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### ABSTRACT

Copious x-ray production is shown to attend the collisions of suprathermal ( $\gtrsim$ MeV) protons with ambient electrons, via a process of inverse bremsstrahlung. Such suprathermal protons have been detected in cosmic rays and in solar flare events; the inverse bremsstrahlung of such cosmic ray protons is expected to make a significant contribution to the diffuse sky background of x-rays and suprathermal solar protons to the x-ray bursts associated with solar flares. The x-ray production in the supernova remnant Tau A (Crab Nebula) is too high to be readily accounted for by the inverse bremsstrahlung of suprathermal protons.

# SUPRATHERMAL PROTON X-RAY BREMSSTRAHLUNG

## INTRODUCTION

For a thermal plasma of electrons and protons, the bremsstrahlung that occurs arises mainly from the collisions of electrons with relatively stationary protons (c.f. Drummond 1961, Maxon and Corman 1967); the laboratory frame of reference approximately coincides with the center-of-momentum system. The purpose of this paper is to show that when there is a highly suprathermal proton component, such as is the case with galactic and solar cosmic rays, there is appreciable radiation associated with the process of inverse bremsstrahlung. It is inverse in the sense that the center of momentum of the proton-electron collision responsible for the radiation is no longer approximately at rest with respect to the laboratory frame but is rather approximately that of the relatively rapid proton. Hence, for the non-relativistic situation ( $\beta \ll 1$ ), the inverse bremsstrahlung associated with a suprathermal proton of kinetic energy  $E$  has the same spectral intensity as that expected from the bremsstrahlung of an electron of kinetic energy  $(m/M)E$ , where  $m$  and  $M$  are the rest masses for the electron and proton respectively.

We consider the x-ray regime, viz  $\lambda \approx (10^{-1}\text{\AA} \rightarrow 10\text{\AA})$ . In this regime, the radiation arising from the electron-proton Coulomb scattering interaction is given by the Bethe-Heitler cross-section (Heitler 1954) as:

$$h\nu \left( \frac{d\sigma}{dh\nu} \right) = \frac{\alpha}{\pi} \sigma_0 \left( \frac{mc^2}{T_0} \right) \ln \left[ \frac{(\sqrt{T_0} + \sqrt{T_0 - h\nu})^2}{h\nu} \right] \quad (1)$$

where

$\alpha = (137)^{-1}$ ; fine structure constant,

$\sigma_0 = 665 \times 10^{-27} \text{ cm}^2$ ; Thompson cross-section,

$h\nu =$  photon energy,

$T_0 =$  electron kinetic energy, in proton rest system, before radiative collision.

By non-relativistic kinematics we identify the  $T_0$  appropriate to inverse bremsstrahlung as:

$$T_0 = (m/M)E. \quad (2)$$

We may now construct the inverse bremsstrahlung photon source function,  $(dq/dh\nu)$  (photons/cm<sup>3</sup>-sec-erg) as

$$h\nu \left( \frac{dq}{dh\nu} \right) = n \int_{(M/m)h\nu}^{\infty} h\nu \cdot \left( \frac{d\sigma}{dh\nu} \right)_{h\nu} \cdot (\beta c) \cdot N(E) \cdot dE \quad (3)$$

where  $n$  is the number density (cm<sup>-3</sup>) of ambient electrons,  $(\beta c)$  is the laboratory velocity of the suprathermal proton, and  $N(E)$  is the spectral density of suprathermal protons (protons/cm<sup>3</sup>-erg).

We substitute (1) and (2) into (3) and obtain

$$h\nu \left( \frac{dq}{dh\nu} \right) = (4\alpha \sigma_0 M c^2) n \int_{(M/m)h\nu}^{\infty} \ln \left[ \frac{\left\{ 1 + \left( 1 - \frac{M}{m} \cdot \frac{h\nu}{E} \right)^{1/2} \right\}^2}{\left( \frac{M}{m} \cdot \frac{h\nu}{E} \right)} \right] \frac{J dE}{E} \quad (4)$$

where  $J \equiv N(\beta c)/4\pi$ , the isotropic spectral intensity of suprathermal protons (protons/cm<sup>2</sup>-sec-sr-erg).

It is instructive to assume a power law spectrum for the suprathermal protons in the explicit evaluation of (4). Therefore, we take

$$J = A E^{-\ell} . \quad (5)$$

This power law spectrum (5) for the evaluation of (4) yields:

$$h\nu \left( \frac{dq}{dh\nu} \right) = (8\alpha \sigma_0 Mc^2) \cdot f(\ell) \cdot n \left[ A \cdot \left( \frac{M}{m} \cdot h\nu \right)^{-\ell} \right] \quad (6)$$

where

$$f(\ell) \equiv \int_0^1 x^\ell \ln \left[ \frac{1 + \sqrt{1-x}}{\sqrt{x}} \right] \frac{dx}{x}$$

(e.g.  $f(1) = 1$ , and  $f(\ell) \approx \ell^{-1.6}$  for  $\ell = 0.5 \rightarrow 5$ ).

For the situation of a relatively neutral gas of hydrogen, this source function (6) for inverse bremsstrahlung (called "inner bremsstrahlung" in this instance (c.f. Hayakawa and Matsuoka 1964)) bears a particularly simple relation to the integral loss function  $Q(E)$ , ergs/cm<sup>3</sup>-sec, for all suprathermal protons of kinetic energy above  $E_0 \equiv (M/m) \cdot h\nu$ . This is obtained from

$$Q(E_0) = 4\pi \int_{E_0}^{\infty} J \cdot \frac{dE}{dx} dE \quad (7)$$

where the ionization loss rate  $dE/dx$  is given (Rossi 1952) by the expression:

$$\frac{dE}{dx} = \frac{3}{4} \sigma_0 n' mc^2 \left( \frac{Mc^2}{E} \right) \left( \ell_n \left\{ \frac{4 \left( \frac{m}{M} E \right)}{V_0} \right\} - \frac{2E}{Mc^2} \right) \quad (8)$$

where

$n'$  is the number density ( $\text{cm}^{-3}$ ) of hydrogen atoms, and

$V_0$  is the ionization potential of hydrogen ( $\sim 15$  ev).

For the x-ray regime we note that  $E \approx 10^6 \rightarrow 10^8$  ev. Therefore, in using (8) for evaluating  $Q(E_0)$  via (7) we may use the approximate expression:

$$Q(E_0) \approx 3\pi \sigma_0 n' Mc^2 \left( \ell_n \left\{ \frac{4 \left( \frac{m}{M} E_0 \right)}{V_0} \right\} \right) \int_{E_0}^{\infty} J \frac{dE}{E} \quad (9)$$

If here again we take the power law (5) for  $J$  we may directly compare  $Q(E_0)$  as given by (9) with  $h\nu \cdot (dq/dh\nu)$  as given by (6) and thereby find

$$h\nu \frac{dq}{dh\nu} \approx \frac{8\alpha}{3\pi} \frac{\ell \cdot f(\ell)}{\ell_n (4h\nu/V_0)} \left( \frac{Q(E_0)}{mc^2} \right) \quad (10)$$

where we have taken  $n' = n$  (i.e. most of the electrons of the hydrogen gas are taken as atomic).

In general, for a partially ionized medium, the energy losses exceed those given by (8) (c.f. Hayakawa and Kitao 1956) and therefore the inverse bremsstrahlung is somewhat less than given by (10). However, even if the medium is completely ionized the inverse bremsstrahlung source function is properly given by (6).

## SOLAR FLARE X-RAYS

An investigation of the role of inverse bremsstrahlung in the generation of solar flare x-rays is motivated by the frequent observation of associated supra-thermal protons of the proper energy. One of the best studied solar flares is that of 28 September 1961 (c.f. Fichtel and McDonald 1967). The spectral index for the source spectrum of the observed associated energetic ( $> \text{Mev}$ ) protons is  $\ell = 2.4^{+0.4}_{-0.2}$  (Krimigis, 1965). The x-rays generated at the sun, at about the temporal vicinity estimated for the ejection of protons, were observed at the earth to have an energy spectral index that varied during the course of the burst from  $\sim 2.3$  to  $\sim 2.8$ , for x-rays above 20 keV (Anderson and Winckler, 1962). Hence, it is spectrally consistent (via Equation 6) to attribute the x-rays to the inverse bremsstrahlung of flare protons at the sun similar to those observed to emerge from the sun during the flare.

The total energy leaving the sun during this flare in the form of energetic protons ( $E > 20 \text{ Mev}$ ) has been estimated (Krimigis 1965) as  $\sim 10^{28}$  ergs. If we assume that a comparable quantity of energy is dissipated by protons of this same flare within the matter of the solar atmosphere (e.g. half the energetic protons driven into the sun), then we may use (10) to estimate that the expected x-ray output above 10 keV is of the order  $10^{24}$  ergs. This expected x-ray output is equal to that inferred from the x-ray observations of Anderson and Winckler (1962) above 20 keV.

The main portion of the solar x-ray burst was of  $\sim 10^2$  seconds duration. This is comparable to the stopping time for energetic protons ( $E \gtrsim 20 \text{ MeV}$ ) at a

level of  $\sim 2000$  km above the solar photosphere (Schatzman 1965). These energetic protons could have been produced high in the rarefied solar atmosphere (e.g. at a level of  $\sim 10^5$  km above the photosphere), directed downward and arrived at the indicated lower level of relatively large density in less than 10 seconds. The "knock on" electrons produced by these protons throughout regions of strong magnetic field would contribute to the synchrotron radio emission associated with the flare (Maxwell 1962).

#### INTERSTELLAR MEDIUM

As already indicated, we expect the inverse bremsstrahlung generation of x-rays to be relevant for astrophysical situations where there are protons of energy  $E \approx 1 - 10^2$  MeV. This is a spectral band where there is an intense observed flux of cosmic ray protons (c.f. Gloeckler and Jokipii 1967). The flux of such cosmic ray protons observed near the earth is clearly a lower limit to the intensity of these particles in interstellar space since their exclusion from the solar system by the solar wind could be severe (Balasubrahmanyam et al. 1967). However, we may obtain an integral measure of these protons by considering that the heating of the hydrogen of interstellar HI regions to the observed temperature of  $\sim 10^2$ °K requires dissipation by suprathermal protons (Balasubrahmanyam et al. 1968, Pikel'ner 1968) at a rate given by:

$$Q(E_0) \approx (3 - 10) \times 10^{-26} n' \text{ ergs/cm}^3\text{-sec} \quad (11)$$

for  $E_0 \leq 30$  MeV.

We may infer from (11) that the source function (10) for inverse bremsstrahlung of interstellar cosmic ray protons is:

$$h\nu \cdot \frac{dq}{dh\nu} \approx (3-10) \times 10^{-23} n \ell \cdot f(\ell) \frac{\text{ergs}}{\text{cm}^3 \text{ sec. erg}} \quad (12)$$

for  $h\nu \leq 15$  kev, where  $n$  is here properly taken as the number density ( $\text{cm}^{-3}$ ) of all electrons, free or bound.

It is interesting to note that the cosmic x-ray background exhibits a power law spectrum above 20 kev with energy index  $\ell \approx 1.4$  (Bleeker et al. 1968), whereas it becomes relatively flat below 20 kev, where  $\ell \approx 0.3$  describes the spectrum down to 2 kev (Boldt et al. 1968). The spectral break occurs at  $h\nu \gtrsim 10$  kev (Henry et al. 1968). As already indicated (12), a relatively flat x-ray background spectrum below  $\sim 15$  kev is to be expected from the inverse bremsstrahlung of interstellar suprathermal protons.

The observed background x-ray spectral intensity at  $\sim 10$  kev is  $I \sim 3$  ergs/erg-cm<sup>2</sup>-sec-sr. (Boldt et al. 1968). We compare this intensity with that expected from the inverse bremsstrahlung of cosmic ray protons, viz

$$I = \frac{1}{4\pi} \int_0^{\infty} h\nu \frac{dq}{dh\nu} \cdot dr \quad (13)$$

when  $r$  is the linear penetration of interstellar space and the integrand is given by (12).

Comparing the intensity observed at 10 keV with (13) gives the required column density of electrons as

$$\int_0^{\infty} n \, dr \gtrsim 4 \times 10^{23} \text{ electrons/cm}^2 . \quad (14)$$

This electron column density (14) is within an order of magnitude of that for directions that transverse most of the galactic disk (c.f. Allen 1963); we have considered all electrons, free or bound. One implication of this would be an enhancement of the x-ray background for observations that examine the galactic disk towards the galactic center. However, the electron column density indicated in the present analysis (14) is also comparable to the electron column density in any direction if one considers the entire metagalaxy (Gould 1968). Hence, we cannot rule out the possibility that much of the isotropic diffuse x-ray background at ~15 keV arises from the inverse bremsstrahlung of a suprathermal proton gas distributed throughout all space, represented in our own galaxy by the interstellar protons responsible for the heating of HI regions. A consistent feature of this possibility is that the lifetime of 30 MeV protons for collisional energy loss within the intergalactic medium ( $\sim 10^{-5}$  electrons/cm<sup>3</sup>) of the metagalaxy would be  $\sim 10^{10}$  years, which is comparable to the age of the metagalaxy. Within the interstellar medium ( $\sim 1$  atom/cm<sup>3</sup>) of our galactic disk this lifetime is  $\sim 10^6$  years, which is comparable to the age of observed galactic cosmic rays.

TAU XR-1

The x-ray emission of the Crab Nebula (Tau XR-1) exhibits a power law spectrum of energy index  $\ell \approx 1$  (Boldt et al. 1968) with an observed total output of  $\sim 10^{37}$  ergs/second,  $1 \rightarrow 10^2$  kev. If we assume that this observed x-ray output from Tau XR-1 is due to inverse bremsstrahlung of suprathermal protons within the supernova remnant, then we may integrate (6) with  $\ell = 1$  to evaluate the required total suprathermal proton kinetic energy content U of the Crab Nebula. For the interval  $E_1$  to  $E_2$ , the formal expression is:

$$U = \frac{\pi \sqrt{\frac{2M}{m}} \left( \sqrt{\frac{E_2}{Mc^2}} - \sqrt{\frac{E_1}{Mc^2}} \right) \left[ \int_{L^3} \int_{(m/M)E_1}^{m/M E_2} h\nu \cdot \frac{dq}{dh\nu} \cdot dh\nu \cdot d^3 x \right]}{2\alpha \sigma_0 c n \ell n (E_2/E_1)} \quad (15)$$

where n is the average electron density ( $\text{cm}^{-3}$ ) in the x-ray emission region of volume  $L^3$ . We consider x-rays of  $1 \rightarrow 10^2$  kev (i.e. this corresponds to  $E_1 = 2$  MeV,  $E_2 = 200$  MeV) for which Tau XR-1 yields a total output of  $\sim 10^{37}$  ergs/second, to be identified with the integral term of (15). The numerical evaluation of (15) then gives

$$U \approx \left( \frac{2 \times 10^{55}}{n} \right) \text{ ergs.} \quad (16)$$

The smallest value of U, for an extended region of the Crab Nebula, would occur for the situation of maximum electron density. The density varies from  $\sim 2 \times 10^3$   $\text{cm}^{-3}$  in the filaments (Woltjer 1958) to  $\sim 1$   $\text{cm}^{-3}$  in the diffuse mass (Minkowski

1968). Hence,  $U \gtrsim 10^{52}$  ergs. It is estimated (Woltjer 1958) that the energy content of electrons required to account for the radio and optical synchrotron emission is  $\sim 10^{48}$  ergs. The proton energy content required for inverse bremsstrahlung x-ray generation is at least  $10^4$  times greater than this electron energy content and would not fit current estimates of the energetics of the Crab Nebula (e.g.  $\sim 10^{49}$  ergs for the kinetic energy of the expanding shell; Woltjer 1958). However, if the protons and electrons were accelerated by the same velocity dependent process, then the energy content in protons would be at least  $10^3$  times greater than the energy content in electrons. Furthermore, the energy content in suprathermal protons could have been entirely generated to its present level during the explosive phase of the supernova rather than during the later evolution of the remnant; the lifetime of suprathermal protons for collisional energy loss is not small compared to the age of the Crab Nebula, even if these protons reside mainly in the relatively dense filaments.

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