10.6 MICRON OPTICAL HETERODYNE COMMUNICATION SYSTEM

PHASE 3 REPORT

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# CONTENTS

<table>
<thead>
<tr>
<th>SECTION I</th>
<th>INTRODUCTION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SECTION II</td>
<td>TECHNICAL DISCUSSION</td>
<td>22</td>
</tr>
<tr>
<td>Systems Considerations</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Frequency Stabilized CO₂ Laser</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Experimental CO₂ Laser</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Vibrations in a Block Laser</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Final CO₂ Laser Design</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Signal and Noise Considerations</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Optical System</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>Configuration</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Detection Mount and Cooling</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>Sighting</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>Piezoelectric Frequency Control and Modulation</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Detector Considerations and Frequency Response</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Automatic Frequency Control Loop (AFC)</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Discriminator</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>AFC Feedback Amplifier</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>System Capabilities</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>

**APPENDIX A**  
PASSIVE FREQUENCY STABILIZATION OF A CARBON DIOXIDE LASER*
### ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Two-Ended 10.6-Micron Optical Communications System</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Experimental CO₂ Laser</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Frequency-Stabilized CO₂ - Laser with Water Cooling</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>Beam Diameter at 3000 Meters</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>H-Frame Configuration of the Optical System</td>
<td>23</td>
</tr>
<tr>
<td>6</td>
<td>Piezoelectric Transducer for Laser Frequency Control</td>
<td>28</td>
</tr>
<tr>
<td>7</td>
<td>Experimental Driver for GA-AS Modulator</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Frequency Response of Experimental Driver for GA-AS Modulator</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>Frequency Response HzCdTe Detector</td>
<td>34</td>
</tr>
<tr>
<td>10</td>
<td>Relative Frequency Stability of Two Passively Stabilized CO₂ Lasers</td>
<td>37</td>
</tr>
<tr>
<td>11</td>
<td>Voltage Output Versus Frequency I.F. Amplifier Discriminator AFC Amplifier</td>
<td>39</td>
</tr>
<tr>
<td>12</td>
<td>Experimental Amplifier, Discriminator and AFC Amplifier</td>
<td>40</td>
</tr>
<tr>
<td>13</td>
<td>50 kHz Subcarrier Generation</td>
<td>41</td>
</tr>
<tr>
<td>14</td>
<td>Conditions for the Tracking Range of Two Lasers</td>
<td>43</td>
</tr>
</tbody>
</table>
SECTION I
INTRODUCTION

This report describes the effort made towards design and construction of a 10.6 micron optical heterodyne communication system during the reporting period from 1 August to 31 October 1967.

During this first phase of the present contract, initial experiments have been made for the design of improved carbon dioxide lasers with a better long-term stability. The design has been proven and construction has started. Assembly and testing will begin during November.

Electronic components for the communication system have been built and tested. These include a complete AFC loop consisting of 10 MHz IF amplifier, limiter, discriminator and a low-pass filter with variable cut-off frequency between 50 Hz to 2 kHz. A second commercial AFC loop was purchased and comparative tests will start as soon as the lasers are built.

Electronic components for CO$_2$ laser modulation were built and tested. A 50 kHz sub-carrier oscillator was built and a 2 MHz oscillator, feasible either for piezoelectric FM modulation or for driving a GaAs modulator, was built and tested.

The optical system in the form of a 4-inch off-axis Cassegrain was designed and components are on order. The mount for the optical system and the heterodyne detector support are under construction.

Signal, noise and bandwidth considerations have been made which indicate that with lasers of short-term stability of 1 part in $10^{10}$ a tracking accuracy of 1 part in $10^{11}$ should be achievable. This implies that the IF frequency is narrow enough to allow the simple baseband modulation.
SECTION II
TECHNICAL DISCUSSION

SYSTEMS CONSIDERATIONS

The two-ended communication system has the big advantage of the quantum noise detection sensitivity in comparison to the shot noise limitation of the one-ended system. Such a system, however, is only useful if the intermediate frequency is stable and the frequency width small in comparison to the information bandwidth. Two-ended communication systems have been built in the visible region (6328Å) by Rabinowitz, et al. (1-3), and very recently by Goodwin (4) for the 3.3 µ He-Ne-transition. The latter system was tested over a 1000-foot range while Rabinowitz et al. worked only over laboratory distances.

The type of modulation that would be employed depends on the resulting beat frequency spectral width that can be achieved with the AFC loop closed. If this frequency width can be held down to 1kHz or less, the more simple baseband modulation can be used on the transmitter laser by piezoelectric mirror motion.

For larger I.F. bands than 1kHz, sub-carrier modulation will be attempted. In this case, the 5kHz information band will be superimposed on a 50kHz carrier frequency. A 50kHz detector follows the discriminator and provides the information output. Initial beam synchronization is obtained by chopping the transmitter laser beam at a low rate (200 Hz) and by AM detection of the signal prior to the limiter discriminator.

The laser local oscillator is shifted 10 MHz away from the transmitter laser. This puts some constraint on the detector since the sensitivity of the HgCdTe detector at 10MHz is reduced by approximately one order of magnitude in comparison to the 1-MHz sensitivity. After I.F. amplification the signal level is limited to make it insensitive to amplitude fluctuations due to atmospheric disturbances. FM discrimination provides a signal to close the loop on the local
oscillator over a low-pass filter. Initial lock-on could be obtained by an automatic locking technique but it was decided for reasons of simplicity to vary manually the lock-on frequency until the AFC loop takes control. The bandwidth of the servoloop will be in excess of 2 kHz, the gain approximately 80 db. A tracking accuracy of 1 part in $10^{11}$ is technically feasible. Figure 1 shows a schematic diagram of the 10.6 micron infrared optical communication system.

The following sections discuss portions of the system in a more detailed manner.

FREQUENCY STABILIZED CO$_2$ LASER

Experimental CO$_2$ Laser

The development of stable CO$_2$ laser oscillators has continued under the present contract. Emphasis was placed on the following aspects of stable CO$_2$ lasers:

1) Improvement of thermal stability by enclosing the laser cavity as well as the discharge tube in a re-circulated water bath.

2) Increase the active discharge length at a given cavity length to increase the power output per cavity length unit.

3) Local oscillator output coupling on a low power level (10-50 mV) through a high reflectivity mirror.

4) Assessment of effects of mechanical vibration to the frequency stability.

Honeywell has continued to use Cer-Vit as a cavity material due to its low thermal expansion coefficient and good mechanical stability. The cavity length will be increased to 35 cm. The structure of the discharge tube has been simplified so that alternately two tubes of variable inside diameter
Figure 1. Two-Ended 10.6-Micron Optical Communications System
(8mm and 12 mm) can be inserted in the Cer-Vit cavity. Two experimental lasers were built before the design of the lasers for the communicator was frozen.

One experimental laser was built with points; 1) and 2) above considered in the design. Figure 2 shows the design of the CO₂ laser. The discharge tube and cavity structure were immersed in a water bath that was circulated by a commercial heat exchanger. This cooling concept removed the power dissipated in the discharge tube quite effectively and will be used in the lasers for the communicator presently under construction. In order to solve the high-voltage insulation problem and to obtain more gain per unit length, hollow internal electrodes were incorporated into this laser design. At equal cavity length the discharge length was increased in this way by 5 cm. From Figure 2 it can be seen that the laser beam goes through both anode and cathode. Power output was quite good at the beginning. An immediate lifetime test that was started revealed that the power as a function of time was reduced significantly. After approximately 100 hours the laser stopped oscillating. After disassembling inspection revealed that the coating of the Irtran mirror located behind the cathode was damaged. It is speculated that with concentric cylinder electrodes some ions can pass the cathode in the center and bombard the mirror. Since there are 0⁺-ions in the discharge they might contribute to the oxidation of the dielectric coating of the Irtran mirror. This electrode configuration therefore was abandoned in favor of electrodes in the tube side arms, as used in previous design configurations.

A second laser was built consisting of a cavity with two Irtran 2 mirrors. One mirror had a dielectric coating with 85 percent reflectivity, the other a dielectric coating with 99.5% reflectivity. The output of the laser was 5W through the low reflectivity front mirror and 50 to 100 mW through the high reflectivity mirror. The local oscillator power was large enough to fulfill all requirements for heterodyning.
An external mirror laser does not have any polarization properties. In order to obtain linear polarization a polarizer consisting of three wires was inserted. The electric vector of the radiation component oscillating parallel to the wire induces a current along the wire and therefore experiences a higher loss than the perpendicular electric vector. The outcoming radiation is therefore linearly polarized with the electric vector oscillating perpendicular to the wires. Experiments in the Honeywell laboratory have shown that insertion of three wires of 0.0001" thickness have linear polarized a CO₂-laser over 97 percent of the frequency spectrum.

**Vibrations in a Block Laser**

The use of Cer-Vit for the basic cavity structure of the linear laser provides high mechanical rigidity and high thermal stability. The degree of mechanical rigidity is determined mainly by:

- Modulus of elasticity $\gamma$ of the cavity material
- Size and shape of the cavity
- Magnitude and frequency of the vibrations

Perturbations in cavity dimensions due to vibrations manifests themself in frequency changes of the laser. For this reason, it is important to assess the effects of vibrations.

The proposed laser cavity configuration (which is basically a prismatic bar) can undergo length perturbations in various modes as the result of applied forces. However, the longitudinal length change $\Delta l$ is the principal factor in determining the laser frequency stability. Transverse bending affects the alignment slightly, and has no first-order coupling into length stability.
The frequency stability of the laser will, therefore, be examined from the point of view of the incremental length change due to the vibration environment. The frequency stability $\frac{\Delta f}{f}$ is identical to the length perturbation. In the discussion below $\frac{\Delta l}{l}$ is examined as a function of a sinusoidal disturbing force and frequency (i.e. the vibrations). In finding this effect, we apply directly the conditions of longitudinal vibrations of prismatic beams, of which the laser cavity is a special case.

If for example $\omega = 2\pi = 1$ Hz, $\frac{\Delta l}{l} = \frac{1}{10^9}$, then the corresponding rigid body motion which causes this perturbation is $\mu_0 = 1.14$ cm.

**Perturbations and Applied Force** -- The vibration response characteristics can also be normalized with respect to the amplitude of the applied force. Below the first resonance point,

$$\frac{\Delta l}{l} \text{ (per gram)} = 3.97 \times 10^{-11} \left( \frac{1}{\text{gr}} \right).$$

If $\frac{\Delta l}{l} = \frac{1}{10^9}$, then the magnitude of the disturbing sinusoidal vibration force is 25 grams. Below the first resonance frequency this effect is independent of frequency.

The mounting arrangement will have a higher transmission of transverse vibrations if the laser is attached parallel to a horizontal surface. The transverse vibrations will have negligible effect on the frequency stability as described above. The longitudinal vibrations do couple into frequency fluctuations, and the effect would have to be considered in designing the shock mount.

Vibration insulation from the environment is restricted from the aspect of high pointing accuracy of the transmitter and good stability of the local oscillator. Both lasers will be supported on rubber feet. The entire cavity will be hermetically sealed with a sound-absorbing material.
The laser shows natural resonance points where the length perturbations become larger, and are defined by the damping of the Cer-Vit material. The natural resonance frequencies are given by

\[ f_n = \frac{n}{2l} \left[ \frac{E}{\gamma} \right]^{1/2} \quad n = 1, 2, 3 \cdots \]

where

- \( f_n \) = resonance frequency, Hz
- \( l \) = length of laser = 35 cm
- \( E \) = modulus of elasticity = \( 1.3 \times 10^{12} \) g cm\(^{-1}\) sec\(^{-2}\)
- \( \gamma \) = density of Cer-vit = 2.5 g/cm\(^3\)

The first resonance point \( (n = 1) \) occurs at 10.3 kHz which is sufficiently high for our application.

**Perturbations and Rigid Body Motion** -- If a sinusoidal force is applied in a longitudinal direction to one end of the laser, the resultant length modulation \( \frac{\Delta l}{l} \) determines the frequency deviation of the laser, \( \frac{\Delta f}{f} \). It is instructive to normalize this perturbation with respect to the rigid body motion \( \mu_o \). Below the first resonance, the effect is given by

\[ \left| \frac{\Delta l}{l} \right| \frac{1}{\mu_o} = 2.22 \times 10^{-11} \omega^2 \]

where

- \( \mu_o \) = rigid body motion, cm
- \( l \) = 35 cm
- \( \omega \) = angular frequency of vibrations, rad/sec

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Final CO\textsubscript{2} Laser Design

The final design of the CO\textsubscript{2} lasers for the communicator is shown in Figure 3. The wall thickness of the basic Cer-Vit yoke has been reinforced in order to increase the rigidity of the cavity structure. The discharge tube has been redesigned such that the O-ring seals are now on the outside of the tube. The yoke is bounded on the upper and lower sides by two plates to form a reservoir which will be filled with recirculated water from a commercial temperature controller. The cooling will provide better long-term stability and also enhance output power. With a temperature control of 0.1°C a long-term stability of 1 part in 10\textsuperscript{9} is obtained. The increase in power output due to cooling in comparison with the free-running case should be approximately 30 percent. Each cavity mirror is attached to a piezoelectric transducer (discussed in more detail in the Piezoelectric Frequency Control Subsection). An aperture is inserted in the Cer-Vit yoke for mode selection which carries also the three 0.0001 inch wires for linear polarization. O-ring seals are provided for the mirror so that no epoxy is being used to obtain a vacuum seal. Both ends of the Cer-Vit yoke are polished flat and parallel to 10 seconds of arc minimum.

In an initial test experiment, laser performance will be tested with 8 mm and 12 mm tubes and output power will be maximized in single transverse mode and single frequency by insertion of an aperture. The influence of flow rate of the water cooling unit will next be assessed. The flow rate will have to be kept at a point where the cooling is sufficient and no vibrations due to turbulence are introduced.

SIGNAL AND NOISE CONSIDERATIONS:

The laser will operate in a single mode with a total output power of at least 2 watts. The output is a 5 mm beam with a beam spread of 3 \times 10^{-3} radians. Assuming a recollimating set of optics the minimum beam diameter \( d \) at a range \( R \) will be defined by the diffraction limit of the system.

12044-PR3
\[ D = \frac{1.22 \, R \lambda}{D} \text{ cm} \]  

Where

\( R \) = range (cm)  
\( \lambda \) = wavelength (cm)  
\( D \) = diameter of collimating optics (cm)

The minimum area covered by the beam at a range \( R \) is therefore

\[ A = \frac{\pi}{4} \left( \frac{1.22 \, R \lambda}{D} \right)^2 \]  

Where

\( A \) = area of the beam (cm²)

For our case where \( R = 3000 \) meters and \( \lambda = 10 \) microns we have

\[ A = \frac{1.05 \times 10^4}{D^2} \text{ (cm}^2) \]  

The irradiance at the receiver optics will be

\[ H = \frac{P}{A \, T_A \, T_o} \left( \frac{\text{Watt}}{\text{cm}^2} \right) \]  

Where

\( P \) = incident power (Watt)  
\( A \) = area of the beam (cm²)  
\( T_A \) = transmissivity of the atmosphere  
\( T_o \) = transmissivity of the optics
Assuming an atmosphere transmissivity of 50 percent (64 mm precipitable H$_2$O at 10.6 microns, 100 percent relative humidity at 72°F over a range of 3000 meters), an optical transmissivity of 50 percent (four reflecting surfaces 84 percent reflectivity each), and an output power of 2 W we have

\[ H = 4.8 \times 10^{-5} D^2 \text{ (watts/cm}^2) \]

This means that with no optics at all, i.e., \( D = 0.5 \) cm we could achieve an irradiance of \( 1.2 \times 10^{-5} \) watts/cm$^2$.

The receiver section will consist of an off axis optical system, a HgCdTe detector supplied by the Radiation Center in Boston, and a CO$_2$ laser local oscillator. The detector supplied will have a sensitive area of approximately 0.025 cm by 0.025 cm and a detectivity, \( D^* \), of approximately $10^{10}$ cm/W sec$^{1/2}$.

The noise equivalent power of the detector can be calculated from

\[ \text{NEP} = \sqrt{\frac{A_D \Delta f}{D^*}} \]  

(5)

Where

\begin{align*}
\text{NEP} & = \text{noise equivalent power (watts)} \\
A_D & = \text{area of the detector (cm}^2) \\
\Delta f & = \text{electrical bandwidth (cps)} \\
D^* & = \text{detectivity cm/W sec}^{1/2}
\end{align*}

In this application with a detectivity $10^{10}$ cm/W sec$^{1/2}$, an area of $(0.025)^2$ cm$^2$ and a bandpass of $10^7$ cps we have

\[ \text{NEP} = \sqrt{(0.025)^2 \times 10^7} = 7.9 \times 10^{-9} \text{ (watts)} \]

(6)
This means that to achieve an $S/N$ of 1 we must collect

$$\frac{\text{NEP}_{\text{system}}}{H} \text{ cm}^2 \text{ of radiation}$$

$$\frac{7.9 \times 10^{-9}}{1.2 \times 10^{-5}} \text{ cm} = 6.6 \times 10^{-4} \text{ cm}^2$$

This means that an aperture of 0.026 cm would be necessary to achieve detection. Since our detector has an 0.025 cm aperture the receiver could also function without optics. By increasing the laser power to 5 W and eliminating the 50 percent optical losses (no optics) for the transmitter and receiver we would achieve an $S/N$ of approximately 8 which should be sufficient for detection.

If a 4 inch optical system were used for both the transmitter and receiver we could achieve a beam diameter of 0.4 meters at a range of 3000 meters. This would mean an irradiance of $4 \times 10^{-4}$ W/cm$^2$. A 4 inch collector would collect approximately 80 cm$^2$ or $3.2 \times 10^{-2}$ W of radiation. Assuming a 50 percent optical transmissivity for the receiver we would have $1.6 \times 10^{-2}$ W incident on the detector. This would result in an $S/N$ out of the detector of

$$\frac{1.6 \times 10^{-2}}{7.9 \times 10^{-9}} = 2 \times 10^6$$

As this type signal may well exceed the dynamic range of the detector, in this case attenuators would be required.

A beam diameter of 0.4 meters is too small for a number of reasons. It would place an unreasonable constraint on sighting and aligning the transmitter and receiver. The beam diameter would also be small with respect to expected image motion due to atmospheric effects. Defocusing of the optics would provide a beam diameter of at least 3 meters. A 3-meter beam would
produce an irradiance of \(7 \times 10^{-5}\) W/cm\(^2\). With a 4-inch primary in the receiver (80 cm\(^2\)) we can collect \(5 \times 10^{-3}\) W. With a 50 percent optical transmissivity and a detector NEP of \(7.9 \times 10^{-9}\) W we would achieve an \(S/N\) of \(3.2 \times 10^5\). Even in this situation it would be desirable to introduce attenuation to bring the signal-to-noise ratio down to approximately \(10^3\).

In order to determine the lowest detectable signal level we must consider sensitivity, frequency response and noise of a photoconductor operated as an optical heterodyne receiver. For an ideal amplifier the noise power spectral density is given by:

\[
\psi(v) = \frac{h\nu}{e^{(h\nu/kT)} - 1} + h\nu. \tag{7}
\]

The first term arises from thermal noise of the source being at the temperature \(T\). The second term arises from spontaneous emission noise in the amplifying medium. If an amplifier with the passband \(B\) (cps) centered at \(\nu >> \frac{kT}{h}\) follows the detector the output SNR is given by:

\[
\frac{S}{N} = \frac{\eta P_s}{h\nu B} \tag{8}
\]

where

\[
P_s = \text{input power to the detector}
\]

\[
\eta = \text{quantum efficiency}
\]

As long as \(h\nu >> kT\) the signal-to-noise ratio of a photomixer can be represented by Equation (8). If we assume a quantum efficiency of 1 we can state that the photomixer can detect a signal of 1 photon/sec in a one-cycle band pass. The heterodyne detection scheme is therefore photon noise limited.
The maximum noise equivalent power for a 1 Hz bandwidth and a quantum efficiency of $\eta = 1/2$ is:

$$P_{\text{NEP}} = \frac{\hbar B}{\eta}$$

$$= 2 \times 6.6 \times 10^{-27} \times 2.7 \times 10^{13} \text{erg/sec} \text{ and } 1 \text{erg/sec} = 10^{-7} \text{W}$$

$$= 3.6 \times 10^{-20} \text{W}$$

For a 10 MHz bandwidth, the noise equivalent power is:

$$P_{\text{NEP}} = 3.6 \times 10^{-13} \text{W}$$

Minimum detectable powers for the heterodyne detection scheme have been measured at 10.6 inches by Teich et al. (12) with a Ge:Au detector and by Buczek et al. (13) for a Ge:Hg detector. Both authors come within a factor of 10 of the photon noise limit.

Thermal or Johnson noise would have to be considered if $\hbar \nu \approx kT$. For optical and infrared frequencies we have calculated the ratio $\frac{\hbar \nu}{kT}$ for different detector temperatures and wavelength of interest and listed below.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>293°K</th>
<th>77°K</th>
<th>4°K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>632 8A</td>
<td>10.6 µ</td>
<td>10.6 µ</td>
</tr>
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</table>
It can be seen that the ratio of $\frac{h\nu}{kT}$ is sufficiently large to assume that the photon noise contribution will be the dominant noise contribution in the heterodyne detector.

The high frequency noise equivalent circuit can be represented as

The local oscillator generates a current level $I_{dc}$ which is large in comparison to the diode dark current. The barrier capacitance is $C_B$ while the conductance for high frequency considerations can be neglected. $I(r_s)$ is the thermal noise current associated with the series resistance. For unit bandwidth we have

$$I^2(r_s) = \frac{4kT}{r_s}$$  \hspace{1cm} (10)
The SNR is then given by:

\[ \frac{S}{N} = \frac{nP_s}{h \nu B \cdot \left(1 + \frac{I^2(r_s)}{2qI_{dc}} \frac{r_s^2 C_B^2 \omega_H^2}{2qI_{dc}} \right)} \]  

\[ (11) \]

Where

- \( \omega_H \) = Heterodyne beat frequency
- \( I_{dc} \) = D-c current due to local oscillator
- \( q \) = Electron charge = \( 1.6 \times 10^{-19} \) amp sec

If the conditions

\[ 2qI_{dc} > I^2(r_s) \frac{r_s^2 C_B^2 \omega_H^2}{2qI_{dc}} \]  

\[ (12) \]

is fulfilled the noise equivalent power can be calculated from Equation (9).

For the considered heterodyne detector we adjust the local oscillator power to 1 mW. At a given responsivity of the HgCdTe detector of approximately 500 V/W and an impedance of 328 \( \Omega \) we obtain for the D-c current arising from the L. O. signal.

\[ I_{dc} = 1.5 \times 10^{-3} \] A

At a 10 MHz bandwidth (\( \omega_H = 6.28 \times 10^7 \) Hz; \( C_B = 2.00 \times 10^{-12} \) Farad) the shot noise term in Equation (12) is 3 orders of magnitude smaller than the D-c term generated by the local oscillator.
If a preamplifier follows the HgCdTe detector the amplifier noise is thermal and the current noise per unit bandwidth can be represented by the equation:

$$I_A^2 (T_A) = \frac{4kT_A}{R}$$  \hspace{1cm} (13)$$

Again we have to postulate for optimum performance that the noise term arising from the preamplifier is small compared to the d-c current from the local oscillator. For the selected preamplifier (Philco video amplifier PA 713) the current noise is 2 orders of magnitude smaller than the noise from the local oscillator, operated under the above described conditions.

With the detector operated under quantum noise limitation and a 10 MHz bandwidth, the smallest heterodyne beat frequency signal that would appear at the input of the preamplifier is

$$U = \sqrt{U_1 U_2} = \sqrt{0.5 \times 3.6 \times 10^{-3} \times 500} \text{ (volt)}$$

$$\approx 10 \, \mu\text{V}$$

under these conditions and with the 4 inch optical system discussed before, communication over a distance of $3.5 \times 10^8$ m is feasible.

OPTICAL SYSTEM

Several optical systems which could be used for the communicator were investigated. These are on-axis Cassegrain, an off-axis concentric spherical system, and an off-axis Cassegrain. The on-axis Cassegrain would be ideal except that a small amount of radiation is reflected back
from the secondary mirror into the laser. Depending on vibrations of the secondary mirror, this back-coupled energy has a variable phase and frequency and increases the frequency spectrum of the stable oscillator. For this reason the on-axis Cassegrain was discarded. An aperture size of 4 inches was selected for optical calculations. The first system analyzed was an off-axis spherical system with a 36 inch focal length and a 4 inch diameter. The theoretical blur circle at 10µ would be approximately $2 \times 10^{-4}$ radians. Since the system is an off-axis system coma and astigmatism are introduced. For a 6 degree off-axis system we have:

- Spherical aberration: $2 \times 10^{-4}$ rad
- Sagital coma: $8 \times 10^{-4}$ rad
- Astigmatism: $18 \times 10^{-4}$ rad

Total aberration = $2.8 \times 10^{-3}$ radians

Since $2.8 \times 10^{-3}$ radians is almost identical to the beam spread of the laser itself, spherical optics would serve no useful purpose.

An off-axis Cassegrain system can be built and operated at 10µ approaching the Rayleigh limit for resolution. In order to achieve this, the primary and secondary must be surfaced to $2\lambda$ at 0.55 micron. An off-axis Cassegrain with the following mirror configuration seems to be most feasible.

- **Primary Mirror:**
  - 4" diameter parabolic surface
  - 36" focal length
  - optical axis 4" off center of primary

- **Secondary Mirror:**
  - 1" diameter hyperbolic surface
  - 3.2" focal length
  - secondary on the optical axis

12044-PR3
Figure 4 illustrates the theoretical beam spread given by optical systems of various size. It should be noted that by defocusing the optical system an increase in beam size can be achieved.

Configuration

Several basic configurations have been investigated. The configurations studied have been a modification of a rigid H frame mount. Figure 5 shows the selected approach. This configuration will provide the necessary structural rigidity and simplicity.

Detector Mount and Cooling

The detector will be mounted in an aluminum holder in an upright position with a mountable aperture in front of the flake. A two-dimensional micrometer positioner will provide xy-positioning. The HgCdTe detector must be cooled to 77°C for proper operation. This temperature can be achieved in a number of ways:

1. Liquid nitrogen can be poured into the detector dewar as needed
2. A direct-transfer liquid nitrogen system can be utilized
3. A Joule-Thompson cryostat can be used with a supply of high pressure nitrogen gas.

Each of the above methods has its advantages and disadvantages. The manual supplying of liquid nitrogen is undoubtedly the simplest cooling method and is also the least expensive. However, the user must rely on a supply of liquid nitrogen. In addition there is no protection from water condensation on the inside of the dewar. This condensation, if sufficient, will crack the dewar and destroy the detector.
Figure 4. Beam Diameter at 3000 Meters
The direct-transfer method which costs roughly $350, will alleviate the problem of continual filling. It will also permit the detector to be effectively sealed from the atmosphere to prevent water condensation. However, the user is still required to maintain a supply of liquid nitrogen.

The Joule-Thomson cryostat offers all the advantages of the direct transfer system with substantially less cost ($195). It also alleviates the problem of maintaining a supply of liquid nitrogen. The only disadvantage of such a system is the cool-down time, several minutes as opposed to several seconds for the liquid nitrogen systems.

After considering all parameters it was decided to use a Joule-Thompson cryostat, although any of the three listed systems would be adequate.

One factor which should be noted in cooling the detector is that in the event cooling is interrupted the detector bias MUST be removed before the sensitive element reaches ambient temperature. If the bias is maintained even with a limiting resistor the detector will burn out. In order to circumvent this a temperature sensor will be placed inside the detector dewar to shut off the bias current when the detector temperature reaches 90°K. A Joule-Thomson cooler and a temperature sensing unit will be purchased from Santa Barbara Research Center.

Sighting

In order to successfully align the transmitter and receiver over a range of 3000 meters a fairly accurate set of sighting equipment must be utilized. It will be necessary to point both the transmitter and receiver to an accuracy of 1 meter at 3000 meters, i.e., 1 min of angle. Since 1 min of angle is the basic resolution of the human eye it will be necessary to utilize auxiliary sighting equipment. Target rifle telescopes are commercially available and would be well suited for the above sighting problem.
Use of a target sight with external adjustments is recommended as opposed to a hunting sight with internal adjustments. Two possible choices would be:

Layman 15X Super Target Spot

Unertel 15X 2" Target Scope

These instruments cost between $100 and $125 each with adjustable external mounts. The mounts provide accurate adjustment to 0.1 min of angle, or approximately 4 inches at 3000 meters.

PIEZOELECTRIC FREQUENCY CONTROL AND MODULATION

The frequency of oscillation of the laser can be controlled by mounting a cavity mirror to a piezoelectric transducer and applying a voltage to the transducer. For a double-ended communication system using two CO₂ lasers in connection with a heterodyne receiver the piezoelectric control has to fulfill the following requirements:

- **Transition Control:** Both lasers must be controlled by a dc bias over a large portion of the frequency spectrum in order to obtain oscillation on one and the same transition for both lasers. Ideally the frequency range c/2t should be scanned.

- **Frequency Modulation:** The transmitter laser must be modulated at a higher frequency in order to provide a sub-carrier for the voice channel. A flat frequency response in excess of 50 kHz is required with a modulation index in excess of 20.

- **Automatic Frequency Control:** The local oscillator laser is frequency locked with respect to the transmitter laser with a time constant given by the frequency response of the AFC-loop. If a tracking accuracy of 1 part in $10^{10}$ is to be obtained, the
AFC-loop must respond at 300 Hz with a time constant of $\tau = 5 \times 10^{-4}$ sec. In addition the transducer must have sufficient tracking range so that the transmitter oscillator can be tracked over a sufficient portion of the doppler profile in order to release stringent requirements on the absolute temperature control of each laser. A tracking range of 10 MHz seems adequate which would allow a temperature variation of the transmitter by 4°C.

All three requirements cannot, of course, be fulfilled with a single piezoelectric element. Construction of a piezoelectric element for each of the above listed functions was decided upon. The basic equation that correlates frequency with length changes is given by:

$$\Delta f = \frac{M \cdot c}{2nL^2} \Delta L$$  \hspace{1cm} (14)$$

For $L = 35$ cm and $M = \frac{2L}{\lambda} = 6.6 \times 10^4$ we obtain

$$\Delta f = 8.05 \times 10^{11} \Delta L$$

Transition control will be accomplished with a piezoelectric cylinder (12.5 mm long) made of lead zirconate/lead titanate (C-type material) with a piezoelectric sensitivity of $4.5 \times 10^{-8}$ cm/V and a total range of 1.4µ (with 2,500V applied) which is equivalent to 110MHz. This range is sufficient to bring three to five transitions into oscillation. The Irtran output coupler is attached to the piezoelectric element and the laser beam is coupled through the piezoelectric cylinder.

Frequency modulation of the transmitter laser will be accomplished with a piezoelectric slab (0.75 mm thickness) of lead zirconate/lead titanate (K-type material) with a piezoelectric sensitivity of $2.5 \times 10^{-8}$ cm/V and a frequency constant for the thickness mode of 175kHz/cm. This element has a cut-off frequency of 2.3MHz. The slab will be ground flat and an
attempt will be made to vacuum deposit a thin layer of gold. With a breakdown potential of 3000 V/cm the maximum applied voltage is 220V and the corresponding frequency tuning range is 4.5MHz. To stay a factor of 2 away from the breakdown potential we still obtain a modulation index ≥40.

The AFC loop will be closed on a piezoelectric stack transducer made of K-type material. This approach was chosen in order to avoid high voltage amplification before closing the AFC loop on the PZT transducer. In this way a Philbrick K2W amplifier with a ±50V output will drive a 5-stack element with a total frequency tuning range of 10MHz. Although the stack has a reduced frequency response in comparison to the single element it will respond flat in excess of 10kHz.

Figure 6 shows the design concept for the piezoelectric elements. Each laser will have a transition control element on one mirror, the transmitter laser a modulator element, and the local oscillator an AFC element on the other mirror. This concept avoids grounding problems that would exist if two functions were performed on one element. Tandem elements have been avoided in order to prevent mechanical vibration coupling from one element into the other element. By using both cavity mirrors for the two elements decoupling over the rigid cavity structure is obtained.

In order to test a piezoelectric element up to several MHz, an experimental high voltage driver has been built which could also be used to drive a gallium arsenide modulator for amplitude modulation.

The information available pertaining to GA-AS modulators disclosed that a typical crystal measures 14 pF and requires about 600 V peak driving signal for good modulation depth. To the 14 pF of the modulator a few pF must be added for the driving source and leads. A total of 22 pF is a realistic figure. For transmitting a high quality television picture a
Figure 6. Piezoelectric Transducer for Laser Frequency Control
bandwidth of 30 Hz to 4 MHz is desirable. As the reactance of 22 pF at 4 MHz is 1800 ohms, instantaneous power at the 600V peak calculates out to be 180W. Fortunately the average power required is considerably lower and some of the modulator capacity reactance can be tuned out by use of peaking coil techniques.

The circuit starts out with a transistor preamp included to build up the signal generator output sufficiently to drive the rest of the circuit to full output. The 6CL6 driver stage is conventional except for the peaking coil used to tune out some of the capacity reactance of the output stage grid. The output stage uses a 4X250D, a tetrode designed for uhf service. Of importance is that the 4X250D plate to cathode capacitance is relatively low for a tube of its power capability. A negative feedback loop from the outer plate to driver cathode is incorporated to reduce the midband gain. Figure 7 shows the circuit diagram.

Figure 8 shows the frequency response of the system. The low frequency fall off can be corrected by increasing the size of the coupling capacitors. The peculiar dip at 1 MHz is probably caused by a resonance condition in one or more of the foil-wound capacitors.

DETECTOR CONSIDERATIONS AND FREQUENCY RESPONSE

For application in the infrared communication system, the photo-detector must satisfy the following specific performance requirements: adequate frequency response, sufficient spectral sensitivity, and simplicity of operation.

For photomixing the detector must respond to the 10.6-micron radiation in order to extract the modulation-induced frequency shifts. Detectors operating beyond 5 to 7 microns usually are of the intrinsic type requiring
Figure 7. Experimental Driver for GA-AS Modulator
Figure 8. Frequency Response of Experimental Driver for GA-AS Modulator

INPUT SIGNAL = 0.38V RMS

OUTPUT MOTS - PEAK TO PEAK

12044-PR3
very precise control over the doping concentration. Such detectors, of the doped-germanium type (such as Ge:Au, Ge:Cu and Ge:Hg) must be operated at liquid helium temperatures (4.2°K) using a double dewar and have spectral response extending to 30 microns.

Another detector type, which requires only cooling to liquid nitrogen temperature (77°K) in a single dewar, can be used in the 1 to 14-micron spectral range. This intrinsic-type detector is the mercury-cadmium-telluride type made from mixed crystals of mercury telluride and cadmium telluride.

Pure HgTe has a long wavelength response limit of 40 microns, while pure CdTe has a limit of 0.8 micron. A crystal composed of 90 percent HgTe and 10 percent CdTe has a 13-micron wavelength limit. These detectors, manufactured at the Honeywell Radiation Center Boston, have a typical spectral response peak in the 8 to 10-micron region.

The detector is, in principle, operable in any of three modes: Photoconductive, photovoltaic, and photoelectromagnetic. For high sensitivity, the photovoltaic and photoconductive modes are the more desirable.

When operating in the photoconductive mode, a small photovoltaic effect is seen superimposed on the response; the response differs slightly depending on the direction of the bias current. Typical $D^*$ values of this detector are $1 \times 10^{10}$, compared with $2 \times 10^{10}$ for the copper-doped germanium type. However, $D^*$ does not carry the usual significance as a figure of merit because signal photon fluctuations may tend to limit the detector application independently of the photon noise from 290°K (earth) background. In the case of signal fluctuations, the minimum detectable power varies linearly with bandwidth, and is independent of detector area (as shown in the Signal...
and Noise Subsection). This behavior does not allow the usual interpretation of $D^*$ which presumes that the minimum detectable power is proportional to the square root of the detector area and the square root of the bandwidth.

The minimum detectable monochromatic power for photon detectors limited by photon noise from a 290°K background decreases rapidly from the visible into the infrared spectrum. Farther into the infrared region, detectors may be limited by signal fluctuations. In addition, operating with large bandwidths tends to cause signal fluctuations to dominate over photon noise from a 290°K background.

Two types of noise are found in the HgCdTe detectors. In the unbiased condition and for only small values of bias, only thermal (Johnson) noise is seen, of a magnitude commensurate with the detector resistance and operating temperature. With increasing bias, current noise with an approximate $1/f$ law dependence becomes dominant. At high frequencies (usually above 1 kHz) the current noise is lost in the frequency independent thermal noise. For wideband operation in the communication system, the greater sensitivity of biased operation is preferred, because the thermal noise mechanism is expected to dominate at these bandwidths.

Apart from the spectral response characteristics, the detector frequency response must encompass the 10MHz offset frequency between transmitter laser and local oscillator. Direct frequency response measurements at 10.6 micron were made by heterodyning two CO$_2$ lasers on the surface of a HgCdTe detector. The beat frequency could be adjusted between 0 and 35 Mhz. Figure 9 shows the relative amplitude of the beat signal as a function of frequency. The signal reduction at 35 MHz amounts to approximately 20 dB. The frequency response can be further improved by reducing coupling and lead capacitance (shorter leads) of the detection system.
Based on the foregoing considerations of spectral response, sensitivity, frequency response and convenience in cooling to only liquid nitrogen temperatures, the Honeywell HgCdTe detector is a good choice for the optical heterodyning detector in the 10.6 micron laser communication system.

AUTOMATIC FREQUENCY CONTROL LOOP (AFC)

Since stable CO₂ oscillators with a high spectral purity are available, their use in a two-ended communication system using coherent detection is feasible. The local oscillator must be able to track the incoming transmitter frequency with a high accuracy so that the low-frequency fluctuations of the transmitter laser are compensated. The frequency fluctuations that remain after the AFC loop has been closed must be small in comparison to the information bandwidth. The following requirements must be imposed on the AFC loop:

- Tracking accuracy and bandwidths -- The tracking accuracy must be a small portion of the information bandwidth. The ultimate tracking accuracy is determined by the relative frequency stability of the two lasers over time intervals \( \tau \) given by the inverse frequency response of the AFC loop. As an example consider the frequency fluctuations of the two free-running lasers delivered to NASA on Contract NAS8-18624. The relative frequency fluctuations of the lasers as a function of time is given by the equation (see Figure 10)

\[
\Delta \nu(t) = 10^4 \sqrt{t} \text{ (Hz)}
\] (15)
An improvement in tracking accuracy can be obtained until the frequency response of the feedback loop $\Delta \nu_{AFC} = \frac{1}{t}$ is equal to the relative frequency fluctuations of the lasers. This occurs for $\Delta \nu_{AFC} = \frac{3}{10^8} \text{ Hz} \approx 460 \text{ Hz}$. For higher frequencies than $\Delta \nu_{AFC} = 460 \text{ Hz}$, the local oscillator would track the fluctuations of the transmitter laser. In addition the AFC bandwidth becomes a substantial fraction of the information bandwidth. This AFC frequency response in connection with a 1-second stability of 10 kHz yields a tracking accuracy of close to 1 part in $10^{11}$. Any further improvement in tracking accuracy can only be obtained by a better relative short-term stability of both lasers.

Loop gain and tracking range -- The gain of the I.F. -amplifier-limiter was laid out such that the smallest expected signal would be amplified to the driving level of the discriminator. The smallest voltage (without optics) at the output of the detector is $3.3 \mu\text{V}$. An amplifier was built for signal levels down to $5 \mu\text{V}$. The amplifier is tunable from 8.5 to 10.5 MHz. When sharply tuned for a 1.5 MHz bandwidth its gain drops to 5000. Unexpectedly, the amplifier exhibits excellent limiting qualities when the input signal exceeds 500 microvolts. The output remains constant as the input signal is varied from 500 microvolts to 0.8 volt. When working into an inductive load the output waveform remains sinusoidal without indication of saturation or clipping.

**Discriminator**

The ratio detector type of discriminator was selected because it requires less driving signal than the conventional discriminator. It also has an inherent noise limiting ability and furnishes a carrier level voltage that is
useful for AGC if required. The ratio detector transformer was especially constructed to provide wide bandwidth by closely coupling the coils and using high L-to-C ratios. It also has a tapped primary for impedance matching to the amplifier. When used by itself, the discriminator has a positive peak to negative peak bandwidth of 2.5 MHz.

**AFC Feedback Amplifier**

A Philbrick KW-2 operational amplifier with a gain of 100 follows the discriminator and provides a total plus-to-minus voltage swing of 130 volts as the frequency is varied across the passband.

A frequency versus output voltage diagram is shown in Figure 11. Linearity is quite good between 9.7 and 10.7 MHz.

Figure 12 shows a circuit diagram of the experimental amplifier, discriminator and AFC amplifier. Figure 13 shows the circuit diagram of the 50 kHz subcarrier generator. In addition to these components the following units have been purchased: 1) 1 commercial RHG linear I.F. amplifier with a 10 MHz center frequency, 2 MHz bandwidth and 80 dB gain to I.F.; 2) 1 RHG wideband limiter discriminator for 10 MHz center frequency and a 1 V/MHz minimum video output. Performance tests of the total system will decide whether the experimental or the commercial AFC components will be used.

The tracking range is determined by the voltage swing of the feedback amplifier and the piezoelectric sensitivity of the 5-stack transducer. With a voltage swing of ±65V and a piezoelectric sensitivity of the 5-stack element of $1.25 \times 10^{-7}$ cm/V the total tracking range in frequency space is 13 MHz.
Figure 11. Voltage Output versus Frequency I.F. Amplifier Discriminator AFC Amplifier

INPUT SIGNAL = 500μV
Figure 12. Experimental Amplifier, Discriminator and AFC Amplifier
Figure 13. 50 kHz Subcarrier Generator
Since the two lasers are set off by 10 MHz both lasers have to stay in single-frequency oscillation over a total frequency range of 23 MHz. This range represents pretty much the upper limit since oscillation on one frequency is on the average obtained over 25 MHz only. Figure 14 shows the conditions for the tracking range of the two lasers. The tracking range can be increased, however, by utilizing pressure dependent shifts in the frequency of oscillation of the CO$_2$ laser. These frequency shifts have been reported on earlier in this work and amount to 6.5 MHz/Torr. Shifting the local oscillator by 2 Torr and proper placement of the cavity mode would increase the tracking range by a factor of 2.

SYSTEM CAPABILITIES

The system as described in this report will have several other uses in addition to the basic one of providing voice communication over a two-mile range. Discussed below are two possibilities which invite further research.

- Determination of the signal-to-noise ratio and minimum detectable power level for heterodyne detection using a mercury-cadmium-telluride detector. While the frequency response of the HgCdTe detector has been determined it has not yet been determined how closely the HgCdTe detector approaches the theoretically perfect photon counter.

- The system makes it possible to determine the influence the atmosphere has on the heterodyne beat spectrum. Amplitude and frequency fluctuations should be readily determined under various atmospheric conditions. Influence of turbulence effects as well as humidity could be investigated. With minor modifications the system could be converted into a homodyne detection system. In this way only the receiver setup would be used and the outgoing
Figure 14. Conditions for the Tracking Range of Two Lasers
beam homodyned after retro-reflection over a certain atmospheric path and combined with the local oscillator power from the same laser. Refined phase noise measurements could be made which would make it possible to determine the amount of destruction of the coherence of the laser beam.
SECTION III
REFERENCES


Detectors and CO$_2$ Laser Stability


PASSIVE FREQUENCY STABILIZATION OF A
CARBON DIOXIDE LASER*

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Passive optical cavities made from fused quartz have been used as reference cavities in laser frequency control systems (1,2). Recently a low expansion coefficient material named Cer-Vit* became available and was used by White (3) as a frequency discriminator. In order to design a CO₂ laser with a good short and long-term frequency stability we have constructed the laser cavity itself from the material Cer-Vit which had an expansion coefficient of +1.0 x 10⁻⁷/°C. Measurements of the short-term and long-term frequency stability are reported in this communication.

The laser cavity is made in the shape of a yoke with a length of 30 cm. A plasma tube is inserted into the Cer-Vit yoke and sealed by two chucks. By polishing the outside end faces of the yoke flat and parallel and attaching two mirrors to these faces of the yoke, an internal mirror system is obtained which is mechanically rigid and stable. Figure A1 shows a photograph of the passive stabilized CO₂ laser.

Two aluminum heat shields prevent the cavity from being heated by the discharge tube. The discharge tube (8 mm diameter) has one anode at each end and a center cathode. The laser was of the flowing gas configuration with an inlet at each anode and a common outlet at the cathode. One curved mirror is made of Irtran 2 and the other mirror is flat with a gold coating and attached to a piezoelectric transducer having a sensitivity of 28KHz/Volt. Oscillation in a single transverse mode (TEM₀₀) is obtained by insertion of an aperture. The laser oscillates on a power level between 3 and 7 Watts with an excitation ranging from 5 to 15 mA.

* Owens-Illinois trade name.
Figure A1. Passively Stabilized Carbon Dioxide Laser
Two lasers were built and tuned in cavity length with the piezoelectric element to oscillate at the same transition of the P-branch of the 00\(^0\)1 - 10\(^0\)0 rotation vibration band of CO\(_2\).

The relative short-term (1/10 sec) and long-term (several hours) frequency stability was measured by heterodyning the two lasers on a liquid nitrogen cooled, mercury-cadmium-telluride detector (4). The beat spectrum was investigated with a Panoramic spectrum analyzer and an oscilloscope.

The short-term stability was obtained by adjusting the sweep rate of the spectrum analyzer to 60 (sec\(^{-1}\)) and by photographing the display screen with an exposure between 1/10 sec up to 1 minute. The halfwidth in frequency at the half-power point yields the frequency stability over a one second time interval. Fig A2(a) shows the beat spectrum on the spectrum analyzer. For example a 10 kHz/sec stability has been obtained for a 1 second observation time. Fig A2(b) shows a one second exposure of a 1MHz beat signal on the oscilloscope operated in the internal trigger mode. This figure represents a time exposure having an overlay of 2 x 10\(^5\) successive sweeps on the oscilloscope screen. The sharp outline of the total trace is indicative of the very small frequency drift in the one second observation time.

Long term frequency stability measurements for time intervals larger than 1 minute were made by observing the maximum frequency drift on the spectrum analyzer during the corresponding time interval. Fig A3 shows the results of the relative frequency stability \(S(t) = \frac{\nu_L}{\Delta \nu(t)}\) and the frequency fluctuations \(\Delta \nu(t)\) for the two free running CO\(_2\) laser oscillators as a function of the time of observation ranging from 10\(^{-1}\) to 10\(^{+4}\) seconds. The functional dependence of the two parameters can be expressed by the equation:

\[
\Delta \nu(t) = 10^4 t^{\frac{3}{8}} \text{ (Hz)}
\]
Figure A2a. Beat Frequency of Two Free-Running Passive Stabilized CO₂ Lasers Horizontal Dispersion 10kHz/cm, Sweep Rate 60 sec⁻¹, Exposure Time 1 Second.

Figure A2b. Oscilloscope Trace of Beat Frequency of Approximately 1 MHz of Two Lasers. Internal trigger operation, 0.5 x 10⁻⁷ Second/cm Horizontal Sweep, 1 second Exposure.
Figure A3. Relative Frequency Stability \( S(t) = \frac{\nu_L}{\Delta \nu(t)} \) and Frequency Fluctuations \( \Delta \nu(t) \) as a Function of Observation Time for Two Free Running Passive Stabilized CO\(_2\) Lasers.
The results obtained indicate that in the free running case a short term stability \((\frac{1}{10} \text{ sec})\) of 1 part in \(10^{10}\) and a long term stability (20 minutes) of 1 part in \(10^8\) has been achieved with only passive stabilization techniques. The long term passive frequency stability approaching 1 part in \(10^9\) can be obtained by controlling the cavity temperature to 0.01 C with a commercially available temperature controller.
LITERATURE

1) A. D. White, J. Quantum Electronics, QE-1, 369(1965).
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