AN ANNOTATED BIBLIOGRAPHY FOR
DISTRIBUTED RC NETWORKS

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by

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Abstract: This bibliography contains a listing of the references to distributed RC networks published from 1958 to mid 1968. A short summary is given describing each of the references.

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INTRODUCTION

This bibliography contains a listing, arranged in alphabetical order, of the papers which have appeared in the electrical engineering literature pertinent to the subject of distributed RC networks. In general, reports which are not readily available, or which contain material similar to that presented in well-known journals, have not been listed. An exception has been made in the case of a few widely referenced papers. A short summary of the content of each paper follows the bibliographic data.

A listing of the journals searched follows:

IEEE International Convention Record (Circuit Theory), vol. 6 (1958) to vol. 15 (1967).

In addition to the papers listed in the bibliography, there are currently available two books which contain a significant amount of material concerning distributed circuits. These are:


Filter synthesis using Heizer's technique (1963) to obtain networks with rational transfer functions is presented.


A procedure for generalizing non-uniform transmission line problems which includes the generalized exponential and bessel lines as special cases is discussed.


Transmission parameters of an arbitrarily tapered RC line are obtained by multiplication of transmission matrices of small sections of the line.


An approach that provides the distributed network's transfer matrix in a form useful for both transient and steady state calculations is used to analyze the n-wire exponential line.


It is shown that the two classes of driving point functions realized by Wyndrum (1963) and by O'Shea (1965), using uniform transmission lines, are identical and further that these driving point functions may always be cascade synthesized. An alternative set of necessary and sufficient conditions for this class of driving point functions is given.


It is shown that the chain matrix parameters of a non-uniform transmission line are related, and that knowing any one of them, the other three may be determined.


For a general class of non-uniform RCG lines with hyperbolic solutions, the Y and Z parameters have been express in terms of those of a uniform RCG line.

Stability criteria are presented for linear, time-invariant networks with uniform RC lines included.


A new type of active network is introduced, combining field effect properties with a distributed RC network. The analysis of the admittance matrix is carried out, and the location of the dominant poles and zeros is discussed.


Flow graph theory is applied to the analysis of non-uniform RC lines.


The indefinite admittance matrix of the R-C-nR, distributed four terminal network with circular geometry is derived.


The indefinite admittance matrix of the uniform distributed four terminal R-C-NR network is derived.


The canonical forms of the network functions of a multi-layer RC distributed network are obtained and expressed in terms of the eigenvalues of a certain "time-constant matrix". The voltage transfer function is found to have a low, high, or band-pass characteristic with an absolute value possibly greater than unity over a range of frequencies.


The selectivity of several types of distributed parameter null networks is determined for null and off null conditions.


The use of an active (semiconductor) resistive film for frequency modulation of phase shift oscillators using RC lines is discussed.


A very technical mathematical analysis of distributed networks using the Bubnov-Galerkin method is presented.
A detailed comparison of various lumped three terminal null networks with their tapered distributed counterparts is given.

Notch networks of various types are discussed with special attention being given to figure of merit.

Characteristics of an RC line with dielectric leakage, are presented.


Design of Oscillators using exponentially tapered RC lines is discussed.

An improved method of computing the Z parameters of 2-port tapered transmission lines is presented. As an example the Z parameters for the "Generalized Transmission Line" are found.

Transfer impedance calculation from knowledge of input impedance under various load conditions is illustrated.

Extending the work of Heizer (1963) to the n-port case, a rectangular structure is developed as an n-port with rational short circuit admittances.

A general method of synthesis of any rational open circuit transfer function using two distributed RC networks, NIC, and a lumped capacitor is presented.

A definition of selectivity for distributed notch filters is given.


A method is presented whereby the delay and rise times for arbitrarily tapered distributed networks may be calculated.


The effects of network shaping and loading on the transient and frequency response of distributed RC networks are examined analytically and experimentally.


Synthesis of irrational driving point impedances using exponential RC lines is presented.


Composite lines composed of various types of non-uniform lines connected in cascade are used to synthesize certain classes of irrational functions as driving point functions.


A new type of RC distributed line, with two resistive layers, is introduced. The Z matrix of the line is derived, and application as a notch filter is considered.


It is shown that if the capacitance of any distributed RC network is suitably divided between two or more conductors rational admittances are obtained. The possible pole positions of the rational transfer functions are shown to be the zeros of the driving point impedance of the prototype network from which the divided conductor is derived.


A new method is presented for finding all RC lines which have exactly solvable differential equations.

Two auxiliary functions are introduced to compare the steady-state characteristics of various non-uniform RC distributed networks.


Analogues of RC lines are deduced, and their chain parameters are expressed in terms of the solutions of the associated difference equations.


An application of the Youla technique (IEEE Trans. on Circuit Theory, Sept. 1964) is used to obtain expressions for the 2-port parameters of non-uniform lines suitable for numerical integration as well as low and high frequency approximations.


Various possibilities of synthesis using network shaping are investigated.


It is shown that for RC structures the impulse response results in a spectrum of Poisson derived functions in the reciprocal time domain. The technique is used to investigate transient response of distributed networks.


Design charts for RC distributed networks based upon desired transient response are constructed by using a lumped model for the network.


Construction of distributed RC lines to have a rational short circuit transfer admittance and one rational short circuit driving point admittance is presented.


A modification of Heizer's earlier structure to increase flexibility is introduced.

Introduction of a "Generalized Hyperbolic Class" of lines, of which the "Generalized Exponential Class" is a subcategory.

It is shown that it is necessary and sufficient that incremental resistance be proportional to incremental capacitance to have a symmetrical network.

A method is presented to calculate equivalent distributed RC networks all of which provide the same electrical performance.

The exponential line is presented as a special case of a general class of exponential line characterized by series impedance per unit length \( z(x) \) and shunt admittance per unit length \( Y(x) \).
The general exponential line is defined by

\[
\frac{1}{r(x)} \frac{\partial \ln K(x)}{\partial x} = C
\]

where \( K(x) = \sqrt{z(x)/Y(x)} \), \( r(x) = \sqrt{z(x)Y(x)} \)

A method of optimal design of distributed RC lines using variational calculus is introduced. The method of solution is based upon the gradient technique.

The RC distributed network is introduced as an effective null device. Analysis is made using a lumped equivalent model, and examples are given. Comparison is made between a distributed bridged-T and a lumped element equivalent.

The characteristics of uniform, exponentially tapered, and linearly tapered distributed networks used as notch filters are extensively investigated.

The concept of the effective dominant pole of a distributed RC network is developed. Key features of the steady-state and transient response such as bandwidth, rise time, and time delay may be closely approximated with minimal effort.


It is shown that the uniform, exponential, hyperbolic, trigonometric and square tapered lines are members of the same class of network, with short circuit admittance parameters given by a single set of equations.


A simple approximation is used to obtain expressions for the 3DB frequency, rise time and delay time of arbitrarily tapered distributed RC networks.


A practical procedure for synthesizing distributed, lumped active (DLA) networks is developed by determining a set of equivalent rational pole positions corresponding to the amplitude response of three specific DLA networks which together allow the realization of filters with left half plane poles and jω axis zeros. An example of the synthesis of a 5-pole, 4 jω axis zero elliptic function low-pass filter is given.


On using a special frequency transformation, a theorem on the synthesis of uniformly distributed RC networks is derived. A network section is introduced which extends the range of realizability of transfer zeros.


An exponential line filter is shown to have maximum notch steepness for a finite degree of taper when loaded by a source and load resistor.


It is shown that the notch steepness of the open-circuit voltage transfer function of an exponential line filter approaches a finite value as the taper degree approaches infinity.

A modification of Wyndrum's method (1963) to minimize the ratio of maximum to minimum capacitance or resistance of individual cascade sections is introduced.


The n-port generalization of a single transmission line is given.


Transfer characteristics of the trigonometric, parabolic, and hyperbolic lines as a generalization of the characteristics of the exponential line are investigated.


A lumped model is used to determine transient behavior. The model is justified in detail.


Three single transistor oscillators with uniform distributed RC networks are presented.


The characteristics of a distributed RC-tunnel diode line are analyzed in detail.


The geometrical shape for an exponentially tapered RC line taking two dimensional current flow into account is given.


Synthesis of driving point and transfer functions using uniform distributed RC networks is presented. The transformation $P = \cosh RC$ is used.


Driving point and transfer function synthesis using uniform RC lines as elements of lumped networks is developed. Use is made of the transformation $P = \cosh RC$. 


PROTONOTARIOS, E. N., O. Wing. "Theory of Non-Uniform RC Lines Part I: Analytic Properties and Realizability Conditions in the Frequency Domain," IEEE Trans. on Circuit Theory, vol. CT-14, pp. 2-12, Mar. 1967. An intrinsic description of the frequency domain response of a non-uniform RC line is presented. The network functions are expressed as ratios of entire functions. Analytic properties of these entire functions are studied. Their order, type, genus, and their asymptotic behavior and their bounds on the real frequency axis are determined. Necessary and sufficient conditions are given for a transcendental frequency function to be a network function of a non-uniform RC line.

PROTONOTARIOS, E. N., O. Wing. "Theory of Non-Uniform RC Lines Part II: Analytic Properties in the Time Domain," IEEE Trans. on Circuit Theory, vol. CT-14, pp. 2-12, Mar. 1967. The time-domain behavior of the general non-uniform RC transmission line is presented. It is shown that the impulse response is a "totally positive" density function. The necessary and sufficient condition on a time function to be the impulse response of a non-uniform RC line is obtained. The general properties, including the bounds on the impulse response and its asymptotic behavior, are given.

PROTONOTARIOS, E. N., O. Wing. "Delay and Rise Time of Arbitrarily Tapered RC-Transmission Lines," IEEE International Convention Record, Part 7, pp. 1-6, Mar. 1965. The transmission parameters of an arbitrarily tapered line are expressed as infinite series. The delay and rise time of the network are given in terms of these parameters. Examples are included.
A method of calculation of step response based on the "Method of Moments" of Chebyshev is presented.

A representation of the exponential transmission line by networks comprised of lumped elements is given.

The exponential line is divided into a number of sections, and the exact transmission parameters for a small section are derived, the transmission matrices of the sections are multiplied to find the transmission matrix of the line.

A method of synthesis is presented which utilizes lumped elements together with uniform RC networks.

Necessary and sufficient conditions for the realizability of a short circuit transfer admittance by means of a grounded uniform RC distributed network are presented. The procedure used in testing for realizability and synthesis is demonstrated by a nontrivial worked example.

The four-port transmission matrix for a constant CRC distributed element is presented. Wyndrum's (1963) hyperbolic complex frequency transformation is used.

Distributed network synthesis using network optimization in both time and frequency domain is investigated. The calculus of variation is employed.

Synthesis using uniform RC lines is presented. The transformation s+tanh(RCs) is employed. A cascade connection of uniform RC networks is used.

Frequency response of distributed lines is investigated by using the transformation \( t = \tanh(\text{RCs})^{1/2} \).


The tunnel diode oscillator in which the negative conductance is distributed along a quarter wave transmission line is analyzed.


A bandpass filter characteristic is obtained by connecting a four layer RC distributed network in the feedback path of a high gain amplifier.


An interesting application of RC distributed networks in transmission line design is illustrated.


Additional types of lines which have exactly solvable differential equations are briefly presented.


The 2-port Z parameters of the hyperbolic line are derived. The hyperbolic line is defined by

\[
\begin{align*}
r &= r_0 \text{sech}^2 x \\
c &= c_0 \text{cosh}^2 x 
\end{align*}
\]


The selectivity of a notch filter is defined, and parameters of the exponential, trigonometric, and hyperbolic lines for different selectivities are given.


An exhaustive analysis of the distributed RC line where

\[
\begin{align*}
r(x) &= r_0 \csc^2 x \\
c(x) &= c_0 \sin^2 x 
\end{align*}
\]

is presented.

The discussion includes applications as a notch filter, and experimental results.

It is shown that the hyperbolic notch filter selectivity is less sensitive to load fluctuation than with exponential or trigonometric lines.


Comments on the letter of M. J. Gay in Dec. 1965 *Electronics Letters* are given.


The Z matrix for the trigonometric line is derived. Application as a notch filter is investigated, and a comparison is made with the exponentially tapered filter.


The relation between the Z parameters of a non-uniform line and its differential equation is presented.


For a distributed RC network a system of linear equations is solved by reducing it to an ordinary differential equation by means of transformation of variables. The equation is solved for the particular boundary conditions

$$V(0, T) = T^n$$

$$V(\infty, T) = 0$$

so that a solution may be found to a power series approximation to a pulse signal input.


D'Alembert's method is applied to the differential equations of a non-uniform line.


Two new transforms are used to solve a generalized Riccati equation.


An analytical approach for exact solutions to non-uniform transmission lines by converting Riccati's equation to a Bernoulli equation is presented.

It is shown that any line with proportional distributions may be transformed into an equivalent uniform line.


A LC-RC transformation is used to derive a lumped equivalent model for the RC line from the lumped model of the LC line.


Feedback design techniques using the uniform distributed RC network are investigated.


A method is presented whereby a closed form solution in the laplace domain for an arbitrarily tapered 2n terminal RC line may be obtained.


Phase shift with an exponentially tapered line is discussed.


An analysis of the four terminal exponentially tapered network, with indefinite admittance matrix is presented. The paper includes applications as a filter and as a component in an oscillator.


A method for synthesizing rational transfer functions with distributed networks and an INIC is presented. The method is compared with lumped element realizations.


Null network tuning is discussed and normalized design curves are given.


The analysis and synthesis of distributed RC networks is treated on the basis of various P-R transformations, thus making it possible to synthesize in terms of lumped LC networks and rational functions and to transform this information into distributed RC specifications.

Four theorems concerning driving point impedances of uniform RC networks are given. The Schlicht nature of the driving point functions is considered to derive more information concerning their character.


Realization of a bistable line by cascading bistable circuits and the use of the line for waveform shaping is discussed.