THE CALCULATION OF ELEVATED-TEMPERATURE CYCLIC LIFE CONSIDERING LOW-CYCLE FATIGUE AND CREEP

by David A. Spera

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ABSTRACT

A general method is presented for calculating the cyclic life of a material subjected to isothermal stress- or strain-cycling at temperatures which may be in the creep range. Failure is assumed to be caused either by the accumulation of creep damage or by conventional, time-independent fatigue. A modification of the life-fraction rule proposed by Robinson and Taira and the Method of Universal Slopes developed by Manson are used to calculate creep and fatigue lives, respectively. The method of cyclic life analysis is evaluated by using data on four high-temperature alloys. Calculated and observed lives are in good agreement.
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SUMMARY

A general method is presented for calculating the cyclic life of a material subjected to isothermal stress- or strain-cycling at temperatures which may be in the creep range. Failure is assumed to be caused either by the accumulation of creep damage or by conventional, time-independent fatigue. Cyclic creep life is calculated by using a modification of the life-fraction rule proposed by Robinson and Taira. Conventional low-cycle fatigue life is calculated by the empirical equations of the Method of Universal Slopes developed by Manson. Equations are presented in sufficient detail to completely define the analytical procedure.

The proposed method of cyclic life analysis is evaluated by using data (mostly from the literature) obtained by (1) stress-cycling of Nimonic 90, (2) strain-cycling of L-605 and wrought Udimet 700, and (3) creep-cycling of Inconel. Generally good agreement was obtained between calculated and observed lives for tests which included variations in temperature, frequency, stress or strain range, wave shape, and mean stress or strain. The proposed method of analysis can be used to define the temperature below which failure occurs in the fatigue mode and above which failure is by cyclic creep rupture, for an arbitrary set of test or service conditions.

INTRODUCTION

At elevated temperatures, time must be considered as an important parameter in the testing of materials. Stress, temperature, and time can combine to cause creep strain and creep damage during almost any type of elevated-temperature strength test, whether cyclic or static. The purpose of this investigation is to demonstrate, first, the importance of creep damage during cyclic testing and, second, how to calculate the effect of creep damage on cyclic life.
Fatigue rules such as those of Basquin (ref. 1), Manson (refs. 2 and 3), and Coffin (ref. 4) are quite useful for predicting the results of low-cycle fatigue tests conducted at room or moderate temperatures. Under these conditions, failure is usually by transcrystalline cracking, and the number of cycles to failure is relatively independent of time parameters such as the frequency of cycling. However, these rules must be modified before they can be used to estimate high-temperature cyclic life. Empirical modifications have been proposed (refs. 5 to 7) to indicate the extent of some high-temperature effects such as intercrystalline cracking and creep damage. These modifications are approximations meant to guide high-temperature fatigue testing and design in the absence of a more rigorous analysis. They are based on the hypothesis that cyclic life at high temperature can be predicted by an extension of room-temperature fatigue experience. However, it has recently been shown (ref. 8) that such an extension is limited and that, as temperatures are increased, a second failure mode separate and distinct from conventional fatigue may become dominant. This failure mode has been called cyclic creep-rupture. Cyclic creep-rupture differs from conventional static creep-rupture in that stress and temperature may vary with time in an arbitrary fashion. Otherwise, these two phenomena are closely related.

One of the first demonstrations of a direct link between cyclic life at high temperature and static creep-rupture was made by Moore and Alleman (ref. 9) in 1931. As shown in figure 1, they expressed the rotating-bending life of structural steel at 1200°F as a function of cycles to failure.

![Figure 1](image-url)
(920 K) both in cycles to failure and in time. The conventional diagram of stress against cycles to failure (fig. 1(a)) indicated that an increase in frequency from 200 to 2500 cycles per minute (3.3 to 42 Hz) caused a substantial increase in life. However, when the same data were replotted as stress against time to failure (fig. 1(b)), one narrow scatterband represented data at both frequencies equally well. Moreover, this same scatterband also included the results of static creep-rupture tests at the peak value of stress. Thus, even under fully reversed loading, creep-rupture was singled out as a possible failure mechanism by these investigators.

Further attempts were made to relate elevated-temperature cyclic strength to static creep strength by Betteridge (ref. 10) and Allen and Forrest (ref. 11). Their work was based on data for several of the Nimonic nickel-base alloys. In figure 2, the cyclic and the static stresses which cause failure in equal times are used as coordinates for each data point. Two distinct curves were obtained experimentally: one for tension-compression tests and the other for rotating-bending tests. For the narrow range of frequencies analyzed, scatter about these two curves was quite small, even for a variety of alloys and temperatures. This rather uniform behavior suggested that a theory could be developed to relate cyclic life at high temperature to static creep-rupture life.

In this investigation, time-dependent cyclic creep-rupture and time-independent conventional fatigue are treated as two separate failure mechanisms which act simulta-
neously. Cyclic life is calculated for each mechanism, and the shorter of the two lives determines the theoretical failure mode. In this way, an upper temperature limit is defined for the application of conventional fatigue equations. This limit depends on such test parameters as the frequency and the strain range. The proposed two-part method of life calculation is evaluated by using the following data, most of which has been obtained from the literature: (1) stress-cycling of the nickel-base alloy Nimonic 90, (2) strain-cycling of the cobalt-base alloy L-605 and the wrought nickel-base alloy Udimet 700, and (3) creep-cycling of the nickel-base alloy Inconel.

METHOD OF ANALYSIS

The proposed method of cyclic life analysis is illustrated in figure 3. It is postulated that at high temperatures cyclic life resulting from any strain or stress cycle can be determined solely from the conventional mechanical properties of a material by calculating lives for each of two distinct and independent failure modes: cyclic creep-rupture and conventional low-cycle fatigue. In the literature, the term "fatigue" is often used to designate any type of cyclic failure. However, in this investigation "fatigue" is used to designate only those cyclic tests which are relatively time-independent and which exhibit the same general characteristics as cyclic tests conducted at temperatures well below the creep limit. "Creep" refers to time-dependent failure by creep mechanisms under nonsteady stress conditions.
As shown in figure 3, life in the cyclic creep-rupture mode is calculated from conventional creep-rupture properties by a modification of the linear cumulative damage rule proposed by Robinson (ref. 12) and Taira (ref. 13). This modified Robinson-Taira theory unifies two apparently different phenomena - cyclic life and static creep-rupture resistance - without the use of empirical constants derived from cyclic tests. Low-cycle fatigue life is calculated from conventional short-time tensile properties by the Method of Universal Slopes developed by Manson (ref. 3). This method relates fatigue life to the strain range experienced by the material. It is assumed that both the stress and the strain histories are known, either by direct measurement or by calculation.

**Calculation of Cyclic Creep Life**

Stress- or strain-cycling. - Cyclic creep life is calculated by using the hypothetical quantity "creep damage," which is zero at the start of cycling and equals 1 at failure. The accumulation of creep damage is described by the following equations (see fig. 4):

\[
\Delta \varphi_c = \sum_{0}^{t_1} \frac{\Delta t}{t_r}
\]

and

\[
\sum \Delta \varphi_c = 1 \text{ at failure}
\]

where

- $\Delta \varphi_c$ = creep damage per cycle
- $\Delta t$ = increment of time
- $t_1$ = duration of one cycle
- $t_r$ = creep-rupture time at $|S|$ and $T$
- $|S|$ = absolute value of average stress during $\Delta t$
- $T$ = average temperature during $\Delta t$

This is the so-called "linear damage rule" or "life-fraction rule" originally proposed by Robinson to account for temperature variations during conventional creep. It was later modified by Taira through the addition of the absolute value sign to the stress in order to give full weight to damage by compressive stress. The role of compressive
stress in causing creep damage is still a matter of investigation and is discussed in the following section, where a tentative hypothesis on the subject is proposed.

Over the years, the simple linear damage rule has been very useful in estimating creep life under nonsteady conditions of stress and temperature. However, the damage fractions \( \Delta t / t_r \) are assumed to be independent of the strain history of the material. Thus, no distinction is made between a test with net strain (such as a conventional creep test) and one without net strain (such as a completely reversed strain-cycling test). A simple modification to equations (1) was proposed in reference 8 to account for some of the observed effects of net strain. The development of this modification is as follows:

The assumption is first made that net strain is potentially detrimental to creep-rupture life because it can lead to increasing true stress and tensile instability. Then, for a given nominal stress and temperature, it is assumed that a smooth creep specimen will have the shortest possible life, because such a specimen experiences the maximum amount of necking and, therefore, the most net strain. On the other hand, it is assumed that a notched specimen (with the greatest amount of notch strengthening) will have the longest possible life, because the triaxial stress state at the notched section severely limits necking and thereby reduces net strain to a minimum. Therefore, on the basis of these assumptions, the rupture time in equation (1a) may vary between limits as follows:

\[
 t_{r,s} \leq t_r \leq t_{r,n}
\]  

(1c)

in which the subscripts \( s \) and \( n \) designate conventional creep-rupture times obtained with smooth and notched specimens, respectively. In this way, upper and lower bounds on life are calculated. Equations (1a) to (1c) define what will be called "the modified..."
Robinson-Taira theory" for calculating cyclic creep-rupture life. These equations depend only on the instantaneous values of stress and temperature, and they can therefore be applied to any cycle, however irregular or intricate.

The application of equations (1) is illustrated in figure 4. The left side of the figure depicts the known stress history, using absolute values when the actual stress is compressive. On the right is a plot of the stress against time to rupture for the material in both the smooth and notched conditions. The increment of creep damage during $\Delta t$ is then assumed to be greater than, or equal to, the fraction $\Delta t/t_{r,s}$ and less than, or equal to, the fraction $\Delta t/t_{r,n}$. The accumulated creep damage at failure is assumed to be 1, so the theoretical number of cycles (each with duration $t_1$) which would cause cyclic creep failure is

$$
N_f \leq \sum_{0}^{t_1} \frac{\Delta t}{t_{r,s}} \quad \text{or} \quad \int_{0}^{t_1} \frac{dt}{t_{r,s}}
$$

$$
N_f \geq \sum_{0}^{t_1} \frac{\Delta t}{t_{r,n}} \quad \text{or} \quad \int_{0}^{t_1} \frac{dt}{t_{r,n}}
$$

where $N_f$ is the number of cycles to failure.

Creep-cycling. - In certain types of cyclic tests at high temperature, specimens are allowed to creep at constant stress or experience stress relaxation at constant strain, in such a way that the amount of creep strain per cycle is known. These tests may be called "creep-cycling" and their lives can be estimated quite simply by using a cumulative creep rule proposed in reference 8. Before proceeding to the derivation of this rule, the question of creep damage under compressive stress must be examined further. Equations (1) for conventional stress- or strain-cycling used Taira's assumption that the magnitude and not the sign of the stress controls the rate of creep damage. However, recent analysis of isothermal and thermal fatigue data (ref. 14) indicates that compressive stress is not damaging if the stress rate is zero (conventional creep) or if the strain rate is zero (conventional relaxation). This hypothesis may be generalized in the single statement that compressive stress is not damaging if the product of the stress
and strain rates is less than, or equal to, zero. Under this hypothesis, damage accumulates only during the tensile stress portions of a creep-cycling test.

To derive a cumulative creep rule for estimating life during creep-cycling, the assumption is first made that creep strain rate and rupture time are inversely proportional at a given temperature. This assumption has been used by several other investigators to develop correlation parameters for creep-rupture from the well-known rate law of Arrhenius. Thus,

\[
\dot{\varepsilon}_c = \frac{K}{t_r}
\]  

(3a)

where

\( \dot{\varepsilon}_c \)  
creep strain rate

\( t_r \)  
rupture time

\( K \)  
temperature-dependent curve-fit constant

Integration of equation (3a) over the duration of the tensile portion of an isothermal cycle \( 0 \leq t \leq t_2 \) produces

\[
\int_0^{t_2} \dot{\varepsilon}_c \, dt = K \int_0^{t_2} \frac{dt}{t_r}
\]

(3b)

The left side of equation (3b) is the tensile creep strain per cycle. The integral on the right side is the creep damage per cycle, which is the reciprocal of the number of cycles to failure. Therefore,

\[
\Delta\varepsilon_c = \frac{K}{N_f}
\]

(4)

where \( \Delta\varepsilon_c \) is tensile creep strain per cycle. When creep-rupture data from smooth and notched specimens are used to determine upper and lower bounds on the rupture time in equation (3a), the cumulative creep strain rule takes the form

\[
K_s \leq N_f \Delta\varepsilon_c \leq K_n
\]

(5)

where
Creep-rupture equation. - In this investigation, the conventional rupture times are calculated from the values of stress and temperature using the following correlation equation developed in reference 8:

\[
\log(t_r) = A_T + B \log(|S|) + C|S| + DS^2
\]  

(6)

where

\begin{itemize}
  \item \(\log\) \: logarithm to base 10
  \item \(A_T\) \: temperature-dependent parameter
  \item \(S\) \: instantaneous stress
  \item \(B, C, D\) \: curve-fit constants, independent of temperature
\end{itemize}

Smooth and notched specimen data are correlated by separate sets of the parameters \(A_T\). However, the curve-fit constants \(B, C,\) and \(D\) may be the same for the two conditions. Other creep-rupture parameters could be used to represent rupture time as a function of stress and temperature.

Calculation of Low-Cycle Fatigue Life

The number of cycles required for conventional low-cycle fatigue failure is calculated from the following empirical equations which comprise the Method of Universal Slopes:

\[
\Delta \varepsilon_t = \Delta \varepsilon_e + \Delta \varepsilon_p
\]

(7a)

\[
\Delta \varepsilon_e = 3.5 \frac{UTS}{E} N_f^{-0.12}
\]

(7b)

\[
\Delta \varepsilon_p = D_t^{0.6} N_f^{-0.6}
\]

(7c)

where
\[ \Delta \varepsilon \] strain range
\[ t, e, p \] total, elastic, and plastic, respectively
\[ \text{UTS} \] ultimate tensile strength
\[ E \] Young's modulus
\[ N_f \] number of cycles to failure
\[ D_t \] tensile ductility, \(-\ln(1 - RA/100)\)
\[ RA \] percent reduction of area

Figure 5 illustrates how equations (7) are used to predict low-cycle fatigue life. Curve A is the predicted relation between the elastic strain range and the cycles to failure, while curve B is the relation between the plastic strain range and cycles to failure. In this simplified analysis, the slopes of -0.12 and -0.6 on full logarithmic coordinates are used for all metals, which leads to the designation "universal" slopes. The sum of the ordinates from curves A and B produces curve C, allowing a prediction of life solely from the total strain range. Generally, the analyst need not determine the individual elastic and plastic components which are often more difficult to compute and subject to greater error than the total strain range.

![Figure 5](image)

Figure 5. Method of universal slopes for calculating conventional low-cycle fatigue life. Ultimate tensile strength, UTS; Young's modulus, E; tensile ductility, D_t.
RESULTS AND DISCUSSION

The proposed method of cyclic life calculation based on the separate consideration of cyclic creep and fatigue modes of failure will now be evaluated by comparing calculated and observed cyclic lives for the following tests:

1. Stress-cycling of the nickel-base alloy Nimonic 90
2. Strain-cycling of the cobalt-base alloy L-605
3. Strain-cycling of the nickel-base alloy Udimet 700 (wrought)
4. Creep-cycling of the nickel-base alloy Inconel

Data for test series 1, 3, and 4 were taken from the literature. Test series 2 was conducted at the Lewis Research Center. The required material property data for these alloys are given in table I.

Stress-Cycling of Nimonic 90

Figure 6 shows the calculated and observed 100-hour stress cycling strength of Nimonic 90 as a function of the test temperature and frequency. Data were obtained from reference 15 for the following test conditions: (1) "sawtooth" stress cycle, (2) frequencies of 0.12 and 10 cycles per minute (0.002 and 0.17 Hz), and (3) zero minimum and zero mean stresses. Fatigue strength is calculated by modifying equation (7b) to relate life to stress range rather than to elastic strain range, with the restriction that maximum stress cannot exceed the ultimate tensile strength. A method similar to that of constructing a modified Goodman diagram was used to account for the effect of mean stress on fatigue life.

Figure 6(a) illustrates the indicated transition from fatigue failure to cyclic creep failure for repeated tension tests with zero minimum stress. As the temperature is increased, this transition occurs between 900°F and 1100°F (750 and 870 K), depending on the frequency. This is in agreement with the reported fracture appearances, which changed from transgranular to intergranular in this temperature range. Transgranular and intergranular fractures are often associated with fatigue and creep failures, respectively. At higher temperatures, the 100-hour cyclic strength is substantially lower than conventional fatigue equations would predict. Also, the strength becomes independent of the frequency when life is measured in time rather than cycles, which is a characteristic of creep failure but not of fatigue failure.

Figure 6(b) shows similar results for tension-compression tests with zero mean stress. The relative amounts of transgranular fracture which were observed for the 10-cycle-per-minute (0.17-Hz) tests were 100 percent at room temperature, 70 percent at 1110°F (870 K), 30 percent at 1290°F (970 K), and none at 1470°F (1070 K) and
The latter temperature is consistent with the calculated crossover temperature of about 1500°F (1090 K) at 10 cycles per minute (0.17 Hz). As expected, the contrast between creep and fatigue failure is more clearly defined for the slower tests. The creep theory predictions in figures 6(a) and (b) are identical because the absolute values of stress were used in the calculation of creep damage. Thus, above the temperature limits for fatigue, the creep theory predicts, and the data verify, that a factor of 2 on stress range and a factor of 85 on frequency have little effect on the 100-hour cyclic strength of Nimonic 90 when the maximum stress is held constant.
Strain-Cycling of L-605

Strain-cycling tests on the wrought cobalt-base alloy L-605 were conducted at Lewis. During strain-controlled cycling, stress response is not necessarily repetitive from cycle to cycle. Therefore, creep damage must be determined for representative cycles throughout the test so that a summation of total damage can be made. Table II and figure 7 illustrate the procedure. In table II, the first column gives the cycle number during which a stress-strain hysteresis loop was recorded; the second and third columns contain the observed maximum and minimum loads; the fourth and fifth columns contain the tensile and compressive loads which occurred as the strain passed through zero. These four loads plus Young’s modulus and the total strain range were used with polynomial curve-fits to define the elastic and plastic segments of the stress-strain hysteresis loop. The strain-time wave was controlled to be sinusoidal. Combining the stress-strain and strain-time relations led to the stress history required for computing creep damage. Creep damage per cycle, given in the last column, is based on smooth-specimen rupture data. Therefore, this is the maximum amount of creep damage calculated for this specimen, leading to a lower limit on life. Table II shows that the loads required to produce a constant strain range continually increased throughout the life of the L-605 specimen until they abruptly decreased as a macrocrack formed between 1150 and 1320 cycles. This cyclic strain-hardening led to rapidly increasing damage rates, as shown in the final column of table II. To determine the cycle number at which the accumulated creep damage exceeds 1, the creep damage per cycle was plotted against the cycle number, as shown in figure 7 for the smooth-specimen or maximum damage rates. Straight line segments have been fitted to the calculated points to define three zones: (1)
"transient" hardening from 1 to 400 cycles, (2) "steady-state" hardening from 400 to 1280 cycles, and (3) macrocrack propagation from 1280 to 1400 cycles. The area under the curve is the accumulated creep damage, and it is equal to 1 at 1270 cycles. This procedure illustrates a limitation of the creep damage theory as applied to strain-cycling tests. If the alloy being tested is strongly cyclic strain-hardening or -softening, the creep damage theory can usually be used only to correlate tests after they have been run. In order to predict the test results, the analyst must be able to predict the amount and rate of the cyclic hardening or softening.

Figures 8(a) to (c) show the variation of calculated and observed cyclic lives with the parameters of strain range, frequency, and temperature, respectively. In figure 8(a), the variation of cyclic life with strain range is presented for specimens at a temperature of 1225° F (940 K) and a nominal frequency of 10 cycles per minute (0.17 Hz), agreeing with the lower bound of the predicted cyclic creep lives. Observed lives are substantially shorter than those calculated by the fatigue theory. Thus, it may be concluded that creep is the dominant failure mechanism even though the strains are fully reversed, the frequency is moderately fast for low-cycle fatigue testing, and the cycles show no evidence of conventional creep or relaxation.

Figure 8(b) shows the variation of cyclic life with frequency for tests at 1180° F (910 K) with nominal strain ranges of 0.9 percent. Frequencies from 0.06 to 120 cycles per minute (0.001 to 2 Hz) were applied. Comparison of these data with calculated lives demonstrates a transition from the creep-failure mode at frequencies below 10 cycles per minute (0.17 Hz) to the fatigue mode at frequencies above 100 cycles per minute (1.7 Hz). In the frequency range of 10 to 100 cycles per minute (0.17 to 1.7 Hz), the two failure modes appear to interact, producing a gradual rather than a clearly defined transition. As a first approximation, a "linear interaction" curve may be constructed as shown in figure 8(c). This curve was obtained by calculating the cycle number at which the sum of the maximum creep damage and the fatigue damage (cycle number divided by fatigue life) was equal to 1. Interaction between the fatigue- and creep-failure modes is not an essential part of the present method of cyclic life analysis. The amount of interaction for a given alloy and the best rule for describing it can only be determined from the experimental data.

Figure 8(c) illustrates the effect of different test temperatures for constant values of strain range and frequency. Temperatures were selected in the range 80° to 1360° F (300 to 1010 K). The calculated transition from the fatigue-failure mode to the creep-failure mode at about 1150° F (900 K) is clearly in agreement with these data. The calculated fatigue life is observed to drop rapidly as the test temperature approaches the range 1400° to 1500° F (1030 to 1090 K), in which L-605 experiences a minimum in ductility. However, the observed lives are still considerably below the fatigue prediction. The linear interaction rule appears to be applicable when the temperature is the variable parameter as well as when the frequency is changed.
Figure 8. - Comparison of calculated and observed lives for strain-cycling of L-605 alloy.

(a) Effect of strain range. Frequency, 10 cycles per minute (0.17 Hz); temperature, 1225°F (652°C).

(b) Effect of frequency. Temperature, 1180°F (910°C); strain range, 0.9 percent.

(c) Effect of temperature. Frequency, 20 cycles per minute (0.33 Hz); strain range, 0.9 percent.
Strain-Cycling of Udimet 700

Strain-cycling tests at 1400° F (1030 K) on the alloy Udimet 700 (wrought) were reported in reference 16. In these tests, specimens were cycled with a mean strain equal to one-half the strain range. This mean strain produced varying amounts of mean stress, depending on the strain range and the number of elapsed cycles. At strain ranges above 1.4 percent, sufficient plastic flow and relaxation were present to effectively eliminate any mean stress. However, for smaller strain ranges with little or no plastic flow, tensile mean stresses were retained which could be as large as one-half the maximum stress. The actual amounts of mean stress and stress amplitude which were measured during the life of each specimen were obtained from reference 16 or by direct communication with the authors. Once a stress cycle was defined, the same calculation procedure was used for this alloy as for L-605, with the exception that no notched-specimen rupture data were available for Udimet 700. Therefore an upper bound on cyclic rupture life could not be established. The results of these calculations are shown in figure 9.

In figure 9, the actual calculated cyclic creep life is observed to follow a transition from hypothetical lives with zero mean stress to those with zero minimum stress. The measured stress cycles also follow this transition as the strain range decreased. The data appear to agree quite well with the lower bound on creep life shown in figure 9. The fatigue prediction shows that the endurance of these specimens falls short of conventional fatigue behavior. Also, it appears that variations in reduction of area from 11 to 31 percent have only a minor effect on the lives of the specimens, which is contrary to conventional low-cycle fatigue experience.
Creep-Cycling of Inconel

Creep-cycling tests on this alloy were reported in reference 17. In these tests, tubular specimens were subjected to alternate equal periods of constant tensile and compressive stress at 1500°F (1090 K) until cracks penetrated the tube walls. This is the simplest and most clearly defined example of cyclic creep-rupture failure. The results of these tests are compared with the predictions of the creep theory in figure 10(a). Test results at three frequencies are displayed as stress amplitude against failure time. The mean stresses were zero. Apparently the test frequency had little effect on the time to failure, as predicted by the cyclic creep theory. The calculated lives in this figure are twice those for static creep-rupture at the same stress level because of the assumption that compressive stress is not damaging if the stress rate is zero. The notch-strengthening factor is 5.9 in time. This was obtained by comparing literature data on

![Diagram of stress vs. time](image1)

(a) "Square" stress cycle; data from reference 17.

![Diagram of cycles vs. strain](image2)

(b) Combined relaxation and creep cycle, 0.5 to 30 minutes per cycle.

Figure 10. - Comparison of calculated and observed lives for creep-cycling of Inconel alloy tubes at 1500°F (1090 K).
smooth and notched bars, the latter having a theoretical stress concentration factor of 7. The test results and the theoretical lives are in good agreement.

Figure 10(b) shows data and calculations for a different type of creep-cycling test. In these tests (ref. 18), elongation and contraction of the specimens were partially restrained by semirigid stops during hold periods in tension and compression. This caused the magnitude of the stress to decrease during the hold, producing both relaxation and creep. Both instantanous plastic strain and viscous creep strain occurred during each half cycle. The range of inelastic strain (plastic plus creep strain) is plotted, in figure 10(b), against the number of cycles to failure for six different frequencies of testing. In this case, cyclic lives were found to be independent of frequency.

The cumulative creep rule (eq. (5)) was used to calculate the cyclic creep-rupture lives shown in figure 10(b) assuming that the amount of plastic strain was negligible in comparison with the creep strain. Creep rate and rupture time data from reference 19 were used to obtain a value of 68.7 percent for $K_S$. The notch-strengthening factor of 5.9 (in time) then gives 405 percent for $K_n$. It can be seen that the simple cumulative creep rule produces a predicted life range which encloses almost all the data, without recourse to any stress calculations. On the other hand, equation (7c) indicates that conventional fatigue lives would have been considerably longer than the observed lives and would also have had a different variation of life with strain range.

CONCLUSIONS

The following conclusions are drawn from the results of this investigation of cyclic life at elevated temperatures:

1. Creep is an important failure mechanism even when loads or strains are fully reversed and frequencies of cycling are moderately fast for low-cycle-fatigue testing.
2. Conventional time-independent low-cycle fatigue can take place at elevated temperatures below an upper limit determined by such test conditions as the frequency, strain or stress range, and mean strain or stress. At temperatures above this limit cyclic creep becomes the dominant failure mode.
3. The temperature which defines the transition from fatigue to cyclic creep failure can be calculated for an arbitrary test cycle by using the methods presented in this report.
4. The method of analysis embodies many simplifying assumptions, but nevertheless, it can be used to calculate cyclic lives which agree well with observed lives for a wide variety of test conditions.

Lewis Research Center,

National Aeronautics and Space Administration,

Cleveland, Ohio, April 24, 1969,

129-03-06-08-22.
REFERENCES


### TABLE I. - MATERIAL PROPERTIES (VARIETY OF SOURCES)

(a) U.S. Customary Units

<table>
<thead>
<tr>
<th>Temperature, °F</th>
<th>Ultimate tensile strength, UTS, ksi</th>
<th>Young's modulus, E, ksi/%</th>
<th>Reduction of area, RA, %</th>
<th>Creep rupture time, ( t_r ), hr</th>
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TABLE I. - Concluded. MATERIAL PROPERTIES (VARIETY OF SOURCES)

(b) SI Units

\[
\log(t_r) = A_T + B \log(|S|) + C|S| + DS^2
\]

(stress, S, kN/cm²)
TABLE II. - SAMPLE CALCULATION OF CYCLES TO CREEP FAILURE

[Alloy, L-605; temperature, 1180°F (910 K); frequency, 10 cpm (0.17 Hz); strain range, 0.9 percent.]

<table>
<thead>
<tr>
<th>Cycle number, n</th>
<th>Maximum load lb</th>
<th>Minimum load lb</th>
<th>Load at zero strain</th>
<th>Maximum creep damage per cycle, Δψ_c</th>
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<tr>
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<td>N</td>
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<td>-620</td>
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 Curve fit:

\[
\Delta \psi_c = \begin{cases} 
0.3 \times 10^{-4} + 0.85 \times 10^{-6} n, \text{ where } n \leq 415 \\
-2.9 \times 10^{-4} + 1.61 \times 10^{-6} n, \text{ where } n \geq 415 
\end{cases}
\]

Therefore, \( \int_0^{N_f} \Delta \psi_c \, dn = 1 \) at \( N_f = 1270 \) cycles.
"The aeronautical and space activities of the United States shall be conducted so as to contribute ... to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— National Aeronautics and Space Act of 1958

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