Fast Fourier Transform Spectral Analysis Program

The problem:
A complete analysis of actual vehicle characteristics has required a program that could perform frequency spectrum analysis of the telemetered data, including a capability to calculate power spectrum rms amplitudes and cross spectrum of sampled parameters at uniform time increments.

The solution:
The Fast Fourier Transform Spectral Analysis Program was developed for use in analyzing post-flight trajectory data emanating from space vehicle telemetered flight data. Except for the input-output formats, the program can be adapted to handle any sampled data at even time increments.

How it’s done:
The Fast Fourier Transform (FFT) algorithm is a method for calculating the finite Fourier transform in approximately \( N \log_2 N \) operations where \( N \) is the number of data points. This is compared to conventional methods requiring \( N^2 \) operations. The accuracy ratio of the FFT method compared to conventional methods is \( N/N \log_2 N \), the same ratio as the speed ratio.

The Runge-Konig derivation of a method of sampling a continuous function at \( N \) points at intervals \( \Delta t \) and performance of a Fourier analysis obtaining \( N \) amplitudes explain the ideas behind the FFT algorithm. Another sampling at the midpoints of the previous sampling intervals yields an \( N \)-point Fourier analysis differing from the previous \( N \)-point Fourier analysis by a phase shift of \( 1/2 \Delta t \). It was noticed by Runge and Konig that one could get an analysis of \( 2N \) points, for the sampling at \( 1/2 \Delta t \), by using the results already calculated. To derive the necessary equations, a sample complex quantity \( X(j) \) is considered.

The method used to analyze a data set \( X(j) \) is to break the set into several segments which can overlap, perform analysis on each segment, and average results from each segment to obtain results for the data set. This method is useful in testing for and measuring nonstationarity.

The program has an option to remove a curve from the input data before performing any spectral analysis on the data. The program can remove up to a fifth order curve of the form \( X(t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3 + A_4 t^4 + A_5 t^5 \). The best curve fit of the order selected is subtracted from the data, and analysis is performed on the residuals.

In general, the Fast Fourier Transform Spectral Analysis Program with the digital algorithm adds to the basic frequency spectrum analysis program the following new capabilities:
1. Transfer function calculation for multiple inputs
2. Cross spectrum analysis
3. Rms amplitude calculations.

Notes:
1. This program was written in FORTRAN IV language for use on the IBM-360 computer.
2. Inquiries concerning the program may be directed to:
   
   COSMIC
   Computer Center
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   Reference: B69-10434

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Patent status:
No patent action is contemplated by NASA.
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