APPLICATION OF A MODIFIED FAST FOURIER TRANSFORM TO CALCULATE HUMAN OPERATOR DESCRIBING FUNCTIONS

by Richard S. Shirley

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SUMMARY

A version of the fast Fourier transform (FFT) is used in a hybrid computer program to permit processing of tracking data to yield the human operator's describing function almost immediately after the period of data-taking. The use of the FFT allows the final calculation time required to process 216 seconds of tracking data to be reduced to 3 seconds from the 10 minutes previously required on the same computer. The algorithm used permits the bulk of the analysis of the data to be performed while the data are being taken, and does not require all the data to be present in core before processing begins.

TABLE OF SYMBOLS

a the index of summation for the additive portion of the FFT

A_k a constant which weights the sinusoids composing the system input, see Table I

A_{x_k} the real part of the truncated Fourier transform of x(t) at the frequency \( \omega_k \), given by

\[
\int_0^T x(t) \cos (\omega_k t) \, dt
\]

B_{x_k} the imaginary part of the truncated Fourier transform of x(t) at the frequency \( \omega_k \), given by

\[
\int_0^T x(t) \sin (\omega_k t) \, dt
\]

c a subscript referring to the output of the human operator at the control stick (see Figures 1 and 4)

c(n\Delta t) the data samples taken at the human operator's output
### TABLE OF SYMBOLS (cont'd)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_k$</td>
<td>an integer devisable by 4 used to determine the input frequencies</td>
</tr>
<tr>
<td>$e$</td>
<td>a subscript referring to the input to the human operator at the oscilloscope (see Figures 1 and 4)</td>
</tr>
<tr>
<td>$e(n\Delta t)$</td>
<td>the data samples taken at the human operator input</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>$F_x(\omega_k)$</td>
<td>the truncated Fourier transform of $x(t)$ at the frequency $\omega_k$, given by $\int_0^T x(t)e^{-j\omega t} , dt$</td>
</tr>
<tr>
<td>$h$</td>
<td>a subscript used to denote frequencies between the input frequencies, equal to 1, 2, 3,...</td>
</tr>
<tr>
<td>$i$</td>
<td>a subscript referring to the system input (see Figures 1 and 4)</td>
</tr>
<tr>
<td>$i(n\Delta t)$</td>
<td>the system input equals $\sum_{k=1}^{14} A_k \sin (\omega_k n\Delta t)$</td>
</tr>
<tr>
<td>$j$</td>
<td>the square root of $-1$.</td>
</tr>
<tr>
<td>$k$</td>
<td>a subscript used to denote the input frequencies, equal to 1, 2, 3,...,14</td>
</tr>
<tr>
<td>$m(n\Delta t)$</td>
<td>the data samples taken at the system output</td>
</tr>
<tr>
<td>$n$</td>
<td>the index of summation for the multiplicative portion of the FFT</td>
</tr>
<tr>
<td>$N$</td>
<td>the number of data samples taken, 10,800</td>
</tr>
<tr>
<td>$T$</td>
<td>the period of data-taking, equal to 216 seconds</td>
</tr>
<tr>
<td>$Y_c(\omega)$</td>
<td>the dynamics of the controlled element (see Figures 1 and 4)</td>
</tr>
<tr>
<td>$Y_p(\omega)$</td>
<td>the linear portion of the quasi-linear describing function</td>
</tr>
</tbody>
</table>
\[ \beta_k = \frac{(N/D_k)}{(-1)} \]

\[ \gamma_k = \frac{(D_k/4)}{(-1)} \]

\[ \Delta t \] the time increment between interrupts, and hence the time between data samples, equals .02 sec

\[ \omega_k \] the frequencies of the sinusoids comprising the system input, see Table I

\[ \omega_h \] frequencies between the \( \omega_k \)'s

\[ \phi_{ic}(\omega) \] the cross power spectral density between the human operator's output and the system input

\[ \phi_{ie}(\omega) \] the cross power spectral density between the human operator's input and the system input

\[ \phi_{nn}(\omega) \] the continuous power spectral density of the human operator's remnant

INTRODUCTION

Only recently have dynamic models of the human operator been used effectively in the design of man-vehicle systems. This is due partially to a lack of understanding of the human operator and also to the difficulty and expense of experimentally determining values for the various parameters of existing models. Improvements in computers and computational techniques are overcoming these difficulties, and already it is possible to bring about significant improvements in a man-vehicle system through the use of pilot models in preliminary design (refs. 1 and 2). This paper describes a computational technique which reduces greatly the cost of obtaining values permitting the use of a current pilot model, i.e., the quasi-linear describing function.

One way to characterize the behavior of a human operator in a continuous tracking task is by a quasi-linear describing function, which consists of a linear describing function and a remnant. The linear describing function is the average frequency response of the human operator, i.e., his amplitude ratio and phase as a function of frequency. The remnant, characterized by a continuous power spectral density, is that portion of the human operator's output which is not linearly correlated with his input. The total output of the human operator is the sum of the remnant and the output of the linear describing function* (see Figure 1).

*Examples of human operator describing functions are shown in Figures 2 and 3.
Figure 1.- Block diagram of the human operator in a compensatory tracking system.

Figure 2.- $Y_p(\omega)$ measured for $Y_C(s) = 1/s$
A direct way to measure describing functions in the laboratory involves the use of a hybrid computer and the method of Fourier coefficients. The method of Fourier coefficients has been extensively investigated and is described in detail (ref. 3). It will be briefly outlined here for completeness. The human operator is placed in a control loop, possibly as shown in Figures 1 and 4. The system input, a sum of sinusoids of known amplitude, phase, and frequency is updated every $\Delta t$ seconds; simultaneously, data are taken at the human operator's input and output. At the end of $T$ seconds, the sampled values of the human operator's input and output are processed as follows:

\[ A_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{N} c(n\Delta t) \cos (\omega_k n\Delta t) \]  
\[ B_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{N} c(n\Delta t) \sin (\omega_k n\Delta t) \]  
\[ A_{ek} = \frac{\Delta t}{T} \sum_{n=1}^{N} e(n\Delta t) \cos (\omega_k n\Delta t) \]  
\[ B_{ek} = \frac{\Delta t}{T} \sum_{n=1}^{N} e(n\Delta t) \sin (\omega_k n\Delta t) \]  
\[ F_c(\omega_k) = A_{ck} - jB_{ck} \]  
\[ F_e(\omega_k) = A_{ek} - jB_{ek} \]  
\[ Y_p(\omega_k) = \frac{F_c(\omega_k)}{F_e(\omega_k)} \]  
\[ \angle Y_p(\omega_k) = -\tan^{-1} \frac{B_{ck}}{A_{ck}} + \tan^{-1} \frac{B_{ek}}{A_{ek}} \]
Figure 3. $Y_p(\omega)$ measured for $Y_c(s) = 1/s^2$

Figure 4. Flow diagram of the human operator in a compensatory tracking system.
\[ |Y_p(\omega_k)| = \left[ \frac{A_{ck}^2 + B_{ck}^2}{A_{ek}^2 + B_{ek}^2} \right]^{1/2} \]  

(9)

\[ \phi_{cc}(\omega_h) = \frac{1}{2\pi T} |F_c(\omega_h)|^2 \]  

(10)

\[ \phi_{nn}(\omega_h) = \phi_{cc}(\omega_h) \left| 1 + Y_p Y_c(\omega_h) \right|^2 \]  

(11)

where the \( \omega_k \)'s are the input frequencies, and the \( \omega_h \)'s lie between the \( \omega_k \)'s.

This paper describes how a version of the fast Fourier transform (FFT) is used to compute human operator describing functions, or more specifically, how a version of the FFT is used to solve Eqs. (1) through (4), while the data samples, \( c(n\Delta t) \) and \( e(n\Delta t) \), are being taken. The FFT is an algorithm which greatly reduces the time required to calculate the truncated Fourier transform, or periodogram, of a sampled time signal. The savings are obtained by replacing calculations which involve trigonometric functions or multiplications with simple additions. The replacement is accomplished by taking advantage of the symmetries of the sine and cosine functions, and by further taking advantage of relationships between the frequencies at which the Fourier analysis is performed.

THE VERSION OF THE FFT USED

The version of the FFT used takes advantage only of the symmetries of the sine and cosine functions. It does not take advantage of the relationships among the frequencies at which the Fourier analysis is performed. By not using the complete version of the FFT, it becomes possible to perform the bulk of the data-processing during the \( \Delta t \) seconds between interrupts while the experiment is still in process. The requirement that the data be in core before processing, or even that the data fit in core, is avoided. The following derivation of the algorithm used will make this point clearer. It should be noted that before the FFT was used it was not possible to perform the calculations between interrupts because the computation time required was over two and a half times greater than that which was available.
It is desired to evaluate Eqs. (1) through (4) using a digital computer. In order to permit the use of the FFT, the input frequencies, $\omega_k$, will be restricted to

$$\omega_k = \frac{2\pi}{D_k \Delta t}$$

where the $D_k$ are chosen from 4, 8, 12, 16, etc. The method of Fourier coefficients further requires that the ratio $N/D_k$ be an integer (where $N$ is the number of data samples taken at intervals $\Delta t$). The derivation for $A_{ek}$ and $B_{ek}$ is identical to the derivation which follows for $A_{ck}$ and $B_{ck}$.

Using the identities

$$\sin (\theta + 2\pi) = \sin \theta, \text{ and}$$
$$\cos (\theta + 2\pi) = \cos \theta$$

or

$$\sin (\omega_k \Delta t) = \sin [(aD_k + 1)\omega_k \Delta t]$$
$$\cos (\omega_k \Delta t) = \cos [(aD_k + 1)\omega_k \Delta t]$$

permits Eqs. (1) and (2) to be rewritten as

$$A_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{D_k} \left[ \cos (\omega_k n \Delta t) \sum_{a=0}^{\beta_k} c [\Delta t(n + aD_k)] \right]$$  \hspace{1cm} (12)$$

$$B_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{D_k} \left[ \sin (\omega_k n \Delta t) \sum_{a=0}^{\beta_k} c [\Delta t(n + aD_k)] \right]$$  \hspace{1cm} (13)$$

where $\beta_k = (N/D_k) - 1$. The identities

$$\sin (\theta - \pi) = -\sin \theta, \text{ and}$$
$$\cos (\theta - \pi) = \cos \theta$$
or
\[
\sin \left[ \omega_k \Delta t \left( n - \frac{D_k}{2} \right) \right] = -\sin \left( \omega_k n \Delta t \right)
\]
\[
\cos \left[ \omega_k \Delta t \left( n - \frac{D_k}{2} \right) \right] = \cos \left( \omega_k n \Delta t \right)
\]

permit Eqs. (12) and (13) to be written as

\[
A_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{D_k/2} \left[ \cos (\omega_k n \Delta t) \sum_{a=0}^{B_k} \left\{ c \left[ \Delta t (n + a D_k) \right] - c \left[ \Delta t (n + \frac{D_k}{2} + a D_k) \right] \right\} 
- c \left[ \Delta t \left( n + \frac{D_k}{2} + a D_k \right) \right] \right] \tag{14}
\]
\[
B_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{D_k/2} \left[ \sin (\omega_k n \Delta t) \sum_{a=0}^{B_k} \left\{ c \left[ \Delta t (n + a D_k) \right] - c \left[ \Delta t (n + \frac{D_k}{2} + a D_k) \right] \right\} 
- c \left[ \Delta t \left( n + \frac{D_k}{2} + a D_k \right) \right] \right] \tag{15}
\]

Finally, the identities \( \sin (-\theta) = -\sin \theta \), \( \cos (-\theta) = \cos \theta \), \( \sin (\pi/2) = \cos (\pi) = 1 \), and \( \sin (\pi) = \cos (\pi/2) = 0 \) permit Eqs. (14) and (15) to be written as

\[
A_{ck} = \frac{\Delta t}{T} \sum_{n=1}^{y_k} \left[ \cos (\omega_k n \Delta t) \sum_{a=0}^{b_k} \left\{ c \left[ \Delta t (n + a D_k) \right] - c \left[ \Delta t \left( n + \frac{D_k}{2} + a D_k \right) \right] \right\} 
- c \left[ \Delta t \left( \frac{D_k}{2} - n + a D_k \right) \right] 
+ c \left[ \Delta t \left( \frac{D_k}{2} - n + \frac{D_k}{2} + a D_k \right) \right] \right] 
- \frac{\Delta t}{T} \sum_{a=0}^{b_k} \left\{ c \left[ \Delta t \left( \frac{D_k}{2} + a D_k \right) \right] - c \left[ \Delta t \left( \frac{D_k}{2} + \frac{D_k}{2} + a D_k \right) \right] \right\} \tag{16}
\]
Atkinson

\[
B_{ck} = \frac{\Delta t}{T} \left[ \sum_{n=1}^{\gamma_k} \sin (\omega_k n \Delta t) \sum_{a=0}^{\beta_k} \left\{ c \left[ \Delta t (n + aD_k) \right] - c \left[ \Delta t \left( n + \frac{D_k}{2} + aD_k \right) \right] \right\} + c \left[ \Delta t \left( \frac{D_k}{2} - n + aD_k \right) \right] - c \left[ \Delta t \left( \frac{D_k}{2} - n + \frac{D_k}{2} + aD_k \right) \right] \right] \\
+ \frac{\Delta t}{T} \sum_{a=0}^{B_k} \left\{ c \left[ \Delta t \left( \frac{D_k}{4} + aD_k \right) \right] - c \left[ \Delta t \left( \frac{D_k}{4} + \frac{D_k}{2} + aD_k \right) \right] \right\}
\]

(17)

where \( \gamma_k = (D_k/4) - 1 \). Equations (16) and (17) represent the algorithm used in the hybrid program. The summation over "a" is performed between interrupts during the experiment and is called the "additive portion" of the FFT. At the end of the data-taking period, the summation over \( n \) (called the "multiplicative portion" of the FFT) and the calculation of the human operator's describing function [using Eqs. (8) and (9)], can be performed in less than three seconds.

The hybrid computer program is written in a Fortran IV language which includes hybrid commands. The program is listed in Appendix A. Table I lists the values of the experimental parameters, including those which characterize the system input. Figure 5 is a flow diagram of the additive portion of the FFT. Figure 6 shows a flow diagram of the hybrid program, and lists the time taken by each part of the program, both for the FFT version and for the version written the old way [directly computing Eqs. (1) through (4)]. As shown in Figure 6, the FFT permits a saving of nearly ten minutes per run, effectively reducing the run time to the time required to take the data and print the results.

RESULTS

An initial check of the hybrid program was made by taking measurements across known filters. The results shown in Figures 7 and 8 are quite accurate, and are repeatable.

Measurements were then taken of the author's tracking performance in a control loop, as shown in Figures 1 and 4. Ten runs were made with each of the controlled elements, 1/s and 1/s². The describing functions shown in Figures 2 and 3 are comparable with established results (ref. 3).
Figure 5.- Flow chart for additive portion of FFT

Figure 6.- Flow diagram and times for computer runs
Figure 7. - Measurement of a known filter

Figure 8. - Measurement of a known filter
**TABLE I**

**PARAMETER VALUES USED FOR THE HYBRID PROGRAM**

<table>
<thead>
<tr>
<th>k</th>
<th>(A_k) volts</th>
<th>(\omega_k) rad/sec</th>
<th>k</th>
<th>(A_k) volts</th>
<th>(\omega_k) rad/sec</th>
</tr>
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<tr>
<td>1</td>
<td>.2</td>
<td>26.18</td>
<td>8</td>
<td>-1.</td>
<td>1.309</td>
</tr>
<tr>
<td>2</td>
<td>-.2</td>
<td>15.71</td>
<td>9</td>
<td>1.</td>
<td>.8727</td>
</tr>
<tr>
<td>3</td>
<td>.2</td>
<td>8.727</td>
<td>10</td>
<td>-1.</td>
<td>.5818</td>
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<tr>
<td>4</td>
<td>-.2</td>
<td>6.545</td>
<td>11</td>
<td>1.</td>
<td>.4363</td>
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<td>5</td>
<td>.2</td>
<td>4.363</td>
<td>12</td>
<td>-1.</td>
<td>.2909</td>
</tr>
<tr>
<td>6</td>
<td>-1.</td>
<td>2.618</td>
<td>13</td>
<td>1.</td>
<td>.1745</td>
</tr>
<tr>
<td>7</td>
<td>1.</td>
<td>1.745</td>
<td>14</td>
<td>-1.</td>
<td>.1164</td>
</tr>
</tbody>
</table>

\(\Delta t = \) time between interrupts = .02 sec

\(T_1 = \) warm-up time before data-taking = 24 sec

\(T = \) period of data-taking = 216 sec

\[i(n\Delta t) = \text{system input} = \sum_{k=1}^{14} A_k \sin (\omega_k n\Delta t)\]

No comparison is made between results for the programs with and without the FFT (on Figures 2, 3, 7, 8) because the results are identical, as is shown analytically in the derivation of Eqs. (16) and (17). The comparison between the computation times for the two programs (Figure 6) however, indicates the substantial savings obtained by using the FFT. The only penalty paid for the reduced computational time is an increase in the complexity of the written Fortran program, as shown in Appendix A.

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Electronics Research Center
Cambridge, Massachusetts, November 1968
125-19-01-11-25
REFERENCES


National Aeronautics and Space Administration
Electronics Research Center
Cambridge, Massachusetts, December, 1968
125-19-01-11-25
APPENDIX A
PROGRAM LISTING

11/13/68
*JOB RSHIRLEY
*DATE 1 NOVEMBER 1968
*TITLE PROGRAM TO MEASURE THE DESCRIBING FUNCTION USING THE FFT
*ASSIGN 1=MT2A, 2=MT3B, 3=MT1A
*ASSIGN 5=CR1A, 6=LP1A
*ASSIGN 7=CPIA
*FORTRAN S,GO

C
C MAIN PROGRAM
C
C SENSE SWITCH FORMAT
C
C **DO NOT SET SENSE SWITCHES DURING COMPILATION**
C SET SWITCH IN, LIGHT ON
C NOT SET=SWITCH OUT, LIGHT OFF
C
C **SWITCH NO. SET**
C ****************************
C 1 * DO NOT READ DATA CARDS * READ DATA CARDS AND CALC. INPUT
C 2 * DO NOT CALC. INPUT * CALC. INPUT
C 3 * DO NOT PROCESS DATA * PROCESS DATA
C 4 * DO NOT USE UNIT 3 TO SAVE DATA * USE UNIT 3 TO SAVE DATA
C 5 * DO A DATA DUMP * NO DATA DUMP
C 6 * DO PUNCH OUT DATA * DO NOT PUNCH OUT DATA
C
C INPUT ERROR * OPERATOR * DYNAMICS
C (PUT) * *** * SYSTEM * (TWO) 
C (ONE) * VEHICLE * ***********
C
C ***#******#************#**#**********#***#*******#**#************#*****#****
C " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " " 

C **F R O M A T S T A T E M E N T S**
100 FORMAT(1H1,3X,1HK,6X,8HFREQ.(K),7X,5MAE(K),10X,5HBE(K),10X,
  15HAR1(K),10X,5HM1(K),10X,5MA2(K),10X,5H2(K),/ / / ,15H,1E15,47)
101 FORMAT(/ / / ,4X,1HK,6X,8HFREQ.(K),7X,8HAREM1(K),7X,8HBREM1(K),7X,
  18HAREM2(K),7X,8HBREM2(K),/ / / /,15H,5E15,47)
102 FORMAT(1E15.8)
103 FORMAT(/ / / ,4X,1HK,8X,8HFREQ.(K),7X,8HAR1(K),9X,7HAR2(K),9X,
  16HAR1(K),9X,7HPHAI(K),/ / /,15H,5E15,41)
104 FORMAT(/ / / /,9X,8HAREM1(K),7X,9X,6HNE(K),7X,6HTWO(K),/ / / /,15H,5E15,
  24))
105 FORMAT(1H1,4X,1H9,9X,6HPUT(K),9X,6HE(K),11X,6HONE(K),9X,6HTWO(K),
  1,/,15H,5E15,4))
106 FORMAT(1H1,4X,1HK,9X,6HPUT(K),/ / /,15H,5E15,4))
107 FORMAT(/ / / ,10X,8HPUTSQ = ,1E15.4 ,/ ,10X,8HERRSQ = ,1E15.4)
114HERRSQ/PUTSQ = ,1E15.4 ,/ ,10X,8HONESQ = ,1E15.4 ,/ ,10X,14HONESQ/PUT
2SQ = ,1E15.4 ,/ ,10X,8HTWOQ = ,1E15.4 ,/ ,10X,14HTWOQ/PUTSQ = ,1E15.4)
1. COMMON PUT(1080) , E(1080), ONE(1080), TWO(1080), KONK, LOP, KANK,
   IN, M, MO, KRINK, PUTSU, ERRSU, ONE, TWO, KLU

2. DIMENSION W(30), AE(15), BE(15), AI(15), BI(15), Z1(15), Z2(15)
   AREM1(15), REM1(15), AREM2(15), REM2(15)

3. DIMENSION AR1(15), AR2(15), PHA1(15), PHA2(15), REM1(15), REM2(15)

4. THE PROGRAM CAN BE RECALLED TO THIS POINT BY IFINITIA AT ANY TIME

5. BY HITTING INTERRUPT 33 AND TYPING A CARRIAGE RETURN

6. CALL IFINITIA

7. CONNECT THE CONSOLE

8. EOM 03120

9. PUT THE ANALOG COMPUTER INTO IC MODE

C SENSE SWITCH4 DETERMINES IF REWIND 3 OR NOT
   IF (SENSE SWITCH4) 98, 99
   99 REWIND 3
   98 CONTINUE

C SENSE SWITCH1 DETERMINES IF FREQUENCY AND REGISTER VALUE CARDS ARE
   READ
   IF (SENSE SWITCH1) 23, 24

C SENSE SWITCH2 DETERMINES IF INPUT CALCULATED
   23 IF (SENSE SWITCH2) 17, 26

C READ IN 29 FREQUENCIES IN ORDER, HIGH FREQUENCIES FIRST,

C IN RADIANS/SECOND, STARTING WITH A REMNANT FREQUENCY AND ALTERNAT-

C TING THEREAFTER WITH THE INPUT FREQUENCIES, THE FREQUENCIES MUST

C ALL BE INTEGER MULTIPLES OF (PI)/(RUNIT), AND THE NUMBER OF

C SECONDS PER CYCLE MUST EQUAL 41(DELTA), WHERE I IS AN INTEGER AND

C DELTA IS THE TIME BETWEEN DATA POINTS.

24 READ(5*102) (W(K),K=1,29)
READ IN THE REGISTER VALUES FOR THE ADDITIVE PART OF THE FFT
READ(5,109) (JA(K), JB(K), JC(K), K=1, 29)
WRITE (6,110) (JA(K), JB(K), JC(K), K=1, 29)

INPUT (CALCULATED EVERY DELT)
CONS1 SCALES THE INPUT
26 CONS1=9.5
REWIND INPUT TAPE PRIOR TO STORING INPUT
REWIND 1
ONW SETS THE STARTING TIME FOR THE INPUT, AND IS NEGATIVE
SO THAT THE HUMAN OPERATOR IS IN A STEADY-STATE TRACKING
CONDITION WHEN THE ONSET OF DATA-TAKING OCCURS
ONW=-1080.
DELT IS THE TIME INCREMENT BETWEEN INPUT VALUES, AND MUST EQUAL
DELT+, THE TIME INCREMENT BETWEEN DATA POINTS.
DELT=.02
THE AM(K) SCALE THE INPUT SINUSOIDS
AM(1)=.2
AM(2)=-.2
AM(3)=.2
AM(4)=-.2
AM(5)=.2
AM(6)=-1.
AM(7)=1.
AM(8)=-1.
AM(9)=1.
AM(10)=-1.
AM(11)=1.
AM(12)=-1.
AM(13)=1.
AM(14)=-1.
AM(15)=1.
THE INPUT IS CALCULATED IN BLOCKS OF 540 VALUES AND STORED ON
MAGNETIC TAPE (UNIT 1).
DO 19 J=1,30
DO 1 K=1,540
PUT(K)=0.
ONW=ONW+1.
T=ONW*DELT
DO 20 L=1,14
PUT(K)=PUT(K)+AM(L)*SIN(T*W(2*L))
20 CONTINUE
PUT(K)=CONS1*PUT(K)
1 CONTINUE
M=1
N=540
CALL BUFFEROUT(1,1,PUT(M),2*(N-M+1),ISTATUS)
CALL GOTO(ISTATUS)
19 CONTINUE

17 CONTINUE
THE INTERRUPT IS CONNECTED, BUT NOT ENABLED
CONNECT (40, INTR)
EOM 020020
POT =00700000

INITIALIZE FOR THE RUN

ZERO THE BUFFER AREAS
DO 92 K=1,1080
PUT(K)=0.
E(K)=0.
ONE(K)=0.
TWO(K)=0.
92 CONTINUE

ZERO THE DATA TAPE
REWIND 2
DO 91 K=1,190
N=540
M=1
CALL BUFFEROUT(2,1,E(M),2*(L-M+1),ISTATUS)
CALL GOTO(ISTATUS)
91 CONTINUE

THE MAGNETIC TAPE UNITS ARE INITIALIZED, UNIT 1 FOR THE INPUT,
AND UNIT 2 FOR THE DATA.
REWIND 1
REWIND 2

ZERO THE REGISTERS WHERE THE FOURIER COEFFICIENTS ARE TO
BE CALCULATED.
DO 31 K=1,15
AE(K)=0.
BE(K)=0.
AI(K)=0.
BI(K)=0.
AZ(K)=0.
BZ(K)=0.
AREM1(K)=0.
BREM1(K)=0.
AREM2(K)=0.
BREM2(K)=0.
31 CONTINUE

INITIALIZE FOR THE ADDITIVE PART OF THE FFT
DO 10 NOW=1,29
KLNOWN)=-1
10 CONTINUE
DO 14 NOW=1,4300
WS(NOW)=0*
14 CONTINUE

INITIALIZE THE COUNTERS FOR THE RUN
KRINK IS A COUNTER TO DETERMINE THE LOCATION FROM WHICH THE
NEXT INPUT VALUE SHOULD BE TAKEN FROM:
KRINK=0
KONK COUNTS THE INTERRUPTS, DETERMINES WHEN THE ONSET OF DATA-
DATA-TAKING SHOUOlD OCCUR, AND WHEN DATA-TAKING IS COMPLETED.

**KONK**=0

**KANK** IS THE HALF REGISTER COUNTER, 1 TO 540

**M0** DETERMINES WHICH HALF OF THE INPUT BUFFER IS BEING USED

**M0**=0

**B** IS A COUNTER ON THE DATA USED DURING THE DATA-PROCESSING

**B**=-1

**LOP** IS THE FLAG SET BY INTR TO END DATA-TAKING

**LOP**=0

**PUTSQ** AND **ERRSQ** ARE THE INTEGRAL SQUARE INPUT AND ERROR

**PUTSQ**=0

**ERRSQ**=0

**ONESQ**=0

**TWOSE**=0

**X1SU**=0

**X2SU**=0

**M** INITIALLY INPUT BUFFER FOR THE RUN, I.e., FILL BOTH HALVES

**N** WITH INPUT VALUES

**M**=1

**N**=540

CALL BUFFERIN(1,1,PUT(M),Z*(N-M+1),ISTATUS)

CALL GOTO(ISTATUS)

**M**=541

**N**=1080

CALL BUFFERIN(1,1,PUT(M),Z*(N-M+1),ISTATUS)

CALL GOTO(ISTATUS)

**M** WAIT TO START RUN ON SIGNAL FROM THE OPERATOR

**PAUSE**

**S** SKS 030000

**S** ORU 5135

GO TO 511

513 CONTINUE

**C** PUT THE ANALOG COMPUTER INTO COMPUTE MODE

**C** CALL COMPUTE

**C** ENABLE THE INTERRUPT

**S** EOM 031032

**C** 11 CONTINUE

CHECK TO SEE IF IT IS THE END OF DATA-TAKING

IF (LOP.EQ.1) GO TO 2

C IT IS NOT THE END OF DATA TAKING, WAIT FOR INTERRUPT

GO TO 11

C IT IS THE END OF DATA TAKING, GO ON

2 CONTINUE

C TURN OFF THE INTERRUPT

**S** EOM 031033

C PUT THE ANALOG COMPUTER INTO THE HOLD MODE
CALL HOLD
C TAKE THE INTEGRAL SQUARE MEASURES
CALL ADL((4,PUTSU,ERRSU,ONESU,TWUSU,X1SU,X2SU))
C PUT THE ANALOG COMPUTER INTO IC MODE
CALL IC
C
C SENSE SWITCH3 DETERMINES WHETHER TO PROCESS THE DATA AND TYPE
C THE RESULTS, OR WHETHER TO RE-INITIALIZE FOR THE NEXT RUN
IF (SENSE SWITCH3) 17, 22
C
C DATA PROCESSING IS DESIRED, GO ON
  22 CONTINUE
C THE FOLLOWING SAVES THE LAST 5~0 DATA POINTS
N=1080
M=541
  CALL BUFFEROUT(2,1,E(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  CALL BUFFEROUT(2,1,ONE(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  CALL BUFFEROUT(2,1,TWO(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
C
C THE FOLLOWING PERMITS A TOTAL OR PARTIAL TAPE DUMP
IF (SENSE SWITCH5) 89, 90
  89 REWIND 1
  REWIND 2
  N=540
  M=1
  CALL BUFFERIN(1,1,PUT(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  N=1080
  M=541
  CALL BUFFERIN(1,1,PUT(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  WRITE(*,106) (J,PUT(J),J=1,1080)
C ROUTINE.
  M=1
  N=540
  DO 16 J=1,3
    CALL BUFFERIN(2,1,E(M),2*(N-M+1),ISTATUS)
    CALL GOTO(ISTATUS)
  16 CONTINUE
C NO DETERMINES EXTNT OF THE DUMP
NO=3
  DC 88 K=1,NO
  CALL BUFFERIN(1,1,PUT(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  CALL BUFFERIN(2,1,E(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  CALL BUFFERIN(2,1,ONE(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  CALL BUFFERIN(2,1,TWO(M),2*(N-M+1),ISTATUS)
  CALL GOTO(ISTATUS)
  L=(K-1)*540
PROCESS THE DATA FOR 10,800 POINTS

MULTIPLICATIVE PART OF THE FFT USED TO CALCULATE THE FOURIER COEFFICIENTS

CALCULATE THE FOURIER COEFFICIENTS FOR THE REMNANT

```
DEZ=3.14159267
DO 21 K=1,29,2
   NE=(K+1)/2
   LEB1=JB(K)+JA(K)-1
   LEE1=JB(K)+2*JA(K)-1
   LEB2=JB(K)+2*JA(K)-1
   LEE2=JB(K)+3*JA(K)-1
   LEM1=LEE1-JA(K)/2
   LEM2=LEE2-JA(K)/2
   AREM1(NE)=0.
   AREM2(NE)=0.
   BREM1(NE)=0.
   BREM2(NE)=0.
   JST=(JA(K)/2)-1
   DO 27 Ji=1, JST
      TOU=TOU/VOU
      SINUS=SIN(P(PDEZ*TOU))
      COSUS=SQRT(1.-SINUS**2.)
      AREM1(NE)=AREM1(NE)+WS(LEB1+Ji)+WS(LEE1-Ji)*SINUS
      AREM2(NE)=AREM2(NE)+WS(LEB2+Ji)+WS(LEE2-Ji)*SINUS
      BREM1(NE)=BREM1(NE)+WS(LEB1+Ji)-WS(LEE1-Ji)*COSUS
      BREM2(NE)=BREM2(NE)+WS(LEB2+Ji)-WS(LEE2-Ji)*COSUS
      27 CONTINUE
   21 CONTINUE
```

CALCULATE THE FOURIER COEFFICIENTS FOR THE DESCRIBING FUNCTION

```
DO 28 K=2,28,2
   NE=K/2
   LEBE=JB((K))-1
   LEEE=JB((K))+JA((K))-1
   LEB1=JB((K))+JA((K))-1
   LEE1=JB((K))+2*JA((K))-1
   LEB2=JB((K))+2*JA((K))-1
   LEE2=JB((K))+3*JA((K))-1
   LEM1=LEE1-JA((K))/2
   LEM2=LEE2-JA((K))/2
   AE(NE)=0.
   BE(NE)=0.
```
A1(NE)=0.
B1(NE)=0.
A2(NE)=0.
B2(NE)=0.
JST=(JA(K)/2)-1
DO 29 JI=1,JST
VOU=FLOAT(JI)
YOU=YOU/VOU
SINUS=SINDEZ#TOU)
COSUS=SQRT(1.-SINUS**2.)
AE(NE)=AE(NE)+(WS(LEB1+JII)+WS(LE1-JI))*SINUS
A1(NE)=A1(NE)+(WS(LEB1+JII)+WS(LEE1-JI))*SINUS
A2(NE)=A2(NE)+(WS(LEB2+JII)+WS(LEE2-JI))*SINUS
BE(NE)=BE(NE)+(WS(LEB1+JII)-WS(LEE1-JI))*COSUS
B1(NE)=B1(NE)+(WS(LEB1+JII)-WS(LEE1-JI))*COSUS
B2(NE)=B2(NE)+(WS(LEB2+JII)-WS(LEE2-JII))*COSUS
29 CONTINUE
C CALCULATE THE HUMAN OPERATORS DESCRIBING FUNCTION AND REMNANT
C DO 32 K=1,14
DENOM=AE(K)*AE(K)+BE(K)*BE(K)
AR(K)=SORT(A1(K)*A1(K)+B1(K)*B1(K)/DENOM)
AR(K)=SORT(A2(K)*A2(K)+B2(K)*B2(K)/DENOM)
PHA1(K)=57.3*ATAN2(B1(K),A1(K))-ATAN2(BE(K),AE(K))
PHA2(K)=57.3*ATAN2(B2(K),A2(K))-ATAN2(BE(K),AE(K))
C USE THE ASSUMPTION THAT THE HUMANS PHASE LEAD IS LESS THAN
C 180 DEGREES TO CORRECT FOR THE LOSS OF PHASE INFORMATION
C IF (PHA1(K).LT.180.) GO TO 93
93 IF (PHA2(K).LT.180.) GO TO 94
94 CONTINUE
C CONS2 SCALES THE REMNANT
C CONS2=(DELT)**2/4(PI)**1
C DELT=.02,T=216
CONS2=1.47E-7
DO 33 K=1,15
REM1(K)=CONS2*(AREM1(K)*AREM1(K)+BREM1(K)*BREM1(K))
REM2(K)=CONS2*(AREM2(K)*AREM2(K)+BREM2(K)*BREM2(K))
33 CONTINUE
C INTERPOLATE FOR THE REMNANT AT THE INPUT FREQUENCIES
DO 230 K=1,14
REM1(K)=REM1(K)+(REM1(K+1)-REM1(K))*(W(2*K)-W(2*K-1))/(W(2*K+1)-
1W(2*K-1))
C 230 CONTINUE
   C CALCULATE THE REMNANT 'POWER'
   REMPW1=0.
   REMPW2=0.
   DO 231 K=1,14
      REMPW1=REMPW1+REMA1(K)*(W(2*K-1)-W(2*K+1))
      REMPW2=REMPW2+REMA2(K)*(W(2*K-1)-W(2*K+1))
   231 CONTINUE

C C WRITE OUT THE FOURIER COEFFICIENTS OF THE REMNANT AT THE HUMANS OUTPUT (ONE), AND AT THE SYSTEM OUTPUT (TWO).
   WRITE(6,101) (K,W(2*K-1),AREM1(K),BREM1(K),AREM2(K),BREM2(K),K=1,15)
   C WRITE OUT THE HUMAN OPERATORS DESCRIBING FUNCTION AND REMNANT AS WELL AS THE SYSTEM OPEN LOOP DESCRIBING FUNCTION AND REMNANT
   WRITE(6,110) (K,W(2*K-1),REM1(K),REM2(K),K=1,15)
   WRITE(6,116) CONS2
   C WRITE OUT THE HUMANS REMNANT AT THE INPUT FREQUENCIES
   WRITE(6,113) (K,W(2*K),REMA1(K),REMA2(K),K=1,14)
   WRITE(6,114) CONS2
   C WRITE OUT THE ERROR SCORES
   ERRSY=ERRSQ/PUTSQ
   ONESY=ONESQ/PUTSQ
   TWOSY=TWOSQ/PUTSQ
   WRITE(6,107) (PUTSQ,ERRSQ,ERRSY,ONESQ,ONESY,TWOSQ,TWOSY)
   WRITE(6,117) X1SQ,X2SQ
   PUTSQ=PUTSQ/216.
   ERRSQ=ERRSQ/216.
   ONESQ=ONESQ/216.
   TWOSQ=TWOSQ/216.

C C WRITE OUT THE REMNANT POWER
   WRITE(6,112) (REMPW1,REMPW2)
   REMPW1=0.
   REMPW2=0.
   DO 240 K=1,14
      REMPW1=REMPW1+REMA1(K)*(W(2*K-1)-W(2*K+1))
      REMPW2=REMPW2+REMA2(K)*(W(2*K-1)-W(2*K+1))
   240 CONTINUE

C SENSE SWITCH 6 DETERMINES WHETHER TO PUNCH OUT THE ANSWERS ON CARDS.
IF (SENSE SWITCH6) 35, 36
PUNCH OUT THE DATA ON CARDS, 14 CARDS WITH AR1, PHA1, AR2, PHA2,
REMA1, AND REMA2, PLUS 1 CARD WITH PUTS0, ERRSO, ONES0, TWO00,
REMPW1, AND REMPW2.
35 WRITE (7, 115) (AR1 (K), PHA1 (K), AR2 (K), PHA2 (K), REMA1 (K), REMA2 (K), K = 1, 114)
WRITE (7, 115) (PUTS0, ERRSO, ONES0, TWO00, REMPW1, REMPW2)
36 CONTINUE

THE FOLLOWING STORES EITHER ONE (M) OR TWO (M) ON UNIT 3 FOR
LATER PROCESSING FOR THE REMNANT.
SENSE SWITCH4 DETERMINES WHETHER TO SAVE ONE (M) OR TWO (M).
IF (SENSE SWITCH4) 95, 97

97 REWIND 2
REWIND 1
M = 1
N = 540
THE FOLLOWING AVOIDS THE SPURIOUS DATA POINTS
DO 521 K = 1, 3
CALL BUFFERIN (2, 1, TWO (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
521 CONTINUE
CALL BUFFERIN (1, 1, PUT (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFERIN (1, 1, PUT (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
DO 96 K = 1, 20
CALL BUFFERIN (2, 1, E (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFERIN (2, 1, ONE (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFERIN (2, 1, TWO (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFEROUT (3, 1, PUT (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFEROUT (3, 1, E (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFEROUT (3, 1, ONE (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
CALL BUFFEROUT (3, 1, TWO (M), 2 * (N - M + 1), ISTATUS)
CALL GOTO (ISTATUS)
96 CONTINUE
95 CONTINUE

RETURN TO INITIALLIZE FOR THE NEXT RUN
GO TO 17
200 STOP

INTERRUPT SUBROUTINE (INTERNAL)
INTR SERVICES THE INTERRUPT
SUBROUTINE INTR
KRINK IS THE REGISTER COUNTER, 1 TO 1080
KRINK = KRINK + 1
KONK IS THE TOTAL COUNTER, 1 ON UP
KONK = KONK + 1
KANK IS THE HALF REGISTER COUNTER, 1 TO 500
KANK = KANK + 1

IF (KONK GT 1060) GO TO 1
2 IF (KANK GE 541) GO TO 3
4 CALL DAL(0, PUT(KRINK))
12 RETURN
1 }F (KONK GT 11880) GO TO 5
IF UANK GE 541 ) GO TO 6
8 CALL DAL(0, PUT(KRINK))
CALL ADL(0, PUT(KRINK), ONE(KRINK), TWO(KRINK))
ADDITIVE PART OF THE FAST FOURIER TRANSFORM

KL= A COUNTER, 0 TO (2*JA(K)*JC(K))-1
JA=HALF THE NUMBER OF CYCLES PER SECOND/DIVISOR
J=REGISTER START FOR EACH FREQUENCY
JC=DIVISOR USED ON HALF THE NUMBER OF SAMPLES PER CYCLE TO GET JA
ADDITIVE PART OF THE FFT

ADD THE DATA INTO THE WS(K)
GO 27 K=1,29
23 KL(K)=KL(K)+1
JNK=JA(K)*JC(K)
IF (KL(K) GE JNK) GO TO 32
KKE(K)=KL(K)/JC(K) + Jb(K)
GO TO 24
32 J=K=2*JA(K)*JC(K)
IF(KL(K) GE JAK) GO TO 33
KLE(K)=(KL(K)/JC(K))-JA(K)+Jb(K)
GO TO 25
33 KL(K)=-1
GO TO 23
24 JD=KLE(K)
JC=KLE(K)+JA(K)
JF=KLE(K)+2*JA(K)
WS(JD)=WS(JD)+E(KRINK)
WS(JE)=WS(JE)+ONE(KRINK)
WS(JF)=WS(JF)+TWO(KRINK)
GO TO 26
25 JD=KLE(K)
JE=KLE(K)+JA(K)
JF=KLE(K)+2*JA(K)
WS(JD)=WS(JD)-E(KRINK)
WS(JE)=WS(JE)-ONE(KRINK)
WS(JF)=WS(JF)-TWO(KRINK)
GO TO 26
20 CONTINUE
27 CONTINUE
RETURN
3 KANK=;
IF (MO GE 1) GO TO 7
Nu=1
N=540
M=1
CALL BUFFERIN(1,1,PUT(M),2*(N-M+1),STATUS)

GO TO 4

7 MO=0
    N=1080
    M=541
    KRINK=1
    GO TO 9

6 KANK=1
    IF (MO+GE.1) GO TO 10
    MO=1
    N=540
    M=1

11 CALL BUFFERIN(1,1,PUT(M),2*(N-M+1),STATUS)
   CALL BUFFEROUT(2,1,E(M),2*(N-M+1),STATUS)
   CALL BUFFEROUT(2,1,ONE(M),2*(N-M+1),STATUS)
   CALL BUFFEROUT(2,1,THO(O),2*(N-M+1),STATUS)
   GO TO 8

10 MO=0
    N=1080
    M=541
    KRINK=1
    GO TO 11

5 LOP=1
    GO TO 12

C

SUBROUTINE GOTO (STATUS)
C
SUBROUTINE (INTERNAL) TO HANDLE TAPE READ AND WRITES

7 GO TO (6,4,5,5,5) STATUS

6 GO TO 7

5 WRITE (102,200) STATUS

4 RETURN

200 FORMAT (5 BUFFERIN STATUS WORD =$12)

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*FIN*
APPLICATION OF A MODIFIED FAST FOURIER TRANSFORM TO CALCULATE HUMAN OPERATOR DESCRIBING FUNCTION

By Richard S. Shirley
Electronics Research Center

ABSTRACT

A modified fast Fourier transform (FFT) is used in a hybrid computer program to permit processing of tracking data during a run to yield the human operator's describing function almost immediately after the data-taking period. The computer processing time is substantially reduced at no cost in accuracy.

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—National Aeronautics and Space Act of 1958

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