MODEL ATMOSPHERES FOR RR LYRAE STARS

S. E. Strom

July 1968

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138
MODEL ATMOSPHERES FOR RR LYRAE STARS

S. E. Strom

Smithsonian Astrophysical Observatory and Harvard College Observatory
Cambridge, Massachusetts

Received

ABSTRACT

Model atmospheres have been computed in the range \(6000 \leq T_{\text{eff}} \leq 7500^\circ\text{K}\) with \(2.0 \leq \log g \leq 3.5\). These parameters are representative of those that obtain for stars in the RR Lyrae gap. Convection is found to play a significant role in the influence of the emergent fluxes predicted by these models. Temperature determinations based on purely radiative models are systematically too low by as much as \(200^\circ\text{-}350^\circ\text{K}\); gravity determinations are not affected. The method of predicting \((B-V)\), and thus reddening, from the strength of the Balmer lines is found to give less than desirable results. The influence of line blanketing and departures from LTE on these models is also discussed.

I. INTRODUCTION

Oke and Bonsack (1960) and Oke, Giver, and Searle (1962), among others, have recently discussed the application of model-atmosphere techniques to the study of the physical properties of RR Lyrae star atmospheres. Sufficient advances have been made since their work, both in the accuracy of models and in the hydrogen-line profiles predicted from them, so that a reexamination of the problem seems in order.

Moreover, the extremely thorough discussion by Christy (1966) of the pulsation mechanism and predicted cyclical atmospheric variations for RR Lyrae variables provides the motivation for a more careful study of these
stars. Specifically, we can now study the influence of convection and
departures from LTE on the observable features of the model as well as the
(small) effects of the most important metal opacities. We have, therefore,
computed a detailed grid of model atmospheres that covers most of the
range in $T_{\text{eff}}$ and $g$ encountered for stars in the RR Lyrae gap. The Paschen
continuum fluxes, the Balmer discontinuities, and the hydrogen-line profiles
predicted by these models are discussed in the context of their effect on the
choice of $T_{\text{eff}}$ and $g$ from observations.

II. DETAILS OF THE GRID

The basic methods outlined by Strom and Avrett (1965) were used in
computing the grid, although some important modifications in the details of
the calculations have been made by R. Kurucz of the Harvard College
Observatory. A new and versatile program, ATLAS, has been written by
him for the IBM 360 and 7000 and the CDC 6000 series machines; the details
of its operation are available on request. Constancy of better than 0.2 per
cent throughout the atmosphere is usually obtained for the flux and the flux
derivative. The sources of opacity included in these calculations were those
arising from H, H$^-$, H$^+_2$, SiI, MgI, Rayleigh scattering for atomic H, and
in some cases, the Balmer and Lyman lines.

Departures from LTE for the first six levels of atomic hydrogen and for
H$^-$ were computed by use of the methods already described in Strom and
Kalkofen (1966), Strom (1967), and Kalkofen (1968).

The expressions employed in the computation of the bound-bound and
bound-free cross sections used in the determination of the H-level populations
have been described by Strom and Peterson (1968). The value $C_- = 1.3 \times 10^{-9}$
cm$^3$ sec$^{-1}$ estimated by Dalgarno and Browne (1967) for the reaction
$H + H^- \rightarrow H_2 + e$ was used in the computation of the H$^-$ departures.
The influence of convective energy transport was estimated by use of a mixing-length theory and the temperature-correction procedure already described by Mihalas (1965). By eliminating the derivative terms in his equation (32) for \( \gamma \), we find that, while convergence is slowed, the resulting temperature corrections and flux errors are considerably smoother than those found by Mihalas.

In Table 1, we indicate the models computed for this study. The notation represents, in compact form \( T_{\text{eff}} \), \( \log g \), \((\text{metal}/\text{H})_{\text{model}}/(\text{metal}/\text{H})_{\text{sun}}\). Space requirements preclude publication of the model details. However, such desiderata as temperature structures, predicted fluxes, and Balmer profiles are available from the author upon request.

III. THE RESULTS

The effective temperature of an RR Lyrae star is usually determined from some measure of the slope of the Paschen continuum or from a Balmer-line profile or equivalent width. The surface gravity is then chosen by a comparison of the observed and computed Balmer discontinuities for a model of the appropriate \( T_{\text{eff}} \).

With these typical procedures in mind, we plot in Figure 1 the relation between \( D_B \), the Balmer discontinuity, and \( S_P \), the slope of the Paschen continuum, predicted by our models. We define

\[
D_B = -2.5 \log \left[ F_v(3650^-)/F_v(3650^+) \right]
\]

and

\[
S_P = -2.5 \log \left[ F_v(4100)/F_v(5800) \right].
\]

The solid and dashed lines in this figure represent the relation found for purely radiative models and that for convective models having \( \ell/h = 1 \), respectively. In Figure 2, the dashed line represents the relation for convective models with \( \ell/h = 2 \). We find that the \( \ell/h = 2 \) models most closely
match the models computed by means of methods that average the influence of upward and downward moving elements (e.g., Latham 1964 and Parsons 1966). A purely local velocity is used in the Mihalas (1965) formulation. To the extent that the former models match reality, the $l/h = 2$ models give more likely estimates of the behavior of the atmosphere in the region of flux formation. In any case, we note two basic conclusions we can reach from Figures 1 and 2:

1) The choice of $g$ is not influenced by the effects of convection.

2) The temperatures estimated from purely radiative models are too small by perhaps as much as $200^\circ - 350^\circ$ K near $7500^\circ$ K and $50^\circ - 100^\circ$ K at $6000^\circ$ K.

In Figure 3, we plot $W(\text{Hy})$, the equivalent width of $H\gamma$ as computed from the most recent Griem (1967) theory, against $S_p$ as predicted from the radiative models. The relation is essentially unchanged for the convective models with $l/h$ values of 1. We conclude from Figure 3:

1) For stars with $\log g \leq 3$ in this $T_{\text{eff}}$ range, an error of $\pm 10$ per cent in $W(\text{Hy})$ results in an error of about $0.06$ mag in $S_p$ or analogous quantities such as $(b-y)$ and $(B-V)$. Therefore, in this range we must be cautious in determining reddening corrections based on observations of $W(\text{Hy})$ and on the associated $S_p$ values predicted from models. (For a discussion of this method, see Searle and Oke 1962.)

2) At least a crude estimate of $g$ must be obtained before $S_p$ can be predicted accurately from $W(\text{Hy})$.

In Figure 4, the dashed lines represent $D_B$ plotted against $S_p$ for some representative models in which we allow for departures from LTE. Again, the solid lines represent the results obtained with the purely radiative models. We note that for a given $S_p$, $D_B$ is smaller for the non-LTE models than for the LTE models. This can be understood as follows: If we define (in the usual way) $b_n$ as the ratio of the true population predicted by the Saha-Boltzmann equation, we find for all models considered here that $b_2 < 1$ and $b_n > 1$ for $n < 3$. Since the second level is underpopulated and
the third and higher levels overpopulated, it follows that the atmosphere is more transparent in the Balmer continuum and less transparent in the Paschen and higher continua than is the case in LTE. As a consequence, we predict a larger emergent flux in the Balmer continuum and a smaller flux in the Paschen and higher continua, and therefore a smaller Balmer discontinuity. In all cases, \( b \), the departure coefficient for H\(^-\) is greater than unity, although not large enough to be more than of academic interest.

In concrete terms, the effect of departures from LTE is to force us to choose lower values of \( g \) by about 0.25 dex than in the case with the LTE models.

For a few points on our grid, the effects of Balmer- and Lyman-line opacities on the models were estimated and found to be negligible. The effects on the results of varying the metal abundances are also negligible.

These results allow us to estimate the effects of the recent changes in the absolute calibration of spectrophotometric observations proposed by Hayes (1967). His new calibration decreases \( S_p \) by 0.11 mag, compared to the calibration suggested by Oke (1964). In this range of \( S_p \) value, the Hayes calibration also increases \( D_B \) by 0.06 mag. As a consequence, the values of \( T_{\text{eff}} \) found from Oke's calibration are about 300°K smaller than is the case if we adopt the calibration of Hayes. Also, as a net result, \( g \) is chosen smaller for Oke's calibration by 0.5 dex.

We can also comment on the helium content of RR Lyrae stars, following Christy's (1966) work. According to his computations, the location of the high \( T_{\text{eff}} \) boundary of the instability strip is a function of the atmospheric helium content; the higher the helium content, the higher the value of \( T_{\text{eff}} \) at the high \( T_{\text{eff}} \) boundary. Since both the new model atmospheres presented here and the recently proposed Hayes calibration act to raise \( T_{\text{eff}} \) (by more than 300°K), we must revise upward any estimates of the atmospheric helium content of these stars (by more than 0.09 by mass fraction).
IV. SUMMARY

The basic results of this work suggest that:

1) If we ignore the influence of convection on RR Lyrae atmospheres, $T_{\text{eff}}$ as estimated from the slope of the Paschen continuum may be underestimated by as much as $200^\circ$-$350^\circ$ K.

2) The gravities chosen from the Balmer discontinuity are unaffected by convection.

3) The effects of departures from LTE act in the direction of decreasing (by as much as 0.25 dex) the values of $g$ chosen from the Balmer discontinuity.

4) The values of Paschen continuum slopes estimated from the observed H$_\gamma$ equivalent widths are in error by as much as $\pm 0.06$ mag if the equivalent widths are known to $\pm 10$ per cent.

5) The helium content of RR Lyrae stars is somewhat higher than that estimated from previous models and observations.

We wish to thank R. L. Kurucz and D. M. Peterson for many useful discussions and assistance in some of the computational details. Dr. I. J. Danziger has also provided some valuable comments. We acknowledge with gratitude the support of NASA grant NGR 22-024-001.
REFERENCES

<table>
<thead>
<tr>
<th>Radiative models</th>
<th>Radiative models</th>
<th>Convective models ($\ell/h = 1$)</th>
<th>Convective models ($\ell/h = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6000, 3, 1)</td>
<td>(7000, 3, 1)</td>
<td>(6000, 3, 0.1)</td>
<td>(6500, 2.5, 0.1)</td>
</tr>
<tr>
<td>(6000, 3, 0.1)</td>
<td>(7000, 3, 0.1)</td>
<td>(6000, 2.5, 0.1)</td>
<td>(6500, 2.5, 0.01)</td>
</tr>
<tr>
<td>(6000, 2.5, 0.1)</td>
<td>(7000, 2.1)</td>
<td>(6000, 2, 0.1)</td>
<td>(7000, 3, 0.1)</td>
</tr>
<tr>
<td>(6000, 2, 1)</td>
<td>(7000, 2, 0.1)</td>
<td>(6500, 2.5, 0.1)</td>
<td>(7500, 3, 0.1)</td>
</tr>
<tr>
<td>(6000, 2, 0.1)</td>
<td>(7000, 2, 0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6000, 2, 0.01)</td>
<td>(7000, 3, 0.1)NLTE</td>
<td>(6500, 2, 0.1)</td>
<td>(7500, 3, 0.1)</td>
</tr>
<tr>
<td>(6000, 3, 0.1)NLTE</td>
<td>(7000, 2, 0.1)NLTE</td>
<td>(7000, 3, 0.1)</td>
<td></td>
</tr>
<tr>
<td>(6500, 3, 1)</td>
<td>(7500, 3, 0.1)</td>
<td>(7000, 2.5, 0.1)</td>
<td></td>
</tr>
<tr>
<td>(6500, 3, 0.1)</td>
<td>(7500, 2.1)</td>
<td>(7500, 3, 0.1)</td>
<td></td>
</tr>
<tr>
<td>(6500, 2, 1)</td>
<td>(7500, 2, 0.1)</td>
<td>(7500, 2.5, 0.1)</td>
<td></td>
</tr>
<tr>
<td>(6500, 2, 0.1)</td>
<td>(7500, 3, 0.1)NLTE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6500, 2, 0.01)</td>
<td>(7500, 2, 0.1)NLTE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1. — The relation between $D_B$, the Balmer discontinuity, and $S_p$, the slope of the Paschen continuum predicted by our models. (See text for definitions of $D_B$ and $S_p$.) The convective models illustrated in this figure have $t/h = 1$. 
Fig. 2. — The relation between $D_B$, the Balmer discontinuity, and $S_p$, the slope of the Paschen continuum predicted by our models. (See text for definitions of $D_B$ and $S_p$.) The convective models illustrated in this figure have $l/h = 2$. 

---

1.20
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
-1.20
-1.40
-1.60
-1.80

$D_B$

$S_p$

---

$\ell/h = 2$

P单纯辐射

---

(7500)

(7000)

(6500)

(6000)
Fig. 3. – The equivalent width of $H_Y$, $W(H_Y)$ plotted against $S_p$. Both $W(H_Y)$ and $S_p$ have been computed from the pure radiative models.
Fig. 4. – A plot of $D_B$ against $S_p$ for LTE (dashed lines) and non-LTE models (solid lines).