Volume II
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Final Report Under Contract NAS 8-4012

Design Criteria For Zero Leakage Connectors
For Launch Vehicles

MATHEMATICAL MODEL
OF INTERFACE SEALING PHENOMENON

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FOREWORD

This is Volume II of a four volume final report covering work accomplished by the Research and Development Center of the General Electric Company, Schenectady, New York from July 1965 to September 1967. This program was sponsored by the Missile and Space Division of the General Electric Company, Philadelphia, Pennsylvania, under National Aeronautics and Space Administration Contract NAS 8-4012 "Design Criteria for Zero Leakage Connectors for Launch Vehicles."

The General Electric technical director was J. A. Bain who replaced F. O. Rathbun, Jr. Mr. C. C. Wood was NASA technical manager.

The four volumes contained in this final report are:

Volume I -- "Computer Programs for Flanged and Threaded Connector Design"

Volume II -- "Mathematical Model of the Interface Sealing Phenomenon"

Volume III -- "Advanced Leakage Tests"

Volume IV -- "Tube Connector with Superfinished Seal"
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Section 1

SUMMARY

During this contract period, a procedure for the calculation of leakage flow through the seal interface of a fluid connector was developed and evaluated.

This is the initial step in an ultimate development of a sufficiently accurate and economical calculation procedure that will qualify as a design tool. The current procedure is not accurate enough for design purposes, but the experimental evaluation did show that the development of a design tool is a realistic goal. If this goal is to be achieved, further development of the procedure is required.

In particular, additional studies must be carried out to understand the displacement of surface asperities during normal loading of the surface. A load to area-of-contact relationship for moderate loads must be developed and the non-continuum flow in the interface analyzed. This future work is of a basic research nature and has broad applicability in the fields of surface mechanics and fluid mechanics. Besides fluid connectors these studies may make significant contributions in such areas as friction, wear, and electrical and thermal contact.

Perfectly flat, randomly lapped identical surfaces were manufactured for the experimental evaluation of the calculation procedure. The leakage flow of helium was experimentally measured and calculated. The following procedures were followed in the calculation:

- Surface roughness measurements were made
- Surface statistics were calculated
- The surfaces were generated mathematically and pressed together mathematically
- The interfacial gap map was generated
- A flow map was generated
- Flow parameters were calculated
- The flow was calculated

Of these steps, all but the flow map generation are programmed for digital computers.
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Section 2

INTRODUCTION

The ultimate goal of the interface studies is the ability to analytically determine the leakage flow between two sealing surfaces with a determinable certainty. It would be desirable to obtain an exact answer rather than one based on probability, but the nature of the surface geometry and the non-continuum flow necessitated the use of statistics.

Throughout the duration of this contract there has been a careful evaluation of the interface modeling and leakage calculation procedure. During the first contract period, the effect of surface finish, surface yield strength, and compressive stress on the leak-tightness of a fluid connector were studied (Ref. 1). An analysis of a one-dimensional channel flow model of leakage was also developed and leakage flow calculations were made (Ref. 2). These studies were adequate to gain a qualitative understanding of the leakage phenomenon, but were not adequate for the direct calculation of leakage. The complexity of the interfacial flow phenomenon necessitates a sophisticated analytical approach based on thorough research studies of the factors influencing the flow. Some of that research was done in the present contract period but the current calculation procedure contains a number of theories which are compromises between simplicity and accuracy. By using them and testing the whole procedure it has been possible to gain a better understanding of its strengths and weaknesses to allow proper direction of further work.

The need for a leakage flow calculation procedure as a research and design tool prompted the undertaking of this study. The analytical determination of the amount of leakage through two sealing surfaces in intimate contact provides a greater insight into the phenomenon than that provided by the experimental investigations alone, and the result of variations in the parameters that affect the flow can be studied without costly and lengthy experimental efforts. Such a procedure would make it possible to calculate directly the leakage flow for many seal designs and to change certain parameters to bring leakage to a desired level.

The first phase of the study was the development of a computer program to mathematically generate the height contours of a mating surface and a map of the gap heights between this surface and a flat surface (Ref. 3). The contour was computed from statistics taken directly from the physical surface under consideration. This program was extended to generate the contour map for both surfaces, the gap map for the interface between these surfaces, and the statistics for the gap map necessary for the flow calculation. A flow model was chosen and analyzed for flow. This completed the leakage calculation procedure, and an experiment was made to investigate the accuracy of the procedure.
The results of the calculation are close enough to the experimental values of leakage to show that the approach is valid. Therefore, it appears possible to develop an accurate calculation procedure. Some of the steps in the procedure are based on untested assumptions. Investigations of these assumptions and their validification or replacement, should lead to the desired improvement in accuracy. It should then be possible to develop a design procedure for calculating the leakage flow for a particular seal, based on the seal geometry, materials, finish, loads, environment, and fluid properties. This will be another important step toward the development of zero-leakage connectors.
Section 3

CONCLUSIONS AND RECOMMENDATIONS

Based on the results of the analytical development and experimental evaluation of a procedure for the calculation of leakage flow, the following conclusions and recommendations are presented.

1. The calculation procedure could be developed into an accurate design tool with sufficient additional effort.

2. The calculation procedure is not sufficiently accurate for design calculations in its present form.

3. Each step of the procedure must be evaluated and improved in order to develop an accurate design tool.

4. Basic research is required to develop an accurate load to area-of-contact relationship.

5. The flow map generation from the gap map must be computerized to remove the element of human judgment from the procedure.
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INTERFACIAL FLOW ANALYSIS

PROBLEM DEFINITION

The goal of this study was to complete the seal leakage calculation procedure and investigate its accuracy by comparing the calculated results with the experimental results for a particular surface. This was an interim goal in the development of a leakage calculation procedure suitable for design. The final procedure should be completely computerized to minimize costs and time. However, the computerization of the current procedure is segmented. Most of the steps have been computerized, but the developmental nature of the procedure requires a flexibility found only in sequence of programs.

The calculation procedure development has required a continual evaluation of the need for accuracy and its subsequent cost. The most difficult task in the analytic approach is establishing a mathematical model which is simple enough to be solvable and yet accurate enough to be meaningful. This is illustrated by noting that a very simple model of a seal can be assumed as was done in work previously reported in the final report for Phase I of this project (Ref. 1). However, while solution of this model was quite simple, the results were inaccurate and could not be extended to explore the effect of such important parameters as seal width. On the other extreme, a specific approach to solving a more accurate model was conceived. It could have handled very general problems and yielded accurate statistical results. Some rough estimates, however, indicated that the cost per solution, using a large digital computer, would be excessive.

The complexity of the situation is easily understood by a brief consideration of two sealing surfaces, their interaction, and the flow between them. Each surface is characterized by a gross surface topography (tool marks, camber, intentional curvature, etc.), and a random array of microscopic surface asperities (scratches, holes, mounds, particles, etc.). The asperities are usually measured in microinches and the gross surface topographic features in mils. A sealing surface might contain millions of asperities and the description of the complete surface is impractical.

The introduction of the mating surface, which may be of a different material and surface topography, pressed against the first surface results in an interfacial gap map which is much more difficult to describe completely. Not only geometric properties of the two surfaces, but also the material properties of the surfaces and environmental conditions (load, temperature, etc.) are involved in the formation of the gap map. As each asperity comes
in contact with the opposite surface, it is first elastically and then plastic-
ally deformed. The degree of deformation depends on the load, material
properties, temperature, and geometry. It determines the amount of
approach of one surface to the other, and the manner in which the material
is displaced.

This complex gap map is not a map of the flow passages of the interface.
It includes pockets and blocked flow channels, neither of which can contribute
to the flow. Thus, it is necessary to identify the flow passages, the sum of
which makes up the flow map. These flow passages are tortuous paths whose
height and width may vary from a few microinches to a few mils, and which
are interconnected in series and in parallel. The identification and descrip-
tion of the flow passages for a whole interface is impractical.

The complexity continues to increase as the fluid flow analysis is
introduced. In addition to the complex geometry, the flow analysis must
contend with continuum and non-continuum flow. The usually low levels of
leakage experienced in connectors for launch vehicles together with the low
ambient pressure means that some part of the flow is non-continuum flow.
The gas becomes rarefied and the actions of individual molecules rather than
slugs of fluid become important. This type of flow requires the introduction
of statistical methods.

Therefore, the calculation procedure developed is a compromise
between accuracy and complexity. The ultimate goal of a design calculation
procedure makes it desirable to characterize the surface geometry with a
few parameters, which may or may not be different from those in current
use. This leads to a statistical approach to the description of the surface
geometry. The approach is extended through the generation of the gap and
flow maps, and the determination of the geometrical parameters for the
flow model. This statistical approach is the most reasonable in light of
the complexity of the problem. However, this approach considers a typical
interface which is statistically representative of the family of interfaces.
Therefore, each individual interface will be slightly different. The
statistical variability is predictable, but unusual events (damage during
handling or assembly) cannot be predicted.

The choice of surfaces for an experimental check on the calculation
procedure is also a careful compromise. Perfectly-flat randomly-lapped
identical surfaces were chosen. These surfaces have a random roughness
that is easily measured, and are smooth enough to achieve low and
predictable leakage rates. The surfaces were first machined until optically
flat and then lapped to the desired roughness. Care was taken to control the
experimental parameters in order to obtain a typical sample.
FLOW CALCULATION

Outline

The flow calculation procedure is presented as a series of steps, each of which contains both the analysis and computations. This is done to fully explain each step of the procedure before continuing to the next step. Nearly all of the computations are programmed for a digital computer and one program may be associated with one or more steps in the calculation.

The calculation is performed in the following sequence.

1. Specify the surface geometry.
2. Calculate the statistical parameters for the surface geometry.
3. Mathematically generate the surface geometry.
4. Compute the gap map.
5. Compute the load corresponding to the gap map.
6. Determine the flow map from the gap map.
7. Compute the flow parameters.
8. Compute the flow.

Specify Surface Geometry

A set of parameters would be used to specify the surface geometry in a design calculation. In this case, the surface characteristics were taken directly from the surfaces to be tested. The lapped surfaces were prepared on the raised annular portions of two mating leak-test specimens of the type previously used in this program (Figure 1). The test specimens were carefully prepared in order to insure controlled test conditions. After finishing the surfaces to a degree of flatness beyond the measuring ability of an optical flat, the surfaces were randomly roughened by a hand lapping operation. The result was a set of surfaces with a random roughness and no regular roughness.

The surface geometry is described by two sets of numbers. The first set describes the larger, regularly varying undulations of the surface, such as the tool marks from machining operations and the camber of the surface. The second set describes the random roughness superimposed on the regular roughness. This is due to the random manner in which the metal tears during manufacturing operations.
Figure 1. Top View of Test Specimen Sealing Surfaces. Mat finished surface on raised portion is sealing surface.
This surface roughness was measured by a "Talysurf" surface measuring instrument. A total of eight traces were taken on the two surfaces in a way that eliminated any directionality in the surface statistics. Each trace was about 0.1 inch in length. These traces were recorded in the usual way on a paper tape and, in a new way, on magnetic tape. This magnetic tape recording was electronically processed to convert the analog signal to a digital signal. The surface height was electronically read off the tape recording every 100 microinches. This spacing is approximately three times the center line average (CLA) roughness measured by the Talysurf. The basis for this factor of three is presented in Reference 3.

During the measurement, a stylus is moved in a straight line across the surface at a constant velocity. The asperities in the surface cause vertical motion of the stylus. Vertical motion of the probe is coupled to an inductance transducer connected in a bridge circuit. An 1800 Hz carrier is modulated in proportion to the displacement of the probe.

Ordinarily the carrier is rectified and fed to the moving coil for a recording galvanometer. The arm of the galvanometer responds only to the low frequency amplitude modulation which is proportional to the displacement of the stylus. By moving the chart paper at a constant speed, a graph of the surface profile is obtained.

For the special tests, the rectified carrier signal was fed to a Tektronix amplifier, type 2A63. This amplifier has a frequency response 0 - 300 kHz. The carrier was filtered out by simply shunting the cathode follower output of the amplifier with a 25 microfarad capacitor. The remaining signal was recorded using a Lockheed Electronics Company FM tape recorder (Figure 2). The frequency response for the tape recorder is 0 - 2500 Hz but the 25 microfarad capacitor caused a break in the effective response of the system at 3 Hz.

The tape recording was processed by playing back the tape into a Hewlett-Packard digital voltmeter and paper tape punch. The signal was sampled for 10 milliseconds at 170 millisecond intervals and the results punched into standard paper tape. The system was calibrated by measuring a standard glass plate with known grooves of two depths, 94 microinches and 14 microinches.

The eight traces were transformed into eight series of numbers. From each series, 400 consecutive numbers were chosen to calculate the surface roughness statistics, thus making a total sample of 3200 numbers. The surface roughness was characterized by the height about the mean for each trace. Because of the possibility of the Talysurf stylus riding up or down on the surface, the mean for each trace was taken to be the least-squares fit of a straight line to the 400 height readings. This is identical to the statistical mean if the Talysurf stylus moved in a plane parallel to the plane of the surface.
Figure 2. Block Diagram of Instrumentation for Recording and Digitizing Surface Measurements
Calculate Surface Parameters

The surface geometry is described by two sets of parameters. One set describes the regular undulations of the surface and is all zero for this lapped surface. The roughness is all of a random nature and is described by a set of cumulative probability distributions.

The surface roughness heights, measured from the mean, were grouped into 20 height groups with equal increments of height (15 micro-inches). The statistics were then calculated for these incremental heights. The mean height is 10.5 increments. The cumulative probability distribution was calculated for the whole surface. This is plotted in Figure 3 and shows the Gaussian nature of the distribution. The coordinate scales of the paper have been transformed so that a Gaussian distribution will plot as a straight line. Only the extreme peaks and valleys of the surface vary from the Gaussian distribution. These results confirm the statement (Ref. 4) that several common surface preparations produce Gaussian distributions.

Conditional cumulative probability distributions were calculated for each incremental height. That is, for each height the cumulative probability distribution for the adjacent height was calculated. These distributions are also nearly Gaussian. The distributions are necessary for the computer program which computes the surface roughness for a small patch typical of the real surfaces.

These twenty one cumulative probability distributions define the surface geometry. Their use to mathematically generate a sample of the seal surface is discussed later in this section.

Mathematical Surface Topography

In this part of the calculation procedure, the topography of the surface is computed from the surface parameters. Starting with the data for the regular roughness and the statistics for the random roughness, the surface topography is calculated for a small patch of each surface. The two patches are brought together and the gaps between them are calculated. The generation of the surface topographies and the gap map are incorporated in the same computer program.

The surface topography generated is only a small rectangular patch of the actual surface. This patch is divided into an array of rectangular spaces. The maximum array size of 120 by 200 is controlled by the computer program. It can be increased, and the controlling factors are whether the sample is large enough to be representative of the surface and small enough to be handled economically in the computer. Further studies are necessary to find the optimum size. The physical dimensions of the spaces are determined by the spacing used in the surface measurements for the conditional cumulative probability distributions. In this
case height measurements were made every 100 microinches. Therefore, each space is 100 by 100 microinches and the total patch size is 0.012 by 0.02 inch. Compared to the sealing surface with an internal diameter of 0.937 inch and an external diameter of 1.169 inches, this is only 0.06 percent of the surface area. However, the patch consists of 24,000 spaces and is considered representative of the surface.

The patch is generated mathematically as a three-dimensional array. Each element of the array has three numbers (h, i, and j) the surface height, h, row, i, and column, j. The random part of the surface topography is generated by randomly entering the ordinate of the cumulative probability distribution, Figure 3, and reading the height off the abscissa. This is the height for (h, 1, 1). If this is considered (6, 1, 1), then the height for (h, 2, 1) is found by randomly entering the ordinate of the conditional cumulative probability distribution for the height 6 and reading the height off the abscissa, (7, 2, 1). The height for (h, 3, 1) is found by first linearly extrapolating from the previous two heights to get an estimate of 8. The height is found by randomly entering the ordinate of the conditional cumulative probability distribution for the height 8 and reading the height off the abscissa. This is continued until the first row and then the first column are completed.

The determination of the remaining heights requires a slight modification of the procedure. Linear extrapolation over a surface, rather than along a line, is used to choose the probability distribution for making the random selection. For example, given (6, 1, 1), (5, 2, 1), and (5, 1, 2), these three points determine a plane which if extrapolated to element (h, 2, 2) gives an estimated height of 4. Using the distribution associated with 4, a random selection is made to find (h, 2, 2). Similarly the heights for the complete patch are determined. The height statistics for the patch are then compared to those for the surface to assure that the patch is typical of the surface. This is discussed in greater detail in Reference 3.

A portion of one of the generated surfaces is shown in Figure 4. The height is given in increments of 15 microinches each and is measured from a reference plane. The sequence is 1, 2, 3, 4, 5, 6, 7, 8, 9, T, A, B, C, D, ------. That is, D is 14 increments or 210 microinches above the reference plane. The computer program is presented in complete detail in Appendix II.

**Interface Gap Map**

After one surface topography is generated, it is stored, and the mating surface topography is generated. These are then brought together until they are just touching, making a reference point. They are pushed together any number of increments (for this case 15 microinches each) and the heights of the gaps remaining between them are determined. This is also done in the computer program described in Appendix II.
Figure 4. Topographic Map Typical of The Sealing Surfaces. Rows and Columns are Indicated, and The Sequence of Height Increments is 19 29 39 ---9, TA O B, -me
The array of gap heights is printed out in the same manner as the surface heights. A portion of a typical gap map used in the calculations is shown in Figure 5. The gap height increments are the same as for the surface topography (15 microinches) and the sequence of heights is also 1, 2, 3, 4, 5, 6, 7, 8, 9, T, A, B, C, D, ---. The height, B, is 12 increments or 180 microinches. Areas of contact are indicated by blanks.

The calculation of the gap heights assumes no deformation of the surfaces. Therefore, it must be assumed that as the surfaces come into contact, the material that is displaced disappears. This is the geometric picture. The physical picture is that the area of contact, as a function of the distance the surfaces are pressed together, is well represented by this assumption for small distances. These are the same assumptions that are used in Abbott’s bearing analysis, Reference 5. However, for large displacements and large areas of contact, the assumption is untested. More research is required to determine a physically realistic assumption that can be readily incorporated in the computer program.

Load - Area of Contact Relationship

The ratio of the area-of-contact to the nominal area is obtained directly from the gap map and this calculation is incorporated in the same computer program. This ratio is related to the load pressing the two surfaces together. The results of a literature search to determine this relationship are summarized in this section.

The initial contact between two surfaces is elastic, and will remain elastic until very large nominal pressures are applied, if the plasticity index is less than 0.6, Reference 4. Further, if the plasticity index exceeds 1.0, plastic flow will occur even at trivial nominal pressures. In the narrow range 0.6 to 1.0, the mode of deformation is in doubt.

The plasticity index is defined as follows:

\[ \psi = \frac{E'}{H} \sqrt{\frac{s}{\sigma}} \]

\[ E' = \left( \frac{1 - \frac{\sigma^2}{E_2^2}}{E_1} + \frac{1 - \frac{\sigma^2}{E_2^2}}{E_2} \right) \]

\[ E - \text{modulus of elasticity} \]

\[ \sigma - \text{Poisson's ratio} \]

\[ H - \text{hardness} \]

\[ s - \text{standard deviation of the asperity heights} \]
Figure 5. Portion of Interfacial Gap Map. Rows and columns are indicated and the sequence of gap height increments is 1, 2, ---, 9, T, A, B, ---. Contact is indicated by a blank.
\( \delta \) - radius assumed for the spherical tops of the asperities

subscripts 1 and 2 refer to the contacting bodies.

For both elastic and plastic contact, the area of contact is proportional to the total load (Ref. 6). However, this relation is only valid for light loads. In the case of light loads where plastic flow occurs, the area of contact is:

\[
A_r = \frac{W}{\sigma}
\]

where \( W \) is the total load and \( \sigma \) is the local plastic yield pressure, which is very nearly constant and is comparable to the indentation hardness of the metal. This relation has been experimentally verified.

There is no experimentally verified relation for moderate to heavy loads. The most plausible relation is that proposed in Reference 7. The proposed relation states that the real area of contact exponentially approaches the nominal area of contact. Also, at light loads, the relation approximates Equation (1). The relation is:

\[
A_r = A_n \left( 1 - e^{-\frac{W}{\sigma A_n}} \right)
\]

where \( A_n \) is the nominal area.

The loads necessary to achieve the low level of leakage desired for this flow calculation are definitely in the moderate load range. Thus Equation (2) is used to relate the load to the area of contact. For each gap map, a corresponding area of contact is calculated. Then the load is calculated from Equation (2). \( \sigma \) is determined from a hardness test or can be taken equal to three times the yield stress, Reference 8.

The hardness of the test specimens, which are made of 347 stainless steel, was measured using Brinell and Vickers hardness testers. The measured hardness is 239,000 psi. The nominal surface area is 0.384 square inch. A value of \( A_r/A_n \) was calculated for each gap map. Thus, using Equation (2), the load was calculated for each gap map. The flow is also related to the gap map and in this way, the flow is related to the load.

**Interface Flow Map**

The gap map Figure 5, is not the flow map. The flow map is that part of the gap map that contributes to the leakage flow. This does not include pockets or blocked passages.
Beginning with the gap map, the flow map is obtained by deleting the non-contributing gaps. This is done manually in this calculation and is a very tedious and time consuming task. It is not computerized, because the logic involved would be very complex, and the cost and time for the development of such a program are not warranted by the present goal. Non-contributing flow gap areas are manually identified by their row and column number. These locating numbers are used by the computer program, Appendix II, to delete the gap heights and thus generate the flow map.

The identification of the areas of the gap map not contributing to the leakage flow is a matter of engineering judgment. First, it is assumed that flow between two individual elements of the gap map is only in the row or column direction and not through a corner. This follows from the chosen method of modeling the flow map in which there is no flow area associated with a corner. Thus in Figure 5 there is flow from element (row 3, column 2) to (3, 3) and also to (4, 2), but not directly to (4, 3). The second assumption is that a blocked passage contributes nothing to the flow area. Consider a river flowing past an inlet or bay with a small entrance. The river flow is affected by the flow area of the river but not by the stagnant water of the inlet. At most there is a widening of the flow area in the region of the entrance to the inlet and this can be determined by tracing the streamline along the edge of the river as it crosses the entrance. This analogy can be applied to the leak paths (rivers) and blocked paths (inlets) of the gap map.

The third assumption is that a leak path is one that connects the top of the gap map to the bottom, or one that connects the left-hand side to the right-hand side or to the bottom. The gap map as produced by the computer is 200 rows by 120 columns. As this is a small patch of the seal interface, it is reasonable to associate the column and row directions with the radial and circumferential directions respectively of the interface. The left side of the patch is taken as the high pressure side. If a flow path connects the left side to the right side this is considered to be a leak path. This assumes that the probability of a flow path connecting one side of the entire seal interface with the other side is the same as that of connecting one side of the gap map to the other. This statistical approach is necessary because of the unmanageable size of a flow map for the whole seal interface.

As only a small patch is considered it is possible that a flow path originating at the left side and terminating at the top or bottom, or originating at the top or bottom and terminating on the right side, may be a leak path. It is assumed that half of these possible leak paths are actual leak paths. This is done by considering a flow path originating on the left and terminating on the bottom or right, or originating at the top and terminating on the right, as a leak path. The sketch of the gap map below is presented to summarize the above discussion. In the sketch, gaps which permit flow are indicated by solid lines and gaps which do not permit flow are indicated by dotted lines.
Based on the above assumptions the gap map of Figure 5 is transformed to a flow map, Figure 6. The elements of the gap map that do not contribute to the leakage flow are deleted in the computer in order to generate the flow map. In Figure 6 the deleted areas have been indicated by cross-hatching so that the non-contributing areas could be identified. Stagnant pockets and blocked passages have been marked. The height increments are the same as those for the gap map. The next step is to relate the flow map to the flow model.

**Flow Parameters**

The flow map of the preceding section has to be related to the flow model. The flow model is described here, with a detailed discussion later in this section.

The levels of leakage flow of interest to this investigation are so low that both continuum and non-continuum flow are expected (Reference 2). For these two flow regimes, different flow analyses are required and different flow models are used. The actual interfacial flow map is too complex to use and it is necessary to meaningfully relate a simplified model to the flow map. In both flow regimes, the flow is assumed to be two-dimensional. The single channel is assumed of sufficient width in relation to its height, so that the effects of the channel sides are neglected (Figure 7). The upper and lower sealing surfaces are assumed sinusoidally varying as a function of the channel length in the continuum flow regime and of constant height in the non-continuum regime. The flow is assumed in a straight line from the inside to the outside of the seal at some angle to the radius. Thus, the length of the flow channel is not the shortest distance between the inside and outside of the seal.

This simplified flow model is statistically matched to the flow map. In this way, each parameter of the flow model is uniquely determined in the most realistic manner. The statistics for the flow map are computed in
Figure 6. Portion of Interfacial Flow Map. Rows and columns are indicated and the sequence of gap height increments is 1, 2, ---, 9, T, A, B, ---. Contact is indicated by a blank.
the computer program that generates the flow map, Appendix I. The flow map statistics are the probability distribution of the flow heights, the probability distribution of the slope of the flow channels in two orthogonal directions (rows and columns of the flow map), and the ratio of the projected flow area (plan view) to the nominal contact area.

From these statistics, the flow parameters D, A, and L of Figure 7, and V, X9, and ϕ of Figure 8 are determined. D and A are determined from the probability distribution of the flow channel heights. For each height increment, there is a corresponding probability, (Fh)i. The incremental heights range from one to a possible maximum of 40. Although the height increments may be of different sizes, this unnecessarily complicates matters and a uniform height increment of Hh is used.

The flow height for the flow model is a cosine function.

\[ h = 2D \left( 1 + A \cos \frac{x}{L} \right) \]  

(3)

D in Equation 3 is equal to one half the mean height.

\[ D = \frac{Hh}{2} \sum_i (Fh)_i \]  

(4)

The A in Equation 3 is chosen so that the probability distribution for the gap heights closely approximates the probability distribution for the gap heights from the flow map. The probability of the height, h, being in the range h1 to h2, is easily found from Equation 3. A bar is used to differentiate the probability for the flow model from that for the flow map.

\[ \overline{F}_{h_1} - h_2 = \frac{1}{\pi} \cos^{-1} \left( \frac{1}{A^2} \left[ \frac{h_2}{2D} - 1 \right] \left[ \frac{h_1}{2D} - 1 \right] \right) \]  

(5)

If \( h_1 = (i - \frac{1}{2}) Hh \) and \( h_2 = (i + \frac{1}{2}) Hh \) are the limits for a height increment for which the probability, \( (Fh)_i \), is known from the flow map, then as D is known, A can be found from Equation 5.

\[ A = \frac{1}{\sin \pi (Fh)_i} \sqrt{\left[ 1 - \frac{1}{2} \frac{Hh}{2D} - 1 \right]^2 + \left[ \frac{1 + \frac{1}{2} Hh}{2D} - 1 \right]^2} \]  

(6)

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Figure 8. Flow Model Plan View

Nominal Area = Radial Width \times Circumferential Length

Direction of Flow

Flow Width (W)

Flow Area

Circumferential Length

Radial Width

Flow Length (X9)
\( A_i \) is calculated for every height increment and \( A \) is the mean of these values.

\[
A = \sum_i A_i (F_h)_i
\]  

(7)

The probability distribution for the flow model is obviously symmetric about the mean. However, the distribution for the flow map is truncated (Figure 9). Therefore, a good match between the two distributions is obtained when the distribution for the flow map is symmetrized by truncating the distribution symmetrically. That is, set \((F_h)_i = 0\) for all \((F_h)_i > (F_h)_1\). This indicates that the flow model will not contain any of the large channel heights found in the flow map. The affect on the flow calculations is small, because it is the small heights (constrictions) that have a large affect and not the large heights.

The \( D \) and \( A \) calculated from the data of Figure 9 are \(1.9152 \times 10^{-5}\) inches and \(0.67548\) respectively. The corresponding channel height probability distribution is shown in Figure 10. The distribution is symmetric, but is convex downward instead of upward like the distribution for the flow map (Figure 9). This is characteristic of the cosine (or sine) function.

The period of the flow model channel height is proportional to \( L \). Differentiating Equation 3 with respect to \( x \) shows that the slope of the channel height is proportional to \( D \), \( A \), and \( L \).

\[
\frac{dh}{dx} = -\frac{2DA}{L} \sin \frac{x}{L}
\]  

(8)

\( D \) and \( A \) have been determined, and it is apparent from Equation 8 that \( L \) can be determined by statistically matching the slopes of the channel height of the flow model to that of the flow map. The slope given by Equation 8 is in the flow direction (\( \phi \) direction in Figure 8). In order to match the slopes, it is necessary to find \( \phi \), but due to the nature of the flow map statistics, \( L \) and \( \phi \) are found simultaneously.

The probability distributions for the channel height slopes of the flow map are readily found in the row and column direction. As the flow map is 120 columns by 200 rows, the radial interface direction is taken in the column direction, and the circumferential direction is taken in the row direction. Let \( r \) be the radial coordinate and \( c \) the circumferential coordinate.

\[
r = x \cos \phi
\]

\[
c = x \sin \phi
\]  

(9)

From Equations 8 and 9, the slopes in the radial (column) and circumferential (row) directions are found to be both functions of \( L \) and \( \phi \).
Figure 9. Flow Channel Height Probability Distribution for the Flow Map
\[
\frac{dh}{dc} = K_c \sin \frac{x}{L} \tag{10}
\]
\[
\frac{dh}{dr} = K_r \sin \frac{x}{L}
\]

where
\[
K_c = - \frac{2DA}{L \sin \phi} \tag{11}
\]
\[
K_r = - \frac{2DA}{L \cos \phi}
\]

From Equations 10, it is possible to obtain the probability of \( \frac{dh}{dc} \) being between \( (\frac{dh}{dc})_1 \) and \( (\frac{dh}{dc})_2 \) and solving for \( K_c \), and the probability \( \frac{dh}{dr} \) being between \( (\frac{dh}{dr})_1 \) and \( (\frac{dh}{dr})_2 \) and solving for \( K_r \). The probability distributions for the increments of channel height slopes computed from the flow map are \( (F_{r})_i \) and \( (F_{c})_i \) respectively in the radial and circumferential direction. The height and distance increments (radial and circumferential) are 15 and 100 micro inches respectively. Each increment of slope is 0.15 and this constant is designated \( H_s \). The limits for each increment, \( i \), are \((i-1/2)H_s\) and \((i + 1/2) H_s\). Therefore, a value of \( (K_r)_i \) and a value of \( (K_c)_i \) correspond to an increment of slope in the radial and circumferential direction respectively.

\[
(K_c)_i = - \frac{H_s}{\sin \pi (F_c)_i} \sqrt{2 \left[ \left( i a + \frac{1}{4} \right) - \left( i a - \frac{1}{4} \right) \cos \pi (F_c)_i \right]}
\]
\[
(K_r)_i = - \frac{H_s}{\sin \pi (F_r)_i} \sqrt{2 \left[ \left( i a + \frac{1}{4} \right) - \left( i a - \frac{1}{4} \right) \cos \pi (F_r)_i \right]}
\]

From these results, the mean values of \( K_c \) and \( K_r \) are calculated using
\[
K_c = \sum_{i} (K_c)_i (F_c)_i \tag{13}
\]
\[
K_r = \sum_{i} (K_r)_i (F_r)_i
\]

The values of \( \ell \) and \( \phi \) are then calculated directly from the following expressions.
Figure 10. Flow Channel Height Probability Distribution for Flow Model
L = \sqrt{\frac{1}{K_C^2} + \frac{1}{K_T^2}} \quad (14)

\cos \varphi = -\frac{2DA}{LK_T}

L is used directly in the flow calculation, while \varphi is used to calculate the flow length, X9, used in the flow calculation.

(X9) = \frac{\Delta r}{\cos \varphi} \quad (15)

where: \Delta r is the radial width of the seal interface

The nominal area of the seal interface is easily calculated from the seal dimensions. The ratio of the projected area of the flow map to the nominal area of the flow map is calculated by the same program that generates the flow map. Thus, the projected area of the flow map, A_f, is easily calculated and then the width, V, of the flow model channel is obtained directly.

V = \frac{A_f}{(X9)} \quad (16)

The values of L, (X9), and V, calculated from the statistics for the flow map of Figure 6, are 3.2308 \times 10^{-5}, 0.1641, and 0.90326 inches respectively. The probability distributions of the channel height slopes are shown in Figure 11 and 12.

Continuum and Non-Continuum Flow

Leakage flow depends primarily on the minimum dimension of a leak path. There are two possible flow regimes of interest, continuum (laminar) and non-continuum; the type of flow in a particular case being determined by the magnitude of the governing dimension. In a real situation, there would generally be flow of both types in parallel and in series in the tortuous leak paths from inner to outer surface. To determine the flow rate through such a network, assuming the leak paths had been found by the computer, would be a very complicated procedure. In addition, it would involve a different solution for each sample interface generated. A flow model which employs certain parameters in a general equation for the flow rate through the seal is needed. These parameters would be derived from individual interfaces or, preferably, a family of interfaces. The model must incorporate the significant geometrical characteristics of the interface and physical properties of the fluid.

Flow in individual leaks in a face seal, when the total flow rate is in the range of "zero leakage", will be in the very low Reynolds number regime.
Figure 11. Probability Distribution of Channel Height Slope in Radial Direction for the Flow Map.
Figure 12. Probability Distribution of Channel Height Slope in Circumferential Direction for the Flow Map.
both regimes are characterized by a preponderance of boundary effects over momentum interchange within the fluid. Therefore, the shape of the boundary is particularly important in setting up a flow equation. The first step in the construction of a flow model is to find a typical flow passage which will realistically represent actual leak paths and, at the same time, lead to a mathematically tractable fluid flow problem. On the basis of representative interface details and flow phenomena, compatibility with the existing analysis, and feasibility of a computer solution, a two-dimensional channel with a periodically undulating ceiling and floor is chosen for the laminar flow model. A two-dimensional channel with smooth ceiling and floor is chosen for the non-continuum flow model.

The flow equations for each regime are derived in Appendix I. The continuum flow analysis is restricted to laminar flow and the solutions are derived from the Navier-Stokes equations with no-slip boundary conditions. The non-continuum flow analysis is restricted to the slip flow regime and the solutions are derived from the Navier-Stokes equations with slip boundary conditions. However, the results are commonly applied to the whole non-continuum regime and are so used in this analysis. The Knudsen number is used to identify the interface between laminar and slip flow. This is arbitrary because there is a gradual transition. This results in some confusion as to which Knudsen number to use. A number of 0.01 has been recommended (Ref. 9), which is used in these calculations.

The calculation of flow rate through the wavy-walled channel, derived to represent a typical leakpath, is composed of a continuum part and a non-continuum part. Starting with an educated first guess for the continuum flow, W, the basic procedure is to calculate the transition point, X8, so that the Knudsen number of the flow (mean free path of the gas/mean channel height) reaches the input transition value, K8. The non-continuum flow equation is then employed to find the flow rate Q8 based on the transition pressure P8, the prescribed final pressure P9, the location of the transition point X8, and the coordinate X9 of the end of the equivalent path. If the flow rates, W and Q8, are not equal, a new W is chosen between the two, and the process repeated iteratively until there is continuity of flow at the transition point. The resulting flow rate represents the rate of flow through the given channel for a fluid which is treated as compressible and viscous up to the transition point. Beyond this point, it is considered a rarefied gas.

The governing equations fail in certain instances which experience indicates will generally not occur for flows of interest in this study. Thus, there is no provision for a shock occurring either within the channel or at the end. A calculation of the maximum Mach number of the compressible flow has been included in the basic procedure, and all calculations have yielded values well below the sonic range. If sonic flow should occur, special steps would need to be taken. If the solution should indicate that the continuum flow extends all the way to the exit of the passage, a shock would presumably develop at the exit which would limit the flow (normally the free
molecule transition represents a limiting condition imposed by molecular motions which is in essence very much like a shock phenomenon). The calculation procedure will indicate the absence of a transition, and yields the completely continuum flow solution which is an upper limit to the actual flow rate.

The entire analysis outlined in this section has been programmed for the General Electric 235 computer Time-Sharing System, and is completely debugged and operational at the present time. A complete listing of the program is included in Appendix I.

A set of calculations were made, using the program, to check the entire calculation procedure. The geometrical parameters were determined, and the gas properties were chosen to match those of the leakage flow experiments. The gas used was helium at 23.1°C, and the viscosity and density are calculated using Sutherland's law and the perfect gas law respectively. The outside pressure for the calculations used is a vacuum of approximately 2.4 x 10^{-7} pounds per square inch. The results are given in Figures 13 and 14. The flow parameters for these curves are given in Table 1.

| Table 1
| FLOW MODEL PARAMETERS USED IN CALCULATIONS OF LEAKAGE FLOW |
|---|---|---|---|---|---|
| Load Pounds | D inch | A | L inch | X9 | V |
| 27940 | 2.69 x 10^{-5} | 0.696 | 4.30 x 10^{-5} | 0.171 | 1.56 |
| 39170 | 2.30 x 10^{-6} | 0.683 | 3.77 x 10^{-6} | 0.171 | 1.26 |
| 52710 | 1.92 x 10^{-6} | 0.675 | 3.23 x 10^{-6} | 0.164 | 0.903 |
| 54820 | 1.88 x 10^{-6} | 0.686 | 3.30 x 10^{-6} | 0.161 | 0.709 |

FLOW EXPERIMENTS

The leakage flow through the seal interfaces shown in Figure 1 was experimentally measured to investigate the accuracy of the flow calculation. The experiments were performed at room temperature. The experimental apparatus used is the same as that described in Reference 10. The specimen was loaded in a standard testing machine, pressurized with helium, and the leakage measured with a mass-spectrometer.

The test sequence began with a load just large enough to sufficiently restrict the leakage at an internal pressure of one atmosphere, to allow flow of the leakage to be measured on the mass-spectrometer. Then the load was increased and the internal pressure incrementally increased at each new load until the leakage exceeded the usable range of the mass-spectrometer. The load was increased a number of increments and, at each load,
Figure 13. Calculated Leakage Flow Curves for Various Total Loads
Figure 14. Calculated Leakage Flow Curves for Various Internal Pressures
the leakage was measured for a range of internal pressures. The results of the experiment are shown in Figures 15 and 16.

DISCUSSION

The calculated leakage flow compares favorably with the experimental results. Although the differences in leakage are as much as two decades on the log scale, these results must be judged in light of the many assumptions and simplifications made in the calculation procedure. The present goal was to verify the feasibility of developing an accurate flow calculation procedure. This has been achieved. The calculated curves, both in their magnitude and shape, are similar to the experimental results.

It is obvious that a great deal of effort is still required to develop a procedure sufficiently accurate for design calculations. The present procedure is not accurate enough and it is uncertain which steps are the major contributing factors. Any continuing effort in this development should be directed toward evaluating and improving the accuracy of each step of the procedure if necessary.

In briefly reviewing the procedure, a number of steps can be identified as possible sources of error. In the generation of the gap map, the displaced material of the contacting surfaces is assumed to disappear. The relation of the area of contact to the load has never been experimentally verified. The generation of the flow map from the gap map involves assumptions about the streamline flow pattern. The flow model is quite simplified from the actual flow map. The flow analyses have not been thoroughly experimentally checked. This is particularly true in the non-continuum flow regime where analysis is very difficult.

The development of a design tool will require generalization of the procedure to cover many types of surfaces and fluids. For instance, a superfinished surface (with asperities less than one micro inch) is so smooth that existing measuring techniques are not adequate. They are unable to generate the surface statistics required for the computerized calculation of the surface topography.

In order to investigate the possible sources of error in the calculation procedure a study was made of the affect on the flow due to changes in the flow model parameters. The calculated flow curve for a load of 52,710 pounds was chosen from Figure 13, and the experimental flow curve for a load of 50,060 pounds was chosen from Figure 15. Recognizing the nearly linear relationship of log-of-leakage versus load, Figure 16, the experimental flow points at 50,060 pounds are easily extrapolated to a load of 52,710 pounds. The results are shown on Figure 17.

Various flow model parameters were varied in order to bring the calculated flow at an internal pressure of 514.7 pounds per square inch down to the experimental value. The result of decreasing the mean flow channel
Figure 15. Experimental Leakage Flow Curves for Various Total Loads
Figure 16. Experimental Leakage Flow Curves for Various Internal Pressures
Figure 17. Leakage Flow Curves for a Total Load of 52,710 Pounds
height, $D$, from $19.2 \times 10^{-2}$ to $2.18 \times 10^{-8}$ inches is shown in Figure 17. At 514.7 pounds per square inch, the new calculated value of leakage is identical to the experimental value. Below and above this pressure, the calculated values are slightly larger than the experimental values. All of the calculated points are in the non-continuum regime of the flow calculation.

While a $8.8:1$ reduction in $D$ results in a $208:1$ reduction in the flow, similar changes in $A$, $L$, $K_8$, or $X_9$ have a much smaller affect on the flow. A $1.38:1$ increase in only $A$ (note $A$ must be less than one) results in a $1.29:1$ decrease in the flow. A $65:1$ decrease in $L$ results in a $1.004:1$ decrease in the flow. An $11.5:1$ increase in the transition Knudsen number $K_8$, results in a $2.66:1$ reduction in the flow. A $6.1:1$ increase in the length of the flow channel, $X_9$, results in a $6.1:1$ decrease in the flow.
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## Section 5

### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Amplitude ratio of sealing surface undulations</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$A_f$</td>
<td>Projected area (plan view) of flow map</td>
<td>inches$^2$</td>
</tr>
<tr>
<td>$A_n$</td>
<td>Nominal area (plan view) of contact</td>
<td>inches$^2$</td>
</tr>
<tr>
<td>$A_r$</td>
<td>Real area (projected in plan view) of contact</td>
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<td>$C_1C_2$</td>
<td>Constants, see Equation 20</td>
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<td>Circumferential coordinate in interface</td>
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<tr>
<td>D</td>
<td>Mean value of $h$</td>
<td>inches</td>
</tr>
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<tr>
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<td>Reynolds number</td>
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</tr>
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<td>$(F_h)_i$</td>
<td>Probability of flow map height being in the $i$th height increment</td>
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</tr>
<tr>
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<td>Probability of flow map height slope in the radial direction being in the $i$th increment</td>
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</tr>
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</tr>
<tr>
<td>G</td>
<td>Dimensionless channel height, see Equation 19</td>
<td>dimensionless</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational constant</td>
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<tr>
<td>H</td>
<td>Hardness</td>
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<td>$H_h$</td>
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</tr>
<tr>
<td>$H_s$</td>
<td>Size of slope increment</td>
<td>inches</td>
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<td>Height of seal surface (of flow model) from midplane of interface</td>
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<td>-------------------------------------------------</td>
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<tr>
<td>K8</td>
<td>Knudsen number at transition</td>
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<td>Distance proportional to period of</td>
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<td>flow model surface undulations</td>
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<tr>
<td>P_9</td>
<td>Exit pressure</td>
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<td>p</td>
<td>Pressure</td>
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<td>( \bar{p} )</td>
<td>Mean pressure</td>
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<td>Q</td>
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<td>see Equations 18 and 23</td>
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<td>q</td>
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<td>of channel, see Equation 21</td>
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<td>Standard deviation of the asperity</td>
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</tr>
<tr>
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<td>heights</td>
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<td>Absolute temperature</td>
<td>°Rankine</td>
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<tr>
<td>U</td>
<td>Dimensionless velocity, ( u ), see</td>
<td>dimensionless</td>
</tr>
<tr>
<td></td>
<td>Equation 17</td>
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</tr>
<tr>
<td>u</td>
<td>Flow velocity in x direction</td>
<td>inches/second</td>
</tr>
<tr>
<td>( \bar{u} )</td>
<td>Mean velocity ( u ), see Equation 22</td>
<td>inches/second</td>
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<tr>
<td>V</td>
<td>Dimensionless velocity, ( v ), see</td>
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<td></td>
<td>Equation 17</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>Flow velocity in y direction</td>
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</tr>
<tr>
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<td>Total load</td>
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<td>Continuum flow rate, ( w = p \ q )</td>
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<td>Dimensionless distance ( x ), see</td>
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<td>x_9</td>
<td>Total length of flow model channel</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>Y</td>
<td>Dimensionless distance $y$, see Equation 17</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$y$</td>
<td>Flow model channel height, coordinate normal to $x$</td>
<td>inches</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Radius assumed for the spherical tops of the asperities</td>
<td>inches</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Ratio of specific heats</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Poisson's ratio</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Mean free path</td>
<td>inches</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Viscosity</td>
<td>pounds seconds/inch$^2$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
<td>inches$^2$/second</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fluid density</td>
<td>pounds seconds$^2$/inch$^4$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Plastic yield pressure</td>
<td>pounds/inch$^2$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle between flow and radial directions</td>
<td>radians</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Plasticity index</td>
<td>dimensionless</td>
</tr>
</tbody>
</table>
REFERENCES


Appendix I.

FLOW ANALYSIS

CONTINUUM FLOW

The continuum flow solution starts with the full Navier-Stokes equations. These are reduced by the introduction of order-of-magnitude considerations completely analogous to the procedure followed in boundary layer theory. The process of reduction and the basic idea of the present solution can be traced to Prandtl's student Blasius (Ref. 11) in 1910. This method was recently extended by R. I. Tanner (Ref. 12) for an axisymmetric flow. The analysis in this appendix is basically an adaptation of Tanner's work to a two-dimensional channel (Ref. 13). The argument leading to the simplified equations based on viscous flow in a narrow gap with slow height variation are given in Ref. 12 and 13. Applied to this study, this is equivalent to the restriction \( A \cdot D \ll L \) (long wave length of constrictions compared to their height), and that viscous effects predominate and geometry changes normal to the flow are sufficiently gradual to permit neglect of the acceleration components in that direction and of \( \gamma \frac{\partial u}{\partial x} \). Thus the Navier-Stokes equations for two-dimensional flow reduce to:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}
\]

and:

\[
0 = \frac{\partial p}{\partial y}, \text{ or } p = p(x).
\]

Following a procedure suggested by Shapiro (Ref. 14, pg. 189), local incompressibility is assumed to integrate the equations of motion across the passage. The density is then allowed to vary with distance along the passage as integration takes place in this direction. Locally, momentum and kinetic energy losses are negligible, but they must be considered in the overall flow. In formulating the expression relating local flow rate and pressure drop, the equation of motion with the condition of incompressibility is supplemented.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.
\]

Dimensionless variables are now introduced to facilitate the parametric expansion method that will be used to obtain a solution to nonlinear equation of motion. Defined is:
\[ Y = \frac{y}{A \cdot D} \]
\[ X = \frac{x}{L} \]
\[ U = \frac{u}{\bar{u}} \]
\[ V = \frac{v \cdot L}{AD\bar{u}} \]
\[ P = p \cdot \frac{A^2 D^2}{\mu uL} \]

where the only new variable introduced is \( \bar{u} \), the mean velocity. The equations of motion and continuity in terms of the new variables become:

\[ \frac{\partial U}{\partial X} = -\frac{e}{U} \left( \frac{\partial U}{\partial Y} + U \frac{\partial U}{\partial X} \right) \]
\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \]

where \( e \) is Reynolds number defined by:

\[ e = \frac{A^2 D^2}{\gamma L}. \]

This method of solution will depend on \( e \) being small, which is consistent with the assumption that \( AD \ll L \) and is confirmed by results obtained from sample calculations.

Consider the expansions:

\[ U = U_0 + e U_1 + e^2 U_2 + ... \]
\[ V = V_0 + e V_1 + e^2 V_2 + ... \]
\[ P = P_0 + e P_1 + e^2 P_2 + ... \]

where \( U_0, U_1, V_0, V_1, P_0, P_1, ... \) are all unknown functions of \( x \) and \( y \). Substituting the expansions into the dimensionless flow equations and equating terms with like powers of \( e \), gives:

\[ \frac{\partial U_m}{\partial X} = -\frac{\partial V_m}{\partial Y} \quad m = 0, 1, 2, ... \]

and

\[ \sum_i e^i \frac{\partial U_i}{\partial Y} = \sum_j e^j \frac{\partial P_j}{\partial X} + e \sum_k e^k \frac{\partial V_k}{\partial Y} e^m \frac{\partial U_m}{\partial Y} + e \sum_m e^m \frac{\partial U_m}{\partial Y} \]
where the sums are interpreted as infinite series. The resulting equations are:

\[
\frac{\partial U_0}{\partial X} = \frac{\partial V_0}{\partial Y}
\]

\[
\frac{\partial^2 U_0}{\partial Y^2} = \frac{\partial P_0}{\partial X}
\]

and

\[
\frac{\partial U_1}{\partial X} = -\frac{\partial V_1}{\partial Y}
\]

\[
\frac{\partial^2 U_1}{\partial Y^2} = \frac{\partial P_1}{\partial X} + V_0 \frac{\partial U_0}{\partial Y} + U_0 \frac{\partial U_0}{\partial X}
\]

and

\[
\frac{\partial U_2}{\partial X} = -\frac{\partial V_2}{\partial Y}
\]

\[
\frac{\partial^2 U_2}{\partial Y^2} = \frac{\partial P_2}{\partial X} + V_0 \frac{\partial U_1}{\partial Y} + V_1 \frac{\partial U_0}{\partial Y} + U_0 \frac{\partial U_1}{\partial X} + U_1 \frac{\partial U_0}{\partial X}
\]

and so on for progressively higher order terms. The solution then follows by simple integration across a channel cross-section, employing the conditions:

\[
U_m = V_m = 0, \text{ on } Y = \pm G(X)
\]

for all \( m \),

\[
\int_{-G}^{G} U_0 \, dY = Q, \text{ the dimensionless volumetric flow rate, (18)}
\]

\[
\int_{-G}^{G} U_m \, dY = 0, \ m = 1, 2, 3, \ldots;
\]

\( G(X) \) is the dimensionless channel height defined by:

\[
G(X) = \frac{h(x)}{AD} = \frac{1}{A} \left( 1 + A \cos X \right).
\]
The zeroth order solution is simply the Poiseuille flow between parallel planes; there is no inertial pressure drop.

$$U_0 = \frac{3}{4} \frac{Q}{G} (1 - \frac{Y^a}{G^a})$$

$$V_0 = \frac{3}{4} \frac{QG'Y}{G^a} (1 - \frac{Y^a}{G^a})$$

$$\frac{dP_0}{dX} = - \frac{3}{2} \frac{Q}{G^a}$$

Here and in the following analysis, primes denote derivatives with respect to $X$.

Substituting these functions in the first order equations, yields a linear system which is directly integrable and which yields:

$$U_1 = - \frac{9}{16} \frac{Q^aG'}{G^b} \left\{ \frac{Y^b}{30} - \frac{G^aY^4}{6} + \frac{11G^bY^3}{70} - \frac{G^b}{42} \right\}$$

$$V_1 = \frac{9}{32} \frac{Q^a}{G^b} \left\{ \frac{Y^b}{105} - \frac{G^aY^3}{15} - \frac{11G^bY^2}{105} - \frac{G^aY^4}{21} \right\}$$

$$\frac{dP_1}{dX} = \frac{27Q^aG'}{70G^b}$$

These functions then permit integration of the second order term and so on. At each step, the error in omitting the higher order terms is of the order of the first term neglected. At each step, the integration and the algebra become considerably more tedious, but no more complex mathematically. Stopping after the second order term, results in the pressure gradient:

$$\frac{dP}{dX} = \frac{dP_0}{dX} + e \frac{dP_1}{dX} + e^a \frac{dP_2}{dX} + 0 (e^a)$$

$$= \frac{3}{2} \frac{Q}{G^a} + e \frac{27Q^aG'}{70G^b} + e^2 \frac{81Q^a}{64G^a} \left\{ C_1 G^a + C_2 G' G'' \right\}$$

where:

$$C_1 = 0.004574$$

$$C_2 = - 0.003430$$

(20)
Now if we were interested in the problem of incompressible flow through our channel \( Y = \pm G(X) \), we could simply substitute our expression for \( G(X) \) and integrate the \( \frac{dP}{dX} \) expression. For the form of \( G(X) \) chosen here and given above, this is integrable in closed form and has been carried out to provide an order of magnitude estimate of flow rate. In fact, this result is used in the computer program to provide the starting value of flow rate, \( w \), in the iterative calculation.

In our general solution, however, at this point we invoke the slowly changing compressible flow approximation and treat the expression which was derived for incompressible viscous flow as valid for compressible flow on the supposition that the changes due to compressibility take place over some distance and are not significant over a length comparable to channel width.

First, the original dimensioned variables are used in following the argument. Since \( Q \) was generated during the solution, its equivalent must be derived in terms of the original variables.

\[
Q = \int_{-G}^{h} U \, dY = \int_{-h}^{\frac{h}{AD}} \frac{u}{\bar{u}} \, \frac{dy}{AD}
\]

\[
= \frac{1}{AD\bar{u}} \int_{-h}^{\frac{h}{AD\bar{u}}} u \, dy = \frac{q}{AD\bar{u}}
\]

But \( \bar{u} = \frac{q}{2D} \), so

\[
Q = \frac{2q}{A}
\]

Also

\[
P = p \frac{A^2D^2}{\mu \bar{u} L} = p \left[ \frac{2A^2D^3}{\mu Lq} \right]
\]

and the Reynolds number

\[
e = \frac{qA^2D}{2\nu L}
\]

Reverting to the original variables, the expression for pressure gradient becomes

\[
\frac{dp}{dx} = \frac{3\mu q}{2h^3} \left\{ -1 + \frac{9qh'}{35\nu} + \frac{27q^2}{32\nu^3} \left[ C_1 h'^2 + C_3 hh' \right] \right\}
\]
Now, as discussed above, this equation is considered to be valid for isothermal, compressible flow. For steady flow, the product \( pq \) is a constant. Letting \( w = pq \) and multiplying the above equation by \( p \), the result is

\[
p \frac{dp}{dx} = \frac{3\mu w}{2h^3} \left\{ -1 + \frac{9wh'}{35\gamma p} + \frac{27w^2}{32\gamma^2 p^2} \left( C_1 h'' + C_2 \right) \right\}
\]

Most gases, and in particular helium which acts much like an ideal gas, have viscosity, \( \mu \), nearly independent of pressure. By the perfect gas law,

\[
\rho = \frac{p}{RT},
\]

and if subscripts \( o \) denote initial conditions, for isothermal flow, therefore,

\[
\frac{\rho}{\rho_o} = \frac{p}{p_o} \quad \text{or,}
\]

\[
\rho = \frac{p}{p_o} \rho_o.
\]

Thus, the kinematic viscosity \( \nu \) can be expressed as:

\[
\nu = \frac{\mu}{\rho} = \frac{\mu p_o}{\rho p_o}, \quad \text{and}
\]

\[
\nu p = \frac{\mu p_o}{p_o}.
\]

Thus, \( \nu p \) is also independent of \( p \) and can be expressed in terms of the initial conditions of the gas.

Thus, the above expression can be substituted for \( \nu p \) in the pressure gradient expression and taking account of the constancy of \( \mu, w, \) and \( \nu p \) in steady, isothermal, compressible flow, the expression can be integrated to obtain:

\[
p(x)^2 = p_o^2 = 3\mu w \left[ -x \int_0^x \frac{dx}{h^3} + \frac{9w\rho_o}{35\mu p_o} \int_0^x \frac{h'' dx}{h^3} \right]
\]

\[
+ \frac{27w^2\rho_o^3}{32\mu^2 p_o^2} \left( C_1 \int_0^x \frac{h'''}{h^3} dx + C_2 \int_0^x \frac{h''' dx}{h^3} \right)
\]

Introducing \( h(x) = D(1 + A \cos(x/L)) \), the integrations can be performed, and the resulting expression is
\[ p(x) = \sqrt{p_o^2 + 3\mu w \left[ -I_1 + \frac{9w_o^3}{35\mu p_o} I_2 + \frac{27w_o^2 c_o^2}{32\mu^2 p_o} \left( C_1 I_3 + C_2 I_4 \right) \right]} \]

where:

\[ I_1 = \frac{L}{D^3} \cdot \frac{1}{2(1-A^2)} \left[ \frac{1}{1+A \cos \frac{x}{L}} + \frac{3}{1-A^2} \right] \left( \frac{-A \sin \frac{x}{L}}{1+A \cos \frac{x}{L}} \right) \]

\[ + \frac{2(2+A^2)}{(1-A^2)^{3/2}} \tan^{-1} \sqrt{\frac{1-A^2}{1+A} \tan \frac{x}{2L}} \]

\[ I_2 = \frac{1}{2D^3} \left( \frac{1}{(1+A)^2} - \frac{1}{(1+A \cos \frac{x}{L})^2} \right) \]

\[ I_3 = \frac{A \sin \frac{x}{L}}{2LD (1+A \cos \frac{x}{L})} + \frac{1}{2} I_4 \]

\[ I_4 = \frac{-A}{DL (1-A^2)} \left\{ \frac{\sin \frac{x}{L}}{(1+A \cos \frac{x}{L})} - \frac{2A}{\sqrt{1-A^2}} \tan^{-1} \left( \frac{x}{(1+A)^2} \right) \right\} \]

The general form of the pressure curve obtained from this solution is a monotonic decreasing function with superimposed periodic fluctuations associated with the constrictions. After some experience with this expression and its use in the total flow model, it was found that an expression for the mean pressure without the periodic variations was more convenient and equally as good. This expression is derived from the above by simply considering points \( x = 2m\pi L \). Substituting this term into the pressure equation, dropping the periodic terms, and then rewriting \( 2m\pi L \) as \( x \), yields the smooth curve which is exact at the equally spaced intervals of \( 2\pi L \).

Thus:

\[ \left[ p(2m\pi L) \right] = p_o^2 + 3\mu w \left( -\frac{L (2+A^2)}{D^3 (1-A^2)^{3/2}} N_{\pi} \right. \]

\[ + \left. \frac{27w_o^2 c_o^2}{32\mu^2 p_o} \left( C_1 /2 + C_2 \right) \frac{2A^2 N_{\pi} \omega}{DL (1-A^2)^{3/2}} \right) \]

The \( N_{\pi} \) terms constitute the crucial part of this operation. Care must be exercised from a computational standpoint to make the arctan terms in the pressure expression, monotonic increasing functions. Thus, if one
considers \( \tan^{-1}(\omega) \) to mean the value of \( \tan^{-1} \alpha \) between \(-\pi\) and \(\pi\), the principal value, then

\[
\tan^{-1}\left(\sqrt{1 - A^2} \tan \frac{2 \pi mL}{1 + A^2}\right) = N\pi + 0
\]

where:

\[
N = \text{INT} \left(\frac{2m\pi + \pi}{2\pi}\right)
\]

= \text{INT} \(m + 0.5\)

the largest integer not exceeding \(m + 0.5\), i.e. \(m\). If \(2\pi mL\) represents a value of \(x\), then:

\[
N\pi = m\pi = x/2L.
\]

The final simplified expression becomes:

\[
[p(x)]^2 = p_o^2 + 3\mu w\left(\frac{-L (2 + A^2)x}{2D^3 (1 - A^2)^{5/2}} + \frac{27w^2 B^2 (0.5C_1 + C_2) A^2 x}{32 DL (1 - A^2)^{5/2}}\right)
\]

where

\[
B = \frac{p_o}{\mu p_o}
\]

NON-CONTINUUM FLOW

The classical expression (Kennard, Ref. 15, pg. 294) for slip and free-molecule flow between parallel walls was used for the non-continuum flow regime of the total flow. Although it is somewhat inconsistent to use a parallel wall model for this part of the flow, it was felt to be a minor shortcoming and a careful search of current technical literature produced no alternative solution which produced better results and was in a form simple enough to be directly applicable. The theoretical reasons for believing that the model would be adequate are that:

- The difference between parallel walls and gently curving walls is slight for the kinetic theory model upon which the solution is based
- Since diffuse reflection is assumed, this difference is further minimized
- The influence of the free molecule portion of the flow is to retard the flow, and using the mean height rather than accounting for the full effect of the constrictions errs on the safe side by predicting slightly higher flow rates.
The expression given by Kennard is then:

\[ pq = \frac{2D^3}{3\mu\Delta x} \left( \bar{p} + \frac{3\xi}{D} \right) \Delta p \]

where, \( \xi \), the slip coefficient, is a wall momentum change parameter analogous to Prandtl's mixing length for turbulent flow. For diffuse reflection \( \xi \) can be taken equal to the mean free path \( \lambda \). Thus, the equation becomes:

\[ pq = \frac{2D^3}{3\mu\Delta x} \left( \bar{p} + \frac{3p\lambda}{D} \right) \Delta p \]

where \( \bar{p} \) is the mean pressure and \( \lambda \) is the mean free path. In a set of units consistent with this expression, the mean free path can be found from (Ref. 16, pg. 28)

\[ \lambda = 1.682 \times 10^3 \frac{\mu}{p} \sqrt{T} \]

where \( T \) is the absolute temperature of the gas, or, combining this with the above gives:

\[ pq = \frac{2D^3}{3\mu\Delta x} \left( \bar{p} + \frac{3p\lambda}{D} \right) \left( 1.682 \times 10^3 \frac{\mu}{\lambda} \sqrt{T} \right) \]

If, from the continuum flow calculation, the transition point \( X_8 \) at which the pressure is \( P_8 \) corresponding to a Knudsen number \( (\gamma/2D) \) of \( K_8 \), is given and the terminal conditions \( P_9 \) and \( X_9 \) is known, the above expression yields a flow rate which can be compared with the continuum flow rate in the iterative solution for the actual rate.

**MACH NUMBER**

Finally, to be assured that the flow does remain subsonic and choking does not occur, it is advisable to calculate the maximum Mach number of the flow. The velocity of sound in a perfect gas, to which helium approximates very closely, is given by:

\[ S = \sqrt{g\gamma RT} \]

for:
- \( g \) = gravitational constant
- \( \gamma \) = ratio of specific heats
- \( R \) = gas constant
- \( T \) = temperature, °R

Since this is dependent only on temperature, for isothermal flow the initial value can be calculated which will then hold for the entire continuum portion.
of the flow. At any cross-section in the continuum portion of the flow, the average velocity of the flow is:

\[ \bar{u} = \frac{w}{p^2h} \]

Since \( w \) is constant for steady flow, \( \bar{u} \) is a maximum at the point of least pressure, that is the transition point. Thus the expression for Mach number will be:

\[ M = \frac{w}{2 \cdot P8 \cdot D \cdot S} \]

where \( h \) has been replaced by \( D \), the average value.

**COMPUTER PROGRAM**

The entire procedure has been programmed for the Time-Sharing computer. The program, which contains complete information on units as well as explanatory notes throughout, is listed at the end of this appendix, followed by typical outputs. Where possible, the same symbols are used as those in the analysis.
1 REM FLOW RATE FOR A CHANNEL WITH PERIODIC CONSTRUCTIONS
2 REM WALLS: Y=(+,-)D*(1+A*COS(X/L)); FLOW PassES FROM LAMINAR TO
3 REM FREE-MOLECULE FLOW AT X=X8 WHERE THE KNUDSEN NUMBER IS K8.
4 REM SOLUTION IS ITERATIVE BASED ON CONTINUITY OF FLOW RATE AT X8.
5 REM INPUT:D-IN,L-IN,A-(NO DIM),P0-PSI,T0-DEG. C,P9-PSI,X9-IN,V-IN
6 REM K8 (NO DIM.,=.MFP/DIAM)
7 REM DENSITY RO(LBF*SEC^2/IN^4) AND VISCOSITY M(LBF*SEC/IN^2) ARE
8 REM CALCULATED FOR HELIUM
9 REM TO OMIT ITER. W VS.QF RECORD,CUT 134,200,202,218,220,222(BUTSAVE)
10 READ T0,K8,P0,P9
11 READ D,L,A,X9,V
12 LET I =1
13 LET RO=PO/(386.2*386.*12.*1.8*(273.+T0))
14 LET M=(.3.95E-7/144.)*(353./(T0+353.))*((273.+T0)/273.)^1.5
15 LET X8=0.
16 REM CALCULATION OF PRESSURE CORRESPONDING TO K8
17 LET NO=M/RO
18 LET Y8=1128*M*SQR(T0+273)/D/KB
19 IF Y8>P0 THEN 22
20 LET PR=P0
21 GO TO 106
22 LET S=(PO-P9)/X9
23 LET Q =2 /A
24 LETA2=A*A
25 LET A3=1-A2
26 LET R=3.14156
27 LET A4=S.QR(A3)/(1+A)
28 LET D2=D*D
29 LET D3=D2*D
30 LET E=RO/(M*PO)
31 LET C1=.004574
32 LET C2=.00343
33 PRINT
34 PRINT
35 REM FIND A STARTING VALUE OF W-INCOMPR. FLOW, GROSS PRESS. DROP
36 REM MUST SOLVE: U*(M1*Et2-M2)+CONST.*DP/DX=0, WHERE E=REY.NO..(SMALL)
37 LET M1=81*Q*Q*A*A*(C1+2*C2)/(64*A3^1.5)
38 LET M2=1.5*Q*A*A*(2+A2)/A3^2.5
39 LET W9=1
40 DEF FNO(T)=T*(M1*A2*A2*D2*D2*T*T/NO/NO/L/L-M2)+2*A2*D2*S/M
41 LET E9=.02
42 REM FIRST TRY FOR U (EXACT IF E GOES TO 0)
43 LET T1=2*A2*D2*S/M/M2
44 GOTO 46
45 LET T1=T2
46 LET T2=T1+E9
47 LET O1=FNO(T1)
48 LET O2=FNO(T2)
49 LET T5=(2*A2*D2*S/M)/(M2-M1*A2*A2*D2*D2/NO/NO/L/L*T1*T1)
50 REM IF FIRST TRY GOOD ENOUGH, GO ON
51 IF ABS(T5-T1)>.01*T1 THEN 54
52 LET T2=.5*(T5+T1)
53 GOTO 70
54 IF W9=0 THEN 58
55 REM OBSERVE CONVERGENCE, METHOD IS INVALID FOR E TOO LARGE.
56 PRINT"TI","T2","01","02-
57 LET W9=0
58 PRINT T1,T2,01,02
59 IF ABS(01)>ABS(02) THEN 65
60 IF 02/ABS(02)=01/ABS(01) THEN 63
61 LET E9=E9/2
62 GOTO 45
63 LET E9=-E9
64 GOTO 45
65 IF O1/ABS(O1)=02/ABS(02) THEN 68
66 LET E9=-E9/2
67 GOTO 45
68 LET E9=E9/2
69 GOTO 45
70 LET W1=0
71 LET UO=T2
72 REM START OF THE COMPRESSIBLE FLOW CALCULATION
73 REM W, THE PV FLOW RATE, IS THE PARAM. IN THE PRESS. DROP EXPRESS.
74 LET W=PO*UO*2*D
75 LET W1=W*1E3
76 REM START, NEW W:
77 LET E=A2*D2*UO/NO/L
78 REM IN CALCULATING P (CALLED Y), USE DIMENSIONLESS X =X/L.
79 REM RATHER THAN A DIRECT PROGRESSIVE TESTING OF VALUES OF P(X)
80 REM TO FIND X8 WHERE P=P8, IT IS MUCH FASTER TO USE A TRIAL AND
81 REM ERROR METHOD. B1 AND B2 ARE THE CONVERGING LIMITS ON CHOICE OF X
82 LET X=X9/L
83 LET B1=0
84 LET B2=X9/L
85 LET G=1+A*COS(X).
86 LET J4=-L*(2+A2)*X/2/D3/A3t2.5
87 LET J5=27*W*W*B*B*(.5*C1+C2)*A2*X/32/D/L/A31,1.5
88 REM IF X IS BEYOND THE COMPR. FLOW ZERO PRESS. POINT, THE NFG. PRESS.
89 REM FROM THE EQUA. IS REPLACED BY ASmall,BUT POS. PRESS.=P9/2
90 IF PO*PO+3*M*W*(J4+J5)<0 THEN 127
91 LET Y=SQR(PO*PO+3*M*W*(J4+J5))
92 LET B8=Y/Y8
93 IF B2-B1<1 THEN 104
94 IF 1-B8 >.05 THEN 97
95 IF 1-B8 <-.05 THEN 100
96 GOTO 104
97 LET B2=X
98 LET X=B1+B8*(X-B1)
99 GOTO 85
100 LET B1=X
101 IF B1>.999*X9/L THEN 156
102 LET X=B2-(B2-B1)/B8
103 GOTO 85
104 LET X8=X*L
105 LET P8=Y
106 REM FREE-MOLECULE FLOW CALCULATION
107 LET L9=2.25E3*M*(T+273)t.5
108 LET Q8=.667*Dt3*(P8-P9)/(X9-X8)*(.5*(P8+P9)+3*L9/D)
109 IF Y8=P8 THEN 118
110 IF W=Q8 THEN 113

I-12
111 LET W1 = W
112 GOTO 114
113 LET W2 = W
114 IF ABS(W-Q8) < .05*W THEN 119
115 LET W = W + (Q8-W)*(W2-W1)*.5/ABS(Q8-W)
116 LET U0 = W/(2*D*PO)
117 GOTO 76
118 LET W = Q8
119 PRINT
120 PRINT 
121 PRINT 
122 PRINT 
123 PRINT "FLOW RATES: COMPRL "W,," MOLEC."Q8"
124 PRINT
125 PRINT "TRANSITION: X8/X9="X8/X9,", P8="P8,", K8="K8*K8/P8"
126 GOTO 136
127 LET Y = .5*Y8
128 GOTO 92
129 REM ITERATION OVERSHOT END... BACK UP A LITTLE AND TRY AGAIN
130 LET W1 = W
131 LET W = W*(1 + .5*ABS(Bg-1)/BF)
132 IF W < W2 THEN 134
133 LET W = .9*W2
134 LET U0 = W/(2*D*PO)
135 GOTO 76
136 LET CO = 1.434*SQR(9*TO/5+491.7)
137 LET MS = W/(PR*2*D*(I-A)*CO)
138 PRINT
139 PRINT "MAX. MACH NO. = "M8," SOUND SPEED-FT/SEC = "CO
140 PRINT
142 PRINT
143 PRINT "ATM. CC/SEC = "1.115*W," PER INCH PATH WIDTH"
144 PRINT
146 PRINT
147 PRINT "D-IN = "D," L-IN = "L," A = "A"
148 PRINT
149 PRINT "P0-PSI = "P0," P9-PSI = "P9," X9-IN = "X9
150 PRINT
151 PRINT "NO-IN+2/SEC = "NO," M-LBF*SEC/INT2 = "M," TO-DEG.C = "TO
152 PRINT
153 PRINT "RO-LBF*SEC+2/IN+4 = "RO
154 PRINT
155 GOTO 156
156 LET W = 2*W
157 GOTO 116
158 READ D,L,A,X9,V
159 GO TO 13
160 DATA 23.1,.01,14.7,2.4E-7
161 DATA 2.6878E-5,4.2955E-5,.6963117084,1.5618
162 DATA 2.3029E-5,3.7694E-5,.68333,17119,1.2646
163 DATA 1.9152E-5,3.2308E-5,.67548,1641,90326
164 DATA 1.8838E-5,3.2982E-5,.68585,16065,70929
165 END
LEAKRA 16:10 JUL 6, 1967

SOLUTION:

FLOW RATES: COMPR \( 7.44067 \times 10^{-3} \) MOLEC. \( 7.44067 \times 10^{-3} \)

TRANSITION: \( x_8/x_9 = 0 \) \( P_R = 14.7 \) \( K_R = 0.142868 \)

MAX. MACH NO. = \( 9.36291 \times 10^{-3} \), SOUND SPEED FT/SEC = 3311.52

FLOW RATE: \( \text{PSI*IN}^3/\text{SEC} = 7.44067 \times 10^{-3} \) \( \text{TORR-LITER/SEC} = 8.79446 \times 10^{-3} \)

ATM. CC/SEC = \( 0.29635 \times 10^{-3} \), PER INCH PATH WIDTH

NET ATM. CC/SEC = \( 1.29572 \times 10^{-3} \) FOR PATHS, WIDTH IN = \( 1.5618 \), NO. = 1

\( D-IN = 2.68780 \times 10^{-5} \) \( L-IN = 4.29550 \times 10^{-5} \) \( A = 0.69631 \)

\( P_0-PSI = 14.7 \) \( P_9-PSI = 2.40000 \times 10^{-7} \) \( X_9-IN = 0.17084 \)

\( NO-IN^2/\text{SEC} = 0.188623 \) \( M-LBF*SEC/IN^2 = 2.90817 \times 10^{-9} \) \( TO-DEG.C = 23.1 \)

\( R_0-LBF*SEC^2/IN^4 = 1.54179 \times 10^{-8} \)
APPENDIX II
Appendix II

MATED SURFACE SIMULATION AND STATISTICS

This appendix was written as an independent report for use by the General Electric Company's Telecommunications and Information Processing Operation (TIPO). It is incorporated in its entirety. The reader who is familiar with computer operations will recognize some material applicable only to TIPO. It should be ignored.
Mated Surface Simulation and Statistics

Program No. RDC/087/01

By, Harold W. Moore
Scientific Applications
Telecommunications & Information Processing Operations
General Electric Company
Schenectady, New York

Prepared for Dr. J. Wallach
Research & Development Center
General Electric Company

Contract NAS-8-4012

June 20, 1967
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Acknowledgments

This program was conceived by the author under the leadership of L. G. Gitzendammer (ATL) in 1964, extended to Gap Maps of Mated Surfaces under F. O. Rathbun (R&DC) in 1965. The Gap and Slope Distribution statistic was added under Dr. J. Wallach (R&DC) in 1967.

The author wishes to thank M. Allison and R. Jordan for their suggestions particularly in the writing and use of the MAZE Threader and Normalization routines. Also D. McKee, M. Ray, J. Alrutz and F. Sindel who programmed various modifications and extensions to produce the present capacity and flexibility.

Grateful thanks is also due Mrs. D. Johnson who deciphered my writing and typed the manuscript.
1.0 IDENTIFICATION

TITLE: MATED SURFACE SIMULATION
PROGRAM: RDC-087-01
LANGUAGE: FORTRAN IV and GMAP - GE 635 Computer
AUTHOR: H. W. Moore - May 10, 1967

Scientific Applications

TELECOMMUNICATIONS & INFORMATION PROCESSING OPERATIONS

This project funded by NASA contract NAS 8-4012

2.0 PURPOSE - Summary

This program will simulate surfaces sampled from a population of surfaces with given roughness characteristics. The user can superimpose on each an arbitrary curve to simulate camber, undulating surface, grooves or tool marks, etc. This arbitrary curve could be applied horizontally, vertically or at an angle. Surfaces with the same or different characteristics are automatically mated. They can be mated (1) directly or (2) flipped end to end or, (3) moved transversely an arbitrary amount. The total disparity or gap is calculated and printed. The surfaces are then pressed together by an arbitrary amount and the disparity or gap recalculated. The process of pressing together is continued step by step until all passages are blocked or, upon option, until all disparities are removed. The total area of contact is printed for each pressing and the user, by option, can have the gap map or matrix of disparities printed. An input option prints contours of the surface instead of the gap map.

There is also an option to delete (set to zero) any portion of the gap maps and obtain the following statistics, (1) distribution of gap heights, (2) distribution of slopes across the surface and (3) distribution of slopes along the length of the surface, (4) void area fraction.
3.0 **METHOD** - Mated Surface Simulation

The simulation of each surface is accomplished in two steps. First the surface roughness which is somewhat random in nature is simulated by combining local surface properties and conditional probability statistics and random numbers. Second the gross surface characteristics such as tilt, camber, grooves or tool marks are simulated by adding (superimposing) an arbitrary table. Both surfaces to be mated can have the same or different characteristics.

In general, a surface may contain any or all of the components of topography which can be described as:

1. Random function in the j direction
2. Random function in the k direction
3. Periodic functions in the j direction
4. Periodic functions in the k direction
5. Type 2 random functions in the m direction

In these descriptions j and k are assumed to be two orthogonal directions and may (but are not restricted to) correspond to directions respectively perpendicular and parallel to the direction of seal width. It is assumed, further, that for analytical purposes all seals may be assumed to be straight with length J and width K, even though in fact all seals must close upon themselves, and most are circular.

The j and k periodic functions noted would, in general, be dependent on the type of machining used and represent tool marks. The j and k random components represent the balance of surface roughness not described by the periodic components or by Type 2 random functions. The Type 2 random functions are intended to represent an overlay of imperfections on a surface of given character and include primarily scratches. They are indicated as being in the m direction to denote that they are not necessarily oriented relative to j and k. It is difficult to conceive of a practical surface which does not have components 1 and 2 to some extent. The last three components are essentially, if not entirely, absent in some surfaces such as are produced by diamond burnishing, lapping, honing, electropolishing, vapor blasting and some forms of casting. For purposes of being able to describe and discuss a surface, consider it to have superimposed on it a grid system with squares of a size consistent with the approximation that height can be considered to be uniform over the area of each square. Figure 3.1 shows such a grid where the seal is assumed to be 5 grid squares in width and some undefined length. Some squares are blank, designating contact or zero gap. Other squares are labeled with a digit or letter designating a finite non-zero gap, and possible leakage, though as indicated not all gaps result in
actual leaks. Here T designates Height 10, A designates Height 11, and B designates Height 12 etc.

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<tr>
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<td></td>
<td>7</td>
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</table>

Figure 3.1. Grid Showing Possible Leak Paths

3.1 Random Roughness

The technique for surface development is as follows: Let us enter a plot of cumulative probability versus quantized gap height; at any specific probability between 0 and 1, inclusive, there will be a corresponding gap height. Further, if our entry point is selected purely at random, the gap height obtained will be representative of the gap height in the (1, 1) grid element of a randomly selected sample surface. Designations such as (1,1) will be used to indicate the row and column numbers, respectively, of a grid square. If the process were repeated a number of times, the heights assigned to the (1, 1) elements should, statistically, have the same frequency of occurrence as would the heights of corresponding elements of randomly selected surfaces taken from the same family.

The assignment of height to the (2,1) element cannot be made in quite the same manner since the prior knowledge of the specific height assigned to element (1,1) biases the probability. Likewise after (1,1) and (2,1) have both been assigned, the probability of any height occurring at (3,1) is biased by knowledge of (1,1) and (2,1).

Several methods of assigning heights to elements (2,1), (3,1), etc. have been considered from two viewpoints. First is the reasonableness of the assigned heights, as compared to those found by measurements on comparable seal surfaces. Second is the suitability of the method for computer solution. Since the operation of assigning height must be repeated many times, computer time can be an important consideration in deciding between two methods that produce comparable results.

The method is that of using linear extrapolation coupled with a distribution or "uncertainty" function. If we assign a number (h, i, j) to an element to designate that the height of
the element in the \( i^{th} \) row and \( j^{th} \) column is \( h \), and a specific first element is given by \((6,1,1)\), then since there is a reasonably high correlation function between \((h,1,1)\) and \((h,2,1)\) there will be a high probability of \((h,2,1)\) being \((6,2,1)\), a somewhat lower probability of \((5,2,1)\) and \((7,2,1)\), etc. Likewise had the first point been \((2,1,1)\) there would be a high probability of \((2,2,1)\), a somewhat lower probability of \((1,2,1)\) and \((3,2,1)\) and a very low probability of \((10,2,1)\). In general, given a specific \((h,1,1)\), to get \((h,2,1)\) we can make a random selection from an appropriate distribution, and there will be needed as many distributions as there are values of \( h \); i.e. ten if there are ten heights.

Having selected and assigned values \((h,1,1)\) and \((h,2,1)\) we can assign \((h,3,1)\). Using a specific illustration to explain the process, assume we have assigned \((4,1,1)\) and \((5,2,1)\). Then by linear extrapolation we conclude \((6,3,1)\) will be a very probable value, but certainly \((5,3,1)\) and \((7,3,1)\) have a significant probability also, and, in general, there will be a complete distribution curve. It is true that to utilize all data previously assigned we should select from one of 100 distribution curves, and later for element \((h,4,1)\) should select from one of 1000 distributions. However, such is neither practical nor deemed necessary. Preliminary work indicates that the loss of accuracy will be unimportant if we select from the same ten distributions used to select \((h,2,1)\). Reducing from the larger numbers of distribution curves that are theoretically needed is essentially equivalent to setting equal to zero the auto-correlations for more distant known points. Some data are neglected and some accuracy is lost in the interest of simplifying calculations.

Using the method just described we can obtain \((h,3,1)\) and all other \( h \)'s in the first row beyond the second column. Likewise the procedure can be used to assign heights to all elements \((h,1,2)\), \((h,1,3)\), etc., in the first column.

The assignment of the remaining heights in the grid requires a slight modification of the procedure in that we need do linear extrapolation over a surface, rather than along a line, to determine which of the ten distributions will be used for making the random selection. For example, given \((6,1,1)\), \((5,2,1)\) and \((5,1,2)\), these three points determine a plane which if extrapolated to element \((2,2)\) gives a height at \((2,2)\) of \( h = 4 \). Using the distribution associated with \( h = 4 \) we may make a random selection and assign a value to \( h \) in square \((2,2)\).

Having assigned \((h,2,2)\) we can in turn assign \((h,3,2)\), \((h,4,2)\), etc., and also \((h,2,3)\), \((h,2,4)\) etc., and by like procedure assign a height to all remaining grid elements. In this manner a surface will have been developed which we intend to be representative of a sample surface selected at

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random from the whole population.

A cross section of such a surface indicating height measurement is shown in Figure 3.2. Note datum plane is one unit below surface. (i.e. minimum height is one)

Actually as is discussed in Reference 1 a weighted linear (plane) extrapolation is used to predict the conditional height in order to avoid diagonal bias. The formula is $d = b \cdot K_b + c \cdot K_c + a \cdot K_a$ where $a$, $b$, and $c$ are known adjacent heights and $d$ is the extrapolated conditional height. See Figure 3.3.

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\[ K_a + K_b + K_c = 1 \] must be satisfied.

\[ K_b = K_c = \frac{\sqrt{2}}{2} \] are used to overcome diagonal bias. Fixed point arithmetic is used exclusively in the table look-up routine and the heights are stored in the matrix \( IH \).

Thus, in using matrix notation the formula becomes

\[
IH(I,J) = \frac{(K_b*IH(I-1,J)+K_c*UG(I,J-1)-K_a*IH(I-1,J-1)+IROUND)}{KD}
\]

Where \( K_a, K_b, K_c, K_D \), and \( IROUND \) are input. \( K_c = K_b = 7071 \), \( K_a = 4142 \), \( K_D = 10000 \), and \( IROUND = 5000 \) could be used. Use of other values could be used to force bias. If \( K_D = 0 \), the \( d = b+c-a \) is used.

### 3.2 Periodic and Other Components

Maps for machined surfaces, in general, can be generated using a combination of periodic functions and maps for random surfaces. For example, a lathe-turning surface may be represented by a periodic function in the direction of the seal width plus a random variation. The periodic function is to represent the slope of the tool and the feed per revolution and, in general, would have a fundamental frequency and harmonics. The random component represents the tendency of the tool to tear metal rather than cut it smoothly to the tool profile.

The "periodic" function to be superimposed with random surface roughness is input as an arbitrary table. Therefore, it can include such things as camber, tilt, or a scratch or groove. \( MTB, KELTA, IDIR \) are control parameters that are read in. \( MTB \) is the number of entries in the periodic table to be read in. If \( MTB \) is zero there is no table and this section is bypassed. If \( KELTA = 0 \), the table is added to each row. If \( KELTA \) is not zero the table is added to \( KELTA \) columns and the table is shifted down on position for the next \( KELTA \) columns, etc. Thus, if \( KELTA = 1 \), the table is applied diagonally with a slope of one, if \( KELTA = 2 \), the slope 2, etc. If \( KELTA \) is larger than the row dimension, say 200, the table is not shifted for any column and could be used with another surface having \( KELTA = 0 \) to get perpendicular grooves.

### 3.3 Mating of Surfaces

After developing a surface, it is stored, its map printed (Figure 4.9) and the program proceeds to simulate a second surface. If the input \( NTABLE \) equals zero the program will use the same distribution curves (i.e. the second surface is to be randomly selected from the same population as the first). If \( NTABLE \) is not equal to zero its value is the number of height used for a new set of distribution curves which are then read in.
The second surface is then generated and printed in the same manner as the first.

The two surfaces are then ready to be mated. If the input IDIR=1 the surfaces are mated in the same position as generated. If IDIR=2 the first surface is switched end to end before mating. Control parameters NSLIP, NRS, IDECR, INCR, ITHRU, MXOUT, and NDELET are read in. The surfaces are then mated or touched together by adding their respective disparities and subtracting 2 (i.e. G(I,J) = H1(I,J) + H2(I,J) -2). The (2) is subtracted because the individual surface disparities were measured from a datum plane one step above or below the surface. (See Figure 3.1). Now the surfaces may be just touching if some of the nearest points of each surface coincided, (Figure 4.13).

The surfaces are then pressed together the number of levels indicated by the parameter IDECR (initial decrement) by subtracting IDECR from each gap height. Negative results are set to zero to indicate contact. No allowance is made for displaced material. The gap map is printed (Figure 4.11) and the percent zeros printed (i.e. the percent of area in contact). The gap map is then further compressed by the amount INCR to obtain a new gap map. At each compression MAZE, the maze threader subroutine, is called to determine if a flow path across the gap or flow map exists. If a flow path exists the compression process is continued by the increment INCR. If flow is blocked ITHRU is tested. ITHRU=1 means stop the compression process. ITHRU=2 means to continue the compression process until all disparities disappear.

If NSLIP is greater than one it means to slip surface one over NRS rows and mate the two surfaces in this new position. NSLIP is reduced by one repeating the process until NSLIP=1. NSLIP=1 indicates the end of the mating and INO is read in. INO=3 ends the run; INO=2 means to select a new pair of surfaces from the same distribution and repeat the whole process; INO=1 means read in a totally new set of distribution curves and repeat the process.

If MXOUT=1 the printing of the Gap Map is suppressed and the output consists of only the percent area of contact. MXOUT=2 prints the Gap Maps.

3.4 Flow Map Statistics

Some areas of the Gap Map do not contribute to or affect the flow paths across a seal. These may be gaps that are completely surrounded by areas of contact such as the element (7,6) in Figure 4.14 or those near the upper right hand corner. They could be Cul de Sacs or any other areas the user may wish to exclude from the flow map. To initiate this option NDELET is changed from 0 to 1 on the mating control card and deletion data inserted immediately following this card. Compare Figures 4.14 and 4.16. The deletion data is entered row-wise. The
first column of data on each card designates the flow map row and the pairs of numbers following designate the flow map columns to be deleted. All columns from the first of each pair through the last of each pair are deleted. On the card immediately following the deletion data a zero terminates the compression process and a negative number indicates the number of levels to compress before the next set of deletion data.

A flow gap map or flow map is printed for each level of compression along with the following statistics:

1. The distribution of gap heights.
2. The distribution of vertical slopes at each gap.
3. The distribution of horizontal slope at each gap. (slopes are not computed where an adjacent height is zero)
4. The void area fraction.

For (1), (2), (3) above both the raw distribution and the normalized distribution are output. See Figure 4.7.
4.0 INPUT/OUTPUT

To run a problem on a TIPO computer a TIPO control card must be filled out as shown in Figure 4.1. It serves as a routing card for the couriers and operators. The job number or sequence number appearing in the upper right hand corner of the TIPO control card must be punched in column 16 through 21 on the SNUMB card as shown in figure 4.2.

4.1 Deck Set-up

The SNUMB card is part of the deck to be read by the computer. It is the first card of the input deck. See Figure 4.3 for a typical deck set-up. The deck consists of (1) System control cards which have a $ punched in column one, (2) the binary or object deck which is inserted between the IDENT card and the OPTION cards, and (3) the problem data deck which is inserted between the INCODE and the ENDDJOB cards.

The second system control card (IDENT card) must have the user's charge number punched beginning in column 16 followed by a comma, then the user's name followed by a comma and finally the code 2B1. If additional details about system control cards is desired, see Reference 8.5.

4.2 Input

The first two cards of input constitute the list of symbols used to label the individual heights in the gap maps. There are 72 characters on the first card and 49 characters on the second card. Thus heights from 0 to 120 can be labeled.

Note: that the first one is blank which is used for the zero height representing contact. Since the label list is input, the user can pick any table to suit his need. For example, if contours of the gap maps are desired, a suitable label might be:

0000bbbbbb11111bbbb22222bbbb33333 etc.

where the b's represent blanks.

The third card is the initial random number. To avoid any repetition, the last random number generated by the previous run should be used (see Figure 4.15). Zero can be used the very first time. The format is I15. As is discussed in Section 7.0, all numbers may be punched anywhere in its field when the modified .FRRD subroutine is used. For standard Fortran all numbers must be right justified in their fields.

With the exception of the deletion data all the rest of the input uses the format 1415. This means there can be 14 fields on a card each 5 columns wide. (i.e. field one is column 1-5, field two is column 6-10, etc.) The deletion
Figure 4.1. TIPO Control Card

Figure 4.2. SNUMB Card
FIGURE 4.3 INPUT DECK
FIGURE 4.4 INPUT DECK-DELETION DATA
$ SNUMB    72751
$ IDENT   RDC/087/01/53,H MOORE,2B1
  *
  *
BINARY DECK
  *
  *
$ OPTION  FORTRAN,NOMAP
$ EXECUTE
$ LIMITS  25,40000,,10000
$ DISC    DI,A1,15R
$ DISC    DJ,A2,15R
$ INCODE  IBMF
123456789TBCDEFGHIJKLMNOPQRSTUVWXYZ+/01234567890ABCDEFGHIJKLMNOPQRSTUVWXYZ+/0
10  0 0 0 0 0 0 0 400 240010000
0 0 0 0 0 0 0 900 4400 8200 980010000
0 0 0 0 0 1800 5400 8500 99001000010000
0 0 0 600 3600 7800 9800100001000010000
0 0 300 2400 8000 950010000100001000010000
0 100 1100 8400 980010000100001000010000
0 300 8800 98001000010000100001000010000
100 8800 970010000100001000010000100001000010000
91001000010000100001000010000100001000010000
100 3700 6400 8200 9100 9600 9800 990010000
50 40 4142 7071 7071 10000 5000
80 5 1
0 1 2 3 4 5 6 5 4 3 2 1 0 1
2 3 4 5 6 5 4 3 2 1 0 1 2 3
4 5 6 5 4 3 2 1 0 1 2 3 4 5
6 5 4 3 2 1 0 1 2 3 4 5 6 5
4 3 2 1 0 1 2 3 4 5 6 5 4 3
2 1 0 1 2 3 4 5 6 5
0 0
50 40 4142 7071 7071 10000 5000
80 5
0 1 2 3 4 5 6 5 4 3 2 1 0 1
2 3 4 5 6 5 4 3 2 1 0 1 2 3
4 5 6 5 4 3 2 1 0 1 2 3 4 5
6 5 4 3 2 1 0 1 2 3 4 5 6 5
4 3 2 1 0 1 2 3 4 5 6 5 4 3
2 1 0 1 2 3 4 5 6 5
1 1 8 1 1 2 0
3
$ ENDJOB

FIGURE 4.5 INPUT DECK-WITH PERIODIC DATA
data (to be described below) uses the format 23I3.

The fourth card is NH, the number of heights used to describe the random roughness. NH=10 in the example shown. NH must be no greater than 20. Following the 4th card are NH tables of Conditional Distribution Curves. Each table begins on a new card. The first table corresponds to height NH. The second table for height NH-1, hence, down to height 1. Following these tables is a similar table of height distribution for the total surfaces. Therefore, there are NH+1 tables. These tables are cumulative probability tables. For example, the first table (Figure 4.3) corresponding to height 10, implies zero probability to go to height 7 or below, 4% probability to go to height 8, 20% probability to go to height 9, and 76% probability of remaining at height 10.

Following the distribution tables the next card contains N, M, KA, KB, KC, KD, IROUND. N is the number of columns, and M is the number of rows of the first surface. (N < 125, M < 200) The other parameters will normally be used as shown.

The next card (0, 0, 0 of Figure 4.3 or 80, 5, 1 of Figure 4.5) contains MTB, KELTA, IDIR. If MTB=0 there is no periodic data for this surface. If MTB≠0 (as in Figure 4.5 where MTB=80), there are MTB values read in (14 numbers per card). The table shown in Figure 4.5 represents a sawtooth curve. The next card is NTABLE which is zero in each example. This signals that the second surface is to use the same distribution data as the first surface. If NTABLE≠0 it is considered to be the new NH for the second surface and a complete set of distribution data must follow.

The next card is N, M, KA, KB, KC, KD for the second surface.

The next card is MTAB, KELTA for the second surface.

The next card contains the mating control parameter; NSLIPS, NRS, IDECR, INCR, ITHRU, MXOUT, NDELET.

NSLIPS=1 means no slipping option.
NSLIPS < 2 means to slip the first surface NRS units and remate the surfaces. This is repeated NSLIPS times.

IDECR is the number of units the two surfaces are to be pressed together for the first compression. INCR is the number of units for each successive compression.

ITHRU=1 means stop compression when flow blocks.
ITHRU=2 means continue compression all the way.
MXOUT = 1 means bypass print of maps
MXOUT = 2 means print out gap maps
NDELET = 0 means no deletion data

If NDELET = 1 as shown in Figure 4.4, deletion data follows the mating control card. The first number of each card is the row number and each additional pair of numbers represent the columns to be set to zero. Compare Figures 4.14 and 4.16. The first card of deletion data eliminates columns 34, 35 and 36 of row 2. Similarly the next card for row 3 and so forth thru row 7 for the example given.

The next card has a zero in the first column which terminates this set of deletions. If this card had been -2 then the next compression would have been two units and the program would have expected a set of deletion data for that level. Note again the deletion data fields are three columns wide, i.e. gap row number in columns 1-3, gap columns in column 4-6, 7-9, etc.

Upon completion of the case the next card containing INO is read.
INO = 3 means quit
INO = 2 means start a new case but use old distribution curves.
INO = 1 means start entirely new case beginning with the NH card.

4.3. Output

Output for the data in Figure 4.3 is given in full in Figure 4.6 through Figure 4.10.

Figure 4.6 is a print out of the distribution data for surface one. In the upper left hand corner is printed the sequence number that was punched on the SNUMB card. The last line is N, M, KA ...

Figure 4.7 records the initial random number prior to developing surface area. The first column on the left is the column number of the gap map. The number in the corresponding line represents the distribution of height for that column. Because of the borders necessary for maze threader the column numbers run from 2-51. At the bottom of the page the first total is the distribution of height for the total surface. The second total represents the same distribution normalized to 10000.

Figure 4.8 prints the periodic data for surface one. In this example this option was bypassed, hence, no table.
Figure 4.9 is the gap map for surface one. Note the first column represents the plenum which is at height 120. At the bottom of the page is a reprint of input data for second case. The zero denotes that no new distribution data was read in. The second surface is to be drawn from the same family as the first.

Figure 4.10, 4.11, and 4.12 are similar to 4.7, 4.8, 4.9 but for the second surface. The last line of Figure 4.12 records the input on the mating control card (i.e. NSLIP, NRS, DECR, INCR, ITHRU, MXOUT, NDELET)

Figure 4.13 is the gap map for the two surfaces. They are just touching. The line on the bottom applies to the next page, i.e. Figure 4.14, it is the output of the maze threader. If J=M there is blockage, if I=N there is flow leakage.

Figure 4.14 is the gap map for the first compression which is at level 6 since IDECR = 6.

Figure 4.15 is the last page of the output giving the last random number generated. This would be used for new runs.

The data deck, Figure 4.4, was run to obtain a deletion. The output is the same as that just described through the first eight pages except NDELET = 1.

Figure 4.16 is the same as Figure 4.14 except a number of deletions in rows 1 through seven have been made.

Figure 4.17 shows the additional statistics that are calculated for deletion runs. The final page for this run is identical to Figure 4.15 and is not reproduced here.
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<th>RANDOM NUMBER</th>
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</thead>
<tbody>
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<tr>
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TOTAL 38 138 444 639 455 157 91 20 11 7

TOTAL 190 880 3100 6295 8570 9355 9810 9910 996510000

Figure 4.7. Output - Page 2

II-23
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</table>

**Figure 4.10. Output - Page 5**

II-26
MTB = 0  KELTA = 0  IDIR = 0
| 2 | 223322112334567765321 | 233 |
| 3 | 2123121234977755311 | 132 |
| 4 | 3121122644334322 | 21 |
| 5 | 23521311 | 11 |
| 6 | 142221311 | 12 |
| 7 | 1 | 11 | 2 | 1 | 22 | 1 | 1 |
| 8 | 1 | 1 | 31 | 12 | 1 | 33 |
| 9 | 1 | 12 | 2 | 123 | 32 |
| 10 | 1 | 21 | 2 | 1111 | 12 | 23 |
| 11 | 21 | 1 | 452331 | 1 | 1111 | 12 | 12112 |
| 12 | 121 | 145321 | 2 | 2112 | 1 | 1 | 112 | 1 |
| 13 | 1112 | 13443211 | 1 | 11212 | 1 |
| 14 | 121 | 2121142333 | 1 | 11 | 335221 | 11 |
| 15 | 2121 | 12212211 | 11 | 17 | 211 | 1 |
| 16 | 2 | 1121223 | 211 | 1311 | 111211 | 1 |
| 17 | 1234343342 | 1 | 1111 | 11 |
| 18 | 2235465432 | 1 | 2111 | 2 |
| 19 | 3423 | 322445333322121 | 11 | 22 | 222111 | 122 |
| 20 | 14763421573211332 | 1 | 2 | 331 | 22 | 21 |
| 21 | 4631 | 321111 | 1212132 | 122 |
| 22 | 452 | 2123313342 | 21 | 122 | 111111 |
| 23 | 1145423322335441 | 221232 |
| 24 | 12431221 | 22331 | 1111 |
| 25 | 111431111 | 1231141 | 11332111 |
| 26 | 121 | 45332212311221 | 11 | 33111 | 12 |
| 27 | 21 | 212122 | 21 | 1121 | 14211 | 2 | 1 | 1422 |
| 28 | 31 | 2341 | 1 | 142 | 1 | 111 | 11 | 12 |
| 29 | 113 | 21211 | 1 | 14 | 31 | 1 | 21 | 11 |
| 30 | 31 | 2131 | 1 | 11 | 3 | 11 | 11 |
| 31 | 2121 | 2 | 13 | 41 | 1 | 1 | 11 |
| 32 | 31 | 1122 | 1 | 1 | 3 | 33 | 31 | 1122 | 2 | 12322 |
| 33 | 6542 | 111 | 3 | 1 | 1 | 1 | 1343 |
| 34 | 77861 | 22 | 1 | 2111 | 1 | 3211 | 111 | 1 | 11 |
| 35 | 87741 | 21 | 1 | 1 | 1 | 3411 | 2331 | 1 | 121 |
| 36 | 96741 | 2 | 14653211 | 1113231 | 11 |
| 37 | AT942 | 3121 | 5842111 | 222423 | 211131 |
| 38 | 13799111211 | 21124411 | 1137531332222113 | 322132 |
| 39 | 1377332233 | 111122432 | 256322 | 322111 | 32 | 324 |
| 40 | 1387541112 | 2211121322 | 1355233152 | 31 | 23 | 345 |

THERE ARE 55.7500 PERCENT ZEROS

Figure 4.14. Output - Page 9
Figure 4.16. Output - Page 9 - Deletion Run
### DISTRIBUTION DATA

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<tr>
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<th>NORMALIZED HEIGHTS</th>
<th>NORMALIZED SLOPES</th>
<th>NORMALIZED SLOPES</th>
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</table>

**TOTALS:**
- 802 values
- 1.00000
- 629 values
- 1.00000
- 610 values
- 1.00000

**VOID AREA FRACTION = 0.43100**

Figure 4.17. Output - Page 10 - Deletion Run
5.0 FLOWCHART - Mated Surface Simulation Gross Logic

START
READ: GAP LABEL
INITIAL RANDOM NO.
ICOUNT = 1
READ
531 NUMBER OF HEIGHTS - NH

ENTRY
7420 NH = NTABL
7430 READ, OUTPUT, AND
NORMALIZE HEIGHT
501 DISTRIBUTION TABLES
ENTRY
542 READ & OUTPUT
DIMENSIONS N, M
AND WEIGHT FACTOR
IDA, IKB, IKC, IKD
IRONOUND

OUTPUT INITIAL RANDOM NO.
SET MODE: LOGIC
FOR SAVING SURFACE
ON DISC
N=N+1, M1=M+1, MAX=M+2

RANDOM SELECT FIRST
ELEMENT, IH(2,2) FROM TOTAL
SURFACE DISTRIBUTION

SET: FIRST COLUMN = 0
543 LAST COLUMN = -1
550 IHIST (I) = 0

DO 570 I = 3, N
COMPUTE FIRST
570 COLUMN (I,2)

SET IHIST = 0

DO 550 I = 3, M1
SET: PLENUM ROW = 120
COMPUTE (FIRST
580 ROW, IH (2,1))

OUTPUT HISTOGRAM
FOR FIRST ROW

DO 590 I = 3, N
SET IHIST = 0

DO 588 J = 3, M1
COMPUTE EACH
588 ROW IH (I,0)

PRINT ROW
HISTOGRAM

600 PRINT TOTAL
HISTOGRAM
PRINT TOTAL
CUMULATIVE
750 HISTOGRAM

810 SET KNZ = 1
IMZ = 2, JMZ = 2 FOR
MAZE THREADER

731 READ & OUTPUT
MTB, KELTA, IDIR

7400 IF (MTB)
7410 READ & OUTPUT: MTB VALUES
OF PERIODIC TABLES
IF (KELTA)
74 ADD TABLE
ROW-WISE
73 ADD TABLE
COLUMNWISE
MOVING DOWN
ONE AFTER EACH
78 KELTA COLUMNS

80 OUTPUT
730 GAP MAP

ICOUNT
1 2
100 190
FLOWCHART - Continued

ENTRY
190 PUT SECOND MATRIX ON DISC

ENTRY
100 PUT FIRST MATRIX ON DISC

IF (IDIR)
+ 110 DIRECT
111 STORAGE

120 REVERSE
121 STORAGE

130 ICONT = 2

READ & OUTPUT: NTABLE

IF (NTABLE)
+ 7420

IF (IDIR)
+ 542

DO 300 K0 = 1, NSLIP

READ EACH MATRIX FROM DISC ROW-WISE.
ADDING THEM TOGETHER AND SUBTRACTING 2 TO GET GAP MAP

250 TO GET GAP MAP

IF (NDELET)

7450 OUTPUT
270 GAP MAP

7460 IDECRT = IDECR

290 KTZER = 0

COUNT NO. OF ZEROS

295

IF (NDELET)

7480 READ DELETION DATA FOR ROW JD

IF (JD)

7494 MAKE DELETIONS

7470 SET LAST ROW NEGATIVE FOR MAZE ROUTINE

CALL MAZE

RESTORE LAST ROW

MXOUT

NO 1 2 YES

901 900
COMMON IH,IMZ,JMZ,KMZ
COMMON IHIST(20),ITHIST(20),IRN,K,NH,I,J,N,M,MAX,M1
COMMON ID0(20),ID1(20),ID2(20),ID3(20),ID4(20),ID5(20)
COMMON ID6(20),ID7(20),ID8(20),ID9(20),ID10(20),ID11(20),ID12(20)
COMMON ID13(20),ID14(20),ID15(20),ID16(20),ID17(20),ID18(20)
COMMON ID19(20),ID20(20)
COMMON N_TAB(200)
DIMENSION LABEL(121),IPRNT(320),IHT(320)
DIMENSION IV(126),IVH(126)
EQUIVALENCE(IV,IVH)
9050 DIMENSION IHEIGHT(240),IHS(240),IVS(240),N1(11),N2(11)
   READ (5,595) (LABEL(I),I=1,72)
595 FORMAT (72A1)
   READ (5,5951) (LABEL(I),I=73,121)
5951 FORMAT (49A1)
540 READ (5,541) IRN
541 FORMAT (I15)
5531 ICONT=1
531 READ (5,530) NH
   GO TO 7430
7420 NH=N_TAB
7430 GO TO (501,502,503,504,505,506,507,508,509,510,511,512,513,514,515,
      516,517,518,519,520,NH)
530 FORMAT (14I5)
520 READ (5,530) (ID20(I),I=1,NH)
   WRITE (6,600) (ID20(I),I=1,NH)
   CALL NORM(ID20)
519 READ (5,530) (ID19(I),I=1,NH)
   WRITE (6,600) (ID19(I),I=1,NH)
   CALL NORM(ID19)
518 READ (5,530) (ID18(I),I=1,NH)
   WRITE (6,600) (ID18(I),I=1,NH)
   CALL NORM(ID18)
517 READ (5,530) (ID17(I),I=1,NH)
   WRITE (6,600) (ID17(I),I=1,NH)
   CALL NORM(ID17)
516 READ (5,530) (ID16(I),I=1,NH)
   WRITE (6,600) (ID16(I),I=1,NH)
   CALL NORM(ID16)
515 READ (5,530) (ID15(I),I=1,NH)
   WRITE (6,600) (ID15(I),I=1,NH)
   CALL NORM(ID15)
514 READ (5,530) (ID14(I),I=1,NH)
   WRITE (6,600) (ID14(I),I=1,NH)
   CALL NORM(ID14)
513 READ (5,530) (ID13(I),I=1,NH)
   WRITE (6,600) (ID13(I),I=1,NH)
   CALL NORM(ID13)
512 READ (5,530) (ID12(I),I=1,NH)
   WRITE (6,600) (ID12(I),I=1,NH)
CALL NORM(ID12)
511 READ(5,530) (ID11(I),I=1,NH)
WRITE(6,600) (ID11(I),I=1,NH)
CALL NORM(ID11)
510 READ(5,530) (ID10(I),I=1,NH)
WRITE(6,600) (ID10(I),I=1,NH)
CALL NORM(ID10)
509 READ(5,530) (ID9(I),I=1,NH)
WRITE(6,600) (ID9(I),I=1,NH)
CALL NORM(ID9)
508 READ(5,530) (ID8(I),I=1,NH)
WRITE(6,600) (ID8(I),I=1,NH)
CALL NORM(ID8)
507 READ(5,530) (ID7(I),I=1,NH)
WRITE(6,600) (ID7(I),I=1,NH)
CALL NORM(ID7)
506 READ(5,530) (ID6(I),I=1,NH)
WRITE(6,600) (ID6(I),I=1,NH)
CALL NORM(ID6)
505 READ(5,530) (ID5(I),I=1,NH)
WRITE(6,600) (ID5(I),I=1,NH)
CALL NORM(ID5)
504 READ(5,530) (ID4(I),I=1,NH)
WRITE(6,600) (ID4(I),I=1,NH)
CALL NORM(ID4)
503 READ(5,530) (ID3(I),I=1,NH)
WRITE(6,600) (ID3(I),I=1,NH)
CALL NORM(ID3)
502 READ(5,530) (ID2(I),I=1,NH)
WRITE(6,600) (ID2(I),I=1,NH)
CALL NORM(ID2)
501 READ(5,530) (ID1(I),I=1,NH)
WRITE(6,600) (ID1(I),I=1,NH)
CALL NORM(ID1)
500 READ(5,530) (IDO(I),I=1,NH)
WRITE(6,600) (IDO(I),I=1,NH)
CALL NORM(IDO)
542 READ (5,530) N,M,IKA,IKB,IKC,IKD,IROUND
WRITE (6,600) N,M,IKA,IKB,IKC,IKD,IROUND
WRITE (6,5540) IRN
5540 FORMAT (1H1,I15,14H RANDOM NUMBER)
IF (IKD) 8,8,9
8     IFORM = 1
GO TO 10
9     IFORM = 2
10     N = N + 1
M1=M+1
MAX=M+2
IF(MAX-160)5541,5542,5543
5542 MODE=4
GO TO 5549
5543 MODE=8
GO TO 5549
5541 IF(MAX-80)5544,5545,5542
5544 MODE=2
5549 CONTINUE
CALL IRAND(IRN)
CALL ITAB(ID0)
IH(2+2)=K
KK=K
KKK=K
DO 543 I=1,N
IH(I+1)=0
  543   IH(I,MAX)=-1
DO 550 L=1,NH
550   ITHIST(L)=0
DO 570 I=3,N
CALL KTRAN
IH(I+2)=K
KT =K+K-KK
KK=K
IF(KT) 5629,562*563
   562   K=1
GO TO 570
563   IF(KT-NH)565,565,564
564   K=NH
GO TO 570
565   K=KT
570 CONTINUE
KK=KKK
DO 551 L=1,NH
   551   ITHIST(L)=0
   ITHIST(KK)=1
   ITHIST(KK)=1
IH(I+2)=120
DO 580 I=3,M1
IH(I+1,I)=120
CALL KTRAN
KT=K+K-KK
KK=K
   ITHIST(K)=IHK(K)+1
   ITHIST(K)=IHK(K)
IH(I+2)=K
IF(KT)572,572,573
   572   K=1
GO TO 580
573   IF(KT-NH) 575,575,574
574   K=NH
GO TO 580
575   K=KT
580 CONTINUE
I=2
WRITE (6,600) I,(IHK(L),L=1,NH)
DO 590 I=3,N
590   DO 552 L=1,NH
   552   ITHIST(L)=0
   K=IH(I+2)
   ITHIST(K)=1
   KT=IH(I+1,2)
   DO 588 J=3,M1
   KKK=IH(I-1,J)
   GO TO (582,583) , IFORM
   582   K = K+KKK-KT
   GO TO 584
   583   K = (K+KK*IKB-KT*IK+A+IORD)/IKD
   584   IF(K) 585,585,586

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585  K=1
GO TO 587
586  IF(K-NH) 587,587,589
588  K=NH
589  CALL KTRAN
      IH(I,J)=K
      IHIST(K)=IHIST(K)+1
590   KT=KKK
      WRITE (6,600) I*(IHIST(L),L=1,NH)
      DO 559 L=1,NH
591   IHIST(L)=IHIST(L)+IHIST(L)
      CONTINUE
      WRITE (6,601) (IHIST(L),L=1,NH)
592   FORMAT (16,2(10I5))
593   FORMAT (6H0TOTAL,2(10I5))
      DO 740 III=2,NH
740   IHIST(III)=IHIST(III)+IHIST(III-1)
      DO 750 III=1,NH
750   IHIST(III)=IHIST(III)*10000/IHIST(NH)
      WRITE (6,601) (IHIST(L),L=1,NH)
594   FORMAT (132H1)
810  KMZ=1
      IMZ=2
      JMZ=2
C     READ THE PERIODIC TABLE, SLOPE FOR ADDITION AND REVERSE ORDER
C     OPTION: IDIR IS ZERO FOR DIRECT OR 1 FOR REVERSE ORDER.
731  READ(5,530) MTB,KELTA,IDIR
      WRITE (6,5530) MTB,KELTA,IDIR
5530  FORMAT (IH1,6HMTB = ,15,5X,8HIDIR = ,15,5X,7HIDIR = ,15)
7400  IF(MTB) 7410*800410
7410  READ(5,530) (MTAB(IZ),IZ=1,MTB)
      WRITE (6,600) (MTAB(IZ),IZ=1,MTB)
595   IF(KELTA.)73v74p73
C     ADD PERIODIC TABLE ROW-WISE
74   DO 75 I=2,N
    DO 75 J=2,M1
75   IH(I,J)=IH(I,J)+MTAB(J-1)
    GO TO 80
C     ADD PERIODIC TABLE COLUMN-WISE OR DIAGONALLY
73   KA=0
      KZ=0
      DO 78 J=2,M1
      DO 76 I=2,N
      KO=KA+I-1
      IH(I,J)=IH(I,J)+MTAB(KO)
596   CONTINUE
      KZ=KZ+1
      IF(KZ=KELTA)78,77,77
77   KA=KA+1
      C III-40
KZ=0
78 CONTINUE
80 WRITE (6,5592)
WRITE (6,592)
WRITE (6,593)
DO 730 J=2,M1
DO 720 I=1,N
L= IH(I,J)
720 IPRNT(I)=LABEL(L+1)
730 WRITE (6,594) J,IPRNT(I),I=1,N)
GO TO (100,190),ICONT
C  PUT SECOND MATRIX ON DISC
190 MOD2=MODE
DO 191 I=1,N
DO 192 J=1,MAX
192 IPRNT(J)=IH(I,J)
CALL WDICs(I,MOD2,IPRNT,6H0000D1)
191 CONTINUE
GO TO 200
C  PUT FIRST MATRIX ON DISC
C  IDIR IS 0 FOR DIRECT OR 1 FOR REVERSE ORDER
100 MOD1=MODE
IF(IDIR)110,110,120
C  DIRECT STORAGE
110 DO 111 I=1,N
DO 112 J=1,MAX
112 IPRNT(J)=IH(I,J)
CALL WDICs(I,MOD1,IPRNT,6H0000D1)
111 CONTINUE
GO TO 130
C  REVERSE STORAGE
120 DO 121 I=1,N
DO 122 J=1,MAX
K0=MAX+1-J
122 IPRNT(J)=IH(I,K0)
CALL WDICs(I,MOD1,IPRNT,6H0000D1)
121 CONTINUE
C  RESET ICONT TO 2 AND READ NEW SURFACE OR ALL NEW INPUT
130 ICONT=2
READ (5,530) NTABLE
WRITE (6,600) NTABLE
IF(NTABLE)7420,542,7420
C  START SLIPPING PROCESS
C  READ NUMBER OF SLIPS AND NUMBER OF ROWS PER SLIP
200 READ (5,530) NSLIP,NRS,IDECR,INCR,ITHRU,MXOUT,DELETE
WRITE (6,600) NSLIP,NRS,IDECR,INCR,ITHRU,MXOUT,DELETE
C  START AT TOP
IVB=2
DO 300 KO=1,NSLIP
IVC=IVB
DO 250 I=2,N
C  BRING I-TH ROW IN OF MASTER MATRIX
CALL RDISC(I,MOD2,IHT,6H0000D1)
C  BRING IVC-TH ROW IN FROM DISC
CALL RDISC(IVC,MODE,IPRNT,6H0000D1)
DO 240 J=2,M1
IV(I,J)= IHT(J)+IPRNT(J)-2
240 CONTINUE
C  GO TO NEXT, ROW
  IVC=IVC+1
250 CONTINUE
  IF(NDELET) 7460,7450,7460
C  PRINT VOID
7450 WRITE(6,592)
  WRITE (6,592)
  WRITE (6,593)
  DO 270 J=2,M1
  DO 260 I=1,N
    L=IV(I*J)
260  IPRNT(I)=LABEL(L+1)
270  WRITE (6,594) J,IPRNT(I),I=1,N
7460  IDECRT=IDECRT
290 KTZR=0
  DO 295 I=2,N
  DO 295 J=2,M1
    JUNK=IV(I,J)-IDECRT
    IF(JUNK)291,292,293
291  JUNK=0
292  KTZR=KTZR+1
293  IV(I,J)=JUNK
295 CONTINUE
  IF(NDELET) 7480,7470,7480
C READ IN DELETION DATA
7480 READ (5,7481) JD(N1(L),N2(L),L=1,11)
7481 FORMAT (23I3)
  IF(JD) 7470,7470,0
C MAKE DELETIONS
7490 J=JD
  DO 7494 L=1,11
    NI=N1(L)
    IF(NI)7480,7480,7491
7491  NJ=N2(L)
    IF(NI-NJ) 7493,7493,7492
7492  NJ=NI
7493  DO 7494 I=NI,NJ
    IH(I,J)=0
7494 CONTINUE
  GO TO 7480
7470 DO 296 J=2,M1
296  IV(N*,J)=-IV(N*,J)
    CALL MAZE(IND)
  DO 297 J=2,M1
    IV(N*,J)=-IV(N*,J)
297  GO TO 901,900,MXOUT
900 WRITE (6,592)
  WRITE (6,592)
  WRITE (6,593)
  DO 285 J=2,M1
  DO 280 I=1,N
    L=IV(I,J)
280  IPRNT(I)=LABEL(L+1)
285  WRITE (6,594) J,IPRNT(I),I=1,N
901 ZERCT=M*N-M
  IZRCK=M*N-M
  IF(NDELET,NE.0) GO TO 7500
  ZERCT=FLOAT(KTZER)/ZERCT
WRITE (6,5593) ZERCT
5593 FORMAT (12HO THERE ARE,2P1F8.4,14H PERCENT ZEROS)
GO TO 9000
7500 DO 7499 I=1,240
IHS(I)=0
IVS(I)=0
7499 IHGHT(I)=0
7500 CONTINUE
C CALCULATION OF STATISTICS FOR DELETION RUN
DO 7505 J=2,M1
DO 7505 I=3,N
C IS THE ELEMENT POSITIVE OR ZERO
IF(IH(I,J)) 7505,7506,7501
C IS THE ADJOINING HORIZON ELEMENT POSITIVE
7501 IF(IH(I,J)) 7503,7503,7502
C CALCULATE HORIZON SLOPE (ADD 120 SO THERE ARE NO NEGATIVES IN IHS VECTOR)
7502 IS=IH(I,J)-IH(I,J-1)+120
C SUM OF INDIVIDUAL HORIZON SLOPES
IHS(IS)=IHS(IS)+1
C IS THE ADJOINING VERTICAL ELEMENT POSITIVE
7503 IF(IH(I-1,J)) 7505,7505,7504
C CALCULATE VERTICAL SLOPE
7504 IS=IH(I-1,J+1)-IH(I,J+1)+120
IVS(IS)=IVS(IS)+1
C SET UP IS FOR NONZERO COUNT OF ELEMENT
7505 IS=IH(I,J)
IHGHT(IS)=IHGHT(IS)+1
7506 CONTINUE
C PICKUP HEIGHT AND VERTICAL SLOPE COUNT FOR I=N
I=N
DO 7508 J=2,M1
IF(IH(I,J)) 7508,7508,7507
7507 IS=IH(I,J)
IHGHT(IS)=IHGHT(IS)+1
IF(IH(I,J+1)) 7508,7508,7520
7520 IS=IH(I,J+1)-IH(I,J)+120
IVS(IS)=IVS(IS)+1
7508 CONTINUE
7509 ISUMHS=0
ISUMVS=0
ISUMHT=0
DO 7510 I=1,240
ISUMHS=ISUMHS+IHS(I)
ISUMVS=ISUMVS+IVS(I)
7510 ISUMHT=ISUMHT+IHGHT(I)
SUMHS=ISUMHS
SUMVS=ISUMVS
SUMHT=ISUMHT
DO 7511 I=1,120
MAXS=121-I
IF(IHGHT(MAXS)) 7511,7511,7513
7511 CONTINUE
WRITE(6,7512)
7512 FORMAT (IH '35ERROR MESSAGE, NO POSITIVE HEIGHTS')
GO TO 9051
7513 THS=0.0
TVS=0.0
THT=0.0
MAXS2=2*MAXS
WRITE (6,8000) IDECR
DO 7516 I=1,MAXS2
LCOL=I-MAXS
ND=120+LCOL
HS=FLOAT(IHS(ND))/SUMHS
THS=THS+HS
VS=FLOAT(IVS(ND))/SUMVS
TVS=TVS+VS
IF(LCOL) 7514,7514,7515
7514 WRITE (6,8010) LCOL, IHS(ND), HS, IVS(ND), VS
GO TO 7516
7515 HT=FLOAT(IHGH(T(LCOL))/SUMHT
THT=THT+HT
WRITE (6,8020) LCOL, IHGH(LCOL), HT, IHS(ND), HS, IVS(ND), VS
7516 CONTINUE
WRITE (6,8030) ISUMHT, THT, ISUMHS, THS, ISUMVS, TVS
C CALCULATE FRACTION OF SURFACE WHICH IS NONZERO
HCT=SUMHT/ZERCT
WRITE (6,8040) HCT
IF(JD*NE.0) GO TO 7518
GO TO 9051
C START NEW DELETION RUN
7518 IDECRT=(-JD)*INCR
C TOTAL COMPRESSION (NEW RUN)
IDECRT=IDECR+IDECRT
GO TO 9001
8000 FORMAT (1H1,20HTOTAL COMPRESSION = ,13//1H0,30X,
135ND I S T R I B U T I O N D A T A/
21H0,34HSLOPES AND HEIGHTS NORMALIZED,8X,
322HORIZONTAL NORMALIZED=9X,21HORIZONTAL NORMALIZED/
412H DISPARITIES=14X,7HEIGHTS=12X,20HSLOPES HORIZONTAL=10X,
519HSLOPES VERTICAL=1H ,56X,6HSLOPES,24X,6HSLOPES///)
8010 FORMAT (1H0,1714X,19,4X,F10,5,7X,19,5X,F10,5)
8020 FORMAT (1H1,1714X,19,4X,F10,5,7X,19,4X,F10,5,7X,19,5X,F10,5)
8030 FORMAT (1H0,3X,7HTOTALS=110,2X,F10,5,6X=110,4X,F10,5,6X=110,
15X,F10,5,7X///)
8040 FORMAT (1H0,21HVOID AREA FRACTION = ,2X,F8.5)
9000 IDECRT = INCR
9001 GO TO (299,290), IND
299 IF(KTZER=I2RCK)998,300,300
998 GO TO (300,290), ITHRU
300 IVB=IVB+NRS
ICONT=1
9051 READ (5,530) INO
GO TO (531,542,999), INO
999 WRITE (6,5540) IRN
CALL EXIT
END
Two subroutines written in Machine Language (GMAP) are required. They are Fortran compatible and may have use in other applications. The first is IRAND, which is an integer random number generator producing integers 0 through 34, 359, 738, 367) from a rectangular distribution. Considerable testing of this generator has been done (see Reference 2). It requires an initial starting value IRN. IRN is replaced by the new number generated. (IRAND is listed in section 7.1) The author has other versions for floating random numbers in the range from 0.0 to 1.0 and from Gaussian or Normal Distribution.

The second GMAP subroutine WDISC writes and reads random records from the disc file. It is listed in Section 7.6.

The standard Fortran input routines may be used provided the data are right justified in their fields. To avoid this and make keypunching much easier the routine .FRDD has been modified to accept the data anywhere in the field. This is available in Binary form and is not listed in this report.

Four subroutines written in Fortran are required. They are NORM, ITAB, KTRAN, and MAZE and are listed in sections 7.2 to 7.5.

NORM is a routine to scale or normalize the height distribution data. The values of these tables represent probabilities which must range from 0 to 1.0000. The tables, however, are entered as integers from 0 to 10000 and must be made to correspond to the range of numbers produced by IRAND the random number generator. NORM, therefore, modifies the tables at input time avoiding the necessity of scaling each random number ever time it is used.

ITAB is an integer table look up subroutine. Given the random number IRN and the address of the proper tables, ITAB searches the table until it locates the smallest entry equal to or exceeding IRN. The index of this entry then corresponds to the level or height desired. This value is stored in K.

KTRAN subroutine, when given a predicted or estimated height K, randomly selects from the corresponding conditional distribution tables a new K, thus providing the randomness desired. KTRAN calls IRAND, selects the proper tables for ITAB, calls ITAB and returns.

MAZE is a subroutine to determine whether or not a flow path exists across a section of a seal. The logic is based on the fact that a maze may be threaded or successfully traversed simply by placing one's right hand on the wall on entering the maze and following that wall. (of course a similar left hand rule also holds) To determine if a path across a gap map exists within a given section one only needs to start at a corner and to apply the right hand rule until he reaches the opposite side or the opposite edge of the gap map. If the edge is reached at least one flow
path exists (perhaps many). If the side is reached then there is no flow path in the given section. The gap map consists of either zeros for contact or positive integers representing various size gaps. If each side of the section under study is bordered by zeros, and the beginning edge is bordered by a plenum, say at a level of 120, and an attempt to thread the maze is made, the program would have to test at every grid point to see if either the last row (farthest edge) or last column (farthest side) had been reached. This extra testing can be avoided by setting the opposite bordering columns to negative ones and negating the last row (farthest edge). The standard Fortran IF test now provides in a "single" test; End for negative, cell blocked for zero, and cell open for positive.
7.1 IRAND

$ GMAP DECK
SYMDEF IRAND
IRAND LDA 2*1
*
* CALL IRAND (IRN)
* RECTANGULAR DISTRIBUTION 000000000000 TO 377777777777
* PERIOD = 2**35
* GE 625 BY H MOORE TIPO DIALCOM 8*235 8352
* SEE TIS R63ASD3 BY A.M. OLESEN A STATISTICAL EXAMINATION
* 
ALS 10
ADLA 2*1*
ADLA CON
ANA CON1
STA 2*1*
TRA 0*1
CON OCT 262035034725
CON1 OCT 377777777777
END

7.2 ITAB

$ FORTRAN DECK
$ INCODE IBMF
CITAB SUBROUTINE ITAB H. MOORE/M. C. RAY TIP BLDG 5 AUG 18, 1965
SUBROUTINE ITAB (ID)
COMMON IH, IMZ, JMZ, KMZ
COMMON IHIST(20), ITHIST(20), IRN, K, NH, I, J, N, M, MAX, M1
COMMON ID0(20), ID1(20), ID2(20), ID3(20), ID4(20), ID5(20)
COMMON ID6(20), ID7(20), ID8(20), ID9(20), ID10(20), ID11(20), ID12(20)
COMMON ID13(20), ID14(20), ID15(20), ID16(20), ID17(20), ID18(20)
COMMON ID19(20), ID20(20)
COMMON nTAB(200)
DIMENSION IV(126*202), IH(126*202)
EQUIVALENCE (IV, IH)
DIMENSION ID(1)
DO 5 K=1, NH
   IF (IRN-ID(K)) 10, 10, 5
5 CONTINUE
   K=NH
10 K=K
RETURN
END
SUBROUTINE KTRAN

COMMON IH, IMZ, JMZ, KMZ
COMMON IHIST(20), ITHIST(20), IRN, K, NH, I, J, N, M, MAX, M1
COMMON ID0(20), ID1(20), ID2(20), ID3(20), ID4(20), ID5(20)
COMMON ID6(20), ID7(20), ID8(20), ID9(20), ID10(20), ID11(20), ID12(20)
COMMON ID13(20), ID14(20), ID15(20), ID16(20), ID17(20), ID18(20)
COMMON ID19(20), ID20(20)
COMMON hTAB(200)
DIMENSION IV(126, 202), IH(126, 202)
EQUIVALENCE (IV, IH)
CALL IRAND(IRN)
GO TO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, K
1 CALL ITAB(ID1 )
    GO TO 21
2 CALL ITAB(ID2 )
    GO TO 21
3 CALL ITAB(ID3 )
    GO TO 21
4 CALL ITAB(ID4 )
    GO TO 21
5 CALL ITAB(ID5 )
    GO TO 21
6 CALL ITAB(ID6 )
    GO TO 21
7 CALL ITAB(ID7 )
    GO TO 21
8 CALL ITAB(ID8 )
    GO TO 21
9 CALL ITAB(ID9 )
    GO TO 21
10 CALL ITAB(ID10)
    GO TO 21
11 CALL ITAB(ID11)
    GO TO 21
12 CALL ITAB(ID12)
    GO TO 21
13 CALL ITAB(ID13)
    GO TO 21
14 CALL ITAB(ID14)
    GO TO 21
15 CALL ITAB(ID15)
    GO TO 21
16 CALL ITAB(ID16)
    GO TO 21
17 CALL ITAB(ID17)
    GO TO 21
18 CALL ITAB(ID18)
    GO TO 21
19 CALL ITAB(ID19)
    GO TO 21
20 CALL ITAB(ID20)
21 RETURN
END
SUBROUTINE NORM (ID)
COMMON IH•IMZ•JMZ•KMZ
COMMON IHIST(20), ITHIST(20), IRN•K•NH•I•J•N•M•MAX•M1
COMMON IDO(20), ID1(20), ID2(20), ID3(20), ID4(20), ID5(20)
COMMON ID6(20), ID7(20), ID8(20), ID9(20), ID10(20), ID11(20), ID12(20)
COMMON ID13(20), ID14(20), ID15(20), ID16(20), ID17(20), ID18(20)
COMMON ID19(20), ID20(20)
COMMON •TAB(200)
DIMENSION IV(126,202), IH(126,202)
EQUIVALENCE (IV, IH)
DIMENSION ID(1)
DO 10 NRM=1•NH
10 ID(NRM)=3435973*ID(NRM)+(8367*ID(NRM))/10000
RETURN
END
SUBROUTINE MAZE(IND)

COMMON IH*IMZ, JMZ, KMZ
COMMON IHIST(20), ITHIST(20), IRN, K, NH, I, J, N, M, MAX, M1
COMMON ID0(20), ID1(20), ID2(20), ID3(20), ID4(20), ID5(20)
COMMON ID6(20), ID7(20), ID8(20), ID9(20), ID10(20), ID11(20), ID12(20)
COMMON ID13(20), ID14(20), ID15(20), ID16(20), ID17(20), ID18(20)
COMMON ID19(20), ID20(20)
COMMON NTAB(200)
DIMENSION IV(126, 202), IH(126, 202)
EQUIVALENCE (IV, IH)
I = IMZ
J = JMZ
K = KMZ

GO TO 202
1 K = 2
   GO TO 207
2 K = 4
   GO TO 204
4 K = 5
   GO TO 204
5 K = 7
   GO TO 206
7 K = 8
   GO TO 206
8 K = 10
   GO TO 201
10 K = 11
   GO TO 201
11 K = 1
   GO TO 207
101 K = 11
   GO TO 208
102 K = 12
   GO TO 208
103 K = 1
   GO TO 201
104 K = 2
   GO TO 200
105 K = 3
   GO TO 200
106 K = 4
   GO TO 207
107 K = 5
   GO TO 203
108 K = 6
   GO TO 203
109 K = 7
   GO TO 204
110 K = 8
   GO TO 205
111 K = 9
GO TO 205
112  K=10
GO TO 206
200  I=I+1
201  J=J+1
202 IF(IV(I,J)) 400,301,300
203  I=I+1
204  J=J-1
205 IF(IV(I,J)) 400,301,300
206  I=I-1
207 IF(IV(I,J)) 400,301,300
208  J=J+1
209  I=I-1
300 GO TO (1,2,2,4,5,5,7,8,8,10,11,11),K
301 GO TO (101,102,103,104,105,106,107,108,109,110,111,112),K
400 WRITE (6,401) I,J,N,MAX
401 FORMAT (3H I=,I4,3H J=,I4,3H N=,I4,3H M=,I4)
402 IND=1
RETURN
404 IND=2
RETURN
END
7.6 WDISC and RDISC Subroutine

The calling sequences:

CALL WDISC (RN, MODE, ARRAY, FC)
CALL RDISC (RN, MODE, ARRAY, FC)

RN  = record number

MODE = 1 to indicate record size of 40 words
= 2 to indicate record size of 80 words
= 4 to indicate record size of 160 words
= 8 to indicate record size of 320 words
= 16 to indicate record size of 640 words
= 32 to indicate record size of 1280 words

ARRAY = Core Storage for data to be transmitted

FC  = File Code (Hollerith)

Since this subroutine uses disc storage, proper disc allocation must be made. (see deck set up section 6.0) This is done by use of $ DISC control card in object deck as follows:

$ DISC DI, A1, 15R
$ DISC DJ, A2, 15R
7.6 WDISC/RDISC

$ GMAP DECK
$ INCODE IBMF
LBL WDISC
SYMDEF WDISC,RDISC

WDISC
LDA WDC
TRA ++2

RDIC
LDA RDC
STA MEME+3
STX1 RTN
LDA =07777,DL
ANA 5,1*
STA FP
LDQ =077,DL
ANQ 3,1*
MPY 2,1*
STQ DSP+1
LDQ =077,DL
ANQ 3,1*
MPY 40,DL
STQ DCP
LDX1 4,1
STX1 DCP

MEME
MME GEINOS

SDIA
ZERO FP,DSP
WDIC
ZERO FP,DCP
ZERO SRW
MME GEROAD
LDA SRW
CANA =0030000,DU

RTN
TZE **
LDQ =02547,DL
MME GEBORT
FP DEC 0
DSP ZERO ++1,1
DEC 0
DCP DEC 0
SRW DEC 0
DEC 0
WDC WDIC
RDC RDIC
END

LOAD WRITE DISC COMMAND
LOAD READ DISC COMMAND
SAVE RETURN
GET FILE CODE
STORE FILE CODE
GET RECORD TYPE
GET STARTING RECORD NUMBER
PUT IN SEEK DCW
GET RECORD TYPE
CALCULATE RECORD SIZE
PUT IN MEMORY DCW
GET ARRAY ADDRESS
READ OR WRITE DISC
WAIT FOR I/O TO FINISH
TEST I/O
GOOD
BAD
PASS THE BUCK
FILE CODE
POINTER
SEEK ADDRESS DCW
DCW FOR COMMAND
STATUS RETURN WORD

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8.0 REFERENCES


8.5 D. S. McKee - "A Fortran Programmer's Introduction to GE-600 Control Cards", May 1965, 65TIP2, General Electric Company