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TECHNIQUE FOR CACULATING OPTIMUM TAKEOFF AND CLIMBOUT TRAJECTORIES FOR NOISE ABATEMENT

by Heinz Erzberger and Homer Q. Lee

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Moffett Field, Calif.

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TECHNIQUE FOR CALCULATING OPTIMUM TAKEOFF AND CLIMBOUT TRAJECTORIES FOR NOISE ABATEMENT

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SUMMARY

A procedure has been developed for determining optimum takeoff and climbout trajectories for jet aircraft that minimize the noise produced during overflight of communities located along the ground track of the climbout path. The procedure enables one to compute optimum takeoff trajectories for noise abatement for a particular aircraft operating from a particular airport subject to prescribed operational constraints.

An important part of the procedure consists in defining appropriate criteria for noisiness of a trajectory. Two criteria are proposed and are used as a basis for calculating noise optimum trajectories. Other criteria could be used instead. The first of these is simply the perceived noise level (in PNdB) measured after a power reduction at the beginning of the noise-sensitive area. The second was taken as the perceived noise level, averaged along the noise-sensitive ground track of a climbout trajectory. Calculating trajectories that minimize the chosen criterion for noisiness is interpreted as an optimum control problem whose state variables are airspeed and altitude and whose control variables are thrust, flight-path angle, and flap deflection. Solutions of this optimum control problem were obtained by implementing the dynamic programming algorithm on a digital computer.

The procedure was applied to the calculation of optimum trajectories for a typical, currently in-service jet transport. Although the optimum trajectories for this type of aircraft were found to depend upon the choice of noise criterion used in calculating them as well as on many other factors, such as the noise characteristics of the jet engines and the length of segments of ground track, some generally valid properties could be discerned. The optimum trajectories calculated have a period of acceleration to a certain climb speed as soon as possible after takeoff. Climb at the climb speed is followed by maximum thrust reduction when the noise-sensitive area or a specified altitude is reached. In the case of the first criterion, the climb speed depends especially on the distance from brake release to the noise-sensitive area; it steadily decreases as this distance decreases. For distances of four miles or greater the climb speed permits full retraction of flaps, whereas for shorter distances, in the case of turbofan-powered aircraft, the climb speed may be insufficient to permit full flap retraction. For the second criterion, the climb speed depends less on this distance and generally falls above the minimum speed for flap retraction. Acceleration rather than a steep climb following lift-off may result in a lower altitude over the noise-sensitive area. The optimum trajectories trade this lower altitude for the

steeper climb angles and the greater thrust reduction obtained in the clean airplane configuration such that the chosen noise criterion is minimized.

INTRODUCTION

Noise produced during takeoff and climbout operation of commercial jet aircraft often causes serious problems in communities located near major air terminals. In this report, we consider the problem of selecting takeoff and climbout trajectories that minimize this noise.

Previous work aimed at establishing improved takeoff and climbout trajectories for noise abatement has utilized noise data obtained from flight tests of different climbout trajectories (refs. 1 and 2). An important result derived from analyzing such experimental noise data was the demonstration of the value of power reduction for noise abatement. But since only relatively few trajectories were flown in flight tests, it is not known if the various tradeoffs that exist between the three main factors affecting perceived noise - namely, thrust, altitude, and airspeed - were optimized for best noise abatement. Moreover, even if one were to succeed in establishing the least noisy climbout trajectory by exhaustive flight testing of a particular aircraft, he would still not know the effects of changes in engine type, aircraft type, or other parameters on the optimum trajectory. These effects would have to be determined by further flight tests.

To alleviate this problem, an analytical approach for determining the optimum climbout trajectory for noise abatement is proposed in this report. The main advantage of this approach is that it allows one to determine quickly and inexpensively the effects of changes in engine noise characteristics, airframe, location of the noise-sensitive area, and operational constraints on the optimum trajectory. Furthermore, the calculated trajectories are optimum in the sense that they minimize specific criteria of noisiness to be discussed later.

It is not suggested that flight testing can be completely eliminated by analysis, but rather than it can be greatly reduced by using the analytically derived properties of the optimum trajectories as a guide. A possible source of error in the analytically calculated trajectories is their dependence on mathematical models for jet noise generation, some of which are not yet well established for certain engine types, notably turbofans. However, if such models are not available, measured noise data may also be used directly in the calculation.

ANALYSIS

Description of the Takeoff and Climbout Trajectory Problem

For the purposes of this study, the ground track of the takeoff and climbout path of an aircraft consists of two major sections. The first section, which is assumed to have low sensitivity to noise but could have a limitation on side-line noise, begins at brake release and ends at the beginning of the noise-sensitive area. Since the length of the section depends on conditions at a particular airport, it will be treated as a parameter in this study. Typical values for its length are three to five miles. The second section of ground track traverses the noise-sensitive area and is typically four to eight miles long. A complete takeoff and climbout trajectory showing the location of the two sections along the ground track is illustrated in figure 1.

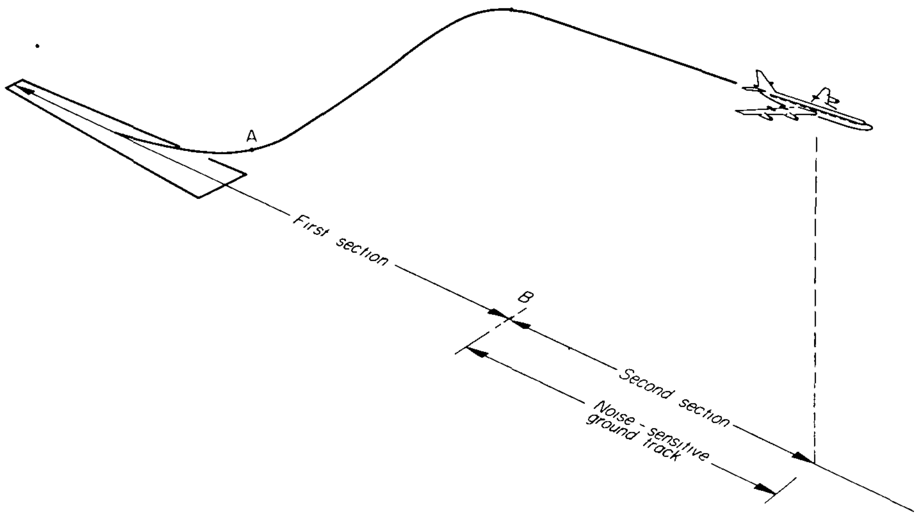


Figure 1.- Typical takeoff and climbout trajectory.

The entire ground track is taken to be a straight line parallel to the runway. This assumption is justified whenever the noise-sensitive area cannot be avoided by early turning maneuvers either because of unfavorable terrain or because the airport is closely surrounded by populated areas all sensitive to noise.

In simplest terms, the objective of this study is to determine the flight path, subject to the operating limits of the aircraft, least annoying to people living in the noise-sensitive area. In determining this optimum flight path, one must take into consideration the earlier stated assumption that the ground track is composed of two adjacent sections, one sensitive to noise, the other insensitive. The implication is that the flight path cannot be optimized independently for each section since the flight path over the first segment strongly affects the noise over the second through the altitude, airspeed, and power setting at the beginning of the second section.

Mathematical Criteria for Noisiness of a Trajectory

The essential component of any mathematical approach to a trajectory optimization is known as the performance function, since it acts as the criterion that ultimately determines the optimum trajectories. Two performance functions for which optimum trajectories are calculated in later sections are described here.

The first, and also the simpler of the two, is the perceived noise level, in units of PNdB, measured at the boundary of the noise-sensitive area (point B in fig. 1 as well as in later figures) after power has been reduced to just sustain level flight at constant airspeed. A summary of important facts concerning calculation of perceived noise level is given in appendix A. To minimize this performance function, only the flight path in the first section needs to be considered.

Though this performance function has the advantage of simplicity, it also has some serious drawbacks that lead one to consider a more comprehensive criterion for noisiness of a trajectory. If point B in figure 1 were the only noise-sensitive point along the ground track, minimizing the perceived noise level at that point, as is done by using the performance function given above, would probably be adequate. However, minimizing the perceived noise level at only one point, which is located at the beginning of the noise-sensitive area, effectively discriminates against other points of equal importance. Furthermore, this performance function does not consider the effect of duration on perceived noise.

The second performance function defined below attempts to correct the difficulties with measuring the noise at only a single point by averaging the noise level along the noise-sensitive second section and by including a penalty on duration of the noise. An approximate, but sufficiently accurate, method for computing the average noise is to divide the noise-sensitive ground track into a number of short segments, to compute the maximum flyover noise for each segment, and then to average these values. Except for the factor $\sqrt{\Delta t_i/\Delta t_{ref}}$, which models duration effects and is discussed in greater detail in appendix A, perceived noise averaged over the length of the second section is the correct interpretation of the second performance function defined below:

$$J = 10 \log_2 \frac{1}{L} \sum_{i=1}^{L} N(F_i, h_i, V_i) \sqrt{\frac{\Delta t_i}{\Delta t_{ref}}} \tag{1}$$

where

J value of the second performance function

L number of short sections

N $2^{(Z-40)/10}$

Z perceived noise level, PNdB

F_i total thrust used to compute noise in the ith section

h_i altitude

V_i airspeed

Δt_i $\frac{\Delta x}{V_i}$

Δt_{ref} reference time for duration of noise perceived in each short section

Δx length of each short section, 750 ft

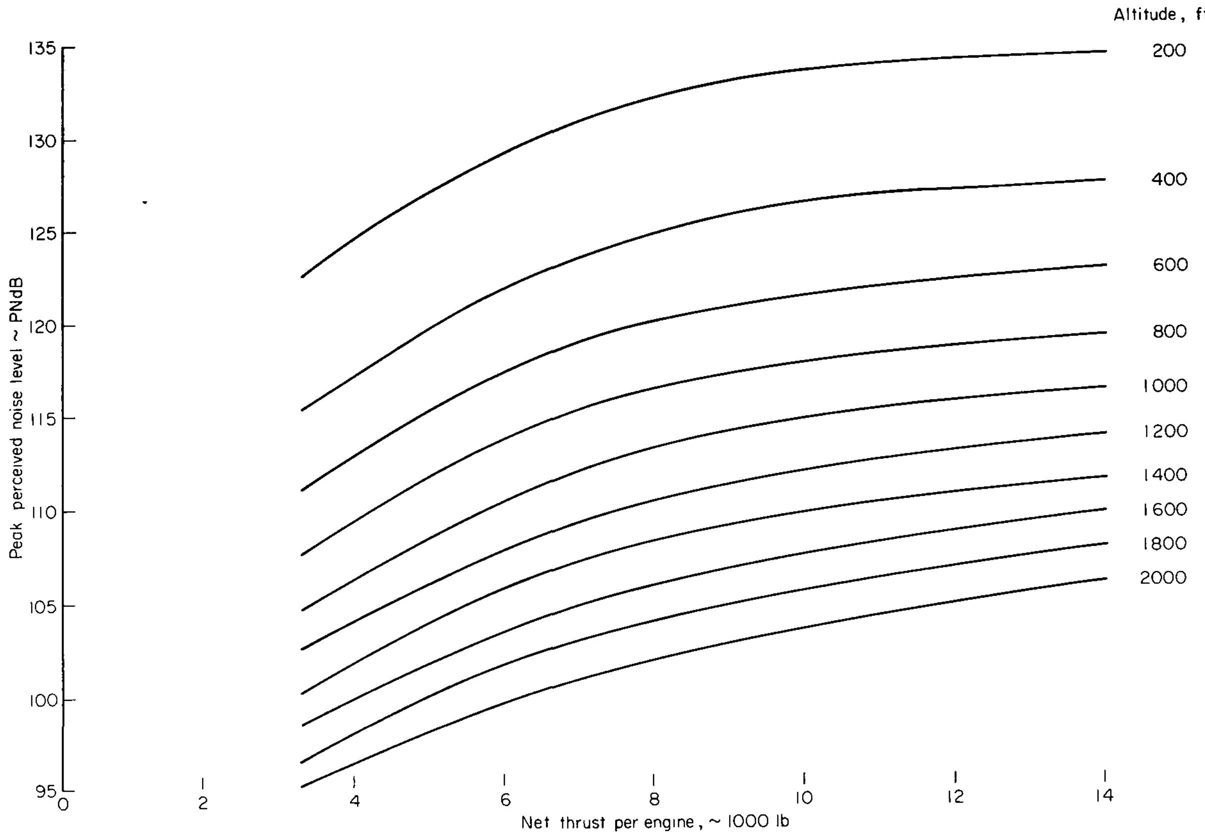
The quantity N defined above, whose unit is the noy, was originally used by psychologists in the development of a scale of noisiness. This quantity is more suitable than Z in equation (1) because, in forming the summation of noise received in each short segment, it penalizes high noise levels more than the logarithmically dependent Z does. For brevity, the quantity J will be referred to as average perceived noise.

The intent of this criterion is to first estimate the noise to which single observers stationed in each short segment of the noise-sensitive ground track are subjected and then to average the noise over all observers. Minimizing this average tends to minimize the number of complaints that would be received from all such observers.

Method for Computing Perceived Noise

The performance functions discussed in the previous section require calculation of perceived noise level as a function of thrust, altitude, and airspeed. For turbojet noise, the SAE noise prediction method, described in references 3, 4, and 5 and summarized in appendix A, can be used to perform these calculations. This method is reasonably accurate for predicting the maximum flyover noise for turbojet engines with standard exhaust nozzles. Fortunately, high accuracy in predicting noise levels is not needed in this study, since, in minimizing the two performance functions for noisiness, the absolute value of the function to be minimized is irrelevant. The relevant items in the minimization are the tradeoffs among thrust, altitude, airspeed, and duration. Hence, a model for jet noise that preserves these tradeoffs, as the SAE model does, is sufficient.

For trajectory calculations involving turbofan noise, however, the currently available SAE noise prediction method, even with refinements introduced specifically to model fan-generated noise, is inadequate. Comparison with measured fan-jet noise data has shown considerable error both in the absolute value of the calculated noise as well as in the accuracy of the tradeoffs between altitude and thrust. As pointed out above, it is the latter type of error that prohibits the use of this method for optimum trajectory calculations. Instead, some noise data as a function of thrust and altitude for a currently used turbofan engine were used. These data are reproduced in figure 2. Their main disadvantage is the unspecified effect of changes in airspeed on perceived noise.



Note Standard day, four engines operating

Figure 2.- Perceived noise as a function of thrust and altitude for a typical turbofan currently in service.

Since the optimum trajectories are influenced mainly by the tradeoffs in the noise measure between altitude, thrust reduction, airspeed, and duration, it is instructive to tabulate the range of change in perceived noise caused by doubling each of these quantities. This is done in table I for turbojet and turbofan generated noise. Particularly noteworthy is the fact that sensitivity of the noise to thrust changes in considerably higher for turbojets than for turbofans. This difference in noise characteristics of the two engine types has important effects on the optimum climbout trajectories, as we

TABLE I.- EFFECTS OF NOISE FACTORS

	Changes in perceived noise caused by doubling the noise factors, PNdB	
	Turbojet	Turbofan
Thrust	9 to 15	4 to 9
Altitude	-9	-9
Airspeed	-2 to -8	Unknown
Duration	2 to 6	2 to 6

shall demonstrate when discussing the results. Although the effect of airspeed on turbofan generated noise is unknown, some negative change with airspeed is expected.

Simplified Equations of Motion

To evaluate either of the two noise measures described earlier, one must be able to generate histories of altitude and airspeed along the ground track for a specific climbout procedure. In this study, it is convenient to describe a climbout procedure in terms of thrust, flight-path angle, and flap deflection angle as a function of the distance along the ground track. These three quantities are the only control variables that need to be considered, since the ground track of the climbout is assumed to be a straight line. Lateral control maneuvers are therefore excluded. The equations of motion are then given as

$$\frac{dV}{dx} = \frac{g}{WV \cos \gamma} \left(F \cos \alpha - \frac{1}{2} \rho S V^2 C_D(\alpha, \delta) - W \sin \gamma\right) \tag{2}$$

$$\frac{dh}{dx} = \tan \gamma \tag{3}$$

where

$C_D(\alpha,\delta)$	drag coefficient
F	total thrust
g	gravitational constant
h	altitude
S	wing reference area
V	airspeed
W	gross weight
x	distance along ground track
α	angle of attack
γ	flight-path angle
δ	flap angle
ρ	air density, slugs/ft^3

The equations are in standard form except that x is the independent variable rather than time, as is usually the case. The angle of attack α, which is needed to solve equation (2), is calculated by solving the following equation for α:

$$\frac{1}{2} \rho S V^2 C_L(\alpha, \delta) - W \cos \gamma + F \sin \alpha = 0 \tag{4}$$

where C_L is the lift coefficient. Equation (4) assumes that the distance along the ground track needed to make a step change in the flight-path angle is small compared to the total length of the ground track. This equation can be solved for α by approximating $\sin \alpha$ with α and C_L with a linear function in α and δ. Equations (2), (3), and (4) allow one to generate climbout trajectories if the initial altitude and airspeed is specified and if the control variables F, γ, and δ are assigned specific functions of x. However, these equations were used only for generating that part of the trajectory beginning at a point where the aircraft has reached an altitude of 400 feet and the takeoff safety speed V_2 (point A in fig. 1). The location of this point can be calculated for a particular aircraft by using the procedure described in its flight manual.

Method of Computing Optimum Trajectories

The foundation has now been laid for the calculation of takeoff and climbout trajectories that minimize either of the two performance functions defined earlier. Moreover, the problem has been formulated in such a manner that techniques from optimum control theory can be brought to bear upon it. In control theory terminology, F, γ, and δ are the controls used to generate takeoff and climbout trajectories. For any initial altitude and airspeed, a specification of F, γ, and δ as a function of x results in generating a particular takeoff and climbout trajectory. For every takeoff and climbout trajectory generated, we can compute the value of the chosen performance function. The objective of the computational procedure then is to select from the infinite set of allowable controls those that minimize the chosen performance function.

In this study, the dynamic programming algorithm, implemented on a digital computer, was used to calculate the optimum controls, thrust, flight-path angle, and flap angle (ref. 6). Dynamic programming, although not as efficient computationally as other methods, has some important advantages in this application. These advantages arise mainly from the simplicity with which inequality constraints on the state variables (airspeed and altitude) and on the controls can be included in the computation of optimum trajectories. Physical, as well as safety, considerations restrict the state variables, altitude and airspeed, to lie within specified bounds. Also, the controls have prescribed lower and upper bounds that arise in the case of thrust and flap angle because of physical limitation and, in the case of flight-path angle, because of operational restriction. A brief discussion of the dynamic programming algorithm can be found in appendix B.

RESULTS

General Properties of the Optimum Trajectories

Optimum trajectories computed for the two performance functions introduced earlier have been found to depend strongly on the engine type - whether turbojet or turbofan - the length of the first section ground track, operational constraints, and, in the case of the second performance function, on the length of the second section ground track. Although influenced by many variables, an optimum trajectory for the second performance function, represented here as a history in altitude-airspeed coordinates, typically has the form shown in figure 3. The constraints used in this calculation are listed in the figure. The near optimum trajectory, which is also shown, will be discussed subsequently. For the purposes of this study, the optimum trajectory is assumed to begin at point A, where the aircraft has achieved the takeoff safety speed V_2 and an altitude of 400 feet, since before this airspeed and altitude is achieved no unusual maneuvers are permitted. Thus, starting at point A, the aircraft accelerates in level flight until a certain climb speed, which is usually close to the minimum drag speed, has been attained. During this acceleration period, flaps are retracted as soon as the minimum speed for flap retraction is achieved. The aircraft then enters an essentially constant airspeed climb until just before the noise-sensitive area is reached, where the climb steepens to become a deceleration climb. This is followed by a large power reduction as the noise-sensitive area is penetrated. The remainder of the trajectory, although depending somewhat on the length of the second section of the ground track, consists here of a slightly decelerating climb at the minimum permissible power setting.

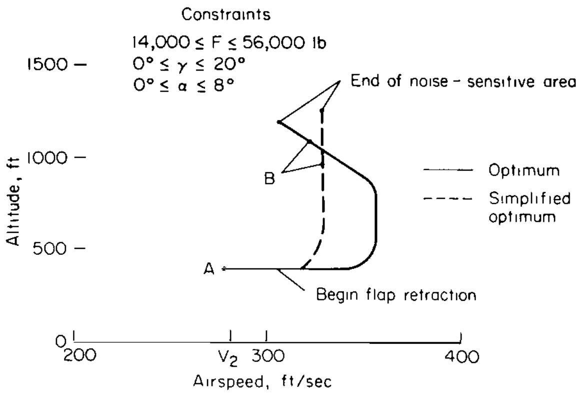


Figure 3.- Optimum and simplified optimum trajectory for the second performance function.

Between points A and B, a typical optimum trajectory for the first performance function has the same general shape as the trajectory just described, and, therefore, requires no additional discussion.

Simplification of the Optimum Trajectories

In assessing the practical value of optimum trajectories, one must consider the difficulty a pilot would experience in flying along them and the number of parameters required to describe them. Examined in this light, the optimum trajectory shown in figure 3 is probably too complicated for operations with current instrument displays. Although a flight director system might permit the optimum trajectories to be flown without difficulty, another approach is to consider simplified trajectories.

We observe that the optimum trajectory between points A and B consists essentially of a period of acceleration followed by a period of steep climb. Therefore, a logical choice for a simplified optimum trajectory would be one that accelerates at level flight to a certain airspeed, then climbs at constant airspeed, and finally enters a reduced power flight near the beginning of the noise-sensitive area. Such a simplified optimum trajectory is indicated in figure 3 by the broken line. In effect, this simplified trajectory needs to be optimized only over two variables, namely, the climb speed and the amount of thrust reduction. This simplifies the computations of the trajectory and produces a trajectory that is easy to fly along.

However, the decisive test of acceptability of the simplified optimum trajectory is given by the penalty measured in terms of the noise generated by it in comparison to the minimum noise. In all cases examined, the noise generated by the simplified trajectories exceeds that of the optimum trajectories by negligible amounts. Hence, only simplified optimum trajectories are presented in the following section and are referred to as "optimum trajectories."

Minimizing Perceived Noise Level at Beginning of Noise-Sensitive Area

Examples of optimum trajectories for this performance function were computed for a typical, currently in-service jet transport powered by either a turbofan or turbojet, at a gross weight of 280,000 pounds. In this section only, the perceived noise level (in PNdB) at fixed thrust and altitude was assumed to be independent of airspeed for both engine types. The absence of dependence of the noise on airspeed represents a limiting case and would occur when the jet exhaust velocity is very large compared to permissible airspeed changes. Also, maximum takeoff thrust was assumed to be the same for both the turbofan and the turbojet. These two assumptions make it possible to attribute differences between optimum trajectories of turbofan- and turbojet-powered aircraft solely to differences in the dependence of the noise on thrust. In determining the turbojet noise as a function of thrust, it was assumed that doubling the thrust causes an increase of 15 PNdB in the noise. This dependence of noise on thrust follows from the SAE prediction procedure for turbojet noise (refs. 4 and 5). The effect of altitude on noise was calculated for standard atmospheric conditions and no wind using the SAE procedure. Turbofan noise as a function of thrust and altitude was obtained directly from figure 2.

In the first section of the ground track, constraints on the trajectory were the same as those listed in figure 3. Just before the ground track of the aircraft penetrates point B, thrust is reduced to permit the aircraft to fly level and unaccelerated at the speed of penetration. The optimum trajectories are calculated to minimize the noise after the thrust reduction has taken effect. The only variable in this minimization is the climb speed.

Before the results are presented, it must be emphasized that the optimum trajectories and the noise reductions predicted for them depend strongly on the engine noise characteristics. These noise characteristics can vary widely for different engine models, particularly if they are turbofans. Hence, it is generally necessary to recalculate the optimum trajectories and the corresponding noise reduction for particular engine, aircraft, and airport conditions.

The altitude-distance histories and the thrust schedules after point B for two optimum trajectories and one nonoptimum trajectory are shown in figure 4 for a first-section length of 19,000 feet. As in figure 3, the trajectories are not shown before point A, located for this example 11,000 feet

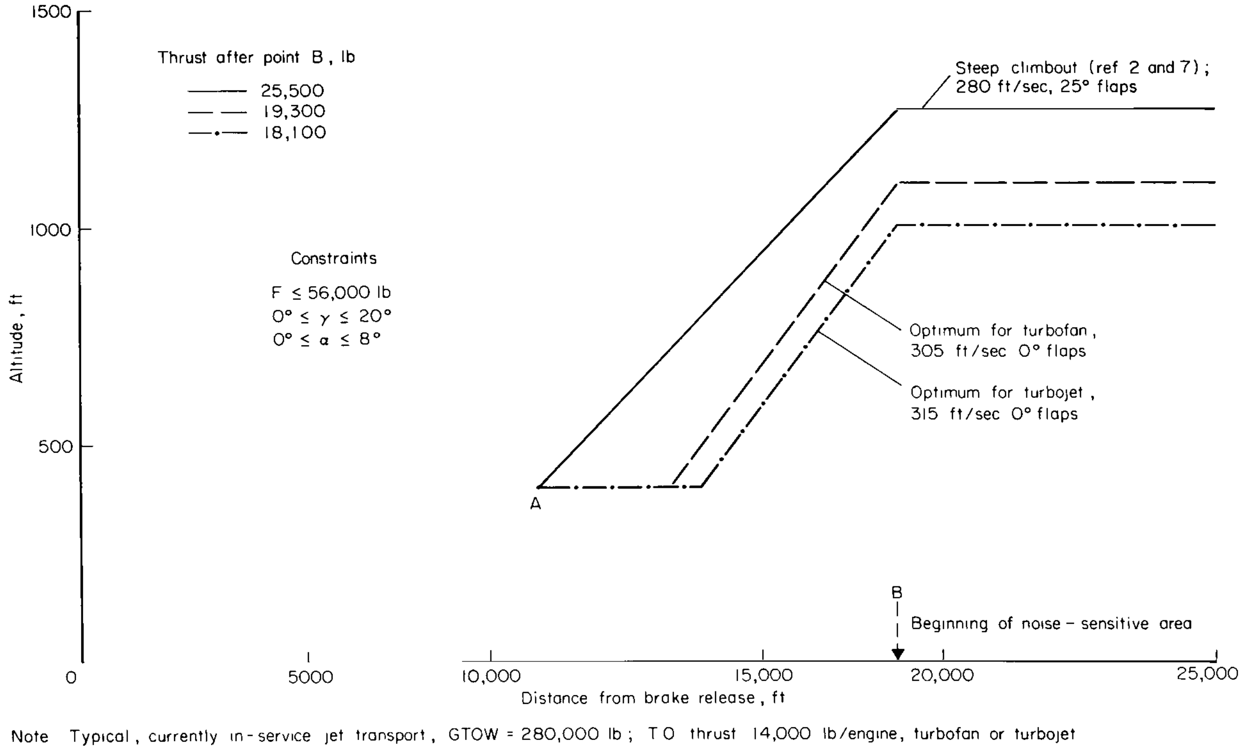


Figure 4.- Trajectories pertaining to first performance function.

from brake release, since the aircraft requires this distance to achieve V_2 and the minimum maneuvering altitude of 400 feet. We note that the optimum climbout speed for the turbojet is 10 ft/sec higher than for the turbofan and that both trajectories require completely retracted flaps. The minimum speed for full retraction of flaps was chosen as 305 ft/sec. For comparison, a steep climbout trajectory that has previously been considered for noise abatement purposes is also shown in figure 4 (refs. 2 and 7). Although this trajectory achieves the highest altitude at the beginning of the noise-sensitive area, it also requires the highest thrust from point B onward. The higher thrust needed to maintain level unaccelerated flight is due to the higher drag associated with the 25° takeoff flap configuration. This higher drag also results in smaller climb angles compared to those of the optimum trajectories. In computing the optimum trajectories, the tradeoff between thrust reduction, airspeed gain, and increase in climb angle on the one hand and altitude gain on the other has been optimized to achieve the greatest noise reduction at point B. This tradeoff also helps to explain the higher climb speed of the turbojet compared to that of the turbofan. According to table I, turbojet noise can be more thrust-sensitive than turbofan noise. This suggests that thrust reduction is more effective for turbojets than for turbofans. Greater thrust reduction at point B is achieved by accelerating early to a higher airspeed where a more favorable lift-to-drag ratio is found.

The dependence of the optimum climb speed on the length of the first section ground track is shown in figure 5. The optimum climb speed for both engine types is seen to decrease as the length of the first section becomes shorter, and, for a length of less than 18,000 feet, is too low to permit full retraction of flaps in the case of turbofan noise. Thus, for short first sections the optimum trajectories become identical to conventional steep climbouts.

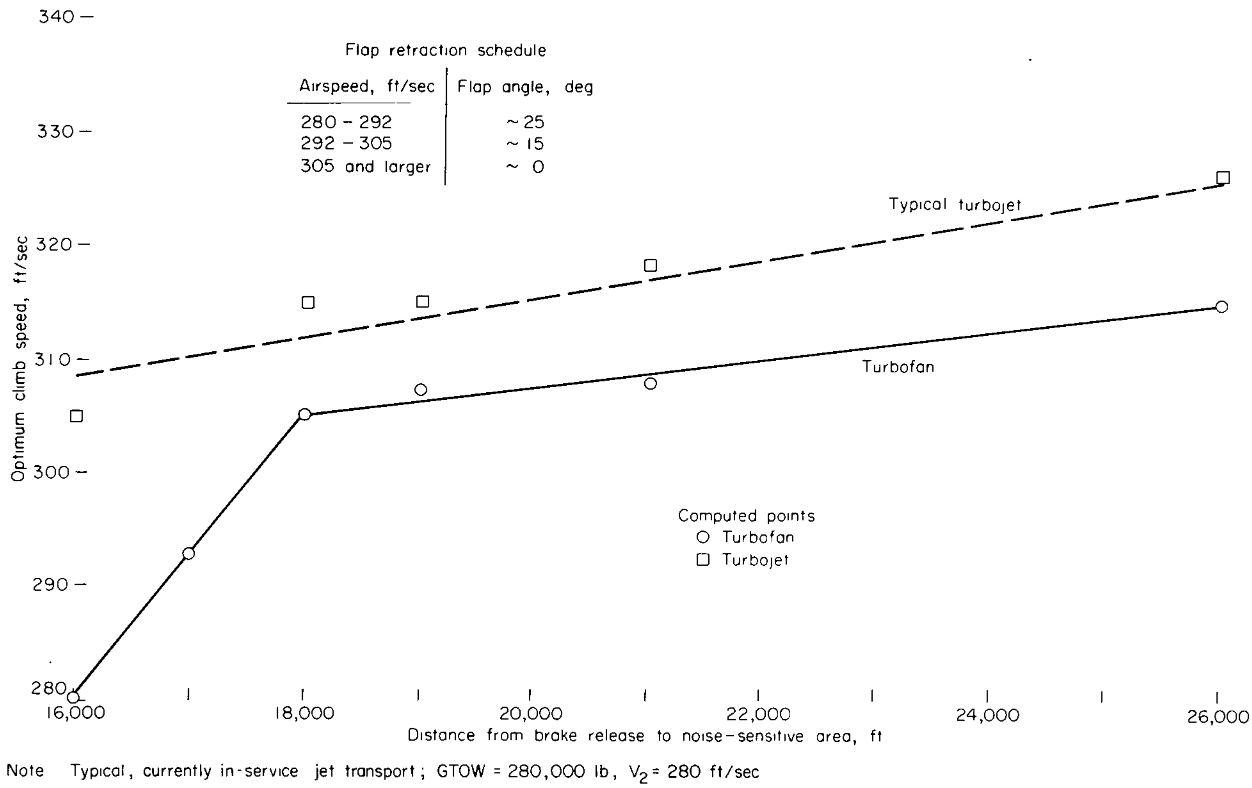


Figure 5.- Optimum climb speed as a function of distance to noise-sensitive area; first performance function.

The optimum trajectories can produce a lower perceived noise level from point B onward than the steep climbout trajectory. The differences in noise between the steep climbout and the optimum trajectories are tabulated in table II as a function of the length of the first-section ground track. These differences are seen to be higher for the turbojet because of the greater effect of thrust reduction on perceived noise for this engine type. It is understood, however, that for engines of the same thrust class, the noise from turbojets is generally higher than from turbofans.

TABLE II.- DIFFERENCE IN NOISE (PNdB) BETWEEN STEEP CLIMBOUT AND OPTIMUM TRAJECTORIES

Distance to noise-sensitive area, ft	Turbojets	Turbofans
16,000	3.0	0
17,000	3.4	.5
18,000	5.0	.8
19,000	5.5	1.2
21,000	6.0	1.5
26,000	7.6	2.5
NOTE: First performance function, typical currently in-service jet transport, GTOW = 280,000 lb, 14,000 lb/engine takeoff thrust.		

Minimizing the Average Noise Over the Noise-Sensitive Area

In this section, we present trajectories that minimize the second performance function defined by equation (1). The noise model used in this section and the next was extended to include the effect of airspeed on perceived noise. Although it is recognized that this effect can vary with engine parameters, for a typical turbojet the SAE noise prediction procedure predicts a decrease of 7 PNdB for a change in airspeed from 200 to 400 ft/sec. This figure was used in both turbojet and turbofan noise calculations. For both engine types, the relationships between noise, thrust, and altitude at a particular airspeed were the same as in the previous section. The length of the first section ground track was chosen as 21,000 feet and that of the second section as 26,000 feet, or approximately 5 miles. In the second section, thrust was constrained to be not less than that needed to maintain level unaccelerated flight at the airspeed at which the second section is entered. Constraints on flight-path angle and angle of attack were the same as before.

Two variables enter into the calculation of optimum trajectories - the speed to which the aircraft is allowed to accelerate before beginning its constant airspeed climb and the thrust used along the noise-sensitive ground track. An optimum trajectory is determined by finding the combination of these two variables that minimizes equation (1). This combination was found by an exhaustive search over the set of these variables. It was found that the optimum thrust for both turbojet and turbofan was always very close to the smallest allowed by the constraint. It should be pointed out, however, that for second section lengths longer than 5 miles, the optimum thrust after power reduction begins to increase above the lower constraint value, resulting in trajectories that are characterized by a gradual climb over the noise sensitive area. Such second section lengths are not considered in this study.

The resulting optimum trajectories for turbojet- and turbofan-powered aircraft are shown in figure 6. The optimum climb speeds for these trajectories are seen to be somewhat higher than those given in the previous section for the first performance function. These differences in the optimum climb speeds are due to the penalty on duration of the noise contained in the present performance function and to the assumed dependence of perceived noise on airspeed. Dependence of the optimum climb speed on the length of the first section ground track is similar to figure 5. But in this case, even for a first section ground track less than 18,000 feet long, the optimum climb speeds for either engine were found to be larger than the minimum speed for complete flap retraction.

Figure 6 also gives the differences in average perceived noise (eq. (1)) between steep climbout and optimum trajectories: 5 dB for the turbofan and 8 dB for the turbojet. The explanation for the greater difference in the case of turbojet noise is the same as in the previous section.

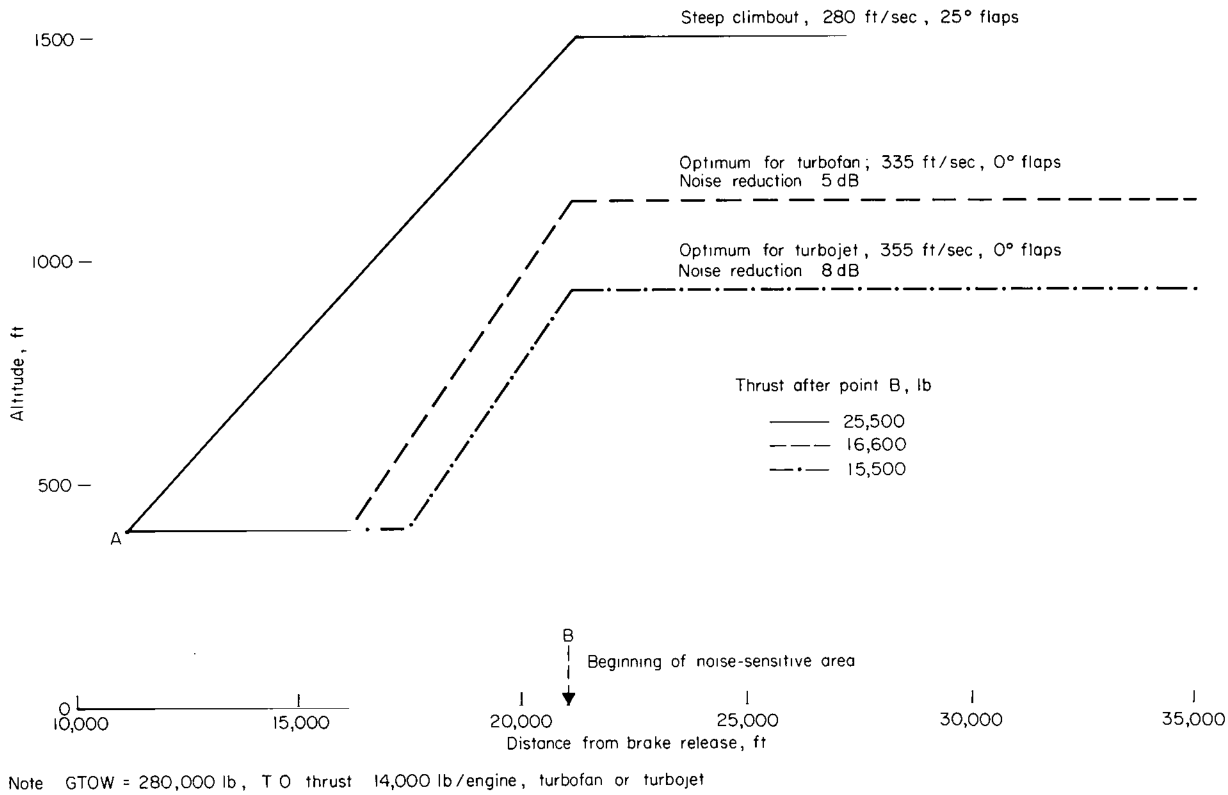


Figure 6.- Trajectories pertaining to second performance function; no altitude constraint.

Minimizing the Average Noise Over the Noise-Sensitive Area With Altitude Constraint

It may be required for operational or safety reasons that the aircraft should first achieve some minimum altitude above ground level before power is reduced in a noise-abatement climbout. In this section, we present trajectories that meet this requirement, that is, they minimize the average noise level given by the second performance function while subject to the constraint that final power reduction not take place until an altitude of 1500 feet is attained. However, some power reduction is assumed to take place at point B if the aircraft is at a lower altitude in order not to violate an upper limit on the perceived noise assumed to exist at the beginning of the noise-sensitive area. Thrust after this initial power reduction was taken as 40,000 pounds, which is assumed to satisfy the maximum noise limitation. Minimum thrust after final power reduction at 1500 feet obeys the constraint described in the previous section.

According to figure 7, the steep climbout trajectory in takeoff flap configuration attains the minimum altitude of 1500 feet just as the noise-sensitive area is penetrated and, therefore, immediately permits maximum power reduction to take place. The optimum trajectories, however, first accelerate to the indicated airspeeds, then climb at full power as far as the boundary of the noise-sensitive area and continue to climb at reduced power to 1500 feet, where the final power reduction occurs. The differences in average perceived noise between steep climbout and optimum trajectories are also given in figure 7: 4 dB for the turbofan and 7 dB for the turbojet.

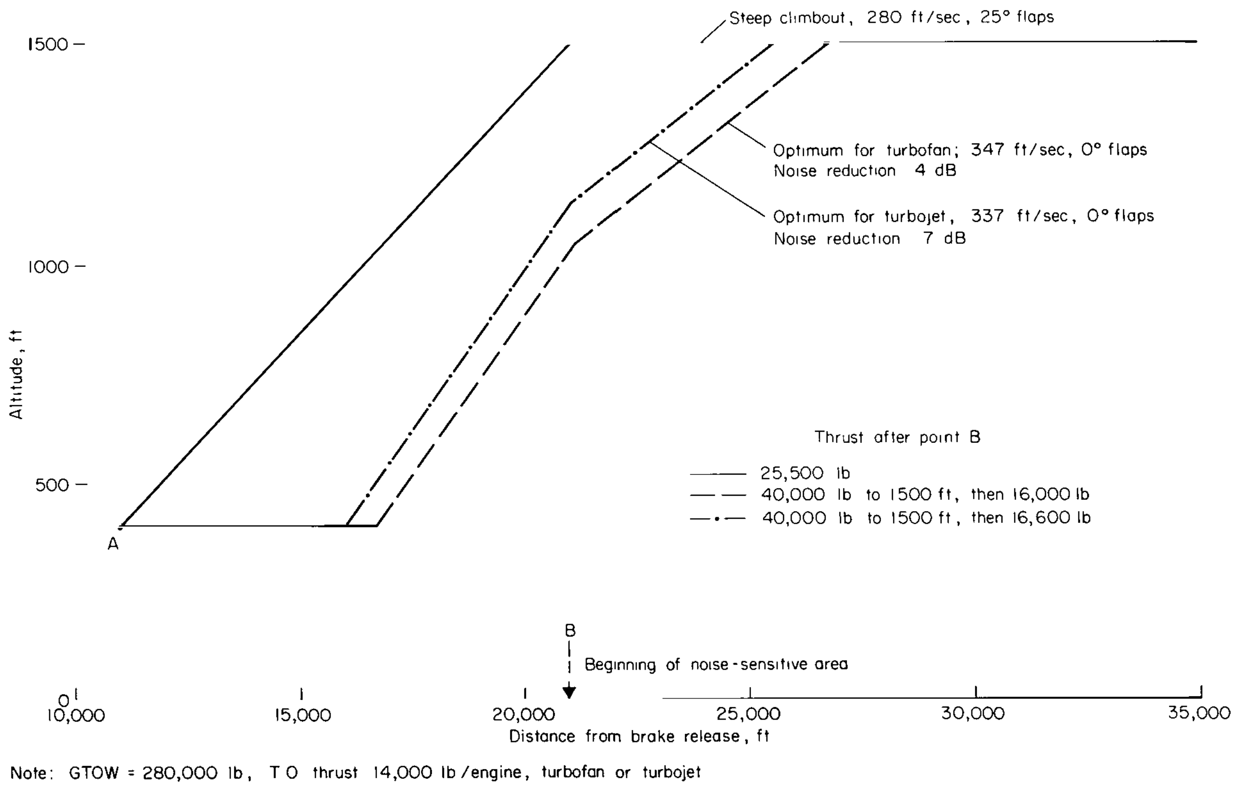


Figure 7.- Trajectories pertaining to second performance function; 1500 feet minimum altitude constraint.

It is interesting to note that the optimum climb speed for the turbofan is here somewhat higher than for the turbojet. That reverses the order of the optimum climb speeds for the two engine types obtained in the previous two sections. Since the minimum altitude constraint is the only difference between calculations in this and the previous section, the reversal must be the result of the constraint. A more complete explanation of this phenomenon has not been found.

CONCLUSIONS

We have established a rational procedure for determining takeoff and climbout trajectories that minimize the annoyance from jet takeoff operations in communities located along the ground track of the climbout path. What distinguishes this procedure from others used in the past is the mathematical formulation of the problem and its solution by purely analytical techniques. The procedure enables one to compute optimum takeoff and climbout trajectories for a particular aircraft operating from a particular airport subject to prescribed operational constraints.

Two mathematically defined criteria were used as a basis for arriving at noise-optimum trajectories. The first of these was simply the perceived noise level measured after a power reduction at the beginning of the noise-sensitive area. The second was taken as the perceived noise level averaged along the noise-sensitive ground track. The latter criterion also includes the effect of duration on the subjectively perceived noise.

The technique was applied to the calculation of optimum takeoff and climbout trajectories for a typical, currently in-service jet transport. Although the optimum trajectories for this type of airplane depend upon the choice of noise criterion used in calculating them, as well as on the noise characteristics of the jet engines and the length of segments of ground track, some generally valid properties of the trajectories can be discerned. The optimum trajectories calculated have a period of acceleration to a certain climb speed as soon as possible after takeoff. Climb at this climb speed is followed by maximum thrust reduction when the noise-sensitive area or a specified altitude is reached. In the case of the first criterion, the climb speed depends especially on the distance from brake release to the noise-sensitive area; it steadily decreases as this distance decreases. For distances of four miles or greater, the climb speed permits full retraction of flaps, whereas for shorter distances, in the case of turbofan-powered aircraft, the climb speed may be insufficient to permit full flap retraction. For the second criterion, the climb speed depends less on this distance and generally falls above the minimum speed for flap retraction. Acceleration rather than a steep climb following lift-off may result in a lower altitude over the noise-sensitive area. The optimum trajectories trade this lower altitude for the steeper angles and the greater thrust reduction obtained in the clean airplane configuration so that the chosen noise criterion is minimized.

Ames Research Center
National Aeronautics and Space Administration
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APPENDIX A

SUMMARY OF PROCEDURE FOR CALCULATING PERCEIVED NOISE

The need to compute jet noise perceived along the ground track from a particular flyover is fundamental to this study. The method employed here for calculating the perceived noise level was adapted from two SAE reports (refs. 3 and 4), and the details of the calculations can be found there. Essentially, the method used in this report predicts the far field noise (in PNdB) for turbojet engines with standard circular nozzles. However, with suitable modifications it applies also to other engine configurations. A brief discussion of the two main steps involved in the noise computation is now given.

In the first step, the acoustic power contained in each of the eight octave bands that fall within the audio frequency range is computed. Reference 4 offers two different procedures for carrying out this computation; the first uses jet parameters and the second is based on measured engine performance characteristics. The second procedure is used here because of its convenience for the purposes of this study.

The parameters that determine the acoustic power and the spectral content of the noise are the relative jet velocity V_R, the jet velocity V_j, the density of the enhaust gases ρ_j, and the cross-sectional area A of the nozzle at the exhaust plane. The last quantity can be measured directly for a given engine, but the first three must be related to airspeed, V, and engine thrust, F, for use in later calculations. This is done partially with the help of the following three elementary relations (ref. 4):

$$V_R = \frac{F}{W_a/g} \tag{A1}$$

$$\rho_j = \frac{W_a}{AV_j} \tag{A2}$$

$$V_j = V_R + V \tag{A3}$$

where W_a is the weight flow of exhaust gases and g is the gravitational constant. If F, V, and A are specified, the three equations still contain four unknowns; hence, an additional relationship involving these variables is needed to solve for them. Such a relationship is provided by engine performance curves that relate net thrust, airspeed, and weight flow. These curves, which also contain other useful information such as fuel flow and operating limits, are supplied by the manufacturer of a given engine.

With the relationship between thrust, airspeed, and weight flow derived from these curves, one can solve for V_R, ρ_j, and V_j and then calculate the sound pressure level in the eight-octave bands of the audio frequency range using the procedure described in reference 4. The eight numbers so obtained define the acoustic power spectrum of the maximum passby noise for a single engine along a line 200 feet from the flight path of the aircraft. These numbers are then adjusted for the number of engines, four in this case, and for the attenuation due to altitude and atmospheric absorption rates. According to reference 4, the noise power spectrum obtained by this method is based on the following simplifying assumptions:

1. The atmosphere is sea level, standard day (59° F, 70 percent relative humidity), homogeneous, and windless.

2. The jet noise follows the empirically determined relationship given in reference 1.

3. The maximum passby noise is generated at 45° to the jet exhaust axis.

4. Aerodynamic noise is negligible.

In addition, two other assumptions of importance were found to be convenient. First, because of the directional characteristics of jet noise, the angle (measured between the horizon and the aircraft) at which maximum passby noise is received depends on the attitude of the aircraft. For an aircraft in a climbing attitude, this angle tends to be larger than for level flight. Therefore, an accurate calculation of jet noise would require some knowledge of the directional characteristics of the noise. It is felt, however, that for the purposes of this study such a refinement of the noise calculation is not necessary, because the optimum trajectories are affected much less by the absolute value of the noise level than they are by the tradeoffs between the main factors entering into the noise calculation, namely, altitude, thrust, and airspeed. Hence, a model for jet noise that preserves the essential characteristics of the tradeoffs is sufficient here. The second assumption consists of ignoring pure-tone components that may be present in the noise spectrum.

Now we can perform the final step in the noise calculation in which the eight numbers obtained from the first step are converted into perceived noise level. For this purpose, tables are available in reference 3 which first weight the values of noise power contained in the octave bands in proportion to the amount of annoyance they produce in humans. The weighted values of acoustic power are then combined by means of a formula into a single number that is the perceived noise level. It is customary to give the perceived noise level of a sound in units of PNdB, a logarithmic unit defined in reference 3.

One additional factor must be considered. It is well known that the subjectively judged noisiness of a sound depends not only on acoustic power and spectral content, but also on duration (ref. 8). The complete relationship between duration and perceived noise is too complex to be considered here, but an approximation sufficient for the purpose of this study is to assume that doubling the exposure time of a noise increases the perceived noise level between 2 and 6 PNdB (ref. 8). Since the N terms in equation (1) are antilogarithmically related to the perceived noise level in

PNdB, an equivalent operation on them to account for a 5-PNdB increase per doubling of duration is to multiply by $\sqrt{\Delta t_i/\Delta t_{ref}}$, where Δt_{ref} is a reference duration. The choice of 5 PNdB is therefore also convenient for computational reasons since other values would require raising $\Delta t_i/\Delta t_{ref}$ to fractional powers other than the square root. The reference duration is arbitrary in this case, since it affects J, the average noise value, only by a constant and thus has no influence on the determination of the trajectories.

APPENDIX B

SUMMARY OF DYNAMIC PROGRAMMING

Dynamic programming has been widely used in theoretical and computational problems occurring in optimum control (ref. 6). The main feature of dynamic programming is the dynamic programming algorithm used to compute optimum control policies. A brief description of this algorithm and its application to the noise minimization problem is given in this appendix. The discussion given here applies specifically to the second performance function (eq. (1)), but, with minor modification, it is also applicable to the first performance function.

A slightly modified form of equation (1) is used as the performance function in this discussion:

$$\hat{J} = \sum_{i=1}^{L} N(F_i,\ h_i,\ V_i) \sqrt{\frac{\Delta t_i}{\Delta t_{ref}}} \tag{B1}$$

where

$$\Delta t_i = \frac{\Delta x}{V_i}$$

This modification of J does not affect the optimum trajectories, but it yields a function that is more convenient for analysis. As explained in the text, the objective of the computation is to find control histories F, γ, and δ that minimize equation (B1). For simplicity, the length of ground track for which such optimum control histories are to be computed is assumed to begin at point B and to terminate at the end of the noise-sensitive ground track. The control histories are a function of the altitude and the airspeed at point B as well as the distance, x, from point B. At all points along the noise-sensitive ground track, the controls and the angle of attack must obey the following constraints:

$$F_{min} \leq F \leq F_{max}$$

$$\gamma_{min} \leq \gamma \leq \gamma_{max}$$

$$0 \leq \delta \leq \delta_{max}$$

$$\alpha_{min} \leq \alpha \leq \alpha_{max}$$

The angle-of-attack constraint is checked by means of equation (4).

In preparation for the computer implementation of the discrete form of dynamic programming used here, it is necessary to quantize the state, control, and distance variables. Quantization of the distance variable, x, into

750-foot segments was already described in the discussion of equation (1). This quantization of the distance variable is now applied to equations (2) and (3) in order to write them as differential difference equations:

$$V_{i+1} = V_i + \Delta x \left[\frac{g}{WV \cos \gamma} (F \cos \alpha - \frac{1}{2} \rho S V^2 C_D - W \sin \gamma) \right] \quad \text{(B2)}$$

$$h_{i+1} = h_i + \Delta x \tan \gamma \quad \text{(B3)}$$

For any i, the state variables V_i and h_i are quantized into L_V and L_h levels as shown:

$$V = V_{min} + \Delta V j_V \qquad j_V \varepsilon \{0, 1, 2, \ldots, L_V\}$$

$$\Delta V = \frac{V_{max} - V_{min}}{L_V}$$

$$h = h_{min} + \Delta h j_h \qquad j_h \varepsilon \{0, 1, \ldots, L_h\}$$

$$\Delta h = \frac{h_{max} - h_{min}}{L_h}$$

Typical values used for the incremental quantities are $\Delta V = 10$ ft/sec and $\Delta h = 130$ ft. Although a similar procedure can also be used to quantize the control variables, it is customary to define a set U of quantized controls as

$$U = \{F^1, F^2, \ldots, F^m; \gamma^1, \gamma^2, \ldots, \gamma^n; \delta^1, \delta^2, \ldots, \delta^q\}$$

Five quantization levels for F, nine for γ, and three for δ, were found to give satisfactory results.

Next, the important concept of the minimum performance function is introduced. This function is defined as

$$I(h_i, V_i, L - i) = \min_U \sum_{k=i}^{L} N(h_k, V_k, F_k) \sqrt{\frac{\Delta t_k}{\Delta t_{ref}}} \quad \text{(B4)}$$

It is interpreted as the minimum value of the performance function, J, for an aircraft that begins the trajectory at the ith segment with altitude h_i and airspeed V_i. The minimization over the quantized control set U refers to the selection for every j $(i \leq j \leq L)$ of a control vector $(F_j, \gamma_j, \delta_j)$

from U such that the minimum value of J is obtained. This selection procedure implies that for any i, 3(L-i) control values must be determined in order to specify I.

The dynamic programming algorithm, which is simply a method for computing I recursively, can now be given:

$$I(h_i, V_i, L - i) = \min_U \left[N(h_i, V_i, F_i) \sqrt{\frac{\Delta t_i}{\Delta t_{ref}}} + I(h_{i+1}, V_{i+1}, L - i - 1) \right] \tag{B5}$$

To compute $I(h_i, V_i, L)$ and the optimum control history with this algorithm, one sets $i = L$ and defines $I(h_{L+1}, V_{L+1}, -1) = 0$. Then $I(h_L, V_L, 0)$ is found by searching the set U for the control vector that yields the smallest N. The search must be repeated for every quantized pair of h and V, and the resulting values of I and the optimum control vector must be stored in the computer for later reference. Next, $I(h_{L-1}, V_{L-1}, 1)$ is computed using equation (B3) and the stored values of $I(h_L, V_L, 0)$. For a given quantized pair of h_{L-1} and V_{L-1}, equations (B2) and (B3) are utilized to compute h_L and V_L. However, since h_L and V_L will not generally fall on the quantized values of these variables, it is necessary to interpolate the value of I. For simplicity in programming, a linear interpolation scheme was used and found to be satisfactory.

Clearly, this process of computing I and the optimum controls for the last two segments can be repeated for the remaining segments of ground track. The result of this computation is the optimum control law that specifies the optimum control vector as a function of the segment number i and the quantized values of the states. This control law, which exists as a stored vector function in three independent variables, can now be used to generate optimum trajectories starting from specified initial values of h and V.

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