \]

ALTITUDE:

\[ h = \frac{\varphi}{\cos \varphi \sin \lambda} - v(\varphi) \]
In the calculation of user latitude, longitude, and altitude a correction is made for earth aspheroidicity. The equations which must be inverted are (Ref. 7).

\[
\begin{align*}
\hat{x} &= (\nu(\phi) + h) \cos \phi \cos \lambda \\
\hat{y} &= (\nu(\phi) + h) \cos \phi \sin \lambda \\
\hat{z} &= \left[1 - e^2\right] \nu(\phi) + h \sin \phi
\end{align*}
\]

Here

\[
\nu(\phi) = \frac{a}{\left[1 - e^2 \sin^2 \phi\right]^{1/2}}
\]

\(e\) is the eccentricity and \(a\) the major axis of the earth ellipsoid.

The values of \(\lambda\) and \(\phi\) are then obtained as shown. When a sufficiently good approximation of \(\phi\) has been obtained, the altitude is solved for as indicated.

3.4 PROCEDURE FOR HAND CALCULATION WITH SIMPLIFIED EQUATIONS

This section presents a sequence of hand calculations from which a simple user can obtain latitude and longitude. The computations may be performed with a desk calculator, trigonometric tables, and a chart from which satellite coordinates may be determined. The equations are essentially those of subsec 3.3, with the exception of the charts for determining the satellite coordinates. The time required for the fix calculation should be of the order of 15 minutes.

3.4.1 Equations to be Solved

The user considered here is assumed to have the following equipment:

1) A desk calculator with square root capability
2) A table of sines, cosines, and tangents
3) A table from which satellite rectangular coordinates may be determined from transmitted ephemeris data.

With the use of this equipment, the user is to determine his latitude and longitude from two range differences and an altitude measurement.

To do this, he performs essentially the calculations stated and explained in subsec 3.3. The only difference is the manner in which the satellite's rectangular coordinates are determined. Rather than use the equations of par. 3.3.2 to calculate satellite X, Y, Z coordinates from transmitted ephemeris data and current time, a table is used containing the X, Y, Z coordinates tabulated as a function of time from the satellite equatorial crossing. In addition, he will iterate only once.

The equations the user must solve are:

\[
\Delta_{ij} = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} - \sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2}
\]

\[
\Delta_{jk} = \sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2} - \sqrt{(x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2}
\]

\[h = \sqrt{x^2 + y^2 + z^2} - R_e\]

This is reduced to two equations in x and y by solving for z.

\[z = \pm \sqrt{(h + R_e)^2 - x^2 - y^2}\]

The plus sign is for the northern hemisphere users, the minus sign for southern hemisphere users. This is then substituted in the first two equations. The partial derivatives used to solve the linearized equations are:

\[
\frac{\partial \Delta_{ij}}{\partial x} = \frac{1}{R_i} \left[ \frac{Z^i}{z} - X_i \right] - \frac{1}{R_j} \left[ \frac{Z^j}{z} - X_j \right]
\]
where the a priori estimate of \(x, y, z\) is used.

The detailed sequence of computations the user must perform is given in the next paragraph.

3.4.2 Sequence of Calculations

1) Receive range measurements \(R_i^*, R_j^*\).

2) Correct range measurements and calculate range differences:

\[
\Delta_{ij} = R_i^* + b_{i0} + b_{i1} \tau_i - R_j^* - b_{j0} - b_{j1} \tau_j
\]

\[
\Delta_{jk} = R_j^* + b_{j0} + b_{j1} \tau_j - R_k^* - b_{k0} - b_{k1} \tau_k
\]

3) Determine satellite coordinates from table.

4) Determine a priori latitude and longitude from map and calculate coordinate estimates:

\[
\hat{x} = (R_e + h) \cos \phi \cos \lambda
\]

\[
\hat{y} = (R_e + h) \cos \phi \sin \lambda
\]

\[
\hat{z} = (R_e + h) \sin \phi
\]
5) Compute a priori range estimates:

\[ \hat{R}_i = \sqrt{(\hat{x} - X_i)^2 + (\hat{y} - Y_i)^2 + (\hat{z} - Z_i)^2} \]

\[ \hat{R}_j = \sqrt{(\hat{x} - X_j)^2 + (\hat{y} - Y_j)^2 + (\hat{z} - Z_j)^2} \]

\[ \hat{R}_k = \sqrt{(\hat{x} - X_k)^2 + (\hat{y} - Y_k)^2 + (\hat{z} - Z_k)^2} \]

6) Resolve ambiguity (app. M)

Round \( \hat{R}_i - \hat{R}_j - \Delta_{ij} \) to nearest multiple of 2,000 (say \( k \times 2,000 \))

Form \( \Delta_{ijR} = k \times 2,000 + \Delta_{ij} - (\hat{R}_i - \hat{R}_j) \)

Calculate \( \Delta_{jkR} \) similarly.

7) Compute partials:

\[ b_{11} = \frac{1}{\hat{R}_i} \left[ \frac{Z_i}{\hat{x}} - X_i \right] - \frac{1}{\hat{R}_j} \left[ \frac{Z_j}{\hat{x}} - X_j \right] \]

\[ b_{12} = \frac{1}{\hat{R}_i} \left[ \frac{Z_i}{\hat{y}} - Y_i \right] - \frac{1}{\hat{R}_j} \left[ \frac{Z_j}{\hat{y}} - Y_j \right] \]

\[ b_{21} = \frac{1}{\hat{R}_j} \left[ \frac{Z_j}{\hat{x}} - X_j \right] - \frac{1}{\hat{R}_k} \left[ \frac{Z_k}{\hat{x}} - X_k \right] \]

\[ b_{22} = \frac{1}{\hat{R}_j} \left[ \frac{Z_j}{\hat{y}} - Y_j \right] - \frac{1}{\hat{R}_k} \left[ \frac{Z_k}{\hat{y}} - Y_k \right] \]
8) Compute corrections to a priori estimate (δx, δy):

\[
\delta x = \frac{1}{b_{11} b_{22} - b_{12} b_{21}} \left( b_{22} \Delta_{ijR} - b_{12} \Delta_{jkR} \right)
\]

\[
\delta y = \frac{1}{b_{11} b_{22} - b_{12} b_{21}} \left( b_{21} \Delta_{ijR} + b_{11} \Delta_{jkR} \right)
\]

\[
\hat{z} + \delta z = \pm \sqrt{(R_e + h)^2 - (x + \delta x)^2 - (y + \delta y)^2}
\]

9) Calculate latitude and longitude:

\[
\lambda = \tan^{-1} \left( \frac{\hat{y} + \delta y}{\hat{z} + \delta x} \right)
\]

\[
\phi = \sin^{-1} \left( \frac{\hat{z} + \delta z}{R_e + h} \right)
\]

This concludes the calculations the user must perform to determine his fix.

3.5 SIMPLEST USER HARDWARE EQUATIONS

The preceding computations are rather involved and the more complex sets require considerable computational equipment by the user. However, it may be observed that for any small region on the earth or in near earth space, very simple functional relationships may be used to derive user position in spherical earth-centered coordinates from the range-difference measurements. Furthermore, these simplified computations for angular position are essentially independent of altitude and, hence, for those cases where conventional methods of measuring altitude are adequate for navigation, only two pairs of satellites (or a minimum of three satellites) are required for a navigation solution. Any additional measurements available from other pairs of satellites can then be used as redundant measurements to increase the accuracy of the computed position fix.
In the simplified satellite hyperbolic navigation scheme described here, the mathematical function that is used to relate the user's measurements to his position is a power series expansion in the range-difference measurements about a reference point of known location. The degree of the polynomial used in this expansion depends on the accuracy required for the navigation fix and the user's distance from the reference point. Navigational accuracy by this scheme is also influenced by the number of sets of range differences used in the solution, i.e., the number of satellites visible, and their geometry.

In the simplest situation, the following equations suffice:

\[ \Delta_{NS} = k_1 + k_2 \Delta R_1 + k_3 \Delta R_2 + \Delta R_3 \]

\[ \Delta_{EW} = k_5 + k_6 \Delta R_1 + k_7 \Delta R_2 + k_8 \Delta R_3 \]

where \( k_1, k_2, k_3, \ldots \) etc. are constants applicable to a particular grid transmitted by a satellite prior to the user's position computation. (These constants are used in lieu of satellite ephemeris and satellite oscillator drift correction data which must be transmitted for the complete hyperbolic solution.) \( \Delta R_1, \Delta R_2, \) and \( \Delta R_3 \) are the measured range differences between three pairs of the four visible satellites. If \( \Delta_{NS}, \Delta_{EW}, \) and \( \Delta R \)'s are given in units of nautical miles, typical values for \( k_1 \) and \( k_5 \) are in the range of 0 to 2000 nmi, and typical values for the other \( k \)'s are between 0.5 and 3. The latter terms are sometimes referred to as the geometric dilution of precision (GDOP) factors since they transform the hyperbolic measurements into map coordinates. This technique can provide 1-nmi accuracy over a 12,000,000 sq mi region of the earth. The data rate required to transmit these constants, assuming users desire a fix every 5 min is only 60 b/sec. This technique provides significant computation reduction and thus has a large cost advantage over the more conventional techniques previously discussed.
4. RELATED STUDIES

4.1 EFFECTS OF GRAVITATIONAL PERTURBATIONS ON STATION-KEEPING AND COVERAGE

A repeating ground-track satellite is subject to orbital disturbances caused by repeated passage over the same features on a planet. The motion caused by these disturbances is libration, a free oscillation of the ascending node about a stable point on the equator, with an amplitude equal to its initial displacement from the stable point (Ref. 8). By means of the RESORB program (app. N), the effect of libration on eight satellites spaced at 45° intervals along a 24-hr circular orbit was determined. The resulting characteristic velocity requirement to maintain position within 5° and 3° deadband limits was computed. The maximum velocity required is about 30 ft/sec, essentially independent of the deadband. Individual corrections are of the order of 1 to 3 ft/sec every 6 to 7 months.

A second cause of orbit perturbation is the out-of-plane gravitational force due to the sun and moon. This can cause a small shift in the orbital inclination, a maximum of about 4° during the 5-yr satellite lifetime. This is acceptable for purposes of the proposed system, and can be held to a lower value by appropriate launch timing.

4.1.1 In-Plane Effects

RESORB runs were made to investigate effects on eight satellites initially distributed uniformly in a 24-hr, 18.5° inclined circular orbit. Figure 26 shows the time history of libration of these satellites. Due to the J_{22}, J_{31}, J_{33}, J_{42}, J_{44} tesseral harmonics, the longitude of the ascending node does not stay constant, but exhibits a libration with amplitude equal to the initial separation from the stable nodes which are at about 77° and 257°. This motion of the longitude of the ascending node can best be understood by imagining a roller-spring-hoop system as

---

*This refers to total travel, not plus or minus; i.e., 5° deadband limits means nominal longitude ±2.5°.
Figure 26. Libration Due to \( J_{22}, J_{31}, J_{42}, J_{44}, J_{33} \)
shown in Figure 27. The lows of the hoop correspond to the stable nodes and represent potential wells in the gravitational field. The number of lows and highs is defined by \( m \) in \( J_{1m} \), but their orientation (with respect to Greenwich) depends on both \( \ell \) and \( m \). For circular 24-hr orbits, \( J_{22} \) dominates, resulting in the rather regular motion shown in Figure 26. This is not at all the case with eccentric orbits. The period of small amplitude libration is shown in Figure 28. For large amplitudes, these periods must be multiplied by a complete elliptical integral of the first kind (modulus = amplitude) to obtain the periods shown in the previous figure.

Figures 29a and b present libration histories up to a maximum of \( 5^\circ \) displacement for eight satellites, with ascending nodes as indicated on the figure and spaced at \( 45^\circ \) intervals. It can be seen that the time to drift \( 3^\circ \) is from 60 to 93 days and to drift \( 5^\circ \) is from 50 to 118 days. The velocity, \( \Delta V \), required to reverse this motion, is shown for both \( 3 \) and \( 5^\circ \) drifts and is repeated in Figures 30 and 31 to indicate the effect of initial longitude on stationkeeping requirements. The total \( \Delta V \) requirement over the 5-year satellite lifetime as a function of longitude of the ascending node is shown in Figure 32. The result is a maximum requirement of 30 ft/sec, with reduced velocities in the vicinities of the stable and unstable nodes.

Figure 33 shows the effect of libration on the relative position of four satellites over a period of 120 orbits. In the absence of resonance, all four satellites would stay at their initial longitude and latitude. In this case, only the fourth satellite stayed close to its initial position, which was very close to an unstable node \( (347^\circ) \). The acceleration at this point is very small; thus, it takes a long time to leave the vicinity of the unstable node. (Given enough time, however, the amplitude of this satellite would be the largest.) The positions when the satellites reach \( 5^\circ \) deviation from their original location are also marked. After 85 days one of the satellites will have shifted \( 5^\circ \) and the others lesser amounts depending on their initial longitudes.
Figure 27. Simulation of the Gross Characteristics of Libration

Figure 28. Libration Periods of 24-Hr Circular Orbits

\begin{align*}
J_{22} &= 1.72 \times 10^{-6} \lambda_{22} = -13^\circ \\
J_{31} &= 1.86 \times 10^{-6} \lambda_{31} = 43^\circ \\
J_{33} &= 0.165 \times 10^{-6} \lambda_{33} = 16^\circ \\
J_{42} &= 0.153 \times 10^{-6} \lambda_{42} = 27^\circ \\
J_{44} &= 0.465 \times 10^{-6} \lambda_{44} = 36^\circ 
\end{align*}
Figure 29a. Libration Histories for 24-Hr 18.5° Inclined Circular Orbits with Various Ascending Nodes
Figure 29b. Libration Histories for 24-Hr 18.5° Inclined Circular Orbits with Various Ascending Nodes
Figure 32. Total Characteristic Velocity Requirement for 5-Yr In-Plane Stationkeeping

Figure 33. Positions of Four Synchronous Satellites After the Same Number of Orbits
The effect on coverage is shown in Figure 34 for time $T_0$. The change in coverage, seen by comparing this figure with Figure 35* (the clear overlay), is small and is primarily in longitude. Similar maps for times $T_{45}$, $T_{90}$, $T_{135}$, and $T_{180}$ were plotted, indicating that the effect of in-plane drift is of the same order of magnitude for these times. The regions where only two satellites are visible expand to a maximum of $5^\circ$ in longitude, with negligible latitude change.

In order to preserve the desired satellite constellation, it is necessary to provide in-plane stationkeeping within some deadband region. With some stationkeeping methods, it is possible for two or more satellites to approach deadband limits simultaneously, which may have an adverse effect on coverage. For example, it was found that if two satellites reached a $5^\circ$ deadband limit simultaneously, the coverage at $T_{45}$ would have regions of indeterminacy (only two satellites visible) extending below $58^\circ$ latitude. Therefore, it may prove desirable to set deadband limits somewhat lower than $5^\circ$ or, alternatively, to use station-keeping logic that prevents two or more satellites from approaching the limits simultaneously.

4.1.2 Out-of-Plane Effects

Earth oblateness, the sun, and the moon exert a torque on the orbital momentum of the satellite. The result is a regression of the line of the nodes and a periodic change of the orbit plane. Figure 36 demonstrates the combined effect of these perturbations on orbits with varying initial inclination. Inclination is plotted along the radius and $\Omega$, the right ascension of the node, in the circumferential direction. All curves start at $\Omega = 180^\circ$, with tick marks at 2-yr intervals. The 10-yr points are connected by dashed lines. Initially, the heliocentric longitude of the ascending node of the moon was $\Omega_M = 0$, which corresponds to Julian date 2440310 (30 March 1969).

*This transparency can be found in the pocket on the inside of the back cover.
Figure 34. In-Plane Resonance Effects On Coverage at T₀.
Figure 37 was obtained from Figure 36 by starting at an inclination of 18.5° at $\Omega = 0$, 90°, 180°, 270° and following the trend for 5 yr. It can be seen that for $\Omega = 0$ and 270°, the inclination increases 4° and 1°, respectively.

Figure 36 was generated with the moon's initial longitude at zero. Similar curves were generated at TRW with $\Omega_M = 90°$, 180°, and 270°. The greatest difference is for $\Omega_M = 180°$ and, on Figure 37, the dashed lines represent regression based on $\Omega_M = 180°$. The variation is rather small. Although the influence of the date can be evaluated with the complete set of charts, it is easier to make a RESORB run for any chosen date and obtain the variations with eight figure accuracy. Figures 36 and 37, however, demonstrate the results of these perturbations rather clearly.

The effect of inclination change on coverage was determined for a slightly pessimistic value of 4.3°. It was assumed that the orbit planes were positioned initially at 2.15° below the 18.5° nominal value (i.e., at 16.34°) and that, after 5 yr, they had drifted apart to final inclinations of 20.65°. Figure 38 indicates the coverage to be expected under these conditions at time $T_0$. Comparison with Figure 34 shows the small effect on coverage. Furthermore, it is possible to attain substantially lower values by selecting appropriate launch times. It is therefore concluded that out-of-plane stationkeeping is not required.

Figure 39 shows how resonance affects satellites whose orbital periods differ slightly from 24 hr. The lower curve corresponds to a repeating ground-track orbit (in the absence of tesseral harmonics); it librates with a period of about 1000 days and an amplitude of about 47°. The next curve corresponds to an orbit whose longitude of the ascending node drifts at a rate of 0.75° per day. It can be seen that the motion (called circulation) is related to that of an overturning pendulum with an amplitude of irregularity of about 2° and a period of about 240 days. The third curve corresponds to an orbit with 1° per day nodal drift rate. The period of circulation is 180 days and the amplitude is about 1°.

Slowly drifting orbits provide the benefit of greatly reduced effects of libration and, hence, require no stationkeeping. A disadvantage of this scheme, however, is the increased difficulty of keeping track of the
Figure 36. Out-of-Plane Perturbation Effects
Figure 37. Luni-Solar Effects on Orbital Inclination Over 5-Yr Satellite Lifetime
system and arranging for hand-over between tracking stations. Also, stationkeeping requirements are not particularly severe for a 24-hr system, so the drifting system has not been considered further.

4.2 SATELLITE ECLIPSE PERIODS

Satellite eclipse duration is important from a satellite design standpoint in that it affects the power supply design and the radiant heat lost through the spacecraft skin. An eclipse of the satellite is defined as the passage of spacecraft through the umbra and/or penumbra created on the dark side of the earth. The eclipse season is defined to be the number of consecutive days that the spacecraft experiences an eclipse during each successive revolution. For a satellite in a circular orbit, there will be no eclipse seasons or there will be two eclipse seasons during the year.

The condition of no eclipses requires specific combinations of spacecraft altitude, orbit-plane inclination, and injection node which do not occur in the TRW navigation satellite system.
An eclipse on every revolution occurs when the inclination of the spacecraft orbit plane to the ecliptic plane is less than the angular radius of the earth shadow at the orbit altitude; as with the completely sunlit orbit, this case requires specific ranges of inclination, altitude, and injection node. For the proposed navigation satellite system, there is a range of injection nodes approximately 41° wide that will produce the continual eclipse cycle. The positions of these bands are dependent upon whether the orbital inclination is positive or negative.

A computer program was used to obtain the eclipse seasons and durations. A spherical earth and unperturbed orbits were used to minimize the cost of obtaining these data. The equations are presented in app. O.

The maximum eclipse duration is the same for all the spacecraft in the system, since for this system it is a function of orbit altitude only. Twice each year each spacecraft experiences a maximum of 70.5 min of eclipse duration per revolution. Since the maximum eclipse durations are all the same, it is necessary only to define the eclipse seasons to see the variation of eclipse duration for each satellite throughout the season. The eclipse seasons and, hence, the duration of eclipses during eclipse season are a function only of the injection node (measured from vernal equinox). This, in turn, makes both the season and eclipse duration functions of time-of-day at injection for any specific date.

The eclipse seasons are presented in Figures 40 and 41 as a function of injection node for +18.5° and -18.5° inclination. There are two eclipse seasons during the year. When one season is less than one-half year (182.7 days), the spacecraft also experiences two seasons of no eclipse during the year. Conversely, for the narrow injection node bands producing half-year eclipse seasons, the spacecraft enters one eclipse season directly from another, with no periods of complete orbital sunlight.

The eclipse durations for eclipse seasons less than 182.7 days are presented in Figure 42 as a function of the fraction of season length into the season. In this manner, Figures 40 through 42 may be combined to produce the eclipse durations as a function of eclipse season time, simply
Figure 40. Eclipse Season as Function of Injection Node for +18.5° Inclined 24-Hr Circular Orbit

Figure 41. Eclipse Season as Function of Injection Node for -18.5° Inclined 24-Hr Circular Orbit
by multiplying the fractional part of the season (Figure 42) by the total season length from Figures 40 and 41. This method of presenting the data eliminates the necessity of presenting data for all possible injection conditions.

Eclipse seasons lasting a full half year require a different method of presentation; the season length is the same for all seasons in this category, whereas the minimum duration of eclipse varies as a function of the injection node. In Figures 43a and b, the minimum eclipse durations are presented as functions of injection node for $+18.5^\circ$ and $-18.5^\circ$ inclination. Figure 44 presents an eclipse duration ratio as a
Figure 43. Minimum Eclipse Duration for Continual Seasons

Figure 44. Eclipse Duration Ratio for Continual Eclipse Season
function of time into the eclipse season. To understand the use of the eclipse duration ratio, the following definitions are made:

\[ T_{\text{MAX}} = \text{maximum eclipse duration (70.5 min for the system as proposed)} \]
\[ T_{\text{MIN}} = \text{minimum eclipse duration, which is a function of the injection node and is obtained from Figure 43.} \]
\[ T_{\text{ECL}} = \text{eclipse duration at any time during the season.} \]

\[
\text{Eclipse duration ratio} = \left( \frac{T_{\text{ECL}} - T_{\text{MIN}}}{T_{\text{MAX}} - T_{\text{MIN}}} \right) = \left( \frac{T_{\text{ECL}} - T_{\text{MIN}}}{70.5 - T_{\text{MIN}}} \right)
\]

With these definitions, Figures 43 and 44 may be combined to produce the eclipse durations at any time during the eclipse season for those injection nodes producing continuous seasons by first finding the value of \( T_{\text{MIN}} \) from Figure 43 for any injection node under consideration. The eclipse duration at any time in the season, then, is found by obtaining the eclipse duration ratio from Figure 44 and

\[ T_{\text{ECL}} = (\text{eclipse duration ratio})(T_{\text{MAX}} - T_{\text{MIN}}) + T_{\text{MIN}} \]

Although this presentation at first appears more awkward to utilize than the method used for the two distinct seasons, it regains some simplicity when it is realized that the ordinate of Figure 44 becomes the fractional part of maximum minus minimum eclipse duration.

To complete the analysis, solar time of injection as functions of time of year and injection node are presented in Figure 45. With this figure, it is possible to specify the time of injection (and, hence, launch time) to meet any eclipse season and/or eclipse duration specified.

The accuracy of this analysis is limited solely by the use of unperturbed orbits and a spherical earth. The effects of an aspherical earth are such that the maximum eclipse duration becomes a function of the time of day of injection, but the variation is less than 5 percent. The effects of the orbit perturbations consist primarily of a slight distor-
Figure 45. Local Solar Time as Function of Day of Year and Nodal Position
tation of the symmetry of the eclipse season. These effects are negligible during this phase of the study, and become factors only when the system requirements become well defined and a launch date approaches.

4.3 SELECTION OF INJECTION NODES

From a spacecraft thermal design standpoint, it is desirable for the eclipse seasons to be as short as possible and for all spacecraft in the system to experience the same eclipse durations and seasons. For the electrical power system, it is desirable to have the orbit planes as close to the plane of the ecliptic as possible to obtain an angle of incidence of the sun's rays as nearly normal as possible. Power system design is also simplified if all spacecraft in the system receive solar radiation at the same angle of incidence.

These factors are affected by the injection nodes chosen; an analysis was made to determine the most favorable injection nodes with respect to the above requirements. With the given constraints of 18.5° orbit-plane inclination and 157.5° nodal separation, it was found that the injection nodes shown in Table XXVIII yielded the minimum inclination angle to the ecliptic plane, with the associated eclipse parameters as shown.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Plane 1</th>
<th>Plane 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right ascension of injection node</td>
<td>281.25°</td>
<td>78.75°</td>
</tr>
<tr>
<td>Inclination to equator</td>
<td>18.5°</td>
<td>18.5°</td>
</tr>
<tr>
<td>Inclination to ecliptic</td>
<td>26.54°</td>
<td>26.54°</td>
</tr>
<tr>
<td>Length of eclipse season</td>
<td>40.7 days</td>
<td>40.7 days</td>
</tr>
<tr>
<td>Duration of maximum eclipse</td>
<td>70.8 min</td>
<td>70.8 min</td>
</tr>
</tbody>
</table>
Figure 46 shows the eclipse duration for these injection nodes; it is identical for all satellites in the system. Figure 47 shows the injection epochs (as functions of the day of the year) for achieving the indicated nodal positioning.

At the present time, there are insufficient subsystem-requirements data to establish launch-window criteria, and a launch-window analysis has not been performed for the system described in this document. It is possible, however, to indicate the effects of off-nominal launch time on the inclination of the orbit planes to the ecliptic (which, in turn, affects the length of the eclipse season). These variations are shown in Figure 48 as a function of deviation of launch time from the nominal.
Figure 47. Injection Epochs for Achieving Desired Nodal Positioning
Figure 48. Variation of Orbit Inclination to Ecliptic from Nominal Inclination as Function of Injection Time off Nominal
REFERENCES


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APPENDIX A

NEW TECHNOLOGY

New technology and innovations developed under this contract are discussed in the appendix to vol. I.
APPENDIX B
WORLDWIDE ACCURACY PROGRAM (MSAT)

1. INTRODUCTION

This appendix contains both a development of the theory for determining navigation accuracies using range-type measurements from satellites and a description of the computer program developed from this theory. The MSAT program provides the capability for a quick analysis of postulated navigation satellite systems, while the NAVSAP program (app. J) provides a more general capability for analysis. In addition, since NAVSAP performs a more complicated operation, the cost is higher for preliminary analysis, and furthermore, NAVSAP is limited to seven satellites.

The MSAT program is applicable to systems employing range-type measurements only. That is, the user obtains estimates of the ranges from his location to the visible satellites or range differences from his location to two satellites.

The problem may be stated as follows. Given the location and inertial azimuth of the satellites in the system, the location of the user, the orbit-plane position uncertainties of the satellites, the measurement noise sigma, the satellite contributed measurement bias sigma, the user contributed bias sigma, and the user visibility constraints, to what accuracy can the position of the user be determined with range measurements to the visible satellites? It is assumed in the development that the user solves for his position and measurement bias; that satellite position errors and bias are "considered"* parameters; and that satellite position errors are independent of other satellite position errors and satellite measurement bias.

The essentials of the theory are covered in sec. 2, and a flow diagram for MSAT is presented as Figure B-1 at the end of this appendix.

* Considered parameters are parameters which are not estimated but whose affects are considered in the error analysis. In this case, user observations not used to solve for satellite positions, but the effects of errors in satellite position on user position are considered.
2. THEORY

Consider a user with ECI coordinates, \(x_u, y_u, \) and \(z_u\). The ECI coordinate system is defined as the \(x\) axis passing through the Greenwich meridian in the plane of the equator and the \(z\) axis through the North Pole. The user receives a range measurement, \(r_{i}^{'}\), from the \(i^{th}\) satellite which has coordinates \(x_i, y_i, \) and \(z_i\). This range measurement is equal to the linear sum of the following: the true range from the user to the satellite, \(\tilde{r}_i\); the user bias, \(b_u\); measurement noise, \(\eta_i\); and minus the satellite bias, \(b_i\). That is,

\[
   r_{i} = \tilde{r}_{i} + b_u - b_i + \eta_i \tag{B-1}
\]

The signs on the biases are chosen for convenience only. From geometry,

\[
   \tilde{r}_{i}^2 = (x_u - x_i)^2 + (y_u - y_i)^2 + (z_u - z_i)^2 \tag{B-2}
\]

From variations in Eq. (B-2), perturbations in the true range can be expressed as a function of perturbations in the user and satellite coordinates as,

\[
   \delta \tilde{r}_i = \cos a_i \left(\delta x_u - \delta x_i\right) + \cos \beta_i \left(\delta y_u - \delta y_i\right) + \cos \gamma_i \left(\delta z_u - \delta z_i\right) \tag{B-3}
\]

where \(\cos a_i = \frac{x_u - x_i}{\tilde{r}_i}\), etc.

Define the solve-for vector and consider vector

\[
   x = (x_u y_u z_u b_u)^T \tag{B-4}
\]

\[
   z_i = (x_i y_i z_i b_i)^T
\]
From all of the visible satellites, the above equations can be combined to yield

\[
\begin{bmatrix}
\delta r_1 \\
\delta r_2 \\
\vdots \\
\delta r_n
\end{bmatrix}
= 
\begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_n
\end{bmatrix}
\delta x
- 
\begin{bmatrix}
0 \\
A_2 \\
\vdots \\
A_n
\end{bmatrix}
\begin{bmatrix}
\delta z_1 \\
\delta z_2 \\
\vdots \\
\delta z_n
\end{bmatrix}
+ 
\begin{bmatrix}
\eta_1 \\
\eta_2 \\
\vdots \\
\eta_n
\end{bmatrix}
\]  

(B-5)

where

\[
A_i = \begin{bmatrix}
\cos \alpha_i \\
\cos \beta_i \\
\cos \gamma_i
\end{bmatrix}
\]

which is in the desired linear form

\[
\bar{Y} = A \bar{X} + B \bar{Z} + \bar{n}
\]  

(B-6)

where \(\bar{Y}\) is the observation vector, \(\bar{X}\) is the vector of parameters to be estimated, \(\bar{Z}\) is the vector of parameters to be considered, and \(\bar{n}\) is the noise vector. The well known covariance matrix of the estimate is

\[
P(\hat{x}) = (A^T P_n^{-1} A + P_o^{-1})^{-1} + (A^T P_n^{-1} A + P_o^{-1})^{-1}.
\]  

(B-7)

\[
A^T P_n^{-1} B P_z B^T P_n^{-1} A (A^T P_n^{-1} A + P_o^{-1})^{-1}
\]

where

- \(P(\hat{x})\) = covariance matrix of estimate
- \(P_o\) = a priori covariance matrix of \(\bar{X}\)
- \(P_z\) = a priori covariance matrix of \(\bar{Z}\)
- \(P_n\) = noise covariance matrix.

With the assumption of independent noise,

\[
P_n = \sigma_n^2 I
\]  

(B-8)
where I is the identity matrix, and the independent satellite errors are

\[
P_z = \begin{bmatrix}
Q_1 & 0 \\
& Q_2 \\
& & \ddots \\
0 & & & Q_n \\
\end{bmatrix}
\] (B-9)

where \( Q_1 \) is the 4x4 covariance matrix of ECI satellite position errors and bias variance

\[
Q_i = \begin{bmatrix}
P_{s_i}^{(3x3)} & 0 \\
0 & Q_{b_i}^2 \\
\end{bmatrix}
\] (B-10)

The covariance matrix of the user's estimate in ECI coordinates is

\[
P^* (\hat{x}) = \left( \frac{1}{\sigma_n} \sum_{i=1}^{n} A_i^T A_i + P_o^{-1} \right)^{-1} \quad \text{(no satellite errors)} \tag{B-11}
\]

\[
P (\hat{x}) = P^* (\hat{x}) + \left( \frac{1}{\sigma_n} \right)^4 P^* (\hat{x}) \left( \sum_{i=1}^{n} A_i^T A_i Q_i A_i^T A_i \right) P^* (\hat{x}) \quad \text{(with satellite errors)} \tag{B-12}
\]
START

1. READ INPUT DATA AND INITIALIZE

2. CALCULATE ECI CO-VARIANCE AND POSITIONS FOR ALL SAT.

3. USER LONGITUDE LOOP, I

4. USER LATITUDE LOOP, II

5. SET NORMAL MATRICES AND VISIBLE SATELLITES TO ZERO

6. SAT LOOP, K

7. IS SATELLITE VISIBLE?

8. STORE VISIBLE SATELLITE NUMBER

9. ACCUMULATE CONSIDER AND SOLVE FOR NORMAL MATRICES

10. NEXT SAT

11. ROTATE USER A-PRIORI COVARIANCE MATRIX INVERSE FROM LOCAL HORIZONTAL TO ECI

12. CALCULATE USER ECI A-POSITION COVARIANCE

13. ROTATE USER COVARIANCE TO LOCAL HORIZONTAL

14. CALCULATE CV5 AND PRINT USER LAT., LON., CV5 AND VISIBLE SATS

15. NEXT LAT.

16. NEXT LON.

Figure B-1. MSAT Flow Diagram
APPENDIX C

PHASED SATELLITE COVERAGE PROGRAM (AT-034)

An analytical computer program is available to analyze the ground coverage of a system of satellites phased in orbit with respect to each other. Circular or elliptical orbits may be considered. Given the initial condition of each satellite, the program determines rise and set times with respect to each ground station. As many as four orbit planes consisting of ten satellites in each plane can be examined with respect to one or two ground stations.

The output quantities include the percentage of time that at least \( n(0 \leq n \leq 10) \) satellites are visible, the probability distribution of satellite "outrate" (not visible) time, and the probability distribution of satellite visibility time.

The running time depends upon the number of satellites in the system and the number of orbits necessary to establish valid statistical data. The typical time is one minute for a case providing statistical data at one pair of ground stations.
APPENDIX D

WORLD MAP GENERATING PROGRAM (AT-86)

AT-86 is a general-purpose program designed to draw maps on the 10 or 30 in. CALCOMP plotter. The program will optionally draw the following:

a) A map of the world
b) Lines of constant latitude and longitude
c) A satellite earth trace
d) Visibility circles for a circular satellite
e) City designations, represented by various symbols on the map.

These options may be utilized one per map or all may be included on one map.

The projections optionally available are:

a) A plate carrée projection (latitude and longitude equally spaced)
b) A satellite map projection on which a satellite in a circular or eccentric orbit traces a straight line
c) A polar projection with an arbitrary point on the earth as the center of the projection.

When a polar projection is selected, an additional option of lines of constant latitude and longitude symmetric about a set of poles of variable position is available.
APPENDIX E
APPLICATION OF NAVSAP TO ESTIMATION OF VELOCITY FROM DOPPLER DATA

The NAVSAP program does not contain the user's velocity components in the state vector. Therefore, several modifications to the normal mode of operation must be made in order to apply the program to velocity estimation. These modifications are based on the fact that the measurement matrix for range measurements used to estimate user position is identical to range-rate measurements used to estimate velocity. That is,

\[
\frac{\partial R}{\partial x_{\text{sat}}} = \frac{\partial R}{\partial x_{\text{user}}} = - \frac{\partial R}{\partial x_{\text{user}}} = \left[ \begin{array}{ccc} x & y & z \\ R & R & R \end{array} \right]
\]

where \( x, y, \) and \( z \) are cartesian coordinates of the relative position \( R \) between the satellites and ground user. Hence, range measurements were simulated, but velocity a priori covariance matrices were inserted in place of position a priori covariance matrices, and the measurement error used was the velocity error of 0.707 ft/sec.

This usage of the program neglects error contributions from user position error uncertainties. These negligible effects are justified below, based on the consideration that user position components are only weakly observable in doppler data. Let

\[
\begin{align*}
\delta y &= \text{variation in range-rate measurement vector } \dot{y} \\
\delta \dot{x} &= \text{variation in user's velocity vector } \dot{x} \\
\delta x &= \text{variation in user's position vector } x \\
\eta &= \text{range-rate measurement error vector} \\
Q_x &= \text{user's a priori velocity error covariance matrix} \\
Q_x &= \text{user's a priori position error covariance matrix} \\
Q_\eta &= \text{measurement noise error covariance matrix} \\
W &= Q_\eta^{-1}
\end{align*}
\]
The linear observation model is

$$\delta \hat{y} = A \delta \hat{x} = B \delta x + \eta$$

where A and B are the appropriate partial derivative matrices. If the program had used the measurements to solve for $\hat{x}$, but considered the errors in $x$, the a posteriori errors in $\hat{x}$ would have been

$$\Sigma_{\hat{x}} = (A^T WA + Q_x^{-1})^{-1} + (A^T WA + Q_x^{-1})^{-1}$$

$$A^T WB Q_x B^T WA (A^T WA + Q_x^{-1})^{-1}$$

The program calculated only the first term of the above. This is equivalent to assuming that $BQ_x B^T$ is small compared to $Q_\eta$, which follows from an alternate form of the above equation:

$$\Sigma_{\hat{x}} = (A^T WA + Q_x^{-1})^{-1} \left[ A^T W (Q_\eta + BQ_x B^T) WA + Q_x^{-1} \right]$$

$$A^T WA + Q_x^{-1})^{-1}$$

that $BQ_x B^T << Q_\eta$ can be seen from a simple hand check using typical standard deviations for $x$ and $\eta$. Since

$$\dot{\hat{R}} = \frac{\dot{x}x + \dot{y}y + \dot{z}z}{R}$$

where $\dot{x}$, $\dot{y}$ and $\dot{z}$ are the components of the relative velocity between the satellite and ground user, it follows that a typical term of B is

$$b \sim \frac{\partial R}{\partial x} : = \frac{- R \dot{x} - \dot{x}R}{R^2} \leq \frac{\dot{x} + \dot{R}}{R} < \frac{3 \times 10^{-5}}{1.2 \times 10^8} = 2.5 \times 10^{-5}$$

Hence, a typical diagonal term of $BQ_x B^T$ is

$$(\sigma_x B^2)_{\text{diag}} \sim (3b)^2 \sim (400)^2 [3(2.5 \times 10^{-5})]^2 = 9 \times 10^{-4}$$
Comparing this to a diagonal term of $Q_{\eta} (\sigma_{\eta}^2 \sim 0.5)$, it is apparent that the approximation is justified.

It is proposed to increase the dimension of the user's state vector in NAVSAP from three to six in order to estimate user position and velocity simultaneously. This modification will permit a more complete treatment of the velocity estimation errors, with or without doppler measurements.
APPENDIX F

RELATIVE NAVIGATION ACCURACY ANALYSIS USING THE NAVSAP PROGRAM

As indicated in subsec. 2.3.4 in the main body of this report, the error covariance matrix of relative error of user 2 with respect to user 1 is given by

\[ \Sigma_R = \Sigma_{11} + \Sigma_{22} - \Sigma_{12} - \Sigma_{21} \]  \hspace{1cm} (F-1)

where

\[ \Sigma_{11} = \Sigma_{1n} + \Sigma_{1s} \]  \hspace{1cm} (F-2)
\[ \Sigma_{22} = \Sigma_{2n} + \Sigma_{2s} \]

The question treated in this appendix is how to compute these component error-covariance matrices using the NAVSAP error analysis program described in (app. J).

The satellite states \( x_1 \) and \( x_2 \) are estimated based on the usual linearized tracking model:

\[ y_1 = A_1 x_1 + B_1 z + \epsilon_1 \]  \hspace{1cm} (F-3)
\[ y_2 = A_2 x_2 + B_2 z + \epsilon_2 \]

where \( y_1, y_2 \) are the observations by users 1 and 2, and \( z \) is the vector of the common error sources of satellite position and clock errors. \( x, y, \) and \( z \) are to be interpreted as deviations from reference values, \( \alpha \) and \( \epsilon_1, \epsilon_2 \) are the measurement errors. The users will estimate their positions from

\[ \hat{x}_1 = (A_1^T W_1 A_1)^{-1} A_1^T W_1 y_1 \]  \hspace{1cm} (F-4)
\[ \hat{x}_2 = (A_2^T W_2 A_2)^{-1} A_2^T W_2 y_2 \]

See par. 2.3.4 for notation and definition of terms.
where

\[ W_1 = \left[ E(\epsilon_1 \epsilon_1^T) \right]^{-1} \]

\[ W_2 = \left[ E(\epsilon_2 \epsilon_2^T) \right]^{-1} \]  \hspace{1cm} (F-5)

These estimates are minimum variance only in the absence of the satellite errors \( z \). The estimation error covariance matrices which account for the effect of \( z \) are

\[ \Sigma_{11} = E(\delta x_1 \delta x_1^T) = (A_1^T W_1 A_1)^{-1} + (A_1^T W_1 A_1)^{-1} A_1^T W_1 B_1 Q B_1^T W_1 A_1 (A_1^T W_1 A_1)^{-1} \]  \hspace{1cm} (F-6)

\[ \Sigma_{22} = E(\delta x_2 \delta x_2^T) = (A_2^T W_2 A_2)^{-1} + (A_2^T W_2 A_2)^{-1} A_2^T W_2 B_2 Q B_2^T W_2 A_2 (A_2^T W_2 A_2)^{-1} \]

where \( Q = E(z z^T) \) is the satellite error covariance matrix and the terms on the right side of Eq. (F-6) are \( \Sigma_{1n} \), \( \Sigma_{1s} \), \( \Sigma_{2n} \), \( \Sigma_{2s} \) as given in Eq. (F-2). The remaining term required for the evaluation of the relative error according to Eq. (F-1) is \( \Sigma_{12} \). This follows from Eqs. (F-4) and (F-3) in the same way that Eq. (F-6) is obtained. The result is

\[ \Sigma_{12} = E(\delta x_1 \delta x_2^T) = (A_1^T W_1 A_1)^{-1} A_1^T W_1 B_1 Q B_2^T W_2 A_2 (A_2^T W_2 A_2)^{-1} \]  \hspace{1cm} (F-7)

Denoting the correlations between user and satellite errors as \( \Sigma_{1z} \) and \( \Sigma_{2z} \) and noting from Eqs. (F-2) and (F-3) that

\[ \Sigma_{1z} = E(\delta x_1 z^T) = (A_1^T W_1 A_1)^{-1} A_1^T W_1 B_1 Q \]  \hspace{1cm} (F-8)

\[ \Sigma_{2z} = E(\delta x_2 z^T) = (A_2^T W_2 A_2)^{-1} A_2^T W_2 B_2 Q \]

it follows that Eq. (F-7) can be written

\[ \Sigma_{12} = \Sigma_{1z} Q^{-1} \Sigma_{2z}^T \]  \hspace{1cm} (F-9)
Consequently Eq. (F-1) becomes

$$
\Sigma_R = \Sigma_{11} + \Sigma_{22} - \Sigma_{1z} Q^{-1} \Sigma_{2z}^T - \Sigma_{2z} Q^{-1} \Sigma_{1z}^T
$$

(F-10)

An alternate form of Eq. (F-10) is obtained from Eqs. (F-6) and (F-8)

$$
\Sigma_R = \Sigma_{1n} + \Sigma_{2n} + \Sigma_{1z} Q^{-1} \Sigma_{1z}^T + \Sigma_{2z} Q^{-1} \Sigma_{2z}^T
$$

\[ - \Sigma_{1z} Q^{-1} \Sigma_{2z}^T - \Sigma_{2z} Q^{-1} \Sigma_{1z}^T \]

(F-11)

The first terms are the estimation errors without satellite errors. The remaining terms tend to cancel as $\Sigma_{2z}$ approaches $\Sigma_{1z}$.

A single run on NAVSAP corresponding to user 1 produces the matrices $\Sigma_{11}$ and $\Sigma_{1s}$. A second run results in $\Sigma_{22}$ and $\Sigma_{2s}$, and $Q$ is input. Furthermore, by setting satellite errors to zero, the individual terms $\Sigma_{1n}$ and $\Sigma_{2n}$ can be computed. In this way the individual columns of the table in subsec. 2.3.4 were computed and assembled into the final relative navigation error covariance matrix of Eqs. (F-10) or (F-11).
1. PROGRAM DESCRIPTION

The AT4 System was designed to support the Able and early Ranger launches. Subsequent development led to a family of orbit-determination programs covering a range of applications from real-time operations to a solution for gravitational harmonics using the simultaneous observations of several satellites. The current ESPOD Orbit Determination Program is the result of 7 years of development effort. This program is the basis for several other closely related special-purpose programs, and has basic characteristics common to the entire family of programs.

The ESPOD program is a precision-trajectory propagation and statistical orbit determination program written in FORTRAN IV language. Versions of the program operate on the IBM 7094, IBM 7030, IBM 360, GE 635, and SDS 9300 computers.

The force model includes a recursive computation of the central body gravitational accelerations, allowing inclusion of harmonics of any desired degree and order. Aerodynamic drag may be computed by using the COESA static, Paetzold, or Lockheed Jacchia (1964) dynamic atmospheres. Gravitational attractions due to other bodies in the solar system are computed by using planetary ephemerides stored on tape. Provision has been made to account for vehicle thrusting, low thrusts due to random venting, and radiation pressure.

The trajectory is computed by numerical integration of the equations of motion using a 10th-order Cowell formulation, with an automatically computed, variable step size.

Integration takes place in the mean of 1950.0 coordinate frame centered at an arbitrary body. All rotations required for proper evaluation of the gravitational potential and representation of observations are performed.
Observation types that are accepted by the program and used to differentially correct the components of the solution vector include the following:

1) Range, azimuth, and elevation
2) Topocentric right ascension and declination
3) Geocentric right ascension and declination
4) Range rate
5) Range acceleration
6) One-, two-, and three-way doppler data
7) Range differences and range-rate differences (interferometer measurements)
8) Rectangular components of estimated position
9) Accelerations as measured by onboard accelerometers

Sensors taking observation types 1 through 8 may be located on the central body, on any other body for which coordinates are available, or onboard the vehicle.

Corrections to the components of the solution vector are computed by using an iterative weighted-least-squares process. Provision for bounding the size of the corrections or any given iteration and automatic convergence logic has been included. The program will compute corrections to the following quantities:

- Initial position and velocity in terms of Cartesian or polar-spherical coordinates, Keplerian elements, or a special set of ξ-variables designed to improve the numerical conditioning of the differential correction process
- Ballistic coefficient
- Burn parameters, including thrust-to-weight ratio, flow rate, body-orientation angles, and body-axis rates
- Potential constants of the central body (any degree and order)
Observational and timing biases
Observation of station locations
Any linear combinations of the above

All program constants, error bounds (e.g., on step-size control), and contributors to the force model may be easily modified on input.

A completely flexible phase logic allows use of several central bodies in succession, interspersed free-flight and powered-flight arcs, and accurate prediction of reentry trajectories. The phase logic, plus several special coordinate transformations, are combined in a version of ESPOD which is designed to track lunar satellites.

The trajectory and the covariance matrices describing uncertainties in the components of the solution vector may be output in any of ten coordinate systems. Provision has been made for updating covariance matrices to any desired epoch. Prior estimates of the solution vector components, along with uncertainties in these estimates, may be input to the program to be combined statistically with the estimate derived from the current observation data. In addition, the effects of uncertainties in parameters not included in the solution vector (e.g., certain gravitational harmonics) may be accounted for in the computation to the covariance matrix for those parameters that have been included.

The ESPOD program has been employed in real data analyses for flight reconstruction of Vela, Minuteman, Gemini, and Apollo. The Gemini and Apollo experiences indicate that a complete revolution of tracking data (approximately 1000 points) can be processed and used to compute a differential correction to the orbital elements in less than 1 min on the IBM 7094-Mod II.

Recent modifications have given the program complete capability for analysis of errors in the estimation process. In particular, the effect of errors in parameters not estimated can be treated in a straightforward manner. Also of interest for navigation satellite error analysis is a modification, currently in progress, that will enable the simultaneous tracking of multiple vehicles.
2. ESPOD GENERAL PRINCIPLES

2.1 Estimation Theory

To introduce and to define the terminology, consider the trajectory estimation problem in the presence of random errors only. Let \( x \) be the actual vehicle state vector of position and velocity at some epoch and let \( n \) be the vector of unbiased Gaussian random noise on the vector of measurements \( y \). Then, if the equation relating the measurements to the state vector, \( y = f(x) \), is expanded in a first-order Taylor's series about a reference trajectory, we have

\[
\delta y = A \delta x + n
\]

where \( A = \delta f/\delta x \). \( \delta y \) is the difference between the observed and computed measurements, and \( \delta x \) is a small deviation from the reference state vector. Then, the weighted-least-squares estimate of \( \delta x \) is

\[
\hat{\delta x} = \left( A^T W A \right)^{-1} A^T W \delta y
\]

and the covariance of the estimate is

\[
\Sigma_x = \left( A^T W A \right)^{-1}
\]

where \( W^{-1} \) is the covariance of the noise, \( n \). The matrix \( A^T W A \) is often referred to as the tracking normal matrix.

As the amount of data increases, the covariance of the estimate \( (A^T W A)^{-1} \) approaches zero. In reality no such simple state of affairs exists. First and most important, the errors or noise on the measurements do not have a zero mean, i.e., the measurement biases, station location errors, etc. (called systematic errors) are not zero. Secondly and less important, since random errors are normally a relatively small magnitude, it is unlikely that the noise is strictly Gaussian distributed. Finally, errors in the modeling of the physical situation will also contribute to uncertainty in the state vector. Thus, one expects that uncertainty will first decrease, but then may level off or increase due to the systematic effects.
It is possible to reduce this uncertainty by solving for systematic errors in the estimation process. Let \( z \) be the vector of systematic errors to be included in the solution vector and let \( B \) be a matrix relating small changes in \( z \) to small changes in the measurements \( y \). Then

\[
\delta y = A \delta x + B \delta z + n
\]

or

\[
\delta y = (A^T B)^{-1} \begin{bmatrix} \frac{\delta x}{\delta z} \\ n \end{bmatrix} + n
\]

The corresponding least-squares estimates of \( x \) and \( z \) are found to be

\[
\begin{bmatrix} \delta x \\ \delta z \end{bmatrix} = \left( \begin{bmatrix} A^T \\ B^T \end{bmatrix} \right) \begin{bmatrix} W \end{bmatrix}^{-1} \begin{bmatrix} A^T \\ B^T \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \delta y
\]

or

\[
\begin{bmatrix} \delta x \\ \delta z \end{bmatrix} = \left( \begin{bmatrix} A^T W A \\ B^T W B \end{bmatrix} \right)^{-1} \begin{bmatrix} A^T \\ B^T \end{bmatrix} \begin{bmatrix} W \end{bmatrix} \delta y,
\]

and the covariance of the solution parameters is

\[
\Sigma = \begin{bmatrix} A^T W A & \begin{bmatrix} A^T \\ B^T \end{bmatrix} \begin{bmatrix} W \\ B^T W B \end{bmatrix}^{-1} \begin{bmatrix} A^T \\ B^T \end{bmatrix} \begin{bmatrix} W \\ B^T W B \end{bmatrix} \end{bmatrix}^{-1}
\]

That is, the solution now converges to an estimate that yields an essentially unbiased noise and residual vector. However, one cannot solve for every one of the large number of parameters that might conceivably affect the solution. Indeed, it is desirable to solve for as few as possible, while including the error resulting from the unsolved parameters. Then, if any of these unsolved parameters cause an intolerable error, one can consider solving for it. The technique for evaluating the uncertainty caused by the unestimated parameters is derived below.
Let \( x \) be the vector of all solved-for parameters, \( z \) be the vector of all unsolved-for parameters (whether their effect be a bias or a time-varying influence), and let \( n \) be the Gaussian random noise on the measurements. As before

\[
\delta y = A\delta x + B\delta z + n
\]

The weighted least-squares estimate of \( \delta x \) is

\[
\hat{\delta x} = \left( A^TWA \right)^{-1} A^T W \delta y ,
\]

and the error in the estimate is

\[
\delta \hat{x} - \delta x = \left( A^TWA \right)^{-1} A^T W (B \delta z + n)
\]

If \( W^{-1} \) is set equal to the covariance of the noise, and the noise is assumed to be independent of the unsolved-for parameters, the following is obtained for the total covariance on the estimate, \( \delta x \):

\[
\text{cov} (\hat{\delta x}) = \Sigma_x = \left( A^TWA \right)^{-1} + \left( A^TWA \right)^{-1} A^T WB \Sigma_z B^TWA \left( A^TWA \right)^{-1}
\]

The first term is a contribution from only the random noise, and the second term contains the contribution from the unsolved-for parameters. \( \Sigma_z \) is the covariance of these unsolved-for parameters. Both of the terms are functions of the amount of tracking data. A characteristic of the \( (A^TWA)^{-1} \) matrix is that it decreases roughly as the square root of the amount of data, while the characteristic behavior of the second matrix is that it increases with time or the amount of tracking data.
2.2 Combining Two Least-Squares Estimates

It is often required to combine two least-squares estimates when it is desired to combine current tracking data with some a priori estimate. Then the new estimate \( \delta x \), resulting from combining the two estimates \( \delta x_1 \) and \( \delta x_2 \), with covariance matrices \( \Sigma_1 \) and \( \Sigma_2 \), respectively, is

\[
\delta x = \left( \Sigma_1^{-1} + \Sigma_2^{-1} \right)^{-1} \left( \Sigma_1^{-1} \delta x_1 + \Sigma_2^{-1} \delta x_2 \right)
\]

and the new covariance matrix is

\[
\Sigma = \left( \Sigma_1^{-1} + \Sigma_2^{-1} \right)^{-1}
\]

2.3 Propagation Matrices

It is often desirable to propagate a least-squares estimate from time \( t_1 \) to time \( t_2 \). The linearized equations for the propagation are

\[
\delta x_2 = \frac{\partial x_2}{\partial x_1} \delta x_1 + \frac{\partial x_2}{\partial z_1} \delta z_1 \quad (G-4)
\]

and

\[
\delta z_2 = \frac{\partial z_2}{\partial x_1} \delta x_1 + \frac{\partial z_2}{\partial z_1} \delta z_1 \quad (G-5)
\]

where it is assumed that \( x_i \) is the vector of vehicle parameters at time \( i \), and \( z_i \) is the vector of systematic errors at time \( i \) for \( i = 1, 2 \). Since the systematic errors are not affected by the orbit parameters,

\[
\frac{\partial z_2}{\partial x_1} = 0
\]
and if it is assumed that the systematic errors are constant, regardless of epoch time, then

$$\frac{\partial z_2}{\partial z_1} = I$$

where $I$ is the identity matrix. Thus, Eqs. (G-4) and (G-5) become

$$\begin{bmatrix}
\delta x_2 \\
\delta z_2
\end{bmatrix} = \begin{bmatrix}
\frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial z_1} \\
0 & I
\end{bmatrix} \begin{bmatrix}
\delta x_1 \\
\delta z_1
\end{bmatrix}$$

when written in matrix form.

The propagated covariance matrix is given by

$$\Sigma_2 = \Sigma_1 \Sigma_1^T$$

where

$$\Sigma_1 = E \begin{bmatrix}
(\delta x_1) \\
(\delta z_1)
\end{bmatrix} \begin{bmatrix}
(\delta x_1)^T \\
(\delta z_1)
\end{bmatrix}$$

$$\Sigma_2 = E \begin{bmatrix}
(\delta x_2) \\
(\delta z_2)
\end{bmatrix} \begin{bmatrix}
(\delta x_2)^T \\
(\delta z_2)
\end{bmatrix}$$

and $E$ is the expectation operator.
2.4 Sensitivity of Solved-For Parameters to Unsolved Parameters

In accuracy analysis studies, it is often desirable to determine the degradation of the estimation accuracy due to unestimated systematic errors. These aspects have been discussed elsewhere and the results are as follows.

We can rewrite Eq. (G-2) as

$$\hat{\delta x} = \delta x + \left( A^T W A \right)^{-1} A^T W (B \delta z + n)$$

where again $\delta x$ is the vector of solved-for parameters, and $\delta z$ is the vector of unsolved systematic errors. Then the partial derivative of the estimate of the solved-for quantities with respect to the unestimated variables can be written as

$$\frac{\partial (\hat{\delta x})}{\partial (\delta z)} = \left( A^T W A \right)^{-1} A^T W B$$
APPENDIX H
APPLICATION OF THE SPIT PROGRAM

1. INTRODUCTION

This appendix contains a description of results of computations made with a special Single Point in Time (SPIT) computer program which considers simultaneous measurements from ground stations and user to a system of satellites. These results have provided valuable information on the influence of correlations in navigation satellite error analysis and have been useful in ground station preliminary design.

The computer program performs the function of determining and propagating the ground-station determined, full satellite covariance matrix into user accuracy at variable locations, given specifications on measurement mode and accuracy (random and bias), a priori satellite and station location uncertainties, and satellite locations. The program is not intended to simulate the process of long-term tracking and data smoothing involved in accurately determining satellite position, but rather to study the influence of satellite/ground station interactions on user accuracy once such a process has been completed. One user area with a fixed four-satellite array representing ±18-1/2° synchronous orbits was considered. Figure H-1 shows this geometry including a set of 5 potential ground station sites.

User and ground station measurements can be represented in the program as either:

a) "absolute" range: that is range with a zero or small finite a priori bias comparable to the random error

b) "relative" range: That is range with a large or essentially infinite a priori bias which, however, is common to all measurements made by that station or user

c) range difference—"uncorrelated": having independent random errors

d) range difference—"correlated": having the intercorrelation structure that obtains by deriving such range differences from basic range measurements by differencing by pairs.
The NAVSTAR system proposed in this report uses type b measurements, which a high-accuracy user will process directly. The intermediate-accuracy user will difference these range measurements to obtain type d measurements, which he will process suboptimally, assuming they are uncorrelated (type c). These distinctions are discussed more fully in sec. 3.

Random measurement errors were taken as 100 ft ($1\sigma$) on range for cases a, b, d, and 100 ft on range difference for case c. The results can be scaled within reason to correspond to other basic measurement errors (Ref. 4, Figure 4-23). Details of the SPIT computer program are included in app. I. Results of the computer study complete this section.

The main topics studied in this error analysis are listed below and discussed in the Preliminary Results, sec. 4, of this appendix. These topics cover the effects of:

1) Measurement mode (range or range difference)
2) Ground station and user making similar measurements
3) Geometric correlations (defined as the correlation effects arising because of the geometrical position of the ground stations with respect to the satellite and independent of the measurement process)
4) Varying the number of ground stations
5) Measurement correlations in range difference measurements.

2. PRELIMINARY RESULTS

The results of the accuracy analyses are position uncertainties over the grid of user locations shown in Figure H-1. Figure H-2 shows the geometrical distribution of user uncertainty for four possible combinations of (absolute) range and (correlated) range-difference measurements by the users and five ground stations. The ranges of user accuracies represented on this and other maps have been condensed into bar graphs in Figures H-3 through -6 for easier interpretation. Each set of numbers in Figure H-2 corresponds to one bar either on Figure H-3 or on Figure H-4.
NOTE: INTERPRETATION OF SETS OF NUMBERS IS:

USER = SATELLITES LINE GROUND STATION

NO. MEASUREMENT - MEASUREMENT

A = GROUNDSTATIONS = USERS

RANGE DIFFERENCES ARE "CORRELATED"
RANGES ARE "ABSOLUTE"

Figure H-2. Typical Results for a Grid of User Locations
COMPARISON OF RANGE VS RANGE-DIFFERENCE MEASUREMENTS BY THE USER

GROUND STATIONS MAKE 3 RANGE MEASUREMENTS EACH

USER MEASUREMENTS:
- - - 3 RANGE DIFFERENCE, CORRELATED
- - - - 4 RANGE, ABSOLUTE

Figure H-3. Comparison of Range Versus Range-Difference Measurements by the User
Comparison of Range vs. Range-Difference Measurements by the User

Ground stations make 4 range-difference measurements each.
User measurements: 
- 3 range difference, correlated
- 4 range, absolute

Figure H-4. Comparison of Range Versus Range-Difference Measurements by the User
COMPARISON OF RANGE VS RANGE-DIFFERENCE MEASUREMENTS BY THE GROUND STATIONS

USERS MAKE 3 RANGE-DIFFERENCE MEASUREMENTS EACH

GROUND STATION MEASUREMENTS
- - - - - 3 RANGE DIFFERENCE, CORRELATED
- - - - - - - 4 RANGE, ABSOLUTE

Figure H-5. Comparison of Range Versus Range-Difference Measurements by the User
Figure H-6. Comparison of Range Versus Range-Difference Measurements by the User
Tradeoffs between measurement modes are shown in Figures H-3 through -6 as is the range of user accuracies as a function of the number of ground stations. The terms "range of user accuracies" is used to denote the interval bounded by the most accurate and the least accurate user within the grid under consideration and is not a range associated with any one user.

As an aid in visualizing the effect of varying the number of ground stations, a tick mark representing the accuracy of a user at longitude 50°W, latitude 40°N (near the center of the grid) has been placed on each bar of the graph. The best range of accuracies obtainable occurs with an infinite number of ground stations and was obtained by setting the satellite covariance matrix equal to zero.

3. MEASUREMENT MODES AND EFFECT OF SIMILAR MEASUREMENTS

In terms of user accuracy, best results are obtained when both ground stations and users measure absolute range. If either ground stations or users measure range differences, it appears to make no difference what the other measures. However, if either ground stations or users measure range, there is a definite advantage to having the others also measure range. This can be seen from the tabulation given below, which is a condensation of some of the data on Figures H-3 and -4 and other runs. The tabulation corresponds to a network of five ground stations viewing 4 satellites. Accuracies given are those for a user at 50°W, 40°N.

<table>
<thead>
<tr>
<th>Accuracies in Feet</th>
<th>Ground Stations Measure:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Users Measure</td>
<td>4 Range (Absolute)</td>
</tr>
<tr>
<td>4 Range (Abs.)</td>
<td>222</td>
</tr>
<tr>
<td>4 Range (Rel.)</td>
<td>611+</td>
</tr>
<tr>
<td>3 Range Dif. (Corr.)</td>
<td>611+</td>
</tr>
</tbody>
</table>
It is clear from an information point of view that 3 Range Differences (Correlated) are equivalent to 4 Range (Relative) from which they are assumed derived. This explains the equalities of the second and third rows and columns in the above table. The apparent equality between the 611 ft terms and the 611+ ft terms appears to be coincidental.

4. EFFECT OF REDUCING NUMBER OF GROUND STATIONS

All the figures examined show the effect of reducing the number of ground stations. Perhaps the most significant fact is that if the best measurement philosophy is used (i.e., both ground stations and users measure range), there is not a great difference in the accuracies obtainable by a relatively modest tracking network and the accuracies obtainable by perfect tracking (or an infinite number of ground stations). This can be seen in Figures H-4 and -6 in which the range of accuracies obtainable by perfect tracking is seen to be from 142 to 210 ft. The range of accuracies obtainable by a system of four ground stations (the existing USBS Network) is from 236 to 285 ft, with no time smoothing. In the present model, which depicts ground stations as making instantaneous single-point-in-time measurements with an a priori constraint, time smoothing may be represented as a smaller a priori satellite covariance matrix, which would lower these figures still further. The effects of time smoothing are investigated in detail in subsec. 2.4.

The bars in Figures H-3 through -6 which denote the accuracies obtainable with one, two, or three ground stations are pessimistic, since no time smoothing was considered. The purpose of the computer runs which generated these data was really to show that underdetermined satellite locations may still lead to quite acceptable user location accuracy. The reasons for this behavior are partly explained in the next section.

5. IMPORTANCE OF CORRELATION IN SATELLITE POSITION (GEOMETRIC CORRELATION)

It is possible to input to the program any desired diagonal satellite covariance matrix. If this matrix corresponds to the diagonal elements of a previously calculated matrix, any change in user accuracy between the two cases may be attributed to the absence of correlation.
This was done, using as the diagonal matrix the diagonal elements of the satellite covariance matrix which yielded the user accuracies shown in Figure H-3 for one ground station. As was expected, the user uncertainty was much larger for the case in which no geometric correlations were considered. Table H-I shows the range of user accuracies with and without geometric correlation for two measurement systems, one in which four ranges were measured and another using three range differences. In each case, the satellite covariance matrix consisted of the diagonal elements of the satellite covariance matrix which resulted from one ground station making three range-difference measurements.

TABLE H-I
RANGE OF USER UNCERTAINTIES

<table>
<thead>
<tr>
<th>Measurement Mode of User</th>
<th>Correlated (ft)</th>
<th>Uncorrelated (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three range-difference</td>
<td>1163-8434</td>
<td>33,956 - 74,187</td>
</tr>
<tr>
<td>Four range</td>
<td>1159-5866</td>
<td>14,062 - 23,115</td>
</tr>
</tbody>
</table>

The improvement assignable to the off-diagonal (correlation) terms in the satellite covariance matrix is of the order of 10 or 20 to 1, with the greatest improvement associated with those user locations which are closest to the network, or particular stations in the network. This is because, relative to the large volume of space encompassed by the satellite network, the ground station and user positions are very close to one another. Consequently, the partial derivatives of ground station measurements with respect to satellite positions are very nearly equal to the negative of the partials of user measurements with respect to user position. Thus, even in an underdetermined ground measurement setup where the complete satellite position cannot be significantly determined, that component of position corresponding to the projection of satellite position on the partial derivative vector may be very well determined, and to the extent that the partial derivative is the same for the user, that is the only component of satellite position that matters. This emphasizes the importance of a complete error covariance matrix propagation from ground measurements, through satellite, to user position.
Another way of verifying the importance of the correlation terms is to change the covariance matrix artificially so that the diagonal terms are essentially unchanged, but the correlations are lower. This can be accomplished by: 1) making few enough measurements so that the locations are underdetermined; 2) choosing an a priori constraint so that the satellite covariance can be determined, but choosing a standard deviation of ground measurement error such that the matrix will be essentially the same size as the a priori matrix; and 3) repeating the procedure with a much larger value of ground station measurement error. Since the matrix was already essentially the same size as the a priori matrix, specifying a degraded ground measurement accuracy does not appreciably change the size of the diagonal elements of the satellite covariance matrix, but it does significantly lower the correlations between the satellites. This was done, and the user accuracies were significantly worse in the case with lower correlations. This illustrates the effect of low-accuracy tracking, which contributes to user inaccuracies out of proportion to the degradation in satellite ephemeris (as measured by the diagonal terms).
APPENDIX I

SINGLE POINT IN TIME ACCURACY PROGRAM (SPIT)

A. Introduction

The SPIT program is designed to evaluate the satellite covariance matrix which results when a system of ground stations makes range and/or range-difference measurements to a network of satellites, and to then use this satellite covariance matrix to determine the covariance matrix of each user of the system. The analyst may specify any combination of range and/or range-difference measurements for the users as well as for the ground stations.

Three other features may be exercised as options. One allows the analyst to specify that the satellite location is known perfectly except for the satellite drift covariance, which may be any diagonal matrix, including the zero matrix.

Another option allows the analyst to account for correlation between the ground station measurements which arises whenever a ground station measures ranges to the several satellites and uses these to form range differences between one satellite and all other satellites. In this case, correlations exist in the random errors in the range differences. This option is only available if the ground measurement random error is the same for each ground station.

The third option allows one to consider relative navigation between pairs of users. This will be explained in more detail later. A flow diagram of the program is shown as Figure I-1.

B. Limitations

The program is limited to a maximum of nine satellites and nine ground stations. In addition, the total number of measurements made by all ground stations cannot exceed fifty. (Forty-five if the correlated measurement option is used).

Each subcase may consist of a maximum of nine users, making a maximum of nine measurements each. Because each user is processed sequentially and is independent of all other users (except for the relative navigation option), there is no other restriction on the total number of measurements made by all users.

Within each subcase all users must make identical measurements. However, there is no limit to the number of subcases which may be processed in one run.
C. Inputs

Inputs to the program are:

1. Ground station, satellite, and user locations, specified as latitude, longitude and range.

2. Number of ground measurements (NGM). A value of zero is interpreted as meaning that the satellite position is perfectly known except for satellite drift, which may or may not be zero. If this is zero, no GMM matrix need be input.

3. Ground measurement matrix (GMM). This is a three-column matrix which specifies which measurements are being made by the ground station. The first columns are the numbers of satellites A and B. If the third column is zero, the measurement being made is a range measurement to satellite A. If non-zero, the measurement being made is the range difference between satellites A and B. The number of rows of the matrix is equal to the number of ground measurements, NGM.

4. Number of user measurements per user (NUM).

5. User measurement matrix (UMM). This is a control matrix similar to GMM. However, since each user within a subcase makes the same measurements there is no reason to have a first column identifying the user by number, as in GMM.

6. Standard deviation of ground station measurement error. There will be one standard deviation per measurement, or NGM total. If the option to consider the correlation between ground measurements is desired, only one value should be input. This will then be used as the standard deviation for all measurements.

7. Standard deviation of user measurement error. Exactly the same comments made above also apply here.

8. Satellite a priori flag (SAF). If zero, the effect is the same as if the equations were written with no regard for any a priori values. Note that this is not equivalent to saying the satellite is perfectly known or that its covariance is infinite, since both the covariance matrix and its inverse have zero values. If the flag is non-zero, the standard deviations of longitude, latitude, range, and bias for each satellite must be input.
9. Ground station a priori flag (GAPF). Exactly the same interpretation as for SAPF, but for the ground stations.

10. User a priori flag (UAPF). Exactly the same as above, but the only variables are longitude, latitude and range. Only one set of these numbers is input, and they are used for every user within the subcase.

11. Satellite drift covariance flag (SDCF). Interpretation is the same as for SAPF and GAPF.

12. Relative navigation flag (RFN). If zero, the program operates in the 'normal' mode discussed previously. If non-zero, the covariance matrix associated with the difference vector between the reference user and all other users is computed and printed. The reference user is always the first user.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAP</td>
<td>Ground Station A Priori Position Matrix</td>
</tr>
<tr>
<td>GAPEL</td>
<td>Square Roots of Elements of GAP Matrix</td>
</tr>
<tr>
<td>GAPE</td>
<td>Ground A Priori Flag</td>
</tr>
<tr>
<td>GLOC</td>
<td>Location Vector of Ground Stations ((\theta, \phi, \rho))</td>
</tr>
<tr>
<td>GMC</td>
<td>Ground Measurement Covariance Matrix</td>
</tr>
<tr>
<td>GMCA</td>
<td>Ground Measurement Covariance Matrix, Augmented</td>
</tr>
<tr>
<td>GMCEL</td>
<td>Elements of GMC Matrix</td>
</tr>
<tr>
<td>GMML</td>
<td>Ground Measurement Matrix (A Control Matrix)</td>
</tr>
<tr>
<td>NGM</td>
<td>Number of Ground Measurements (Number of Rows of GMM)</td>
</tr>
<tr>
<td>NGS</td>
<td>Number of Ground Stations</td>
</tr>
<tr>
<td>NS</td>
<td>Number of Satellites</td>
</tr>
<tr>
<td>NU</td>
<td>Number of Users</td>
</tr>
<tr>
<td>NUM</td>
<td>Number of User Measurements per User (Number of Rows of UMM)</td>
</tr>
<tr>
<td>FGMG</td>
<td>Partial Derivatives of Ground Measurements with Respect to the Ground Station</td>
</tr>
<tr>
<td>PGMS</td>
<td>Partial Derivatives of Ground Measurements with Respect to the Satellites</td>
</tr>
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<td>SAP</td>
<td>Satellite A Priori Position Matrix</td>
</tr>
<tr>
<td>SAPEL</td>
<td>Square Roots of Elements of SAP Matrix</td>
</tr>
<tr>
<td>SAPF</td>
<td>Satellite A Priori Flag</td>
</tr>
<tr>
<td>SC</td>
<td>Satellite Covariance Matrix</td>
</tr>
<tr>
<td>SCA</td>
<td>Satellite Covariance Augmented (Includes Effects of SAP, SDC)</td>
</tr>
<tr>
<td>SDC</td>
<td>Satellite Drift Covariance Matrix</td>
</tr>
<tr>
<td>SDCEL</td>
<td>Square Roots of Elements of SDC</td>
</tr>
<tr>
<td>SDCF</td>
<td>Satellite Drift Covariance Flag</td>
</tr>
<tr>
<td>SLOC</td>
<td>Satellite Location ((\theta, \phi, \rho))</td>
</tr>
<tr>
<td>UAP</td>
<td>User A Priori Matrix</td>
</tr>
<tr>
<td>UAPEL</td>
<td>Square Roots of UAP</td>
</tr>
<tr>
<td>UAPF</td>
<td>User A Priori Flag</td>
</tr>
<tr>
<td>UC</td>
<td>User Covariance Matrix</td>
</tr>
<tr>
<td>ULOC</td>
<td>User Location ((\theta, \phi, \rho))</td>
</tr>
<tr>
<td>UMC</td>
<td>User Measurement Covariance Matrix</td>
</tr>
<tr>
<td>UMCEL</td>
<td>Square Roots of Elements of UMC</td>
</tr>
<tr>
<td>UMM</td>
<td>User Measurement Matrix (A Control Matrix)</td>
</tr>
</tbody>
</table>
**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>Longitude, Degrees</td>
</tr>
<tr>
<td>φ</td>
<td>Latitude, Degrees</td>
</tr>
<tr>
<td>ρ</td>
<td>Geocentric Range, Nautical Miles</td>
</tr>
</tbody>
</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>GS</td>
<td>Ground Station</td>
</tr>
<tr>
<td>S</td>
<td>Satellite</td>
</tr>
<tr>
<td>U</td>
<td>User</td>
</tr>
</tbody>
</table>

*Note: Primed quantities are only the results of intermediate calculation and have no real meaning.*
Figure A-1. SPIT Flow Diagram
APPENDIX J
NAVIGATION SATELLITE ACCURACY
PROGRAM (NAVSAP)

1. INTRODUCTION

The logical structure of the TRW SVEAD program, delivered to ERC under a separate contract, has been utilized in the development of the Navigation Satellite Accuracy Program. This program (NAVSAP) performs an error analysis for a given satellite configuration and user positions. The analysis is based on minimum variance estimation of the state vector consisting of user position and other parameters of interest, such as measurement bias and satellite positions and velocities. The results are presented in terms of the "C 95", the radius of a circle containing the user with probability 0.95. Range, range difference, or range sum measurements can be considered.

Inputs to the program are the following:

a. The first partition of the state vector comprised of the positions and velocities for as many as 7 satellites

b. The second partition of the state vector comprised of the user latitude, longitude, and altitude (actually a grid of user positions is prescribed in terms of the boundary values of latitude and longitude and the latitude-longitude spacing between users)

c. The error covariance matrix of the uncertainty in the satellites' positions and velocities

d. The error covariance matrix of the uncertainty in the a priori estimate of the user positions

e. The variance of the measurement noise.

For the first user position the program computes the partial derivatives of the observations with respect to the elements of the state vector. The filter equations are then used to adjust the covariance matrix of the state vector to account for the first observation. The C95 is then computed and printed out. This process is repeated until all measurements have been processed, and the final C95 for that position is printed out. This process is repeated until all measurement have
been processed, and the final C95 for that position is printed out. The program then proceeds to the next user position, incrementing first latitude and then longitude, until a C95 is computed for each point in the grid.

The program can also consider a user moving at constant altitude along a great circle path. Measurements are taken at prescribed intervals, and the estimate is continually updated as a result of the new measurements. In this mode it is possible to consider the effect of random perturbations in the user flight path by inserting noise (state noise) on the velocity vector.

The program employs a Runge-Kutta integration package to integrate the satellites' trajectories, based on input initial positions and velocities. As many as seven satellites may be integrated simultaneously and the state vectors stored at specific measurement times for use in the subsequent error analysis.

This appendix contains a complete engineering description of the program. An accompanying document contains the detailed program implementation and subroutine descriptions. An overall flow diagram appears in Figure 1.
INTTEGRATOR
Construct satellites' ephemerides for all measurement times

INPUTS:
1. Satellite initial conditions
2. Ground user information
3. Measurement types and biases
4. Consider parameters
5. State Noise

Updating Matrices
Compute:
\[ U = \begin{bmatrix} U_1 \\ \phi \\ \phi \\ U_k \end{bmatrix} \]
where \( U_k \) depends on user conditions. (\( U = I \) for \( t = 0 \))

Update Error Covariance Matrices
\[ J_{i+1/i} = U_{i+1/i} U^T + R_i \]
\( R_i \) is state noise

User Location
Pick next position:
1. Grid Method
2. Flight Path Method

Visibility Check
Label all "visible" satellites

Measurement Matrices
Compute:
\[ M = [M_1 : M_2] \]
for current type of measurement

Kalman Filter
Compute:
\[ B = J_{i+1/i} M_i^T (M_i + L_i M_i^T + W)^{-1} \]
\[ J_{i+1/i+1} = (I - BM) J_{i+1/i} \]

Finished with all measurements of current measurement type?

Increment time
\[ t_{i+1} = t_i + \Delta t \]

Finished with all measurement types?

YES

YES

Figure 1-1. NAVSAP - General Flow Diagram
2. STATE VECTOR

The state vector is partitioned into two sections. The first section $X_1$ contains the satellite states while the second $X_2$ contains the user position and measurement biases. These vectors are constructed as:

$$X_1^T = \begin{bmatrix} x_1 & y_1 & z_1 & \dot{x}_1 & \dot{y}_1 & \dot{z}_1 & x_2 & \ldots & z_N \end{bmatrix} \text{1 x 6N}$$

$$X_2^T = \begin{bmatrix} x_u & y_u & z_u & b_1 & \ldots & b_M \end{bmatrix} \text{1 x (3+M)}$$

where $N$ is the number of satellites (input) and $M$ is the number of measurement biases. The superscript $T$ denotes transpose; the numerical subscript refers to the satellite and $u$ refers to the user.

The error covariance matrix is correspondingly partitioned

$$J = E[\delta X \delta X^T]$$

$$= E \begin{bmatrix} \delta X_1^{T} & \delta X_1^{T} & \delta X_1^{T} & \delta X_2^{T} & \delta X_2^{T} & \delta X_2^{T} \end{bmatrix} = \begin{bmatrix} J_1 & J_3^T \ \ J_3 & J_4 \end{bmatrix}$$

Since $J$ is symmetric, it is only necessary to compute and store the partitions, $J_1$, $J_3$, and $J_4$. 

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3. COORDINATE SYSTEMS

All quantities in the program are referenced to one of the following four coordinate systems (Figure 3-1):

- \( \mathbf{X}_o = (x_o, y_o, z_o) \) Earth-centered inertial (ECI) Cartesian system
- \( \mathbf{X} = (x, y, z) \) Earth-centered fixed (ECF) Cartesian system
- \( \theta = (\theta, \lambda, r) \) ECF spherical system
- \( \mathbf{U} = (u, v, w) \) Satellite-centered inertial (SCI) Cartesian system (radial, in-track, cross-track)

The time origin is selected at the first measurement time. At that instant \( \mathbf{X}_o \) and \( \mathbf{X} \) are colinear, \( z \) passes through the North Pole and \( y \) is in the equatorial plane, passing through the prime meridian (Greenwich). In the \( \theta \) system, latitude \( \theta \) is measured positive north from the equator and longitude \( \lambda \) is measured positive west from Greenwich. In the \( \mathbf{U} \) system, \( u \) is directed along the radius vector to the satellite, \( w \) is in the direction of the satellite angular momentum vector, and \( v \) completes the orthogonal set.

Using the notation that \( \partial \mathbf{A}/\partial \mathbf{B} \) is the matrix that maps coordinate system \( \mathbf{B} \) into \( \mathbf{A} \), the following transformation matrices are defined:

\[
\frac{\partial \mathbf{X}_o}{\partial \mathbf{X}} = \left( \frac{\partial \mathbf{X}}{\partial \mathbf{X}_o} \right)^{-1} = \left( \frac{\partial \mathbf{X}}{\partial \mathbf{X}_o} \right)^T = \begin{bmatrix} \cos \omega t & -\sin \omega t & 0 \\ \sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}
\] (4)

\[
\frac{\partial \mathbf{X}}{\partial \theta} = \begin{bmatrix} -z y & \gamma & x \\ y & -x & \gamma \\ \gamma & 0 & z \end{bmatrix}
\] (5)
Figure 3-1. Coordinate Systems

\[ \omega = \text{ROTATIONAL RATE OF EARTH} \]
\[
\frac{\partial \Theta}{\partial X} = \begin{bmatrix}
\frac{xz}{\gamma r} & \frac{yz}{\gamma r} & \frac{y}{r} \\
\frac{y}{\gamma^2} & \frac{x}{\gamma^2} & 0 \\
\frac{x}{r} & \frac{y}{r} & \frac{z}{r}
\end{bmatrix}
\]

where

\[
\gamma = \sqrt{x^2 + y^2}
\]

\[
r = \sqrt{\gamma^2 + z^2}
\]

\[
\frac{\partial X_o}{\partial U} = \left(\frac{\partial U}{\partial X_o}\right)^{-1} = \left(\frac{\partial U}{\partial X_o}\right)^T = \begin{bmatrix}
u_x & v_x & w_x \\
u_y & v_y & w_y \\
u_z & v_z & w_z
\end{bmatrix}
\]

using the following equations which define \(U\)

\[
u = \frac{r}{|r|}
\]

\[
w = (r \times \mathbf{v}) \left[ (rv)^2 - (r \cdot \mathbf{v})^2 \right]^{-1/2}
\]

\[
v = w \times u
\]

where \(\mathbf{v}\) is the satellite velocity vector, we find

\[
u_x = \frac{x}{r}, \quad u_y = \frac{y}{r}, \quad u_z = \frac{z}{r}
\]

\[
w_x = \frac{1}{D} (y \dot{z} - \dot{y}z)
\]

\[
w_y = -\frac{1}{D} (x \dot{z} - \dot{x}z)
\]

\[
w_z = \frac{1}{D} (x \dot{y} - \dot{xy})
\]

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\[ v_x = w_y u_z - w_z u_y \]
\[ v_y = w_z u_x - w_x u_z \]
\[ v_z = w_x u_y - w_y u_x \]

where

\[ D = \sqrt{r^2 V^2 - (x\dot{x} + y\dot{y} + z\dot{z})^2} \]

and \( x, y, z \) denote ECI coordinates.
4. INTEGRATOR

If measurements are taken at times other than \( t = 0 \), the program integrates the satellite trajectories to the specified measurement times and constructs an ephemeris. Measurement times are every \( \Delta t_m \) seconds until time is greater than final time \( t_f \). \( \Delta t_m \) is specified as an integral multiple of the integration step size.

The program uses a fourth order, self-starting, Runge-Kutta procedure with a point-mass, two-body force model. The constraint equations for the \( i \)th satellite are:

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{z}_i \\
\end{bmatrix} = \begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{z}_i \\
\end{bmatrix} + \begin{bmatrix}
-\mu x_i / r_i^3 \\
-\mu y_i / r_i^3 \\
-\mu z_i / r_i^3 \\
\end{bmatrix}
\]

where \( \mu \) is the earth's gravitational constant.
5. ERROR ANALYSIS

5.1 FILTER EQUATIONS

At time \( t_i \) after \( i-1 \) measurements, let the \( i^{th} \) observation \( \xi_i \) be linearly related to the column state vector of unknowns \( X_i \) by the relation

\[
\xi_i = M_i X_i + w_i
\]

where \( M_i \), the measurement vector, is a row vector of the partial derivatives of \( \xi_i \) with respect to the components of \( X_i \), and \( w_i \) is zero mean, uncorrelated, random noise. That is,

\[
E(w_i) = 0
\]
\[
E(w_i w_j) = 0 \quad i \neq j
\]
\[
E(w_i^2) = W
\]

where \( E \) is the expectation operator.

Let \( J_{i/j} \) be the error covariance matrix of \( X_i \) based on \( j \) measurements \( (j \leq i) \); define the measurement weighting matrix.

\[
B_i = J_{i/i-1} M_i^T (M_i J_{i/i-1} M_i^T + W)^{-1}
\]

and let

\[
C_i = I - B_i M_i
\]

The general formula for the current error covariance matrix is then

\[
J_{i/i} = C_i J_{i/i-1} C_i^T + B_i W B_i^T
\]
which, when $B$ is given by Equation (14), assumes it minimum value

$$J_{i/i} = C_i J_{i/i-1}$$  \hspace{1cm} (17)

Let $U_{i/j}$ be the state transition matrix for $X_j$ from time $t_j$ to $t_i$. Then $J_{i/i}$ is propagated to the next measurement time $t_{i+1}$ according to the relation

$$J_{i+1/i} = U_{i+1/i} J_{i/i} U_{i+1/i}^T + R_i$$

where $R_i$ is a random disturbance covariance matrix (state noise). These equations are programmed in NAVSAP in partitioned form corresponding to the partitioning of the overall program state vector.

5.2 CONSIDER OPTION (SUBOPTIMAL FILTERING)

The program can compute the estimation errors caused by errors in parameters which are not estimated. These parameters may include the state vector of any satellite and any of the measurement biases. This option requires that two stacked cases be run on NAVSAP. In the first case, all portions of the measurement vector $M$ pertaining to the considered parameters are zeroed out. The measurement weighting vector $B$ computed using this $M$ is then stored on tape. In this phase, the program uses the optimal error covariance matrix computation given by Equation (17). In the second case, the full $M$ along with the corresponding $B$ computed in the first case is used in the suboptimal error covariance matrix computation according to Equation (16).

5.3 MEASUREMENT TYPES

The program can process several types of measurements, taken in any desired order. It is assumed that all measurements of all specified types are taken simultaneously at each measurement time.
5.3.1 Range and Range Rate Measurements

The range $R$ from the $s$th satellite to the user is defined to be the magnitude of the separation vector $R$ (see Figure 5-1).

$$R = \left| \mathbf{R} \right| = \left| \mathbf{R}_s - \mathbf{R}_u \right|$$  \hspace{1cm} (18)

The components of $R$ are

$$x = x_s - x_u, \quad y = y_s - y_u, \quad z = z_s - z_u$$  \hspace{1cm} (19)

The range is then

$$R = \sqrt{x^2 + y^2 + z^2}$$  \hspace{1cm} (20)

from which the range rate is

$$\dot{R} = \frac{\mathbf{R} \cdot \mathbf{R}}{R} = \frac{x \dot{x} + y \dot{y} + z \dot{z}}{R}$$  \hspace{1cm} (21)

From these equations it follows that the partials of $R$ and $\dot{R}$ with respect to the components of $\mathbf{R}_s$ are

$$\frac{\partial R}{\partial (x_s, y_s, z_s)} = m_s = \frac{1}{R} \left[ x, \ y, \ z \right]_{1 \times 3}$$  \hspace{1cm} (22)

$$\frac{\partial \dot{R}}{\partial (x_s, y_s, z_s)} = \dot{m}_s = \frac{1}{R^2} \left[ (R \dot{x} - x \dot{R}), \ (R \dot{y} - y \dot{R}), \ (R \dot{z} - z \dot{R}) \right]$$  \hspace{1cm} (23)

$$\frac{\partial R}{\partial (\dot{x}_s, \dot{y}_s, \dot{z}_s)} = \left[ 0, \ 0, \ 0 \right]_{1 \times 3}$$  \hspace{1cm} (24)

$$\frac{\partial \dot{R}}{\partial (\dot{x}_s, \dot{y}_s, \dot{z}_s)} = m_s$$  \hspace{1cm} (25)
Figure 5-1. Satellite-User Geometry
Hence, the measurement vector corresponding to $X_1$ has the form

$$M_1 = \begin{cases} \begin{bmatrix} \phi & m_s & \phi \end{bmatrix}_{1 \times 6N} & \text{(range)} \\ \begin{bmatrix} \phi & \dot{m}_s & \dot{m}_s & \phi \end{bmatrix}_{1 \times 6N} & \text{(range-rate)} \end{cases}$$

where the first possible nonzero element of $M_1$ is the $(6S - 5)^{th}$ term and $\phi$ denotes the appropriate null array. The measurement vector corresponding to $X_2$ has the form

$$M_2 = \begin{cases} \begin{bmatrix} -m_s & m_b \end{bmatrix}_{1 \times (3+M)} & \text{(range)} \\ \begin{bmatrix} -\dot{m}_s & \dot{m}_b \end{bmatrix}_{1 \times (3+M)} & \text{(range-rate)} \end{cases}$$

where $m_b$ is the vector of appropriate measurement bias partials (see Section 5.4).

### 5.3.2 Sums and Differences of Range or Range-Rate Measurements

A range (range rate) sum measurement to satellites I and J is defined to be the sum of the individual ranges (range rates) to these satellites. Similarly, a range (range rate) difference measurement to these satellites is defined to be the difference between the ranges (range rates). Using the notation $\Sigma$ for sums and $\Delta$ for differences, the measurements are:

$$\begin{align*}
\Sigma R_{ij} &= R_i + R_j \\
\Sigma \dot{R}_{ij} &= \dot{R}_i + \dot{R}_j \\
\Delta R_{ij} &= R_i - R_j \\
\Delta \dot{R}_{ij} &= \dot{R}_i - \dot{R}_j
\end{align*}$$

(28)
Hence, the measurement vector corresponding to $X_1$ has the form

$$M_1 = \begin{bmatrix} \phi & m_i & \phi & \pm m_j & \phi \\ \phi & \dot{m}_i & m_i & \phi & \pm \dot{m}_j & \pm m_j & \phi \end{bmatrix}_{1 \times 6N}$$  \hspace{1cm} \text{(range)} \hspace{1cm} (29)$$

$$M_1 = \begin{bmatrix} \phi & m_i & \phi & \pm m_j & \phi \\ \phi & \dot{m}_i & m_i & \phi & \pm \dot{m}_j & \pm m_j & \phi \end{bmatrix}_{1 \times 6N}$$  \hspace{1cm} \text{(range rate)} \hspace{1cm} (30)$$

where $m_i$ in Equation (29) and $\dot{m}_i$ in Equation (30) start at the $(6i-5)^{\text{th}}$ term and $m_j$ in Equation (29) and $\dot{m}_j$ in Equation (30) start at the $(6j-5)^{\text{th}}$ term. The plus signs correspond to sum measurements, and the minus to differences. The measurement vector corresponding to $X_2$ has the form

$$M_2 = \begin{bmatrix} -(m_i \pm m_j) \dot{m}_b \\ -(\dot{m}_i \pm \dot{m}_j) \dot{m}_b \end{bmatrix}$$  \hspace{1cm} \text{(range)} \hspace{1cm} (31)$$

$$M_2 = \begin{bmatrix} -(m_i \pm m_j) \dot{m}_b \\ -(\dot{m}_i \pm \dot{m}_j) \dot{m}_b \end{bmatrix}$$  \hspace{1cm} \text{(range rate)}$$

5.4 MEASUREMENT BIASES

These biases reflect constant measurement errors originating in the user equipment. The number and order in which they are to be included at the end of $X_2$ are specified by input quantities. As mentioned in Section 5.2, they can be individually solved for or considered in the error analysis.

The partials of an observation with respect to these biases depend only on the type of measurement being taken. The quantities input to $m_b$ are

$$\frac{\partial (\text{observation})}{\partial (\text{bias})} = \begin{cases} 1 \text{ for } R, \dot{R} \\ 0 \text{ for } \Delta R, \Delta \dot{R} \\ 2 \text{ for } \Sigma R, \Sigma \dot{R} \end{cases}$$  \hspace{1cm} (32)$$

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6. USER POSITION SELECTION

The program has two methods of sequentially selecting user positions. In either case, this is done after all combinations of all desired measurement types have been processed for the current user position as indicated in Figure 1-1.

6.1 GRID METHOD

In this method, the area over which user positions are to be selected is defined by boundary values of latitude and longitude and the actual positions by the latitude-longitude spacing between users. This information is input in the following form (see Figure 6-1).

\[
\begin{align*}
\theta_i &= \text{initial latitude} \\
\theta_f &= \text{final latitude} \\
\Delta \theta &= \text{incremental change in latitude} \\
\lambda_i &= \text{initial longitude} \\
\lambda_f &= \text{final longitude} \\
\Delta \lambda &= \text{incremental change in longitude}
\end{align*}
\]

6.2 FLIGHT PATH METHOD

This method sequentially selects user positions along a great circle arc at every measurement time. The user is assumed to be moving at a constant speed and altitude. The positions are determined from the initial position, velocity, and heading by use of the following identities from spherical trigonometry (Figure 6-2).

\[
\begin{align*}
\sin \theta &= \cos \omega \sin \theta_i + \cos \alpha \cos \theta_i \sin \omega \\
\cos \alpha &= \cos \alpha_i \cos \beta - \sin \beta \sin \alpha_i \sin \theta_i \\
\sin \omega &= \frac{\cos \theta_i}{\sin \alpha} = \frac{\cos \theta}{\sin \alpha_i}
\end{align*}
\]

\[\text{(33)}\]

\[\text{(34)}\]

\[\text{(35)}\]
Figure 6-1. Grid Method
Figure 6-2. Flight Path Method
where \( \theta, \theta_i \) are the final and initial latitudes, \( \alpha, \alpha_i \) are the final and initial headings (positive east of north), \( \beta \) is minus the change in west longitude and \( \omega \) is the angle traversed along the great circle. If \( V \) is the user's velocity, and \( \Delta t_m \) is the time between measurements, then \( \omega \) is given by

\[
\omega = \frac{V \Delta t_m}{R}
\]

where \( R \) is the distance from the user to the center of the earth. In Equations (33) and (34) the angles are restricted to the following ranges:

\[-\frac{\pi}{2} \leq \theta \leq +\frac{\pi}{2}\]

\[\omega \geq 0\]

\[-\pi \leq \alpha \leq +\pi\]

Since the initial conditions are known and \( \omega \) is readily computed from Equation (36), the latitude \( \theta \) can be computed from Equation (33) and \( \beta \) follows from Equation (35) which in turn leads to the longitude

\[
\lambda = \lambda_i - \beta
\]

\( \alpha \) also follows from Equations (34) and (35) for use at the next measurement point where \( \theta, \alpha, \lambda \) become \( \theta_i, \alpha_i, \lambda_i \).

6.3 VISIBILITY CHECK

At each measurement time, the program checks to see which satellites are visible. The criterion is that each satellite's elevation above the user's local horizon be greater than or equal to an input minimum elevation angle \( \epsilon_m \). This check is performed in the following manner (see Figure 6-3).
Figure 6-3. Visibility Geometry
Let $\mathbf{R}_u$ be the user position vector and $\mathbf{R}_s$ be the satellite position vector. Then the relative range vector $\mathbf{R}$ is

$$\mathbf{R} = \mathbf{R}_s - \mathbf{R}_u$$

(38)

Hence,

$$\cos \theta = \frac{\mathbf{R} \cdot \mathbf{R}_u}{|\mathbf{R}||\mathbf{R}_u|}$$

(39)

The visibility criterion requires that

$$\frac{\pi}{2} - \theta \geq \epsilon_m$$

(40)

or equivalently

$$\cos \theta \geq \sin \epsilon_m$$

(41)

Hence, the program tests on the following relation

$$\frac{\mathbf{R} \cdot \mathbf{R}_u}{|\mathbf{R}||\mathbf{R}_u|} \geq \sin \epsilon_m$$

(42)
7. UPDATE SECTION

To propagate the error covariance matrices between measurement times \( t_i \) and \( t_{i+1} \), an updating routine is required. Two updating (state transition) matrices are calculated, one for each partition of the state vector.

7.1 SATELLITE UPDATE

The updating matrix \( U_1 \) for \( X_1 \) is calculated from an analytic solution to the variational equations for the satellites.

Let

\[
R_j = \sqrt{x_j^2 + y_j^2 + z_j^2}
\]  

be the magnitude of the relative range of the \( j \)th satellite from the user. Define \( k_j \) to be

\[
k_j = \frac{k}{R_j^5}
\]

\[
\begin{bmatrix}
3x_j^2 - R_j^2 & 3x_jy_j & 3x_jz_j \\
3y_j^2 - R_j^2 & 3y_j^2 & 3y_jz_j \\
3z_j^2 - R_j^2 & 3z_j^2 & 3z_j^2
\end{bmatrix}_{3 \times 3}
\]

Let

\[
\overline{k}_j = \frac{1}{2} \left[ k_j(t_{i+1}) + k_j(t_i) \right]
\]  

be an approximation to the time averaged value of \( k_j \) over the updating interval. Using \( I \) to denote the identity matrix and \( \Delta t \) to denote the updating interval \( (= t_{i+1} - t_i) \) define
Using the above, \( U_1 \) can be written as a block diagonal matrix of the form:

\[
K_j = \begin{bmatrix}
I + \frac{\Delta t^2}{2} \frac{1}{k_j} & (\Delta t)I + \frac{\Delta t^3}{6} \frac{1}{k_j}
\
(\Delta t)\frac{1}{k_j} & I + \frac{\Delta t^2}{2} \frac{1}{k_j}
\end{bmatrix}_{6 \times 6}
\]

Using the above, \( U_1 \) can be written as a block diagonal matrix of the form:

\[
U_1^{i+1/i} = \begin{bmatrix}
K_1 & \phi \\
& K_2 \\
& & \ddots \\
& & & \phi \\
& & & & K_N
\end{bmatrix} \quad \text{6x6N}
\]

where \( \phi \) denotes the appropriate array of zeros.

### 7.2 USER UPDATE

The updating matrix \( U_4 \) for the second partition \( X_2 \) is computed in one of two ways corresponding to the mode of selecting user positions (see Section 6). In either case, \( U_4 \) is a block diagonal matrix of the form:

\[
U_4 = \begin{bmatrix}
U_{4\text{user}} & \phi \\
\phi & I_{(3+M) \times (3+M)}
\end{bmatrix}
\]

where \( U_{4\text{user}} \) is the state transition matrix for the user position coordinates.

#### 7.2.1 Grid Method

In the grid method, the user is stationary in an ECF system. Hence, his associated position error covariance matrix remains constant that system. Since this matrix is computed in an ECI system, the
The corresponding updating matrix is the product of two coordinate transformations as follows: (see Sections 3 and 5 for notation)

\[
U_{4\text{user}, \ i+1/i} = \frac{\partial X_{o,i+1}}{\partial X_{o,i}} = \frac{\partial X_{o,i+1}}{\partial X_{i+1}} \frac{\partial X_{i+1}}{\partial X_i} \frac{\partial X_i}{\partial X_{o,i}} = \frac{\partial X_{o,i+1}}{\partial X_{i+1}} \frac{\partial X_i}{\partial X_{o,i}} \tag{49}
\]

### 7.2.2 Flight Path Method

In the flight path method of selecting user positions, the user motion is specified in the ECF system. The program calculates \(U_{4\text{user}}\) from

\[
U_{4\text{user}, \ i+1/i} = \frac{\partial X_{o,i+1}}{\partial X_{i+1}} \frac{\partial X_{i+1}}{\partial \theta_i} \frac{\partial \theta_i}{\partial X_i} \frac{\partial X_i}{\partial X_{o,i}} \tag{50}
\]

where

\[
\frac{\partial \theta_i+1}{\partial \theta_i} = \begin{bmatrix}
\frac{\partial \theta_i+1}{\partial \lambda_i} & \frac{\partial \theta_i+1}{\partial \lambda_i} & \frac{\partial \theta_i+1}{\partial r_i} \\
\frac{\partial \lambda_i+1}{\partial \lambda_i} & \frac{\partial \lambda_i+1}{\partial \lambda_i} & \frac{\partial \lambda_i+1}{\partial r_i} \\
\frac{\partial r_i+1}{\partial \lambda_i} & \frac{\partial r_i+1}{\partial \lambda_i} & \frac{\partial r_i+1}{\partial r_i}
\end{bmatrix}
\]

From the equations in Section 6.2, it follows that

\[
\frac{\partial \theta_i+1}{\partial \theta_i} = \begin{bmatrix}
\frac{\partial \theta_i+1}{\partial \lambda_i} & 0 & 0 \\
\frac{\partial \lambda_i+1}{\partial \lambda_i} & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \tag{51}
\]

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where

\[
\frac{\partial \theta_{i+1}}{\partial \theta_i} = \frac{1}{\cos \theta_{i+1}} (\cos \theta_i \cos \omega - \sin \theta_i \sin \omega \cos \alpha_i)
\]

(52)

\[
\frac{\partial \lambda_{i+1}}{\partial \theta_i} = \frac{-\sin \omega \sin \alpha_i \sin \theta_{i+1}}{\cos (\lambda_i - \lambda_{i+1}) \cos^2 \theta_{i+1}} \left[ \frac{\partial \theta_{i+1}}{\partial \theta_i} \right]
\]

Using Equations (47) through (52) to define \(U_1\) and \(U_4\), the updating section calculates the following:

\[
J_{1,i+1/i} = U_{1,i+1/i} J_{1,i/i} U_{1,i,i+1/i}^T
\]

\[
J_{3,i+1/i} = U_{4,i+1/i} J_{3,i/i} U_{4,i,i+1/i}^T
\]

(53)

\[
J_{4,i+1/i} = U_{4,i+1/i} J_{4,i/i} U_{4,i,i+1/i}^T + R_i
\]

The random disturbance covariance matrix \(R_i\) adds the effect of state noise to the updated user position error covariance matrix \(J_{4,i+1/i}\). State noise results from random disturbances in the user's speed and heading. \(R_i\) is computed as follows:

\[
R_i = \begin{bmatrix}
J_R(3x3) & \phi \\
\phi & \phi
\end{bmatrix}_{(3+M) \times (3+M)}
\]

(54)
where

\[ J_R = \left( \frac{\partial \mathbf{X}_o}{\partial \psi} \right)^T J_\psi \left( \frac{\partial \mathbf{X}_o}{\partial \psi} \right) \]  

(55)

\( J_\psi \) is a 2 x 2 input error covariance matrix of speed and heading and \( \frac{\partial \mathbf{X}_o}{\partial \psi} \) is the 3 x 2 transformation matrix that maps these errors into the user's position coordinates.

That is

\[ \frac{\partial \mathbf{X}_o}{\partial \psi} = \frac{\partial \mathbf{X}_o}{\partial \mathbf{X}} \frac{\partial \mathbf{X}}{\partial \theta} \frac{\partial \theta}{\partial \psi} \]  

(56)

\[ \frac{\partial \theta}{\partial \psi} = \begin{bmatrix} \frac{\partial \theta}{\partial \mathbf{V}} & \frac{\partial \theta}{\partial \mathbf{a}_i} \\ \frac{\partial \lambda}{\partial \mathbf{V}} & \frac{\partial \lambda}{\partial \mathbf{a}_i} \\ \frac{\partial r}{\partial \mathbf{V}} & \frac{\partial r}{\partial \mathbf{a}_i} \end{bmatrix} \]  

(57)
From the equations in Section 6.2, it follows that

\[ \frac{\partial \theta}{\partial V} = \frac{\Delta t}{R \cos \theta} (\cos \theta \cos \omega \cos \alpha_i - \sin \theta \sin \omega) \]

\[ \frac{\partial \theta}{\partial \alpha_i} = -\sin \omega \sin \alpha \]

\[ \frac{\partial \lambda}{\partial V} = -\frac{1}{\cos (\lambda_i - \lambda)} \left[ \frac{\Delta t}{R \cos \theta} \frac{\cos \omega \sin \alpha_i}{\cos \theta} + \frac{\sin \omega \sin \alpha_i \sin \theta}{\cos^2 \theta} \frac{\partial \theta}{\partial V} \right] \] (58)

\[ \frac{\partial \lambda}{\partial \alpha_i} = -\frac{1}{\cos (\lambda_i - \lambda)} \left[ \frac{\sin \omega \cos \alpha_i}{\cos \theta} + \frac{\sin \omega \sin \alpha_i \sin \theta}{\cos^2 \theta} \frac{\partial \theta}{\partial \alpha_i} \right] \]

\[ \frac{\partial r}{\partial V} = 0 \quad \frac{\partial r}{\partial \alpha_i} = 0 \]
8. EFFECT OF SATELLITE ESTIMATION ERRORS

The user of a navigation satellite system does not estimate the satellite positions. The satellite orbits are determined by prior tracking from ground stations, and are transmitted to the user after being computed at a central site. Errors in the satellite locations are determined by orbit determination studies. In a least squares analysis this is done by appending to the inverse normal matrix an additional term accounting for the satellite errors. In the filter analysis of NAVSAP, it is necessary to do the computation twice as described in Sec. 5.2. In the first pass, the gain is computed assuming the satellite errors are zero; in the second, the gain is used in a fictitious attempt to solve for the satellite locations. The resulting error covariance matrix of user position contains the desired effect of satellite errors.

This multiple pass through the filter is undesirable for parametric studies and it turns out to be unnecessary. Since the satellite locations are essentially not observable in the user data (tracking with a single station of unknown location), it can be postulated that, even when the satellite position is included in the regression vector, the satellite errors are only slightly reduced, and the effect on user position errors is essentially the same as when the satellite positions are not solved for. Hence, error analysis runs can be made, assuming that satellite positions are estimated, with the results showing only the effect of the initial satellite position uncertainties.

In more concrete terms, consider the linear observation model

\[ y = Ax + Bz + \epsilon \]  \hspace{1cm} (59)

where \( y \) is the observation vector, \( x \) is the user position vector, \( z \) is the satellite position vector, and \( \epsilon \) is an error vector with uncorrelated components. \( A \) and \( B \) are matrices of partial derivatives relating the deviations \( x \) and \( z \) to the observation deviations \( z \). Assuming the satellite positions are known perfectly \( (z = 0) \), the minimum variance estimate of \( x \) is

\[ \hat{x} = (A^T W A)^{-1} A^T W y \]  \hspace{1cm} (60)
where

\[ W = \left[ E(\epsilon \epsilon^T) \right]^{-1} \]  \hspace{1cm} (61)

The covariance matrix of the error in this estimate, considering the
effect of a priori satellite errors \( z \), is

\[ \text{Cov} (\hat{x} - x) = (A^T W A)^{-1} + (A^T W A)^{-1} A^T W B Q B^T W A (A^T W A)^{-1} \]  \hspace{1cm} (62)

where \( Q \) is the a priori satellite error covariance matrix

\[ Q = E(zz^T) \]

On the other hand, if the satellite position is included in the
regression vector, the estimate becomes

\[
\begin{pmatrix}
\hat{x} \\
\hat{z}
\end{pmatrix}
= \begin{pmatrix}
A^T W A & A^T W B \\
B^T W A & B^T W B + Q^{-1}
\end{pmatrix}^{-1}
\begin{pmatrix}
A^T \\
B
\end{pmatrix} W y
\]  \hspace{1cm} (63)

with error covariance matrix

\[
E\left[ \begin{pmatrix}
\hat{x} - x \\
\hat{z} - z
\end{pmatrix} \begin{pmatrix}
\hat{x} - x \\
\hat{z} - z
\end{pmatrix}^T \right]
= \begin{pmatrix}
A^T W A & A^T W B \\
B^T W A & B^T W B + Q^{-1}
\end{pmatrix}^{-1}
\]  \hspace{1cm} (64)

Partitioning Equation (64) leads to the individual results for \( x \) and \( z \)

\[
\text{Cov} (\hat{x} - x) = (A^T W A)^{-1} + (A^T W A)^{-1} A^T W B \left[ B^T W B + Q^{-1} - B^T W A (A^T W A)^{-1} A^T W B \right]^{-1}
\]

\[
\cdot B^T W A (A^T W A)^{-1}
\]

\[
= \left[ A^T W A - A^T W B (B^T W B + Q^{-1})^{-1} B^T W A \right]^{-1}
\]  \hspace{1cm} (65)

\[
\text{Cov} (\hat{z} - z) = \left[ B^T W B + Q^{-1} - B^T W A (A^T W A)^{-1} A^T W B \right]^{-1} = Q^*
\]  \hspace{1cm} (66)

Using Equation (66) in (65) leads to the desired expression

\[
\text{Cov} (\hat{x} - x) = (A^T W A)^{-1} + (A^T W A)^{-1} A^T W B Q^* B^T W A (A^T W A)^{-1}
\]  \hspace{1cm} (67)

Assuming the satellite positions are solved for is equivalent to using
Equation (67) in place of Equation (63), which is a good approximation if
\( Q^* \approx Q \). This will be true if the satellite positions are only weakly
observable in the data.

The correctness of this hypothesis is shown by comparison of
the computer results presented in Table 1. The first column shows re-
sults for no satellite errors. The second column considers, satellite
position errors, requiring two passes through the filter as described in Section 5.2. The third column results when satellite positions are solved for.

The last two columns are nearly equal, which demonstrates conclusively that solving for satellite positions is essentially equivalent to considering them. Hence, the extra pass through the filter is not required.

Table 1. Comparison of Solving for and Considering Satellite Errors.

<table>
<thead>
<tr>
<th>East Longitude</th>
<th>North Latitude</th>
<th>No Satellite Errors</th>
<th>Consider Satellite Errors</th>
<th>Solve for Satellite Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>30° 0</td>
<td>347</td>
<td>448</td>
<td>448</td>
<td></td>
</tr>
<tr>
<td>30 0</td>
<td>468</td>
<td>588</td>
<td>586</td>
<td></td>
</tr>
<tr>
<td>60 0</td>
<td>327</td>
<td>442</td>
<td>441</td>
<td></td>
</tr>
<tr>
<td>30 341</td>
<td>455</td>
<td>454</td>
<td>453</td>
<td></td>
</tr>
<tr>
<td>60 364</td>
<td>474</td>
<td>473</td>
<td>473</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Measurement noise = 100 ft (1σ)
User altitude error = 150 ft (1σ)
Satellite position errors from Appendix K based on 72 hours tracking from three stations.
APPENDIX K

SUPPORTING ORBIT DETERMINATION ANALYSIS

In order to assess the magnitudes and importance of certain variables in the satellite tracking and orbit determination process, several runs were made with the TRW Systems ESPOD computer program. The purpose of the program was to determine the effects of the following: of using angle measurements and range-rate measurements in addition to range measurement, of using two tracking stations or three, of tracking for 72 hours or 36 hours, and of solving or not solving for the earth's gravitational constant and two or more harmonics.

A single satellite whose ground trace is centered at 75° west longitude and inclined at 18.5° to the equatorial plane was chosen for the first series of tracking analyses. The orbit ground trace and tracking station locations are shown in Figure K-1. Stations 1 and 2 are used in all cases where only two-station tracking is specified. These locations were chosen to provide good tracking geometry while conforming to geographical realities. During the course of the study a recommended tracking network was finalized. A three station, single-satellite tracking arrangement was selected from this network for the final case considered.

Table K-1 lists the values and sources of the errors used in the study. The measurement errors are considerably in excess of the expected errors summarized in subsec. 2.4 in the main body of this report, and the data rate was less (1 point/min). The resulting errors turn out, therefore, to be greater than those presented in subsec. 2.4. These preliminary analyses were nevertheless quite adequate to provide the answers to the questions posed.

The study consisted of the following cases:

1) Seventy two-hours tracking with two stations, RAER measurements.
   a) Solve for satellite state. Consider measurement bias errors, survey errors and μ, J₂ uncertainties.
   b) Solve for satellite state and above parameters.
Figure K-1. Tracking Configuration
TABLE K-I
ERROR SOURCES (1-SIGMA)

<table>
<thead>
<tr>
<th>Measurement Errors</th>
<th>Noise</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>60 ft</td>
<td>120 ft</td>
</tr>
<tr>
<td>A</td>
<td>1.4 mr</td>
<td>3.2 mr</td>
</tr>
<tr>
<td>E</td>
<td>1.4 mr</td>
<td>3.2 mr</td>
</tr>
<tr>
<td>( \dot{R} )</td>
<td>0.06 ft/sec</td>
<td>0.06 ft/sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station Location Errors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude</td>
<td>100 ft</td>
</tr>
<tr>
<td>Latitude</td>
<td>100 ft</td>
</tr>
<tr>
<td>Altitude</td>
<td>100 ft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gravitational Potential Uncertainties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( 1.06 \times 10^{11} ) ft³/sec²</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>( 2.0 \times 10^{-7} )</td>
</tr>
<tr>
<td>( J_{22} )</td>
<td>( 2.0 \times 10^{-7} )</td>
</tr>
<tr>
<td>( J_{33} )</td>
<td>( 2.6 \times 10^{-7} )</td>
</tr>
</tbody>
</table>
c) Range measurements only.

2) Seventy two-hours tracking with three stations, range measurements.
   a) Solve for satellite state. Consider parameter errors.
   b) Solve for satellite state and parameters.
   c) Effect of \( J_{22} \) and \( J_{33} \).

3) Thirty six-hours tracking with three stations.
   a) Solve for satellite state. Consider parameter errors.
   b) Solve for satellite state and parameters.

Results can be scaled to apply to other data rates and noise variances. If \( n \) data points are taken in a short time interval \( \Delta t \), with measurement error variance \( \sigma_n^2 \), then the results will be very nearly the same as for \( m \) data points in the same interval with variance

\[
\sigma_m^2 = \frac{m}{n} \sigma_n^2
\]

Results

The tracking analyses are based on minimum variance estimation. The numerical computations were performed utilizing the TRW System's ESPOD computer program series. A brief description of the methods involved is given in (app. C).

Case 1 - 72-Hr Tracking - Two Stations

Table K-II shows the satellite position and velocity standard deviations in a \( u, v, w \) coordinate system, a right-handed set with \( u \) in the direction of the geocentric radius vector, \( w \) in the direction of the angular momentum vector, and \( v \) completing the orthogonal set. The predominant error source is seen to be the uncertainty in the earth gravitational constant \( \mu \). This error affects significant period errors, which are manifest in large \( v \) (in track) errors.

Solving for the parameters leads to a considerable improvement particularly in the \( v \) (downrange) direction. Only the results for range
<table>
<thead>
<tr>
<th>Position (ft)</th>
<th>Solve for State and Parameters</th>
<th>Contribution of ( \mu ) Uncertainty</th>
<th>Total Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_u )</td>
<td>189</td>
<td>0.233</td>
<td>0.0516</td>
</tr>
<tr>
<td>( \sigma_v )</td>
<td>3250</td>
<td>0.0342</td>
<td>0.00687</td>
</tr>
<tr>
<td>( \sigma_w )</td>
<td>816</td>
<td>0.0340</td>
<td>0.00705</td>
</tr>
<tr>
<td>( \alpha )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The table compares errors for different measurements from two stations at 1/10000 scale.
measurements are not tabulated since there is no significant change from
the previous results. Hence, there is no benefit in taking angle measure-
ments.

Case 2 - 72-Hr Tracking - Three Stations

The results are shown in Table K-III. As in case 1, there is con-
siderable benefit in solving for the parameters. The additional station
provides the primary benefit of reducing the in-track error from 720 to
452 ft. For shorter tracking periods, it can be expected that this effect
would be more pronounced, with a substantial reduction occurring also
in the cross track errors.

The last columns show the effect of the uncertainty in the $J_{22}$ earth
potential coefficient. The results show a substantial increase in the
satellite errors, indicating the need to solve for $J_{22}$. When this was done,
the errors were essentially reduced to their previous values. A further
run was made to examine the effect of $J_{33}$; however, in that case there
was no appreciable contribution to the total error.

Case 3 - 36-Hr Tracking - Three Stations

The results, given in Table K-IV, again show the need to solve for
the parameters. Comparing with Table K-III shows, furthermore, that
there is very little to be gained from increasing the tracking period from
36 to 72 hours.
TABLE K-III
ERRORS FOR CASE 2 - 72-HR TRACKING FROM THREE STATIONS
WITH RANGE MEASUREMENTS AT 1/MIN

<table>
<thead>
<tr>
<th></th>
<th>Solve for Satellite State Only</th>
<th>Solve for State and Parameters</th>
<th>Consider Effect of $J_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Error</td>
<td>Contribution of $\mu$</td>
<td>Total Error</td>
</tr>
<tr>
<td>Position (ft)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>141</td>
<td>138</td>
<td>79</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>3500</td>
<td>3200</td>
<td>452</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>338</td>
<td>295</td>
<td>150</td>
</tr>
<tr>
<td>Velocity (ft/sec)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.152</td>
<td>0.144</td>
<td>0.0330</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.0354</td>
<td>0.0351</td>
<td>0.00573</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>0.0254</td>
<td>0.0231</td>
<td>0.00576</td>
</tr>
</tbody>
</table>
TABLE K-IV  
ERRORS FOR CASE 3 - 36-HR TRACKING FROM THREE STATIONS  
WITH RANGE MEASUREMENTS AT 1/MIN

<table>
<thead>
<tr>
<th>Position (ft)</th>
<th>Solve for Satellite State Only</th>
<th>Solve for State and Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Error</td>
<td>Contribution of $\mu$ Uncertainty</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>98</td>
<td>61</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>2550</td>
<td>2340</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>316</td>
<td>46</td>
</tr>
<tr>
<td>$\sigma_{\dot{u}}$</td>
<td>0.184</td>
<td>0.169</td>
</tr>
<tr>
<td>$\sigma_{\dot{v}}$</td>
<td>0.0365</td>
<td>0.00003</td>
</tr>
<tr>
<td>$\sigma_{\dot{w}}$</td>
<td>0.0442</td>
<td>0.00002</td>
</tr>
</tbody>
</table>
APPENDIX L

EFFECT OF APPROXIMATING ELLIPTICAL ORBITS WITH CIRCLES

This section gives analytic expressions for the in-track and radial errors caused by neglecting the ellipticity of the orbit, as is done in the user computations described in sec. 3 in the main body of this report. These expressions can be used to provide updated satellite errors for use in the NAVSAP error analysis program.

It is assumed that at some instant $t_0$, radar tracking has determined the position of the satellite in question and the parameters of its (elliptical) orbit.

The plane of the elliptical orbit is defined by:

1) the angle $\lambda$ of the orbit ascending node with the positive $x$-axis of some earth-centered inertial system

2) the inclination $i$ of the orbit plane with the equatorial plane.

The approximate satellite position computed by the user is then given by the following equations:

$$X = \frac{1}{2} \rho [(1 + \cos i) \cos \lambda + (1 - \cos i) \cos (2\omega \tau - \lambda)] \quad (L-1)$$

$$Y = \frac{1}{2} \rho [(1 + \cos i) \sin \lambda - (1 - \cos i) \sin (2\omega \tau - \lambda)] \quad (L-2)$$

$$Z = \rho \sin i \sin \omega \tau \quad (L-3)$$

where $\omega = \frac{2\pi}{24}$ rad/hr, $\tau = t - t_1$ and $t_1$ will be defined below. This definition of the approximate orbit causes its plane to coincide with the plane of the actual orbit.

The orbit parameters $\lambda$ and $i$ may be expressed as perturbations about nominal parameters $\lambda_0$ and $i_0$ as:

$$i_0 + \Delta i = i$$

$$\lambda_0 + \Delta \lambda = \lambda$$
The radius of the approximate orbit is chosen so that at time $t_0$, the position of the satellite computed from the equations of the approximate orbit coincides with the position of the satellite in the actual orbit as determined by the orbit determination program. The time $t_i$ of nodal crossing of the approximate orbit is chosen to facilitate this; i.e.,

$$t_i = t_o - \frac{\theta_o}{\omega}$$

$\theta_o$ is the angle between the radius vector $r_o$ of the actual orbit at time $t_o$ and the line of nodes.

The time $t_i$ of nodal crossing is:

$$t_i = t_o - \frac{\theta_o}{\omega}$$

The radius of the approximate orbit is thus:

$$\rho = \rho_o + \Delta \rho = r_o$$

(Observe that this approximate orbit is not actually a physical orbit, since circular orbits of radius $\rho$ will not in general have 24-hr periods).

The procedure is consequently the following: the ground system tracks for some time interval, and determines the satellite orbit parameters and position and velocity at the end of this interval. From these it determines $\Delta \rho$, $\Delta \lambda$, $\Delta i$, and $t_i$ as described above and transmits them to the users via the satellite. The user then uses these values in Eqs. (L-1) through (L-3) to determine the satellite position at this measurement time $t$. Of interest is the error in this estimate of position at $t$ as compared with the elliptical orbit.

It is most convenient to derive the desired expressions in terms of perturbations from circular orbits. The situation is most easily explained with the aid of Figure L-1 below.

The quantities on Figure L-1 are defined as follows:

Arc DAB - circular approximation to true satellite orbit
Arc DC - true (elliptical) orbit

---

Figure L.1. Geometry of Orbital Perturbations
D - position of satellite at time $t_0$
C - position of satellite at time $t = t_0 + \Delta t$
A - position of satellite in circular orbit of radius $\rho$ at time $t + \Delta t$, starting at D at time $t_0$
B - position of satellite at time $t_0 + \Delta t$ as calculated from Eqs. (L-1) through (L-3).

Then the in-track error $\phi$ is given by

$$\phi = \phi_f + \delta\phi_f - \phi_c$$

$\phi_f = \text{angle traversed by a satellite in a circular orbit of radius } \rho \text{ in time } \Delta t.$

$$= \sqrt{\frac{\mu}{\rho^3}} \Delta t \text{ (} \mu \text{ is proportional to the gravitational constant).}$$

$\phi_c = \text{angle traversed by satellite in the approximate circular orbit in time } \Delta t.$

$$= \frac{2\pi}{24} \Delta t \text{ radians. (} \Delta t \text{ in hours).}$$

$\delta\phi_f = \text{difference in angular displacement between circular orbit and perturbed circular orbit}^2.$

$$= (-3\phi_f + 4 \sin \phi_f) \frac{\delta v}{v_0} + 2 (1 - \cos \phi_f) \delta\beta_o$$

where $\delta v_0$ is the magnitude of the change in tangential velocity at D which produces the elliptical orbit DC, and $\delta\beta_o$ is the change in the angle of the tangential velocity at D. The angular in-track error is thus:

$$\phi = \left\{ \sqrt{\frac{\mu}{\rho^3}} - \frac{2\pi}{24} \right\} \Delta t + (-3\phi_f + 4 \sin \phi_f) \frac{\delta v_0}{v_0}$$

$$+ 2 (1 - \cos \phi_f) \delta\beta_o$$

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242 Skidmore and Braham, op. cit.
\( \phi_f \) will usually be small (for a one hour prediction, \( \phi_f = 15 \) deg), so that this expression becomes:

\[
\phi = \left\{ \sqrt{\frac{\mu}{\rho^3}} - \frac{2\pi}{24} \right\} \Delta t + \phi_f \frac{\delta v_o}{v_o}
\]

\[
= \left\{ \sqrt{\frac{\mu}{\rho^3}} \left( 1 + \frac{\delta v_o}{v_o} - \frac{2\pi}{24} \right) \right\} \Delta t
\]

\[
= \left\{ \sqrt{\frac{\mu}{\rho^3}} - \frac{2\pi}{24} \right\} \Delta t
\]

The radial error is obtained directly from reference as:

\[
\frac{\delta r}{r_o} = 2(1 - \cos \phi_f) \frac{\delta v_o}{v_o} - \sin \phi_f \delta \beta_o
\]

For \( \phi_f \) small, this reduces to:

\[
\frac{\delta r}{r_o} = \phi_f \delta \beta_o \]

\[
= -\sqrt{\frac{\mu}{\rho^3}} \Delta t \delta \beta_o
\]

(Clockwise angles are positive in the above expressions.)

In-track and radial error can thus be calculated from the tracking interval \( \Delta t \) if the parameters of the (true) elliptical orbit are known.
APPENDIX M

RESOLUTION OF RANGE AND RANGE-DIFFERENCE MEASUREMENT AMBIGUITIES

1. RANGE AMBIGUITY

Let $R_i^*$ be the range measurement from the $i^{th}$ satellite, corrected for satellite oscillator drift. Then $R_i^*$ has the form:

$$R_i^* = R_i + B_o + w_i - K_i \times 2,000$$

and $B_o$ is a number which is unique modulo 2,000. The problem is to determine the quantity $R_i + B_o + w_i$, given the measurement $R_i^*$. The problem will be solved if we can determine a procedure for adding some multiple of 2,000 to each measurement $R_i^*$ (say $K'_i \times 2,000$) so that the quantity

$$R_i^* + K'_i \times 2,000 - R_i - w_i$$

is the same for all $i$. This quantity will then be the bias common to all range measurements. We proceed as follows:

Suppose $R_i^*$ is the first-range measurement received. Let $\hat{R}_i$ be the computed range based on the a priori estimate of the user's position and the computed satellite position. Let $\epsilon_i$ be the error in this computed range, i.e.,

$$R_i = \hat{R}_i + \epsilon_i$$

Since

$$R_i^* = R_i + B_o + w_i - K_i \times 2,000$$

(Recall that $K_i$ is arbitrary, but to each different $K_i$ there corresponds a different value of the bias $B_o$.)

We have

$$R_i^* = \hat{R}_i + \epsilon_i + B_o + w_i - K_i \times 2,000$$
Expand

\[ \hat{R}_i = \Delta \hat{R}_i + K_i \times 2,000, \text{ } K_i \text{ an integer,} \]

Then,

\[ R_i^* = \Delta R_i + K_i \times 2,000 + \varepsilon_i + B_0 + w_i - K_i \times 2,000 \]

Purely for convenience, choose \( K_i = K_i' \). This uniquely determines a value of \( B_0 \). We must modify a subsequent measurement, say \( R_j^* \), so that the quantity

\[ R_j^* + K_j \times 2,000 - R_j - w_j \]

is the same as the quantity

\[ R_i^* - \Delta R_i - \varepsilon_i - w_i \]

We do this as follows:

For the first measurement, with \( K_i = K_i' \), the bias is

\[ B_0 = R_i^* - \Delta \hat{R}_i - \varepsilon_i - w_i \]

An estimate of the bias is

\[ \hat{B}_o = R_i^* - \Delta \hat{R}_i \]

(The error in the estimate is

\[ B_o - \hat{B}_o = -\varepsilon_i - w_i \]
We will now use the estimate $\hat{B}_0$ to correct the measurement $R_j^*$. To do this, we select $K_j$ so that

$$R_j^* + K_j \times 2,000 - \hat{R}_j = R_i^* - \Delta R_i = \hat{B}_0$$

Or select $K_j$ so that

$$K_j \times 2,000 = \hat{B}_0 + \hat{R}_j - R_j^*$$

To do this, simply compute $\hat{B}_0 + \hat{R}_j - R_j^*$, and round to the nearest 2,000. The probability of selecting the wrong $K_j$ is found as follows:

The actual bias is

$$B_o = R_i^* - \Delta R_i - \epsilon_i - w_i$$

The correct $K_j$ is the one that satisfies

$$R_j^* + K_j \times 2,000 = R_j + B_o + w_j$$

$$= R_j + R_i^* - \Delta R_i - \epsilon_i - w_i + w_j$$

$$= \hat{R}_j + \hat{B}_o - \epsilon_j - \epsilon_i - w_i + w_j$$

$$\therefore K_j \times 2,000 = \hat{R}_j + \hat{B}_o - R_j^* + \epsilon_j - \epsilon_i - w_i + w_j$$

So, we will select the wrong $K_j$ if

$$|\epsilon_j - \epsilon_i - w_i + w_j| \geq 1,000$$

The probability of this happening can be directly calculated, knowing the distributions of $\epsilon_j$, $\epsilon_i$, $w_i$, $w_j$. 

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2. RANGE DIFFERENCE AMBIGUITY

The $i^{th}$ and $j^{th}$ range measurements (corrected for satellite clock drift) have the form:

\[ R_i^* = R_i + B_0 + w_i - K_i \times 2,000 \]
\[ R_j^* = R_j + B_0 + w_j - K_j \times 2,000 \]

(Ki, Kj are integers)

hence

\[ \Delta = R_i^* - R_j^* = R_i - R_j + w_i - w_j - (K_i - K_j) \times 2,000 \]

Let $\hat{R}_i$, $\hat{R}_j$ be the values of $R_i$ and $R_j$ computed on the basis of an a priori estimate of the user's position. Let $\varepsilon_i$ and $\varepsilon_j$ be the errors in these computations, i.e.,

\[ R_i = \hat{R}_i + \varepsilon_i \]
\[ R_j = \hat{R}_j + \varepsilon_j \]

Substituting in the above:

\[ \Delta = \hat{R}_i - \hat{R}_j + \varepsilon_i - \varepsilon_j + w_i - w_j - K_{ij} \times 2,000 \quad (K_{ij} = K_i - K_j) \]

i.e.,

\[ K_{ij} \times 2,000 = \hat{R}_i - \hat{R}_j - \Delta + \varepsilon_i - \varepsilon_j + w_i - w_j \]

So, if we round $\hat{R}_i - \hat{R}_j - \Delta$ to the nearest multiple of 2,000, the probability of rounding to an incorrect $K_{ij}$ equals the probability that $|\varepsilon_i - \varepsilon_j + w_i - w_j| > 1,000$. This probability may be calculated, knowing the distributions of $\varepsilon_i$, $\varepsilon_j$, $w_i$, $w_j$. 

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APPENDIX N

THE RESORB PROGRAM

RESORB is a special computer program designed for simulation of commensurate and near-commensurate orbits (synchronous and super or subsynchronous orbits) with or without station keeping. These orbits are subject to resonance due to the longitude dependent (tesseral) harmonics of the potential field. The resonance manifests itself in the libration of the groundtrack with periods measured in years. Since a libration cycle contains many hundred orbits, numerical integration of the accelerations by Cowell's or Encke's method requires hours of machine time.

RESORB integrates Lagrange's planetary equations. The potential field is expressed by the Keplerian elements (Kaula's formulation) and as long as only the long-periodic (critical) terms are introduced, the integration steps can reach many times the orbital period. Thus, several hundred orbits are integrated in a second. (Long-periodic luni-solar effects are also included in the perturbations.)

This program handles orbits with any inclination except that of exactly zero (for example, even 0.1° can be handled) and any eccentricity from zero to about 0.8. Deviation from exact commensurability slows down the program, noticeable only when the orbit is so far off resonance (period off more than 0.1 percent) that it does not librate any more, but these cases are out of the range of RESORB application.

The RESORB program contains an optional subroutine which determines from an initial estimate the correct semimajor axis for the nearest commensurate orbit. Stationkeeping is also optional. If deadband width is given, the program prints out the exact date when the satellite reaches the bottom of the limit cycle. It also prints out the ΔV required for stationkeeping and changes the semimajor axis correspondingly.
Output of RESORB consists of the following:

- Mean elements and their time derivatives
- Longitude of the ascending node of the mean satellite and its time derivative
- Groundtrace and coverage of the $n^{th}$ orbit and its integer multiples plus the orbits where stationkeeping was applied
- Look angles for any ground station during the $n^{th}$ orbit and its integer multiples.
APPENDIX O

SATELLITE ECLIPSE PROGRAM

This analytic (as opposed to integrating) program was used to compute the results presented in subsec. 4.2 in the main body of this report. This program applies to circular orbits and neglects all gravitational perturbations.

Figure O-1 presents a satellite passing along its path P1 to P4.

As the satellite passes the point P1, it enters the penumbra or the zone of partial shadow. While passing point P2 the satellite enters the total shadow or umbra. The umbra shadow angle, $s$, is given by the relationship:

$$ s = 1.02 \left[ \sin^{-1} \left( \frac{1}{R} \right) - 16' \right] $$
where

\[ s \text{ is in degrees} \]
\[ r \text{ is the radius (in earth radii) of the satellite orbit.} \]
\[ 1.02 \text{ is the factor due to the refraction of the atmosphere} \]

The earth-centered angle, \( \psi \), between points \( P_1 \) to \( P_2 \) and \( P_3 \) to \( P_4 \) is the angular width of the penumbra. Regardless of the orbit altitude, this angle is almost constant and is approximately equal to \( 0.54^\circ \).

For a circular orbit, the maximum time per revolution that the satellite is in shadow (umbra and penumbra) is given by

\[ T_{\text{MAX}} = \frac{2(s+\psi)R^{3/2}}{57.3 \sqrt{\mu}} \]

where

\[ T_{\text{MAX}} \text{ is in minutes} \]
\[ S \text{ and } \psi \text{ are in degrees} \]
\[ R \text{ is in earth radii} \]
\[ \sqrt{\mu} = 0.0744 \text{ earth radii}^{3/2}/\text{min} \]
\[ 57.3 \text{ is the degree to radian conversion.} \]

The eclipse season, \( N_{\text{MAX}} \), is defined as the number of consecutive days that the satellite passes through the shadow. Neglecting regression of the line of nodes, \( N_{\text{MAX}} \) becomes:

\[ N_{\text{MAX}} = \frac{2}{0.986} \sin^{-1} \left( \frac{\sin(s+\psi)}{\sin i_e} \right) \]

where

\[ i_e \text{ is the inclination of the orbit plane to the ecliptic plane} \]
\[ 0.986 \text{ is the mean motion of the sun in degrees per day} \]
It is noted that there are two such eclipse seasons per year. The variation of the time, $T$, that the satellite is in the shadow per revolution as a function of the time, $N$, of the eclipse season is computed as:

$$T = T_{\text{MAX}} \sin \left[ (180^\circ) \left( \frac{N}{N_{\text{MAX}}} \right) \right]$$

when the inclination of the orbit plane to the ecliptic plane is greater than the angular size of the shadow, $S + \psi$. When the orbit plane inclination to the ecliptic is less than the angular size of the shadow, the satellite will again experience two eclipse seasons during the year, but each season will be continuous for the full half-year. Therefore, the season will last for a half-year, and will be bounded by minimum eclipse durations rather than periods of no eclipse. For this case, the time, $T$, the satellite is in shadow per revolution as a function of the time, $N$, if the eclipse season is computed as:

$$T = T_{\text{MAX}} \cos \phi$$

where

$$\phi = \phi_{\text{MAX}} \left( \frac{2N}{1 - N_{\text{MAX}}} \right)$$

and

$$\phi_{\text{max}} = \sin^{-1} \left[ \frac{\sin i_e}{\sin(S+\psi)} \right]$$

$$= \tan^{-1} \left[ \frac{\sin i_e}{\sqrt{\sin^2 (s+\psi) - \sin^2 i_e}} \right]$$

The inclination of the orbit plane relative to the ecliptic plane, $i_e$, as used above is directly related to the inclination of the orbit plane relative to the equatorial plane, $i_a$, through the relationship

$$\cos i_e = \cos 23.45^\circ \cos i_a + \sin 23.45^\circ \sin i_a \cos \Omega_a$$
where $23.45^\circ$ is the inclination of the equatorial plane relative to the ecliptic plane.

and $\Omega_a$ is the longitude of the ascending node measured from the vernal equinox to the orbit crossing of the equatorial plane in degrees.