INCOMPRESSIBLY LUBRICATED
RAYLEIGH STEP JOURNAL BEARING

I - Zero-Order Perturbation Solution

by Bernard J. Hamrock and William J. Anderson
Lewis Research Center
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ABSTRACT

A theoretical analysis of the pressure distribution, load capacity, load angle, and friction force for a single-step concentric journal bearing is performed. The resulting expressions were evaluated on a digital computer. The maximum load capacity is obtained when the ratio of the step clearance to ridge clearance is $1.7$ and the ratio of the angle subtended by the ridge to the angle subtended by the pad is $0.35$. Finally, for relatively small radius-to-length ratios, the load capacity of the stepped journal, while concentric, is higher than the load capacity of a finite-length Sommerfeld bearing operating with an eccentricity ratio of $0.1$. 
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SUMMARY

A theoretical analysis of the pressure distribution, load capacity, load angle (analogous to an attitude angle in an eccentric bearing), and friction force for a single-step concentric Rayleigh step journal bearing is performed. The analysis is presented in two parts, the ridge and the step regions. The pressure in the two regions is expressed in terms of an asymptotic series expansion in the eccentricity ratio and only the zero-order term is retained. The pressure in the ridge and step regions is then obtained from the Reynolds equation. The load capacity, load angle, and friction force can be derived once the pressure is known. The resulting equations are a function of the radius-to-length ratio $\xi$, the ratio of the step clearance to the ridge clearance $k$, the ratio of the angle subtended by the ridge to the angle subtended by the pad $\psi$, and the angle subtended by the lubrication groove $\delta$. The range of applicability of the results covers the entire range of radius-to-length ratios.

A consideration of load capacity showed the results to yield an optimum step-to-ridge-clearance ratio of 1.7 and an optimum ratio of the angle subtended by the ridge to the angle subtended by the pad of 0.35. The results indicated that in the Rayleigh step journal bearing, a positive pressure is developed completely around the journal, whereas in a plain journal bearing, the pressure theoretically becomes negative over a portion of the circumference. Also, for small radius-to-length ratios ($\xi < 0.2$) the load capacity of the stepped journal, while concentric, is higher than the load capacity of a plain bearing of the same radius-to-length ratio operating with an eccentricity ratio of 0.1. Finally, the dimensionless friction force of a Rayleigh step journal bearing proved to be less than that of a plain bearing.
INTRODUCTION

Lord Rayleigh (ref. 1) illustrated that, when side leakage is neglected, the step film shape has the greatest load capacity in a slider bearing lubricated with an incompressible fluid. A calculus-of-variations approach was used in obtaining these results. In a recent paper by Maday (ref. 2), a bounded variable approach was used to arrive at a conclusion similar to Rayleigh’s. Archibald (ref. 3) found that when side leakage is considered, the optimal step shape changed considerably from that obtained by Rayleigh, and the load capacity decreased somewhat. Archibald applied his finite stepped slider results from reference 3 to the journal bearing problem (ref. 4). In the present report, however, the journal bearing problem is considered directly. Expressions are derived for the radial and tangential load components, as well as for the load angle and friction force.

Instability and low load capacity, more than any other factors, limit the usefulness of fluid film bearings when lubricants of low viscosity, such as liquid metals, are used. It is a known fact (ref. 5) that discontinuous films, such as those produced by steps and grooves, tend to stabilize the bearing. Finally, the stepped journal is a relatively simple bearing as opposed to a tilting pad bearing, for example.

Figure 1 shows the stepped journal bearing in a concentric position. The term "pad" in this report refers to the angle subtended by the ridge, step, and lubrication groove. In this report, the bearing and journal will remain concentric. With this assumption, it can easily be seen that, if more than one step is placed around the journal, the resulting load is zero. Therefore, the single step case is the only one of interest when the journal is concentric. The single-step bearing supports a finite load in a unique direction.

In a recent patent disclosure (ref. 6), a single-step Rayleigh journal bearing design capable of supporting an arbitrarily directed load is presented. The subject invention (part of which is shown in fig. 2) utilizes the single-step configuration, but in three sections. Each section is displaced 120° circumferentially with respect to the others to achieve symmetry. The concept resulted from the analysis of a gas-lubricated stepped journal bearing (ref. 7), which indicated that the optimal number of steps around the journal tended toward one.

The objective of this report is to obtain the optimal step configuration for maximum load capacity for an incompressibly lubricated one-step concentric journal bearing.

SYMBOLS

A, B, D, E    integration constants
C              radial clearance
e              journal and bearing eccentricity
2
\(F\) dimensionless friction force, \(fC/2\pi RL\mu U\)

\(f\) friction force

\(H\) dimensionless film thickness, \(h/c\)

\(h\) thickness of lubricating film

\(I_m\) Fourier coefficient

\(j\) order of perturbation

\(K\) separation constant

\(k\) film thickness ratio, \(C_s/C_r\)

\(L\) length of bearing

\(m\) odd positive integers

\(N\) number of steps placed around journal

\(P\) dimensionless pressure, \(C^2(p - p_a)/6\mu UR\)

\(p\) pressure

\(p_a\) ambient pressure

\(Q\) flow rate

\(R\) radius of bearing

\(U\) velocity of bearing rotation

\(W\) dimensionless total load, \(w/p_a L R\Gamma\)

\(WR\) dimensionless radial load, \(wr/p_a L R\Gamma\)

\(WT\) dimensionless tangential load, \(wt/p_a L R\Gamma\)

\(wr\) radial load component orientated in \(\theta = 0^\circ\) direction

\(wt\) tangential load component orientated in \(\theta = 90^\circ\) direction

\(Z\) dimensionless axial coordinate, \(z/L\)

\(z\) axial coordinate

\(\beta\) angle subtended by step

\(\Gamma\) dimensionless parameter, \(6\mu UR/p_a C_r^2\)

\(\gamma\) angle subtended by ridge

\(\delta\) angle subtended by lubrication groove

\(\epsilon\) eccentricity ratio, \(e/C_r\)

\(\zeta\) radius-to-length ratio, \(R/L\)
\[ \theta \quad \text{angular coordinate} \]
\[ \mu \quad \text{viscosity of lubricant} \]
\[ \tau \quad \text{shear stress} \]
\[ \Phi \quad \text{load angle} \]
\[ \psi \quad \text{proportion of ridge angle to total angle of ridge-step-groove combination, } \gamma/(\gamma + \beta + \delta) \]

Subscripts:

0 zero-order perturbation solution
r ridge region
s step region

\section*{ANALYSIS}

The steady-state, incompressible Reynolds equation for a journal bearing in cylindrical coordinates (ref. 8) can be written as

\[ \frac{\partial}{\partial \theta} \left( h^3 \frac{\partial p}{\partial \theta} \right) + R^2 \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = 6 \mu UR \frac{\partial h}{\partial \theta} \quad (1) \]

This equation can be written in dimensionless form for the ridge and step regions of the Rayleigh step journal bearing as

\[ \frac{\partial}{\partial \theta} \left( H_r^3 \frac{\partial P_r}{\partial \theta} \right) + \zeta^2 H_r^3 \frac{\partial^2 P_r}{\partial Z^2} = - \frac{\partial H_r}{\partial \theta} \quad (2) \]

where

\[ P_r = \frac{C_r^2 (p_r - p_a)}{6 \mu UR} \]

\[ H_r = \frac{h_r}{C_r} \]

\[ Z = \frac{Z}{L} \]
\[ \zeta = \frac{R}{L} \]
and

\[ \frac{\partial}{\partial \theta} \left( H_s^3 \frac{\partial p_s}{\partial \theta} \right) + \xi^2 H_s^3 \frac{\partial^2 p_s}{\partial Z^2} = - \frac{\partial H_s}{\partial \theta} \quad (3) \]

where

\[ p_s = \frac{C_r^2 (p_s - p_a)}{6 \mu U R} \]

\[ H_s = \frac{h_s}{C_r} \]

The reason for the term on the right side of equations (2) and (3) to appear negative where it appeared positive in equation (1) is that in a Rayleigh step journal bearing (fig. 1) the velocity \( U \) is taken to be positive in the negative \( \theta \)-direction. This orientation is necessary in order for a positive pressure to develop at the step within the bearing. In equations (2) and (3), and in figure 1, the subscripts \( r \) and \( s \) refer to ridge and step regions, respectively. These symbols will be used throughout the analysis.

The equations for the circumferential flow rate in the ridge and step regions may be written as

\[ Q_r = - \frac{Uh_r}{2} - \frac{h_r^3}{12\mu R} \frac{\partial p_r}{\partial \theta} \]

\[ Q_s = - \frac{Uh_s}{2} - \frac{h_s^3}{12\mu R} \frac{\partial p_s}{\partial \theta} \]

In terms of the dimensionless pressures and film thicknesses, the flow rates are

\[ Q_r = - \frac{UC_r}{2} \left( H_r + H_r^3 \frac{\partial p_r}{\partial \theta} \right) \quad (4) \]
\[ Q_s = -\frac{UC_r}{2} \left( H_s + H_s^3 \frac{\partial P_s}{\partial \theta} \right) \]  

(5)

The equations for the dimensionless film thickness in the ridge and step regions may be written as

\[ H_r = \frac{h_r}{C_r} = 1 + \epsilon \cos \theta \]  

(6)

\[ H_s = \frac{h_s}{C_r} = k + \epsilon \cos \theta \]  

(7)

where \( \epsilon = \frac{\theta}{C_r} \) and \( k = \frac{C_s}{C_r} \).

The dimensionless pressure in the ridge and step regions may be written as an asymptotic expansion in \( \epsilon \):

\[ P_r = \sum_{j=0}^{\infty} \epsilon^j P_{rj} \]

\[ P_s = \sum_{j=0}^{\infty} \epsilon^j P_{sj} \]

If only the zero-order perturbation solution is considered, that is, the case where the journal is in the concentric position, the dimensionless pressure and film thickness equations in the ridge and step regions are

\[ P_r = P_{r0} \]

\[ P_s = P_{s0} \]

\[ H_r = 1 \]

\[ H_s = k \]
Substitution of these expressions into equations (2) to (5) gives the following equations:

\[
\frac{\partial^2 \theta^{2} P_{r0}}{\partial \theta^2} + \zeta^2 \frac{\partial^2 \theta^{2} P_{s0}}{\partial Z^2} = 0
\]  

(8)

\[
\frac{\partial^2 \theta^{2} P_{r0}}{\partial \theta^2} + \zeta^2 \frac{\partial^2 \theta^{2} P_{s0}}{\partial Z^2} = 0
\]  

(9)

\[
Q_r = -\frac{\text{UC}_r}{2} \left( 1 + \frac{\partial P_{r0}}{\partial \theta} \right)
\]  

(10)

\[
Q_s = -\frac{\text{UC}_r}{2} \left( k + k^3 \frac{\partial P_{s0}}{\partial \theta} \right)
\]  

(11)

Applying the standard technique of separation of variables to equations (8) and (9) gives the following equations:

\[
P_{r0} = [A_r \sinh(K_r \theta) + B_r \cosh(K_r \theta)] \left[ D_r \sin \left( \frac{K_r Z}{\zeta} \right) + E_r \cos \left( \frac{K_r Z}{\zeta} \right) \right]
\]  

(12)

\[
P_{s0} = [A_s \sinh(K_s \theta) + B_s \cosh(K_s \theta)] \left[ D_s \sin \left( \frac{K_s Z}{\zeta} \right) + E_s \cos \left( \frac{K_s Z}{\zeta} \right) \right]
\]  

(13)

The boundary conditions are as follows:

1. \( P_{r0} = 0 \), when \( \theta = 0 \).
2. \( P_{s0} = 0 \), when \( \theta = 2\pi - \delta \).
3. \( \frac{\partial P_{r0}}{\partial Z} = \frac{\partial P_{s0}}{\partial Z} = 0 \), when \( Z = 0 \).
4. \( P_{r0} = P_{s0} = 0 \), when \( Z = 1/2 \).
5. \( P_{r0} = P_{s0} = \sum_{m=1,3,\ldots}^{\infty} I_m \cos(m\pi Z) \), when \( \theta = 2\pi \psi \), where \( I_m \) is a Fourier coefficient and \( \psi = \gamma/2\pi \).

This last boundary condition assumes that the dimensionless pressure at the common boundary is given by the Fourier cosine series. This technique was initially used by Archibald (ref. 3) in his thrust bearing work.

Substituting these boundary conditions into equations (12) and (13) gives
\[
P_{r0} = \sum_{m=1, 3, \ldots}^{\infty} \frac{I_m \cos(m\pi Z) \sinh(m\pi \xi \theta)}{\sinh(2m\pi^2 \xi \psi)} \quad (14)
\]

and

\[
P_{s0} = \sum_{m=1, 3, \ldots}^{\infty} \frac{I_m \cos(m\pi Z) \sinh[\frac{m\pi \xi (2\pi - \delta - \theta)]}{\sinh[\frac{m\pi \xi (2\pi - 2\pi \psi - \delta)]} \quad (15)
\]

In order to solve for the Fourier coefficient of equations (14) and (15), the flow rate must be equated at the common boundary of the ridge and step regions. Therefore, from equations (10) and (11), the following equation can be written:

\[
\left( \frac{\partial P_{r0}}{\partial \theta} \right)_{\theta=2\pi\psi} - k^3 \left( \frac{\partial P_{s0}}{\partial \theta} \right)_{\theta=2\pi\psi} = k - 1
\]

Equations (14) and (15) can be used to write the preceding equation as

\[
\sum_{m=1, 3, \ldots}^{\infty} m\pi \xi I_m \cos(m\pi Z) \left\{ \cosh(2m\pi^2 \xi \psi) + \frac{k^3}{3} \cosh[\frac{m\pi \xi (2\pi - 2\pi \psi - \delta)]} \right\} = k - 1 \quad (16)
\]

The right side of the equation (16) can be expanded in terms of a Fourier cosine series by use of the standard cosine series for \( \frac{\pi}{4} \) (ref. 9), so that the following expression can be written:

\[
k - 1 = \sum_{m=1, 3, \ldots}^{\infty} \frac{4(k - 1)\sin\left(\frac{m\pi}{2}\right) \cos(m\pi Z)}{m\pi}
\]

Substituting this equation into equation (16) and solving for the Fourier coefficient give
\[ I_m = \frac{4(k - 1)\sin\left(\frac{m\pi}{2}\right)}{m^2\pi^2\zeta \left\{ \cosh(2m\pi^2\zeta) + k^3 \cosh[m\pi\zeta(2\pi - 2\psi - \delta)] \right\} } \]  

(17)

Therefore, equations (14), (15), and (17) define the dimensionless pressure in the ridge and step regions.

The dimensionless radial and tangential load components in the ridge region are

\[ WR_{r0} = \frac{w_{r0}}{p_a L r} = -2 \int_0^{2\pi} \int_0^{1/2} P_{r0} \cos \theta \, dZ \, d\theta \]

\[ WT_{r0} = \frac{w_{t0}}{p_a L r} = 2 \int_0^{2\pi} \int_0^{1/2} P_{r0} \sin \theta \, dZ \, d\theta \]

where \( \Gamma = 6 \mu U R/p_a C_r^2 \). Figure 3 illustrates the radial and tangential load components. The radial load component \( w_r \) is defined as the load acting in the fixed \( \theta = 0^\circ \) direction. The tangential load component \( w_t \) is the load acting perpendicular to the radial load component. Substituting equation (14) into the immediately preceding equations gives the following:

\[ WR_{r0} = \sum_{m=1, 3, \ldots}^{\infty} \frac{2I_m [m\pi \zeta - m\pi \zeta \cos(2m\pi \zeta) \cosh(2m\pi^2 \zeta)]}{m\pi \sin\left(\frac{m\pi}{2}\right) \sinh(2m\pi^2 \zeta)(1 + m^2\pi^2\zeta^2)} \]

(18)

\[ WT_{r0} = \sum_{m=1, 3, \ldots}^{\infty} \frac{-2I_m [\cos(2m\pi \zeta) \sinh(2m\pi^2 \zeta) - m\pi \zeta \sin(2m\pi \zeta) \cosh(2m\pi^2 \zeta)]}{m\pi \sin\left(\frac{m\pi}{2}\right) \sinh(2m\pi^2 \zeta)(1 + m^2\pi^2\zeta^2)} \]

(19)

The dimensionless radial and tangential load components in the step region can be written as
Substitution of equation (15) into the immediately preceding equations gives

\[
\begin{align*}
WR_{s0} &= \frac{w_r s_0}{p_a L R' \Gamma} = -2 \int_{2\pi\psi}^{2\pi-\delta} \int_{0}^{1/2} P_{s0} \cos \theta \, dZ \, d\theta \\
WT_{s0} &= \frac{w_t s_0}{p_a L R' \Gamma} = 2 \int_{2\pi\psi}^{2\pi-\delta} \int_{0}^{1/2} P_{s0} \sin \theta \, dZ \, d\theta
\end{align*}
\]

The total dimensionless load capacity and load angle can be written as

\[
W_0 = \frac{w_0}{p_a L R' \Gamma} = \left[ (WR_{r0} + WR_{s0})^2 + (WT_{r0} + WT_{s0})^2 \right]^{1/2}
\]

Therefore, the dimensionless load capacity and load angle are functions only of \( k, \psi, \zeta, \) and \( \delta. \) For given values of radius-to-length ratio \( \zeta \) and angle subtended by the lubrication groove \( \delta, \) values of the ratio of the step-to-ridge clearance \( k, \) and the ratio of the angle subtended by the ridge to the angle subtended by the pad \( \psi \) may be found which maximize the load supported by the bearing. Appendix A presents a simple analytical procedure for finding the optimal \( k \) for maximum load while \( \psi \) is held constant. Because of the way \( \psi \) appears in the load expressions, the optimum value of \( \psi \) must be determined numerically.
The shear stress at the moving member in the ridge and step regions can be written as

\[ \tau_{r0} = \frac{\mu U}{C_r} - \frac{C_r}{2R} \frac{\partial p_{r0}}{\partial \theta} \]

\[ \tau_{s0} = \frac{\mu U}{C_s} - \frac{C_s}{2R} \frac{\partial p_{s0}}{\partial \theta} \]

The friction force at the moving member in the ridge and step regions is then

\[ f_{r0} = 2RL \int_0^{2\pi} \int_0^{1/2} \tau_{r0} \, dZ \, d\theta = 2RL \left[ \frac{\pi \mu U \Psi}{C_r} - \frac{C_r}{2R} \left( \frac{6 \mu UR}{C_r^2} \right) \int_0^{2\pi} \int_0^{1/2} \frac{\partial p_{r0}}{\partial \theta} \, dZ \, d\theta \right] \]

\[ f_{s0} = 2RL \int_0^{2\pi} \int_0^{\frac{1}{2}} \tau_{s0} \, dZ \, d\theta = 2RL \left[ \frac{\mu U (2\pi - 2\pi \psi - \delta)}{2C_s} - \frac{C_s}{2R} \left( \frac{6 \mu UR}{C_r^2} \right) \int_0^{2\pi} \int_0^{\frac{1}{2}} \frac{\partial p_{s0}}{\partial \theta} \, dZ \, d\theta \right] \]

Equations (14) and (15) can be used in integrating the immediately preceding equations. Then the dimensionless friction force in the ridge and step regions can be written as

\[ F_{r0} = \frac{f_{r0}C_r}{2\pi RL \mu U} = \psi - \frac{3}{\pi^2} \left[ \sum_{m=1}^{\infty} \frac{I_m \sin \left( \frac{m\pi}{2} \right)}{m} \right] \]

\[ F_{s0} = \frac{f_{s0}C_r}{2\pi RL \mu U} = \frac{1}{k} \left( 1 - \psi - \frac{\delta}{2\pi} \right) + \frac{3k}{\pi^2} \left[ \sum_{m=1}^{\infty} \frac{I_m \sin \left( \frac{m\pi}{2} \right)}{m} \right] \]
The total dimensionless friction force is the sum of the two components, or

\[ F_0 = \frac{f_0 C_r}{2\pi RL\mu U} = \psi \left(1 - \frac{1}{k}\right) + \frac{1}{k} \left(1 - \frac{\delta}{2\pi}\right) + \frac{3(k - 1)}{\pi^2} \sum_{m=1, 3, \ldots}^{\infty} \frac{I_m \sin \left(\frac{m\pi}{2}\right)}{m} \]  

(24)

**DISCUSSION OF RESULTS**

Numerical solutions of equations (14) to (24) were obtained on a digital computer for a number of cases of interest. For all the data presented, the angle subtended by the lubrication groove \( \delta \) was 2°. Tables I to VIII show the dimensionless load capacity, load angle, and dimensionless friction force, while varying the step configuration for values of radius-to-length ratios \( \zeta \) of 0.0625, 0.125, 0.25, 0.5, 1.0, 1.5, and 2.0. The following observations can be made from the data in the tables:

1. Optimal configurations for maximum load capacity occur for all the radius-to-length ratios investigated.
2. The observations made in appendix A are substantiated, that is, for \( \psi = 1.0 \), the optimal \( k \) for maximum load capacity is 1.5, and for \( \psi = 0.5 \), the optimal \( k \) for maximum load capacity is 1.68. This is true for all the radius-to-length ratios evaluated.
3. The dimensionless load capacity is substantial, especially when the radius-to-length ratio is small, even though the bearing is in a concentric position.
4. The load angle is only slightly affected by the film thickness ratio. Furthermore, for \( \psi = 0.5 \), the load angle is essentially zero.
5. For \( \psi = 0.99 \) and \( k = 1.01 \) the Rayleigh step journal bearing approaches a Sommerfeld bearing. Comparison of the dimensionless friction force of a Sommerfeld bearing (\( F = 1.0 \)) with that of a Rayleigh step journal shows that the friction force of the stepped journal is lower.

The limiting equations from appendix B were utilized to obtain the data in table I. The reason is that, as the radius-to-length ratio \( \zeta \) approaches zero, the expressions for the load capacity, load angle, and friction force approach zero divided by zero. L'Hôpital's rule was used to solve for this limiting case.

From the tables it can be concluded that the optimal step configuration over the entire range of radius-to-length ratio considered (0 ≤ \( \zeta \) ≤ 2) is \( k = 1.7 \) and \( \psi = 0.35 \). In support of this conclusion, figures 4 to 6 are presented. Figure 4 shows the effect of the step location on dimensionless load capacity for various values of film thickness ratio for the infinite length bearing (\( \zeta = 0 \)). It is seen that \( k = 1.7 \) does indeed yield the maximum dimensionless load capacity. Figure 5 shows the effect of step location \( \psi \) on the dimen-
Dimensionless load capacity for various values of radius-to-length ratio $\zeta$, while holding the film thickness ratio $k$ fixed at 1.7. It is seen that for $\zeta \geq 0.5$, $W_0$ changes very little with $\psi$, and as $\zeta = 0$, $\psi = 0.35$ is optimal. Figure 6 shows the effects of film thickness ratio on dimensionless load capacity for various values of $\zeta$, while holding $\psi$ at 0.35. It is shown in this figure that $k = 1.7$ is an optimal for the entire range of $\zeta$.

Figure 7 shows the midplane circumferential pressure distribution in a single-step journal bearing. The step configuration used here is that which is optimal for load capacity. This figure shows a significant dropoff of pressure as the radius-to-length ratio is decreased. Furthermore, as the radius-to-length ratio approaches zero, the pressure distribution becomes linear as is demonstrated by the pressure equations in appendix B. Finally, this figure shows that a unique feature of the Rayleigh step journal is a positive pressure developed completely around the journal.

Figure 8 compares the concentric stepped journal bearing loads with those of the Sommerfeld bearing operating at an eccentricity ratio of 0.1 for various radius-to-length ratios. The optimal step configuration ($\psi = 0.35, k = 1.7$) is used for the Rayleigh step journal bearing data. The finite-length Sommerfeld bearing load capacity results were obtained from Donaldson (ref. 10). Figure 8 shows that for small radius-to-length ratios ($\zeta < 0.2$), the dimensionless load capacity of a concentric single-step journal bearing is higher than the dimensionless load capacity of a finite-length Sommerfeld bearing with an eccentricity ratio of 0.1. Also, the change in load with radius-to-length ratio of a Rayleigh step journal bearing is greater than that for a finite Sommerfeld bearing. Side leakage, therefore, is more pronounced in a Rayleigh step journal bearing. The load capacity of a Rayleigh step journal bearing, with realistic radius-to-length ratios ($0.5 \leq \zeta \leq 2$) could be increased by placing side rails at the ends of the bearing so that the axial flow is restricted. This approach is also discussed in the trisector patent disclosure (ref. 6). Finally, it should be pointed out that in the comparison of the Rayleigh step journal bearing with the Sommerfeld bearing, the negative pressure region contributes to a load capacity for the Sommerfeld bearing whereas the Rayleigh step journal bearing, as was shown in figure 7, contains a positive pressure profile completely around the bearing.

**SUMMARY OF RESULTS**

An analysis of a single-step concentric Rayleigh step journal bearing was performed. The resulting expressions for the dimensionless pressure, load, load angle, and friction force were evaluated on a digital computer. A consideration of load capacity showed the results to yield an optimum film thickness ratio of 1.7 and an optimum ratio of the angle subtended by the ridge to the angle subtended by the pad of 0.35. The results indicated
that a feature of the Rayleigh step journal is a positive pressure developed completely around the journal. Also, for small radius-to-length ratios ($\zeta < 0.2$), the load capacity of the stepped journal, while concentric, is higher than the load capacity of a finite-length Sommerfeld bearing operating with an eccentricity ratio of 0.1. Finally, the dimensionless friction source of a Rayleigh step journal bearing proved to be less than that of a finite Sommerfeld bearing.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 28, 1968,
129-03-13-05-22.
From equations (18) to (22), the dimensionless load capacity is seen to be a function of the film thickness ratio $k$ only through the Fourier coefficient $I_m$. Maximizing the dimensionless load capacity with respect to $k$ may be achieved by maximizing the Fourier coefficient with respect to $k$; that is, the $k$ for which $\frac{\partial I_m}{\partial k} = 0$ is the same for which $\frac{\partial W_0}{\partial k} = 0$. Therefore, from equation (17) the following equations can be written:

\[
\frac{\partial I_m}{\partial k} = \left[ \frac{4 \sin \left( \frac{m\pi}{2} \right)}{m^2 \pi^2 \xi} \right] \left[ \frac{\text{ctnh}(2m\pi^2 \xi \psi) + (3k^2 - 2k^3)\text{ctnh}[m\pi \xi (2\pi - 2\pi\psi - \delta)]}{\{\text{ctnh}(2m\pi^2 \xi \psi) + k^3 \text{ctnh}[m\pi \xi (2\pi - 2\pi\psi - \delta)]\}^2} \right]
\]

Therefore

\[
\frac{\partial I_m}{\partial k} = 0
\]

implies

\[
0 = \alpha + 3k^2 - 2k^3 \quad (A1)
\]

where

\[
\alpha = \frac{\text{ctnh}(2m\pi^2 \xi \psi)}{\text{ctnh}[m\pi \xi (2\pi - 2\pi\psi - \delta)]} \quad (A2)
\]

Since the angle subtended by the lubrication groove $\delta$ is usually very small compared with $2\pi - 2\pi\psi$ (for all the data presented in this report $\delta = 0.035$), equation (A2) can be rewritten as
\[ \alpha \approx \frac{\text{ctnh}(2m\pi^2\xi\psi)}{\text{ctnh}[2m\pi^2(1 - \psi)]} \quad \text{(A3)} \]

The range of values of \( k \) and \( \psi \) imposed on the problem by physical considerations of the bearing is the real numbers defined by the following inequalities:

\[ k \geq 1 \]

and

\[ 0 \leq \psi \leq 1 \]

Observing the optimal value of \( k \) for two fixed values of \( \psi \) results in the following cases:

Case 1 - \( \psi = 1.0 \):
From equation (A3), it is apparent that \( \alpha = 0 \). Then, from equation (A1), \( k = 1.5 \); that is, for \( \psi \) fixed at 1.0, the optimal \( k \) for a maximum load condition is 1.5.

Case 2 - \( \psi = 0.5 \):
This indicates that \( \alpha = 1.0 \), and from equation (A1), \( 3k^2 - 3k^3 + 1 = 0 \). The real number for \( k \) which satisfies this equation is approximately 1.68; that is, for \( \psi \) fixed at 0.5, the optimal \( k \) for maximum load is approximately 1.68.
APPENDIX B

EQUATIONS FOR SPECIAL CASE OF INFINITE-LENGTH BEARING

Hospital's rule was employed twice to obtain the limiting case when \( \xi \to 0 \) for the pressure, load, load angle, and friction force (eqs. (14) to (23)). The results are shown in the following equations:

\[
\lim_{\xi \to 0} P_{r0} = \frac{\theta(k - 1)(1 - \psi - \frac{\delta}{2\pi})}{1 - \frac{\delta}{2\pi} + \psi(k^3 - 1)} \tag{B1}
\]

\[
\lim_{\xi \to 0} P_{s0} = \frac{\psi(k - 1)(2\pi - \delta - \theta)}{1 - \frac{\delta}{2\pi} + \psi(k^3 - 1)} \tag{B2}
\]

\[
\lim_{\xi \to 0} W_{0} = \left(\frac{k - 1}{2}\left[1 - \frac{\delta}{2\pi} - \cos(2\pi \psi)\right] + 2\psi(1 - \cos \delta)(\psi - 1 + \frac{\delta}{2\pi}) + \psi\left(1 - \frac{\delta}{2\pi}\right)[\sin(2\pi \psi)\sin \delta + (1 - \cos \delta)\cos(2\pi \psi)]\right)^{1/2}
\left[1 - \frac{\delta}{2\pi} + \psi(k^3 - 1)\right] \tag{B3}
\]

\[
\lim_{\xi \to 0} \Phi_{0} = \tan^{-1} \left[\frac{\psi \sin \delta + \left(1 - \frac{\delta}{2\pi}\right)\sin(2\pi \psi)}{(1 - \frac{\delta}{2\pi})(1 - \cos(2\pi \psi)) - \psi(1 - \cos \delta)}\right] \tag{B4}
\]

\[
\lim_{\xi \to 0} F_{0} = \psi + \frac{\left(1 - \psi - \frac{\delta}{2\pi}\right)}{k} + \frac{3\psi(k - 1)^2\left(1 - \psi - \frac{\delta}{2\pi}\right)}{1 - \frac{\delta}{2\pi} + \psi(k^3 - 1)} \tag{B5}
\]
REFERENCES


**TABLE I. - EFFECT OF STEP CONFIGURATION ON DIMENSIONLESS LOAD CAPACITY, LOAD**

| Ratio of ridge to pad angles, $\psi$ | 1.01 | 1.20 | 1.40 | 1.60 | 1.80 | 2.00 | 2.20 | 2.40 | 2.60 | 2.80 | 3.00 | 3.20 | 3.40 | 3.60 | 3.80 | 4.00 | 4.20 | 4.40 | 4.60 | 4.80 | 5.00 | 5.20 | 5.40 | 5.60 | 5.80 | 6.00 | 6.20 | 6.40 | 6.60 | 6.80 | 7.00 | 7.20 | 7.40 | 7.60 | 7.80 | 8.00 | 8.20 | 8.40 | 8.60 | 8.80 | 9.00 |
|-------------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ | $W_0$ | $\varphi_0$ | $F_0$ |
| 0.10 | 0.0001 | -72.11 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 0.20 | 0.0277 | -54.30 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 | 0.0277 |
| 0.30 | 0.0161 | -36.30 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 | 0.0161 |
| 0.40 | 0.0128 | -18.40 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 | 0.0128 |
| 0.50 | 0.0097 | -5.45 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 | 0.0097 |
| 0.70 | 0.0033 | -87.21 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 |
| 0.90 | 0.0008 | -87.21 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0008 |
### Angle, and Dimensionless Friction Force for Radius-Length Ratio of 0

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### Angle, and Dimensionless Friction Force for Radius-Length Ratio of 0.0625

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### Angle, and Dimensionless Friction Force for Radius-Length Ratio of 0.125

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### Table IV - Effect of Step Configuration on Dimensionless Load Capacity, Load

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### Table V - Effect of Step Configuration on Dimensionless Load Capacity, Load

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### Table VI - Effect of Step Configuration on Dimensionless Load Capacity, Load

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**Note:** \( \theta \) is the eccentricity ratio, \( \psi \) is the angle of the lubrication groove, \( L_0 \) is the number of steps placed around the journal, \( \psi \).
### ANGLE, AND DIMENSIONLESS FRICTION FORCE FOR RADIUS-LENGTH RATIO OF 0.25

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### ANGLE, AND DIMENSIONLESS FRICTION FORCE FOR RADIUS-LENGTH RATIO OF 0.90

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### ANGLE, AND DIMENSIONLESS FRICTION FORCE FOR RADIUS-LENGTH RATIO OF 1.0

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### TABLE VII. EFFECT OF STEP CONFIGURATION ON DIMENSIONLESS LOAD CAPACITY, $L_{op}$

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### TABLE VIII. EFFECT OF STEP CONFIGURATION ON DIMENSIONLESS LOAD CAPACITY, $L_{op}$

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Film thickness
ANGLE, AND DIMENSIONLESS FRICTION FORCE FOR RADIUS-LENGTH RATIO OF 1.5

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ANGLE, AND DIMENSIONLESS FRICTION FORCE FOR RADIUS-LENGTH RATIO OF 2.0

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Figure 1. - Concentric Rayleigh step bearing.

Figure 2. - Three-section Rayleigh step bearing.

Figure 3. - Rayleigh step and Sommerfeld journal bearings.
Figure 4. - Effect of step location on dimensionless load capacity for various values of film thickness ratio. Radius-to-length ratio, 0.

Figure 5. - Effect of step location on dimensionless load capacity for various values of radius-length ratio. Film thickness ratio, 1.7.
Figure 6. - Effect of film thickness ratio on dimensionless load capacity for various values of radius-length ratio.
Step location parameter, 0.35.
Figure 7. - Mid-plane circumferential pressure distribution in a single-step Rayleigh step journal bearing. Eccentricity ratio, 0.
Concentric Rayleigh step journal bearing
\(N = 1, k = 1.7, \phi = 0.39, \delta = 2^\circ\)

Finite-length Sommerfeld bearing with eccentricity ratio of 0.1 (obtained from Donaldson, ref. 10)

Figure 8. Effect of radius-to-length ratio on dimensionless load capacity.
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—National Aeronautics and Space Act of 1958

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