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THE RESPONSE OF HARD-LIMITING BAND PASS LIMITERS TO PM SIGNALS

HASD 822228

LOCKHEED ELECTRONICS COMPANY

Telecommunications Department

Systems Analysis Section

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PREFACE

The Systems Analysis Branch of the Information Systems Division performs the task of communication systems engineering and analysis at the Manned Spaceflight Center. This NASA organization provides support to the Apollo program in the areas of performance specifications, testing, tradeoff studies, and predictions.

The Systems Analysis Section, a part of the Houston Aerospace Systems Division of the Lockheed Electronics Company, aids in the accomplishment of these tasks by maintaining and operating a developmental, electronic and mechanical production and testing capability. The capability includes provisions for testing prototype models of telecommunication subsystems and components. In addition, a subsystem and component analysis capability is provided. The Systems Analysis Section performs this function as a part of the broader responsibilities of the Telecommunications Systems Department.

The analysis described in this report was accomplished to provide additional information concerning the behavior of specific elements of the Apollo Communication System. It forms the basis for evaluating results that are to be obtained from laboratory measurements. The plan for making these measurements is described in HASD 822227.

The analysis contained herein concerns the performance of bandpass limiters, when PM signals are received. A continuation of this analysis will be reported in HASD 822229. In that document the analysis is extended to a cascade connected bandpass limiter and a phase detector.

This report has been prepared for the Information Systems Division of NASA's Manned Spacecraft Center under Contract NAS 9-5191. It is distributed for information purposes only. The conclusions and recommendations contained herein should neither be regarded as representing a firm position of NASA's Manned Spacecraft Center, nor should they obligate or commit the Center in any way.

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SUMMARY

The output signal-to-noise ratio of a hard-limiting bandpass limiter is calculated for a phase-modulated signal. A direct approach using first order statistics of the random phase variable is employed for all input SNR levels. The computer program and the calculated results are given in this report. The resulting limiter SNR transfer characteristics are in close agreement with Blachman's result.

THE RESPONSE OF HARD-LIMITING BANDPASS LIMITERS TO PM SIGNALS

I. Introduction

The bandpass limiter is a non-linear device which is insensitive to the noisy irregularity of the amplitude of an input waveform. The effect of hard limiting of an AM or CW signal plus noise has been analyzed by Davenport (1). Other people (2) and (3) have done similar work concerning the performance of the limiter for AM signals.

Bandpass limiters are also used extensively in PM and FM receivers to remove amplitude fluctuations from the signal. Recently, Kuhar and Schilling (4) obtained a closed form approximation for the bandpass limiter signal-to-noise ratio characteristic, using Rice's approach (5). The work of Kuhar and Schilling agrees reasonably well with Davenport's result in the region of high signal-to-noise input. However, their result differs from that of Davenport in the threshold and very low input signal-to-noise regions. Sevy (6) has analyzed the effect of hard-limiting an angle-modulated signal and his conclusion is that the output signal-to-noise ratio of a limiter and zonal bandpass filter combination is the same for an angle-modulated signal and a CW signal. Lately, Blachman (7) gave a plot of the bandpass limiter output signal-to-noise ratio curve which is originally derived for a power law device (8).

The purpose of this report is to evaluate the concepts of Blachman and Sevy by performing the actual calculation. The calculation of the output signal-to-noise ratio of hard-limiting, bandpass limiters is accomplished in this report by the use of first order statistics. This direct approach will also be used advantageously in the calculation of the output signal-to-noise ratio of a limiter-phase detector combination in another report.

II. PROBABILITY DENSITY FUNCTION FOR THE RANDOM PHASE VARIABLE

As shown in Figure 1, we shall consider the effect of passing a sinusoidal signal and narrow band random noise through a limiter and zonal filter combination. The signal s(t) and noise n(t) can be expressed respectively as:

$$s(t) = a(t) \operatorname{Sin} \omega_c t + b(t) \operatorname{Cos} \omega_c t \tag{1}$$

$$n(t) = x(t) \operatorname{Sin} \omega_c t + y(t) \operatorname{Cos} \omega_c t \tag{2}$$

Where

$a = \sqrt{2P_s} \operatorname{Cos}\theta = E_s \operatorname{Cos}\theta$

$b = \sqrt{2P_s} \operatorname{Sin}\theta = E_s \operatorname{Sin}\theta$

$E_s(t)$ = rms value of signal $= \sqrt{a^2(t) + b^2(t)}$

P_s = signal power

$\theta(t)$ = signal phase $= \tan^{-1}\left(\frac{b(t)}{a(t)}\right)$

$x(t) = \sigma(t) \operatorname{Cos}\eta(t)$

$y(t) = \sigma(t) \operatorname{Sin}\eta(t)$

$\sigma(t)$ = rms value of noise $= \sqrt{x^2(t) + y^2(t)}$

η = noise phase angle $= \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$

Equations (1) and (2) can also be expressed as:

$$s(t) = E_s \left(\cos\theta \sin \omega_c t + \sin\theta \cos \omega_c t\right)$$

$$= E_s \sin \left(\omega_c t + \theta\right) \qquad (3)$$

$$n(t) = \sigma(t) \left(\cos\eta \sin \omega_c t + \sin\eta \cos \omega_c t\right)$$

$$= \sigma(t) \sin(\omega_c t + \eta) \qquad (4)$$

The input signal plus noise of the limiter is:

$$e_{in}(t) = s(t) + n(t)$$

$$= \left[a(t) + x(t)\right] \sin \omega_c t + \left[b(t) + y(t)\right] \cos \omega_c t$$

$$= R(t) \sin \left(\omega_c t + \phi\right) \qquad (5)$$

Where

$$R(t) = \sqrt{[a(t) + x(t)]^2 + [b(t) + y(t)]^2}$$

$$\phi(t) = \tan^{-1} \left(\frac{a(t) + x(t)}{b(t) + y(t)}\right)$$

In equation (5), we have the expression of the input signal plus noise, $e_{in}(t)$, in the terms of two orthogonal components, i.e.,

$$e_{in}(t) = X(t) \cos \omega_c t + Y(t) \sin \omega_c t \qquad (6)$$

Where

$$X(t) = x(t) + a(t)$$

$$Y(t) = y(t) + b(t)$$

The probability density functions of X(t) and Y(t) are:

$$P(X) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(X - a)^2}{2\sigma^2}} \tag{7}$$

$$P(Y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(Y - b)^2}{2\sigma^2}} \tag{8}$$

Where

σ^2 = variance of random variables X and Y

= noise power

The joint probability density function of the two independent random variables, X and Y, is:

$$P(X, Y) = P(X)\,P(Y)$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{(X - a)^2 + (Y - b)^2}{2\sigma^2}} \tag{9}$$

We want to find the joint probability density function, $P(R,\phi)$, of the envelope R and the random phase ϕ, put

$$e_{in}(t) = X(t) + Y(t)$$

$$= R(t)\,\mathrm{Cos}[\phi(t)] + R(t)\,\mathrm{Sin}[\phi(t)] \qquad (10)$$

Where

$$R(t) = \sqrt{X^2(t) + Y^2(t)}$$

$$\phi(t) = \tan^{-1}\left[\frac{Y(t)}{X(t)}\right]$$

Since

$$P(X, Y)dX\,dY = P(X, Y)\,R\,dR\,d\phi$$

$$= P(R, \phi)\,dR\,d\phi \qquad (11)$$

From equation (11), we have

$$P(R, \phi) = R(t)\,P(X, Y)$$

$$= \frac{R}{2\pi\sigma^2}\,e^{-\frac{(X-a)^2 + (Y-b)^2}{2\sigma^2}} \qquad (12)$$

Where

$$X(t) = R(t)\ \mathrm{Cos}[\phi(t)] \tag{13}$$

$$Y(t) = R(t)\ \mathrm{Sin}[\phi(t)] \tag{14}$$

Substituting equations (13) and (14) into equation (12), we get,

$$P(R,\phi) = \frac{R}{2\pi\sigma^2}\ e^{-\frac{(R\ \mathrm{Cos}\phi - a)^2 + (R\ \mathrm{Sin}\phi - b)^2}{2\sigma^2}}$$

$$= \frac{R}{2\pi\sigma^2}\ e^{-\frac{[R^2 + a^2 + b^2 - 2aR\mathrm{Cos}\phi - 2bR\mathrm{Sin}\phi]}{2\sigma^2}} \tag{15}$$

Notice that

$$R^2 + a^2 + b^2 - 2aR\ \mathrm{Cos}\phi - 2bR\ \mathrm{Sin}\phi$$

$$= R^2 + E_s^{\ 2} - 2RE_s\ (\mathrm{Cos}\phi\ \mathrm{Cos}\theta - \mathrm{Sin}\phi\ \mathrm{Sin}\theta)$$

$$= R^2 + E_s^{\ 2} - 2RE_s\ \mathrm{Cos}(\phi - \theta) \tag{16}$$

where

$$E_s^{\ 2} = a^2(t) + b^2(t)$$

$$a(t) = E_s\ \mathrm{Cos}\theta$$

$$b(t) = E_s\ \mathrm{Sin}\theta$$

Thus equation (15) becomes

$$P(R,\phi) = \frac{R}{2\pi\sigma^2} e^{-\frac{[R^2 + E_s^2 - 2RE_s \operatorname{Cos}(\phi - \theta)]}{2\sigma^2}} \tag{17}$$

The probability density function $P(\phi)$ of the random phase variable ϕ is:

$$P(\phi) = \int_0^\infty P(R,\phi)dR$$

$$= \int_0^\infty \frac{R}{2\pi\sigma^2} e^{-\frac{[R^2 + E_s^2 - 2RE_s \operatorname{Cos}(\phi - \theta)]}{2\sigma^2}} dR \tag{18}$$

Since

$$R^2 + E_s^2 - 2RE_s \operatorname{Cos}(\phi - \theta) = [R - E_s \operatorname{Cos}(\phi - \theta)]^2$$

$$+ E_s^2[1 - \operatorname{Cos}^2(\phi - \theta)] ,$$

equation (18) can be expressed as:

$$P(\phi) = \frac{e^{-\frac{E_s^2[1-\operatorname{Cos}^2(\phi-\theta)]}{2\sigma^2}}}{2\pi\sigma^2} \int_0^\infty Re^{-\frac{[R - E_s \operatorname{Cos}(\phi-\theta)]^2}{2\sigma^2}} dR$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{E_s^2[1 - \operatorname{Cos}^2(\phi-\theta)]}{2\sigma^2}}$$

$$\left\{ \int_0^\infty 2\sigma^2 \left[\frac{R-E_s \operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}\right] e^{-\frac{[R - E_s\operatorname{Cos}(\phi-\theta)]^2}{2\sigma^2}} \, d\left[\frac{R - E_s\operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}\right] \right.$$

$$\left. + E_s\operatorname{Cos}(\phi-\theta) \int_0^\infty e^{-[R - E_s\operatorname{Cos}(\phi-\theta)]^2/2\sigma^2} \, dR \right\}$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{E_s^2[1 - \operatorname{Cos}^2(\phi-\theta)]}{2\sigma^2}} \left\{ \int_{-\frac{E_s^2\operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}}^{\infty} \sigma^2 e^{-\alpha^2} \, d\alpha^2 \right.$$

$$\left. + \sqrt{2}\,\sigma E_s \operatorname{Cos}(\phi-\theta) \int_{-\frac{E_s\operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}}^{\infty} e^{-\alpha^2} d\alpha \right\} \tag{19}$$

The error function erf(x) , is defined as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha \tag{20}$$

and,

$$\operatorname{erf}(-x) = -\operatorname{erf}(x) \tag{21}$$

Using the relation of equations (20) and (21), equation (19) becomes,

$$P(\phi) = \frac{1}{2\pi\sigma^2} e^{\frac{-E_s^2[1 - \cos^2(\phi-\theta)]}{2\sigma^2}} \left\{ \sigma^2 e^{\frac{-E_s^2\cos^2(\phi-\theta)}{2\sigma^2}} \right.$$

$$\left. + \sqrt{\frac{\pi}{2}}\, \sigma\, E_s \cos(\phi-\theta) \left\{ 1 + \operatorname{erf}\left[\frac{E_s \cos(\phi-\theta)}{2\,\sigma} \right] \right\} \right\}$$

$$= \frac{e^{-\frac{E_s^2}{2\sigma^2}}}{2\pi} + \frac{E_s \cos(\phi-\theta)}{2\sqrt{2\pi}\,\sigma} e^{\frac{-E_s^2[1-\cos^2(\phi-\theta)]}{2\sigma^2}}$$

$$\left[1 + \operatorname{erf}\left(\frac{E_s \cos(\phi-\theta)}{\sqrt{2}\,\sigma} \right) \right] \tag{22}$$

Now, put

$$Z = \frac{\frac{E_s^2}{2}}{\sigma^2}$$

= Input Signal-to-Noise Ratio

and we have

$$P(\phi) = \frac{e^{-Z}}{2\pi} + \frac{\cos(\phi-\theta)}{2} \sqrt{\frac{Z}{\pi}}\, e^{-Z(1 - \cos^2(\phi-\theta))}$$

$$[1 + \operatorname{erf}(\sqrt{Z}\cos(\phi-\theta))] \tag{23}$$

Equation (23) is the probability density function of the random phase variable ϕ with input signal-to-noise ratio Z and signal phase angle θ.

III. Limiter Output Signal-to-Noise Ratio

Suppose we pass the input signal plus noise e_{in} through a limiter, the output e_L of the limiter is:

$$e_L = \begin{cases} +V_L & \text{Sign of } e_{in} \text{ is positive} \\ -V_L & \text{Sign of } e_{in} \text{ is negative} \end{cases} \tag{24}$$

In general, e_L is a series of square waves with period T and random phase ϕ as shown in Figure 2. The Fourier series expansion of e_L is:

$$e_L(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \operatorname{Cos} n\omega t + b_n \operatorname{Sin} n\omega t) \tag{25}$$

Where

$$a_n = \frac{2}{T} \int_{-t_\phi}^{T-t_\phi} e_L(t) \operatorname{Cos}(n\omega t)dt \tag{26}$$

$$b_n = \frac{2}{T} \int_{-t_\phi}^{T-t_\phi} e_L(t) \operatorname{Sin}(n\omega t)dt \tag{27}$$

From equation (26), we have

$$a_1 = \frac{2}{T} \int_{-t_\phi}^{\frac{T}{2} - t_\phi} V_L \operatorname{Cos}\omega t dt - \frac{2}{T} \int_{\frac{T}{2} - t_\phi}^{T-t_\phi} V_L \operatorname{Cos} \omega t \, dt \tag{28}$$

Since $T = \frac{2\pi}{\omega}$, equation (28) becomes

$$a_1 = \frac{V_L}{\pi} \int_{-\phi}^{\pi-\phi} \text{Cos}\phi \, d\phi - \frac{V_L}{\pi} \int_{\pi-\phi}^{2\pi-\phi} \text{Cos}\phi \, d\phi$$

$$= \frac{4V_L \, \text{Sin}\phi}{\pi} \tag{29}$$

Likewise, from equation (27) we get

$$b_1 = \frac{4V_L \, \text{Cos}\phi}{\pi} \tag{30}$$

The statistics of the random phase variable ϕ can be assumed stationary (in wide sense) and ergodic. Therefore, the first zonal filter output voltage can be expressed as

$$e_{out} = \frac{4V_L}{\pi} [\text{Sin}\phi \, \text{Cos} \, \omega_c t + \text{Cos}\phi \, \text{Sin} \, \omega_c t] \tag{31}$$

where the random phase variable ϕ is described by its probability density function $p(\phi)$.

The in phase part of $e_{out}(t)$ is $\frac{4V_L \, \text{Cos} \, \phi}{\pi}$

and the quadrature part is $\frac{4V_L \, \text{Sin} \, \phi}{\pi}$

The output signal, S_o may be represented as the ensemble average of $e_{out}(t)$, i.e.

$$V_o = E\left\{e_{out}(t)\right\}$$

$$= \frac{4V_L}{\pi}\left[E\left\{\mathrm{Cos}\phi\right\} \ \mathrm{Sin}\ \omega_c t + E\left\{\mathrm{Sin}\phi\right\} \ \mathrm{Cos}\ \omega_c t\right] \quad (32)$$

Where

$$E\left\{\mathrm{Cos}\phi\right\} = \int_o^{2\pi} \mathrm{Cos}\phi \ P(\phi)\ d\phi$$

$$E\left\{\mathrm{Sin}\phi\right\} = \int_o^{2\pi} \mathrm{Sin}\phi \ P(\phi)\ d\phi$$

$$P(\phi) = \frac{e^{-Z}}{2\pi} + \sqrt{\frac{Z}{\pi}}\ \frac{\mathrm{Cos}(\phi-\theta)}{2}\ e^{-Z(1-\mathrm{Cos}^2(\phi-\theta))}$$

$$\left[1 + \mathrm{erf}(\sqrt{Z}\ \mathrm{Cos}(\phi-\theta))\right]$$

Since $P(\phi)$ is an even function of ϕ,

$E\{\mathrm{Sin}\phi\} = 0$. This fact has been proven in the actual calculation of $E\{\mathrm{Sin}\phi\}$.

Thus equation (32) becomes

$$V_o = \frac{4V_L}{\pi}\ E\{\mathrm{Cos}\phi\}\mathrm{Sin}\ \omega_c t \quad (33)$$

Mean square signal, S_o is then

$$S_o = \frac{1}{2}\left(\frac{4V_L}{\pi}\right)^2 \quad (E\{\text{Cos } \phi\})^2 \tag{34}$$

The noise at the output of the first zonal filter consists of whatever is left after the signal portion is subtracted out, i.e.,

$$\begin{aligned} e_{no} &= \frac{4V_L}{\pi} \Big[(\text{Cos}\phi - E\{\text{Cos}\phi\}) \text{ Sin } \omega_c t \\ &\qquad + (\text{Sin}\phi - E\{\text{Sin}\phi\}) \text{ Cos } \omega_c t\Big] \\ &= e_{ni} + e_{nq} \end{aligned} \tag{35}$$

From equation (35), we can get the mean square noise power, i.e.,

$$\begin{aligned} E\{e_{no}{}^2\} &= \frac{8V_L{}^2}{\pi^2} \; E\left\{(\text{Cos}\phi - E\{\text{Cos}\phi\})^2\right\} \\ &\qquad + \frac{8V_L{}^2}{\pi^2} \; E\left\{(\text{Sin}\phi - E\{\text{Sin}\phi\})^2\right\} \\ &= N_{oi} + N_{oq} \end{aligned} \tag{36}$$

Where

N_{oi} = in phase noise power

$$= \frac{8V_L^2}{\pi^2} \; E\left\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\right\} \tag{37}$$

N_{oq} = noise power in quadrature

$$= \frac{8V_L^2}{\pi^2} \; E\left\{(\mathrm{Sin}\phi - E\{\mathrm{Sin}\phi\})^2\right\}$$

And

$$E\left\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\right\} = \int_0^{2\pi} (\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2 \, P(\phi) \, d\phi \tag{38}$$

$$E\left\{(\mathrm{Sin}\phi - E\{\mathrm{Sin}\phi\})^2\right\} = \int_0^{2\pi} (\mathrm{Sin}\phi - E\{\mathrm{Sin}\phi\})^2 \, P(\phi) \, d\phi$$

$$= \int_0^{2\pi} \mathrm{Sin}^2\phi \, P(\phi) \, d\phi \tag{39}$$

The noise in quadrature affects the phase parameter ϕ much more than the in-phase noise at high input signal to noise ratio. N_{oi} and N_{oq} are equally important in the very low input signal to noise ratio region.

The effective output noise is:

$$N_o = N_{oq}\left[1 + \left(\frac{N_{oi} \quad N_{oq}}{[S_o + N_{oi}]^2}\right)^{\frac{1}{2}}\right] \tag{40}$$

Substituting equations (34), (37) and (38) into equation (40), we have

$$N_o = \frac{8V_L^{\,2}}{\pi^2} \; E\left\{(\mathrm{Sin}\phi - E\{\mathrm{Sin}\phi\})^2\right\}$$

$$\left[1 + \frac{\left[E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\}\; E\,\{\mathrm{Sin}^2\phi\}\right]^{\frac{1}{2}}}{\left[(E\{\mathrm{Cos}\phi\})^2 + E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\}\right]}\right] \tag{41}$$

From equations (34) and (41), we obtain the output signal to noise ratio of the limiter, i.e.,

$$\frac{S_o}{N_o} = \frac{(E\{\mathrm{Cos}\phi\})^2}{N_{oq}\left[1 + \frac{\left[E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\}\; E\{\mathrm{Sin}^2\phi\}\right]^{\frac{1}{2}}}{\left[(E\{\mathrm{Cos}\phi\})^2 + E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\}\right]}\right]} \tag{42}$$

Where

$$E\{f(\phi)\} = \int_0^{2\pi} f(\phi)\; P(\phi)\; d\phi$$

$$= \text{ensemble average of } f(\phi).$$

$$N_{oq} = E\{(\mathrm{Sin}\;\phi - E\{\mathrm{Sin}\;\phi\})^2\}$$

IV. A COMPARISON OF COMPUTATIONS

A computer program has been compiled to calculate the output signal-to-noise ratio of the limiter (equation (42)). The computer program is listed in Appendix A. The results of the calculation are shown in Appendix B. For each input signal-to-noise ratio Z, the Appendix tabulation gives the calculated results of the output signal-to-noise ratio $\frac{S_o}{N_o}$ and the ratio of $\frac{S_o/N_o}{Z}$ given in dB.

The calculation also shows us that $E\{\sin \phi\} = 0$.

A plot of $\frac{S_o/N_o}{Z}$ versus Z (input signal-to-noise ratio) is shown in Figure 3. Figure 3 includes the results of Davenport, Springett, and Blachman. Our calculated result matches pretty well with Blachman's result.

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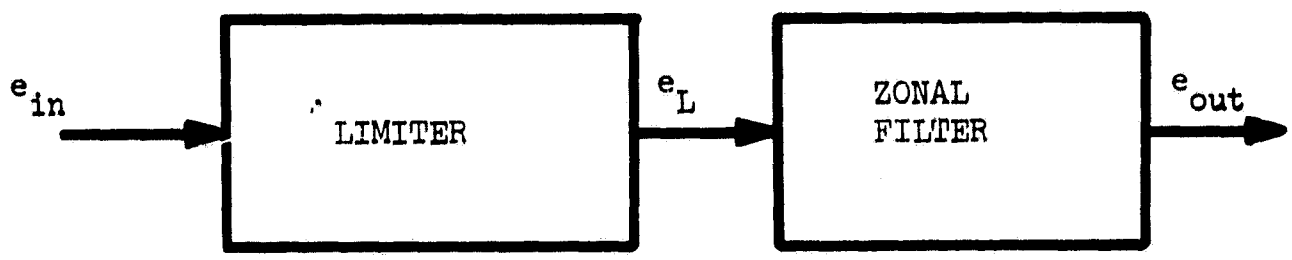

FIGURE 1 LIMITER-ZONAL FILTER COMBINATION

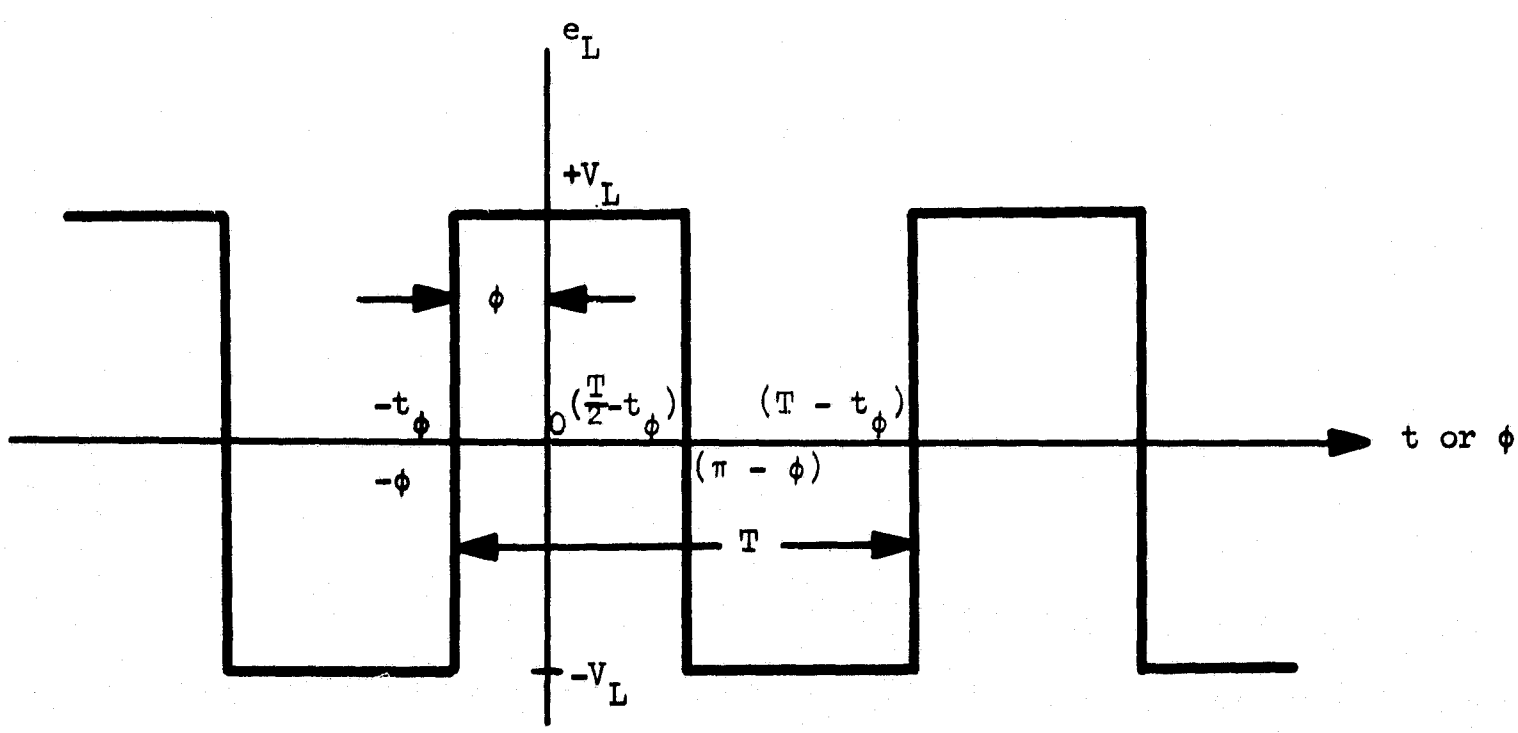

FIGURE 2 LIMITER OUTPUT WAVEFORM

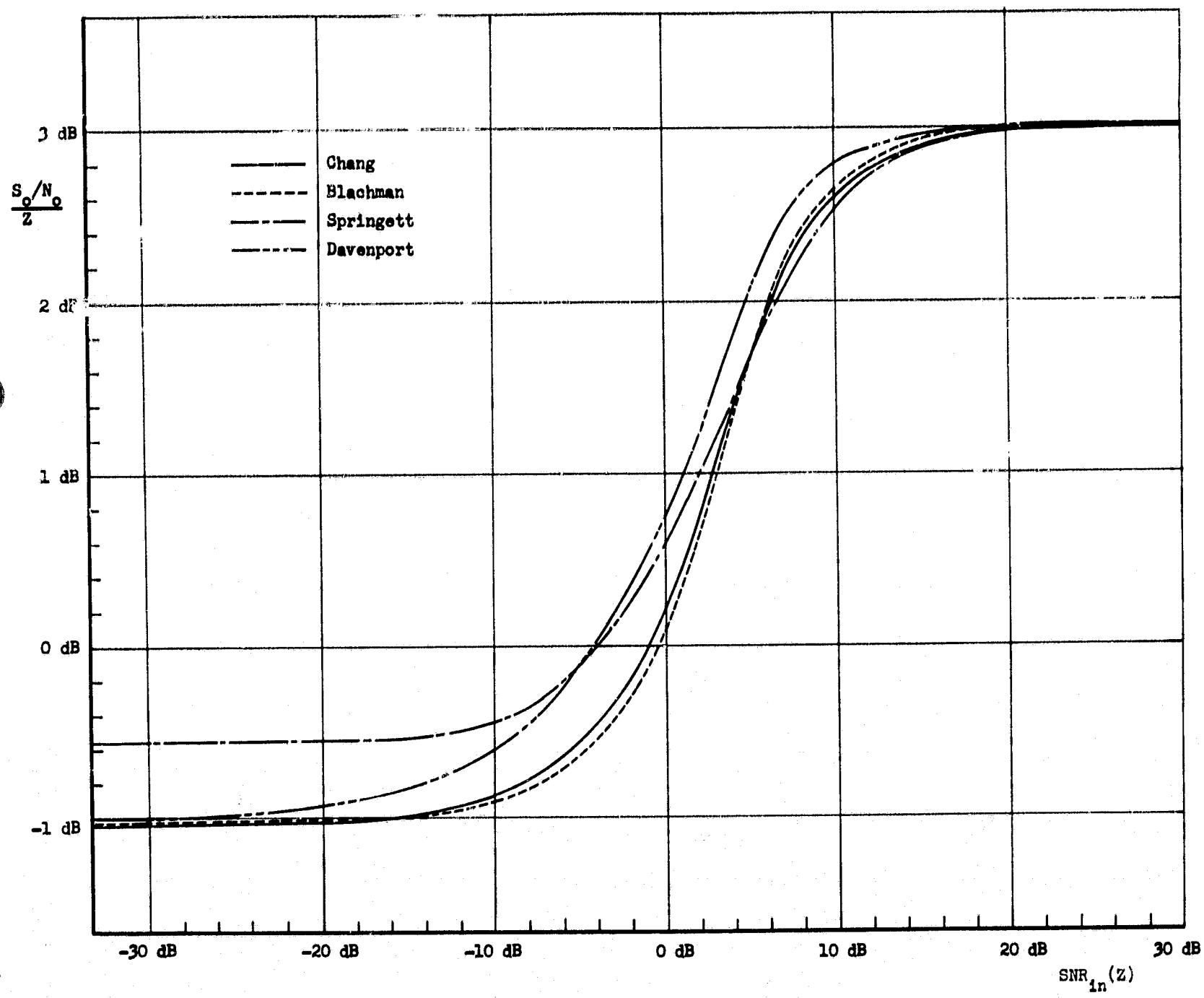

FIGURE 3: LIMITER SNR TRANSFER CHARACTERISTICS

APPENDIX A

COMPUTER PROGRAM

"REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR"

```
FORTRAN IV     MODEL 44  P5        VERSION 2,  LEVEL 1    DATE  68234

0001                DOUBLE PRECISION THETA,XCOS(6000),DEL,YSNRI,XCOS2,X7,X8,X9,X10,X11,X12,
                   1 Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,DELT,XCOS3,X1,X2,X3,X4,X5,X6,X13,X14,
                   2  B8,SNR2,Y9,X9,DEL1,DEL2,DELT1,DELT2,AA1,AA2,AA3,XSIN(6000),
                   3 DELT3,S P4,AA4,A4,Y10,Y11,YSNR,Y12,X10,X11,XSNR,X12,AA2,AA4
0002                DO 100 I1=1,2880
0003                THETA=I1 3.1415926535897932/1440.D0
0004                 XCOS(I1)=DCOS(THETA)
0005                 XSIN(I1)=DSIN(THETA)
0006            100  CONTINUE
0007                DO 800 I8=1,10
0008                J8=I8-6
0009                 B8=10.D0**J8
0010                 DO 300 I3=1,10
0011                 DEL=0.D0
0012                  DEL1=0.D0
0013                 DEL2=0.D0
0014                YSNRI=I3*1.D0*B8
0015                DO 400 I4=1,2880,2
0016                 IF(YSNRI.GE.60.0)GO TO 2
0017                 Y10=DEXP(YSNRI)
0018                Y11=1.D0/Y10
0019                GO TO 4
0020              2  Y11=0.D0
0021              4  YSNR=DSQRT(YSNRI)
0022                 I7=I4+1
0023                 XCOS2=XSIN(I4)*XSIN(I4)
0024                 Y6=YSNRI*XCOS2
0025                 IF(Y6.GE.60.0)GO TO 10
0026                 Y1=YSNR*XCOS(I4)
0027                 Y2=DEXP(Y1)
0028                Y3=3.1415926535897932
0029                Y4=DSQRT(Y3)
0030                 Y5=Y4*(1.D0+Y2)*Y1
0031                 Y7=DEXP(Y6)
0032                 Y12=Y5/Y7
0033                 GO TO 20
0034             10  Y12=0.D0
0035             20 Y8=(Y11+Y12)*XSIN(I7)
0036                Y9=(Y11+Y12)*XCOS(I7)
0037                 DEL2=DEL2+Y8/1440.D0
0038                DEL1=DEL1+Y9/1440.D0
0039            400  CONTINUE
0040                 DELT=0.D0
0041                 DELT1=0.D0
0042                 DELT2=0.D0
0043                DO 500 I5=1,2880,2
0044                 IF(YSNRI.GE.60.0)GO TO 12
0045                 X10=DEXP(YSNRI)
0046                X11=1.D0/X10
```

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0047              GO TO 14
0048        12    X11=0.D0
0049        14    XSNR=DSQRT(YSNRI)
0050              I9=I5+1
0051              XCOS3=XSIN(I5)*XSIN(I5)
0052              X6=YSNRI*XCOS3
0053              IF(X6.GE.60.D) GO TO 40
0054              X1=XSNR*XCOS(I5)
0055              X2=DERF(X1)
0056             X3=3.141592653589793D2
0057             X4=DSQRT(X3)
0058              X5=X4*(1.D0+X2)*X1
0059              X7=DEXP(X6)
0060              X12=X5/X7
0061              GO TO 50
0062        40    X12=0.D0
0063        50   X13=XSIN(I9)-DELT2
0064             X14=XCOS(I9)-DELT1
0065             X8=(X11+X12)*X13*X13
0066             X9=(X11+X12)*X14*X14
0067              DELT1=DELT1+X8/1440.D0
0068             DELT2=DELT2+X9/1440.D0
0069       500    CONTINUE
0070              DEL=DELT1+DELT2
0071              DELT=DELT1+DELT2
0072             SNR=DEL*DEL/DELT
0073             SNR2=DEL*DEL/DELT1
0074             SNR3=DEL*DEL/DELT2
0075             AA1=SNR/YSNRI
0076             AA2=SNR2/YSNRI
0077              AA3=SNR3/YSNRI
0078              A42=DELT2/(DELT1*DELT1+DELT2)
0079              A44=DSQRT(A42)
0080              DELT3=DELT1*(1.D0+A44)
0081               SNR4=DEL1*DEL1/DELT3
0082             AA4=SNR4/YSNRI
0083              A4=10.*DLOG10(AA4)
0084             WRITE(6,3000)YSNRI,DEL1,DELT,DELT1,DELT2,DEL2,DEL,SNR,SNR2,
                1 SNR3,AA1,AA2,AA3,DELT3,SNR4,AA4,A4
0085      3000    FORMAT(1X,5D23.16,/,1X,5D23.16,/,1X,3D23.16,/,1X,4D23.16,//)
0086       400    CONTINUE
0087       800    CONTINUE
0088              END
```

APPENDIX B

CALCULATED DATA

APPENDIX B

CALCULATED DATA

Z	$\frac{S_o}{N_o}$	$\frac{S_o/N_o}{Z}$ in dB
1×10^{-5}	0.785401×10^{-5}	-1.049084
2×10^{-5}	0.157081×10^{-4}	-1.049067
3×10^{-5}	0.235622×10^{-4}	-1.049050
4×10^{-5}	0.314164×10^{-4}	-1.049033
5×10^{-5}	0.392707×10^{-4}	-1.049016
6×10^{-5}	0.471250×10^{-4}	-1.048999
7×10^{-5}	0.549794×10^{-4}	-1.048982
8×10^{-5}	0.628338×10^{-4}	-1.048965
9×10^{-5}	0.706883×10^{-4}	-1.048948
1×10^{-4}	0.785429×10^{-4}	-1.048931
2×10^{-4}	0.157092×10^{-3}	-1.048760
3×10^{-4}	0.235647×10^{-3}	-1.048589
4×10^{-4}	0.314209×10^{-3}	-1.048419
5×10^{-4}	0.392776×10^{-3}	-1.048248
6×10^{-4}	0.471350×10^{-3}	-1.048078
7×10^{-5}	0.549930×10^{-3}	-1.047908
8×10^{-5}	0.628516×10^{-3}	-1.047737
9×10^{-5}	0.707108×10^{-3}	-1.047567
1×10^{-3}	0.785707×10^{-3}	-1.047396
2×10^{-3}	0.157203×10^{-2}	-1.045693
3×10^{-3}	0.235897×10^{-2}	-1.043990
4×10^{-3}	0.314652×10^{-2}	-1.042289
5×10^{-3}	0.393469×10^{-2}	-1.040589
6×10^{-3}	0.472348×10^{-2}	-1.038890
7×10^{-3}	0.551288×10^{-2}	-1.037193
8×10^{-3}	0.630290×10^{-2}	-1.035496
9×10^{-3}	0.709353×10^{-2}	-1.033801

CALCULATED DATA (Continued)

Z	$\frac{S_o}{N_o}$	$\frac{S_o/N_o}{Z}$ in dB
1×10^{-2}	0.788477×10^{-2}	-1.032107
2×10^{-2}	0.158309×10^{-1}	-1.015234
3×10^{-2}	0.238382×10^{-1}	-0.984784
4×10^{-2}	0.319063×10^{-1}	-0.981840
5×10^{-2}	0.400348×10^{-1}	-0.965317
6×10^{-2}	0.482237×10^{-1}	-0.948907
7×10^{-2}	0.564725×10^{-1}	-0.932610
8×10^{-2}	0.647810×10^{-1}	-0.916423
9×10^{-2}	0.731489×10^{-1}	-0.900345
1×10^{-1}	0.815760×10^{-1}	-0.884374
2×10^{-1}	0.169045	-0.730272
3×10^{-1}	0.262162	-0.585509
4×10^{-1}	0.360708	-0.449045
5×10^{-1}	0.464480	-0.320027
6×10^{-1}	0.573292	-0.197748
7×10^{-1}	0.686968	-0.081614
8×10^{-1}	0.805337	-0.028879
9×10^{-1}	0.928237	0.134162
1	0.105551×10	0.234609
2	0.253507×10	1.029609
3	0.429088×10	1.554252
4	0.620156×10	1.904412
5	0.818429×10	2.140110
6	0.101935×10^{2}	2.301706
7	0.122087×10^{2}	2.415710
8	0.142227×10^{2}	2.498932
9	0.162339×10^{2}	2.561815

CALCULATED DATA (Concluded)

Z	$\frac{S_o}{N_o}$	$\frac{S_o/N_o}{Z}$ in dB
10	0.182425×10^2	2.610843
2×10	0.382725×10^2	2.818563
3×10	0.582800×10^2	2.883980
4×10	0.782834×10^2	2.916099
5×10	0.982934×10^2	2.935192
6×10	0.118287×10^3	2.947848
7×10	0.138288×10^3	2.986854
8×10	0.158288×10^3	2.963589
9×10	0.178289×10^3	2.968817
1×10^2	0.198289×10^3	2.972992
2×10^2	0.398291×10^3	2.991760
3×10^2	0.598292×10^3	2.997917
4×10^2	0.798292×10^3	3.001018
5×10^2	0.998292×10^3	3.002876
6×10^2	0.119829×10^4	3.004115
7×10^2	0.139829×10^4	3.004999
8×10^2	0.159829×10^4	3.005663
9×10^2	0.179829×10^4	3.006178
1×10^3	0.199829×10^4	3.006591
2×10^3	0.399829×10^4	3.008446
3×10^3	0.599829×10^4	3.009064
4×10^3	0.799829×10^4	3.009373
5×10^3	0.999829×10^4	3.009558
6×10^3	0.119983×10^5	3.009682
7×10^3	0.139983×10^5	3.009770
8×10^3	0.159983×10^5	3.009837
9×10^3	0.179983×10^5	3.009888
10×10^4	0.199983×10^5	3.009929
	0.399983×10^5	3.010115