NASA TECHNICAL NOTE

NASA TN D-5398

c. 1

NASA TN D-5398

THE RED-SHIFT—MAGNITUDE RELATION AND OBSERVATIONAL DATA

by Windsor L. Sherman

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • AUGUST 1969

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-5398		

4. Title and Subtitle
THE RED-SHIFT—MAGNITUDE RELATION AND OBSERVATIONAL DATA

5. Report Date
August 1969

6. Performing Organization Code

7. Author(s)
Windsor L. Sherman

8. Performing Organization Report No.
L-5648

9. Performing Organization Name and Address
NASA Langley Research Center
Hampton, Va. 23365

10. Work Unit No.
188-41-01-01-23

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D.C. 20546

13. Type of Report and Period Covered
Technical Note

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The red-shift—magnitude relation for uniform, zero-pressure models of the universe is one of the stronger relationships connecting theory and observation in cosmology. This relationship has been used to determine models for the universe from the observed apparent magnitudes and red shifts of galaxies. The problems associated with the use of the red-shift—magnitude relation to determine a model universe are discussed. In particular, the effects of evolution and selection on the use of the red-shift—magnitude relation are considered. Selection is the tendency of observers to select intrinsically brighter sources for observation as the distance to the sources increases. The results indicate that the red-shift—magnitude relation should be used with caution when determining a model for the universe because the variation of the intrinsic luminosity of sources must be taken into account.

17. Key Words Suggested by Author(s)
Cosmology
Observational data
Models for the universe
Evolution
Selection

18. Distribution Statement
Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	25	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

THE RED-SHIFT—MAGNITUDE RELATION AND OBSERVATIONAL DATA

By Windsor L. Sherman
Langley Research Center

SUMMARY

The red-shift—magnitude relation for uniform, zero-pressure models of the universe is one of the stronger relationships connecting theory and observation in cosmology. This relationship has been used to determine models for the universe from the observed apparent magnitudes and red shifts of galaxies. The problems associated with the use of the red-shift—magnitude relation to determine a model universe are discussed. In particular, the effects of evolution and selection on the use of the red-shift—magnitude relation are considered. Selection is the tendency of observers to select intrinsically brighter sources for observation as the distance to the sources increases. The results indicate that the red-shift—magnitude relation should be used with caution when determining a model for the universe because the variation of the intrinsic luminosity of sources must be taken into account.

INTRODUCTION

The theory of uniform models of the universe based on general relativity provides several connections between theory and observation for models in which the pressure is equal to zero. Sherman (ref. 1), McVittie (ref. 2), Solheim (ref. 3), and Sherman and Wallace (ref. 4) have studied the connections between theory and observation and derived exact and simplified relationships for the analysis of observational data. Although the use of this connection between theory and observation has been illustrated, there has been little discussion of the problems of calculation and interpretation of the results.

In this report some of the problems connected with the use of the red-shift—magnitude relation to obtain a model of the universe from observational data are discussed. Consideration is given to the effects of evolution and selection on the model. Selection is the tendency of observers to select intrinsically brighter sources for observation as the distance to the sources increases. The results obtained from the red-shift—magnitude relation are compared and interpreted with respect to models based on other observational data and different methods of model determination. The symbols used in the analysis are defined in appendix A.

THE RED-SHIFT—MAGNITUDE RELATION

The most general form of the red-shift—magnitude relation for zero-pressure relativistic models of the universe is (ref. 4)

$$m - K = 5 \log_{10}\left[R_o(1 + z)S(\omega)\right] + M + \Delta M - 5 \tag{1}$$

where

$$R_o = \frac{c}{H_o\sqrt{\left|3\sigma_o - q_o - 1\right|}} \tag{2}$$

$$\left.\begin{aligned} S(\omega) &= \sinh \omega & (k = -1)\\ S(\omega) &= \omega & (k = 0)\\ S(\omega) &= \sin \omega & (k = +1)\end{aligned}\right\} \tag{3}$$

and the curvature constant k is given by

$$\left.\begin{aligned} k &= -1 & \left(\left(3\sigma_o - q_o - 1\right) < 0\right)\\ k &= 0 & \left(\left(3\sigma_o - q_o - 1\right) = 0\right)\\ k &= +1 & \left(\left(3\sigma_o - q_o - 1\right) > 0\right)\end{aligned}\right\} \tag{4}$$

The variable ω is given by an elliptic integral of the first kind as

$$\omega = \sqrt{\left|3\sigma_o - q_o - 1\right|} \int_0^z \left[2\sigma_o z^3 + \left(3\sigma_o + q_o + 1\right)z^2 + 2\left(q_o + 1\right)z + 1\right]^{-1/2} dz \tag{5}$$

The correction for evolutionary effects ΔM is given by

$$\Delta M = -\dot{M}\left(t_o - t\right)$$

and the time of light travel $t_o - t$ is

$$t_o - t = H_o^{-1}\int_0^z \left\{(1 + z)^2\left[2\sigma_o z^3 + \left(3\sigma_o + q_o + 1\right)z^2 + 2\left(q_o + 1\right)z + 1\right]\right\}^{-1/2} dz \tag{6}$$

In the foregoing equations, m is the apparent magnitude, K is the red-shift correction, R_o is the present value of the scale factor, z is the red shift, M is the absolute magnitude, c is the speed of light in a vacuum, H_o is the Hubble parameter, σ_o is the density parameter, q_o is the deceleration parameter, $\dot{M}$ is the rate of change of absolute magnitude, t_o is the present time, and t is any other time. When the red-shift—magnitude relation is used to obtain a model of the universe, the unknown parameters are

H_o, σ_o, q_o, and M. However, use of the method of differential corrections showed that M and H_o are too interrelated and highly correlated to permit these two parameters to be determined. The parameters H_o and M are usually combined into a single parameter, and when this is done, equation (1) is written as

$$m - K = 5 \log_{10} \frac{1 + z}{\sqrt{|3\sigma_o - q_o - 1|}} S(\omega) + \Delta M + C \tag{7}$$

where

$$C = 5 \log_{10} \frac{c}{H_o} + M - 5 \tag{8}$$

Combining H_o and M into a single parameter means that only C, σ_o, and q_o need be determined from the observed data. After C, σ_o, and q_o have been obtained from observational data, they are used to specify a model for the universe in terms of the curvature constant k and the cosmical constant Λ. The constant k is given by equations (4) and

$$\Lambda = 3H_o^2\left(\sigma_o - q_o\right)$$

The parameter C (eq. (8)) is made up of the Hubble parameter H_o, the speed of light in a vacuum c, and the absolute magnitude of the source M. The Hubble parameter which is determined from the observation of nearby galaxies is assumed to be a constant. The speed of light in a vacuum is also a constant. Thus, the absolute magnitude is the only quantity that can vary in the expression for C. The absolute magnitude is a function of the physical process that is supplying energy for the source, the state of the process, and the size and composition of the source. Because none of these factors are a function of the position of the source, the absolute magnitude is independent of the model universe. In a complicated structure like a galaxy where the emission is the integrated output of many stars and nebulae, the absolute magnitude depends on the number of emission sources and the state of the energy process of each source. Inasmuch as the size of a galaxy is a rough indication of the number of sources, the absolute magnitude of galaxies varies. Thus, if n sources are being used to determine a model for the universe, it may be necessary to determine n values for C in addition to the parameters σ_o and q_o. Because it is not possible to determine n + 2 values of the parameters C, σ_o, and q_o, an assumption must be introduced to render the problem determinant. Inasmuch as σ_o and q_o are the parameters that determine the model, the usual assumption introduced is that all sources in the set of data being used have the same value of C. This assumption is equivalent to saying that all sources have the same absolute magnitude.

ANALYSIS OF OBSERVATIONAL DATA BY USE OF THE RED-SHIFT—MAGNITUDE RELATION

The form of the red-shift—magnitude relation given in equation (7) and the method of differential corrections were used to obtain the parameters σ_o, q_o, and C from the observational data for clusters of galaxies. The best values of the parameters σ_o, q_o, and C were obtained when the sum of the squares $\sum_{i=1}^{n}(m_c - m_{cal})_i^2$ was a minimum. The values of σ_o, q_o, C, and the root mean square (rms) on the magnitude residuals were obtained. The rms is given by $\sqrt{\frac{\sum_{i=1}^{n}(m_c - m_{cal})_i^2}{n}}$ where m_c is the corrected observed apparent magnitude, m_{cal} is the calculated apparent magnitude obtained from the red-shift—magnitude relation, and n is the number of sources in the group being analyzed. The observational data used to determine σ_o, q_o, and C are presented in table I, discussed in appendix B, and plotted in figure 1.

TABLE I.- OBSERVATIONAL DATA FOR CLUSTERS OF GALAXIES

Cluster	Red shift, z	Corrected apparent magnitude, m_c
Virgo	0.004	9.27
0316 + 4121	.018	12.51
1257 + 2812	.022	12.91
1603 + 1755	.036	14.12
2308 + 0720	.043	14.78
2322 + 1425	.044	15.04
1145 + 5559*	.052	15.71
0106 - 1536	.053	15.21
1024 + 1039	.065	15.88
1239 + 1852*	.072	15.22
1520 + 2754*	.072	15.86
0705 + 3506	.078	16.26
1431 + 3146*	.131	17.31
1055 + 5702	.134	17.20
0025 + 2223	.159	17.39
0138 + 1840	.173	17.16
0925 + 2044	.192	17.56
0855 + 0321	.202	17.84
0024 + 1654	.290	18.88
1448 + 2617*	.350	18.68
1410 + 5224	.460	19.48

*Does not fall nicely on a simple curve.

Figure 2 shows the results obtained when the data were processed to determine σ_o, q_o, and C. In this calculation, $\dot{M}$ was set equal to zero and it was assumed that all sources in the group had the same value of the parameter C. The values of σ_o and q_o found for this set of data give a curvature constant k of +1 and a cosmical constant Λ that is greater than zero. In addition to the σ_o, q_o, and C values obtained from processing the data, the present value of the scale factor R_o, the time since the beginning of expansion T, the variation of R/R_o with T, the variation of $t_o - t$ with z, the time of light travel, and the variation of m with $\log_{10} z$ are also given in figure 2. The equations and method for determining T, R_o, and R/R_o are given in reference 4. The information presented in figure 2 gives a complete description of the model universe. The values of σ_o and q_o of this model agree well with the values of σ_o and q_o of the Solheim model (ref. 3). The values of σ_o and q_o found by Solheim were 4.53 and -0.06, respectively. The small differences between σ_o and q_o given in figure 2 and those obtained by Solheim probably arise from the omission of the Virgo cluster from Solheim's data. The model given in figure 2 was obtained from observational data and should

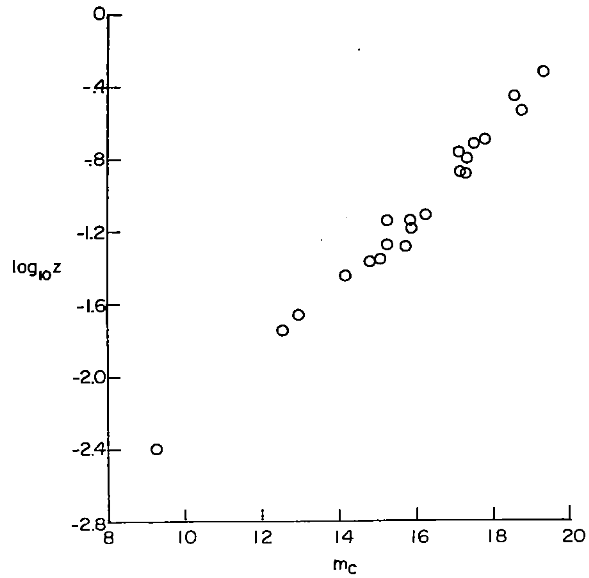

Figure 1.- Observational data for clusters of galaxies (from table I).

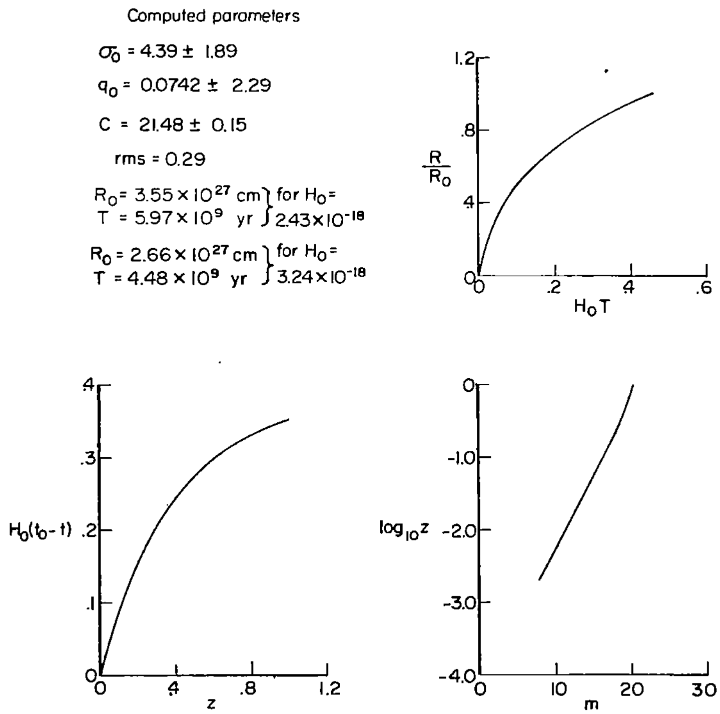

Figure 2.- Model for the universe obtained when the observational data from table I were analyzed by using the red-shift—magnitude relation with parameter C assumed to be a constant and $\dot{M} = 0$.

represent the observed universe if general relativity is the theory of gravitation that applies to the universe. It is possible to obtain independent estimates of the density parameter and the time since the beginning of expansion through the study of the Galaxy and clusters of galaxies. Because the data from which the model obtained by means of the red-shift—magnitude relation come from the same universe as the independently determined values of σ_o and T, reasonable agreement should exist between the two sets of values. Lastly, the model should have a feature or features that can be used to account for the 3^o K blackbody radiation that appears as an isotropically distributed background in the universe.

Comparison of Red-Shift—Magnitude Model Parameters With Independently Determined Values

The independently determined values of density parameter and time since the beginning of expansion are based on estimates of the present density of the universe and the age of the Galaxy. These data and the 3^o K blackbody radiation are discussed in appendix B.

Density.- For the model given in figure 2, $2.503 \leqq \sigma_o \leqq 6.277$ with the most likely value of the density parameter being 4.39. The independent determination gives $0.0045 \leqq \sigma_o \leqq 0.45$ with the most likely value being 0.045. The density parameter interval for the model of figure 2 is completely different from the density parameter interval based on observational results, and the most likely values differ by a factor of 97.6.

Time since beginning of expansion.- In a model, like that shown in figure 2, where the expansion started at zero radius, the initial state was extremely compact and it is generally hypothesized that the galaxies formed very soon after expansion started. This hypothesis means that age of all galaxies is about the same; thus, the time since the beginning of expansion should be greater than or at least equal to the age of the Galaxy. The time since the beginning of expansion, based on the variance of σ_o and q_o, falls in the interval $3.65 \times 10^9 \leqq T \leqq 6.86 \times 10^9$ years with the most likely value being 4.48×10^9 years. These times are for $H_o = 3.24 \times 10^{-18}$ sec^{-1}. The age of the Galaxy A_g falls in the interval $1.2 \times 10^{10} \leqq A_g \leqq 2.0 \times 10^{10}$ years. The most likely value of the age of the Galaxy is 1.5×10^{10} years. Comparison of these data show that the time interval for the age of the Galaxy and the time interval for the time since the beginning of expansion are completely different. The most likely age of the Galaxy is about $3\frac{1}{3}$ times the most likely time since the beginning of expansion. In a model universe such as that shown in figure 2, a galactic age greater than T is not admissible.

The 3^o K blackbody radiation.- An isotropically distributed 3^o K blackbody radiation has been observed in the universe. The present hypothesis concerning this radiation is that it is the remnant of the initial state of the universe. In this initial state the universe was an extremely compact, hot, dense body that expanded into the present observed universe, the so-called big bang model. The model given in figure 2 has a zero radius (radius is proportional to R) at zero time. This feature would give an extremely compact initial state and, thus, the conditions for the development of the 3° K blackbody radiation are found in this model.

Discussion of the comparison.- Except for the 3° K blackbody radiation, the model universe presented in figure 2 does not have a density parameter and time scale that are compatible with independently determined values of these quantities. This lack of compatibility could be due to (1) use of an incorrect theory of gravitation, (2) too much simplification of the basic model, (3) incorrect data, and (4) incorrect use of the data leaving out the effects of evolution and selection or use of an incorrect assumption when the data were analyzed. Because there is insufficient evidence for either item (1) or item (2) and saying that the data are incorrect as in item (3) amounts to questioning the validity of a large amount of astronomical data without a good basis, it was decided to follow the course set up in item (4).

Uncertainties in Red-Shift—Magnitude Relation

Besides the assumptions made in the derivation of the red-shift—magnitude relation, the most important assumption is that the parameter C is a constant. This assumption implies that all galaxies have the same absolute magnitude. The absolute magnitude of all galaxies is not the same. For instance, the galaxies marked with an asterick in table I could not have the same absolute magnitude as the adjacent galaxies. See reference 5 (p. 272) for absolute magnitudes of galaxies determined from Cepheid variables. Because all galaxies do not have the same absolute magnitude, a study was made to determine the effect of assuming constant absolute magnitude on the model when the actual absolute magnitudes were not the same. The study was made with the red-shift—magnitude relation and a synthetic set of data. In the synthetic data each source had a different value of C and this parameter was assumed to be a constant in the red-shift—magnitude relation. It was found that when intrinsic luminosity increased with distance, the assumption of constant intrinsic luminosity in the red-shift—magnitude relation caused the extracted value of σ_o to be much greater than the value of σ_o used in the construction of the synthetic data; that is, a high-density universe was predicted instead of the low-density universe used to obtain the data. The evolution of galaxies and selection, both observational and instrumental, could produce a set of data in which the distant sources were brighter than the near sources.

Effect of evolution on model for the universe.- The evolutionary effect is based on the change of the luminosity of galaxies with time and the time of light travel from the source to the observer. Evolution has been discussed by several authors (for instance, refs. 2 and 6). A value of $\dot{M}$ of 0.3^{m} per 10^9 years was selected to be a value of $\dot{M}$ large enough to illustrate the effect of evolution and small enough not to be unrealistic. The data were analyzed to determine σ_o and q_o with $\dot{M}$ equal to the foregoing value. The results of this calculation are presented in figure 3. Comparison of the data presented in figure 3 with those presented in figure 2 shows that evolution reduces both σ_o and q_o. The density parameter now lies in the interval $0.22 \leqq \sigma_o \leqq 3.0$ with the most likely value being 1.61. The change in σ_o reduces the discrepancy with the independently obtained value. The time since the beginning of expansion now lies in the interval 4.79×10^9 years with the most likely value being 6.59×10^9 years, an increase in T that reduces the discrepancy with the age of the galaxy. These times are for $H_o = 3.24 \times 10^{-18}$ sec^{-1}. The model of figure 3 has a zero radius at $t = 0$; thus, the 3^o K blackbody radiation can be accounted for in this model. The introduction of evolution caused changes in σ_o and q_o which apparently moved the model toward the observed universe; this indicates the importance of accounting for evolution when determining a model for the universe. This result agrees with the result of reference 6 concerning the importance of evolution. Even though the changes in σ_o and q_o improved the model, this change is not reflected in the rms of the magnitude residuals. The rms

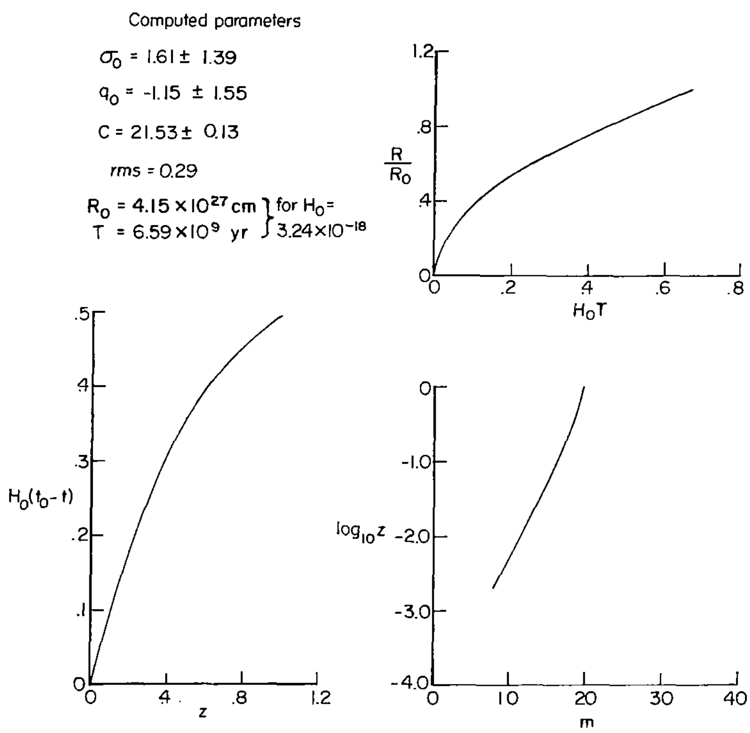

Figure 3.- Model for the universe obtained when the observational data from table I were analyzed by using the red-shift—magnitude relation with $\dot{M} = 0.3$ per 10^9 years.

is the same as that for the model given in figure 2. The lack of change in the rms is more than likely caused by a high dependence of the evolutionary effect on σ_o and q_o. This dependence of the evolutionary effect on σ_o and q_o apparently absorbs the effects of evolution on the rms.

Selection effect.- Selection is a complicated process and involves such factors as the instruments, observers, and available sources. Both the size and physical state of the available sources enter into the problem of selection. In observational cosmology, such factors as instrument sensitivity, recording time, reading of survey plates, and available sources influence the observer to select the brightest of the available sources for study. At large distances only the brighter sources are available; therefore, there is considerable chance that the distance sources will be intrinsically brighter than the nearer sources. A discussion of selection and how it can affect the model for the universe is presented in reference 7; however, the effects of selection have not yet been detected in the observational data.

A study of observational data was made to determine whether any trends in the data might indicate the presence of a selection effect. The apparent magnitudes and red shifts of 223 galaxies, principally from reference 8 were separated into groups as shown in table II. In order to reduce and if possible to eliminate random errors, the data for the galaxies in each group were averaged by determining the mean apparent magnitude and mean red shift for the group. The mean apparent magnitude is given by

$$\overline{m}_k = \frac{\sum_{i=1}^{n} (m_i)}{n}$$

and the mean red shift is given by

$$\overline{\log_{10} z_k} = \frac{\sum_{i=1}^{n} \log_{10} z_i}{n}$$

where n indicates the number of galaxies in the group and k refers to the group. In addition, the variance σ^2 was also determined, that is,

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(m_i - \overline{m}_k\right)^2}{n}$$

The results of the calculations are also given in table II.

TABLE II.- MEAN APPARENT MAGNITUDE, VARIANCE, AND RED SHIFT OF 223 GALAXIES

Red-shift interval	$\overline{m}$	σ^2	$\log_{10} cz$	No. of sources
$0.0133 \leqq z < 0.0167$	13.64	0.81	3.65350	62
$0.0167 \leqq z < 0.025$	14.28	.75	3.77759	64
$0.025 \leqq z < 0.0333$	14.49	.70	3.93171	18
$0.0333 \leqq z \leqq 0.05$	15.51	.32	4.07583	16
$0.05 \leqq z \leqq 0.0667$	15.93	.70	4.22897	11
$0.0667 \leqq z \leqq 0.1$	16.96	1.12	4.38533	18
$0.1 < z \leqq 0.1833$	18.28	.76	4.63838	19
$0.1833 \leqq z$	19.11	.41	4.83982	15

The mean magnitudes given in table II were corrected for the effects of red shift (ref. 8). The corrected mean magnitude data are plotted in figure 4 with $\log_{10} z$ as the ordinate. In addition to $\overline{m}_{k_c}$ the variation of m with $\log_{10} z$ for a model universe in which $\sigma_o = q_o = 1.0$, the linear model, is also shown in the figure. The bars through the symbols for $\overline{m}_{k_c}$ indicate the 1σ variation for the computed mean at these points. For small values of z, the curve of m plotted against $\log_{10} z$ for the linear model lies within the error bars of $\overline{m}_{k_c}$. However, as $\log_{10} z$ (and, hence, distance) increases, the separation of $\overline{m}_{k_c}$ caused by the fact that $\overline{m}_{k_c}$ is brighter than m at the same $\log_{10} z$, keeps the model line out of the error bars. The difference between $\overline{m}_{k_c}$ and the model line is made up of differences due to the model (that is, σ_o and q_o different from 1.0), evolution, and selection. The dashed curve in figure 4 shows the effect of model and evolution in the interval $-1.8236 \leqq \log_{10} z \leqq -0.6373$. The upper limit corresponds to a red shift of about 0.23. This curve is for the model given in figure 3 and is considered typical for a model with evolution in the interval for $\log_{10} z$. There is not an inconsiderable difference between the dashed curve and the $\overline{m}_{k_c}$ points, and as the upper limit is approached, the error bars no longer touch the model curve. These differences can be attributed to selection. In fact, figure 4 shows that if selection is

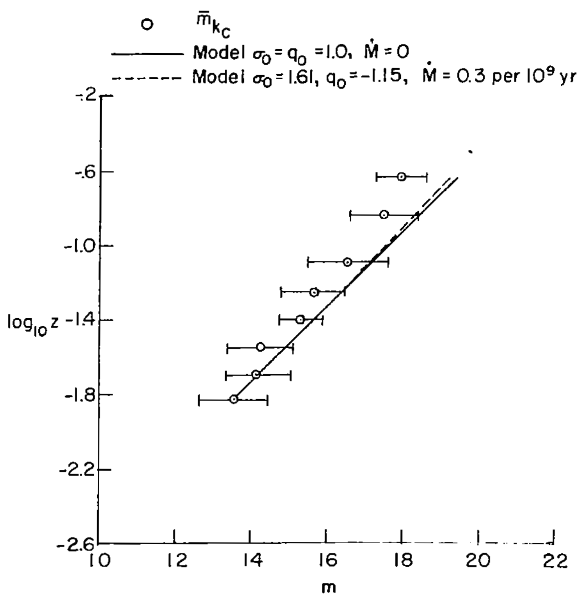

Figure 4.- Comparison of corrected mean apparent magnitude with 1σ error bars and models for the universe.

present, it may be greater than or equal to the model increment; this means that with present data the red-shift—magnitude relation should not be used to determine a model for the universe unless the parameter C can be determined by independent means for each source.

An Alternate Approach to Determination of σ_0 and q_0

Several methods have been studied for making an independent estimate of the parameter C. The most satisfactory method depends on the noise in the recorded data and estimates of the possible range of values of σ_0 and q_0. This method, discussed in appendix C, was used in conjunction with data given in table I to estimate parameter C. The results of the estimation of C are given in table III. The method used to estimate the parameter C gives rise to many possible combinations. At least 272 combinations involving C are possible but only one is given in table III. The combination presented in table III was selected so that the most distant sources were intrinsically brighter than the nearer sources. To make use of these values of C, m_{11}, the apparent magnitude for the model $\sigma_0 = q_0 = 1.0$, was determined for each source by use of

$$m_{11} = 5 \log_{10} z + C \tag{9}$$

and was then subtracted from the observed apparent magnitude. This procedure gives the incremental apparent magnitude Δm.

TABLE III.- POSSIBLE VALUES OF THE PARAMETER C AND OF Δm FOR OBSERVATIONAL DATA FOR CLUSTERS OF GALAXIES FROM TABLE I

Cluster	z	$5 \log_{10} z$	m_c	C	Δm
Virgo	0.004	-11.990	9.27	21.26	0
0316 + 4121	.018	-8.725	12.51	21.26	-.025
1257 + 2812	.022	-8.290	12.91	21.26	-.06
1603 + 1755	.036	-7.220	14.12	21.26	.08
2308 + 0720	.043	-6.835	14.78	21.66	-.045
2322 + 1425	.044	-6.785	15.04	21.66	.165
1145 + 5559	.052	-6.420	15.71	22.06	.07
0106 - 1536	.053	-6.380	15.21	21.66	-.07
1024 + 1039	.065	-5.934	15.88	21.66	.154
1239 + 1852	.072	-5.715	15.22	20.86	.075
1520 + 2754	.072	-5.715	15.86	21.66	-.085
0705 + 3506	.078	-5.540	16.26	21.66	.14
1431 + 3146	.131	-4.415	17.31	21.66	.065
1055 + 5702	.134	-4.365	17.20	21.66	-.095
0025 + 2223	.159	-3.995	17.39	21.26	.125
0138 + 1840	.173	-3.810	17.16	20.86	.11
0925 + 2044	.192	-3.585	17.56	20.86	.285
0855 + 0321	.202	-3.475	17.84	21.26	.055
0024 + 1654	.290	-2.690	18.88	21.26	.31
1448 + 2617	.350	-2.280	18.68	20.86	.10
1410 + 5224	.460	-1.685	19.48	20.86	.305

The incremental apparent magnitude Δm and the red shift for each source were then used in the equation

$$\Delta m = 5 \log_{10} \frac{1 + z}{z\sqrt{|3\sigma_0 - q_0 - 1|}} S(\omega) \tag{10}$$

to determine σ_0 and q_0. This equation was developed in the paper for which the abstract is presented in reference 9. The results of this calculation are shown in figure 5. It was found that σ_0 falls in the interval $0 \leqq \sigma_0 \leqq 0.779$ with a most likely value of 0.217. The most likely value of σ_0 based on observation now falls in this interval. The results in figure 5 show that q_0 falls in the interval $0.0212 \leqq q_0 \leqq 0.782$ with the most likely value being 0.402. Compared with the values for the models in figures 2 and 3, σ_0 has decreased and q_0 has increased. The decrease in σ_0 is in the correct direction to increase the time since the beginning of expansion. This parameter now lies in the interval $5.92 \times 10^9 \leqq T \leqq 9.71 \times 10^9$ years with the most likely value of 7.29×10^9 years, which moves the time since the beginning of expansion closer to the age of the Galaxy. These times are for $H_0 = 3.24 \times 10^{-18}$ sec^{-1}. In the model presented in figure 5, the rms of the magnitude residuals has improved by a factor of more than 3

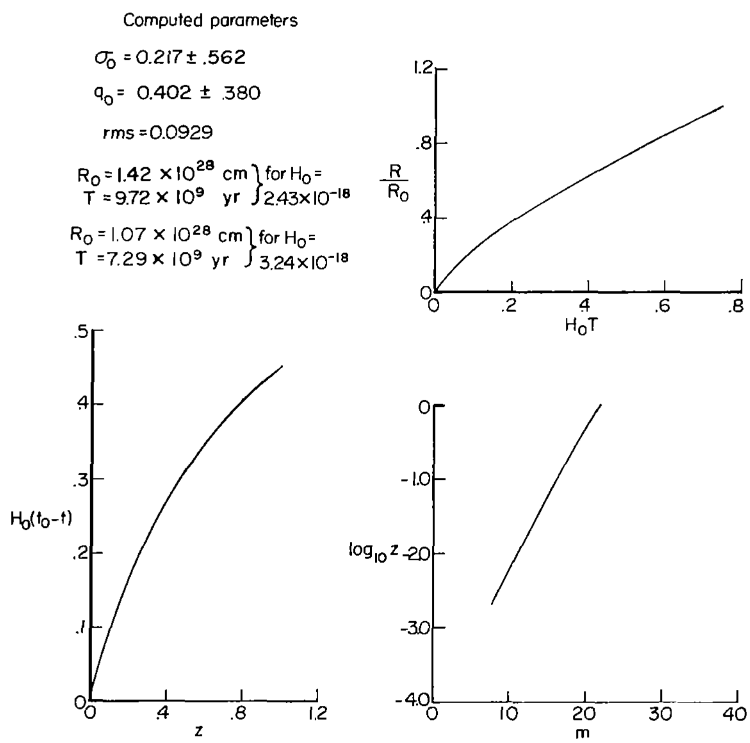

Figure 5.- Model for the universe obtained when the data in table III were analyzed by using the incremental red-shift—magnitude relation (eq. (10)).

over the previous models. The fact that this model has a zero radius at $t = 0$ means that it is simple to account for the 3^{o} K blackbody radiation in this model. These results indicate that the inconsistency between the independently determined values of the density parameter and the age of the universe is much reduced when the parameter C is independently determined and it is assumed that selection effects are present. The model of figure 5 is remarkably close to the model found by a different method of model determination presented in reference 10. The results obtained for a model for the universe when C is independently determined confirms the possible presence of selection in the observational data and that the assumption that the parameter C is the same for all sources in the group being studied should not be used.

NEW DATA

Subsequent to completion of research for this report in mid 1968, new data on the age of the Galaxy have become available. Dicke (ref. 11) has published new ages based on nuclear cosmochronology, and new information relative to stellar ages, some not formally published, has been made available. A review of all information relative to the age of the Galaxy indicates that this age should now be considered to be greater than 7.0×10^9 years and possibly greater than 7.5×10^9 years. These new lower boundaries for the age of the Galaxy give a much better fit with the time since the beginning of expansion for the model presented in figure 5 for which $\sigma_o = 0.217$ and $q_o = 0.402$.

CONCLUDING REMARKS

The use of red-shift—magnitude relation to analyze observed red shift and apparent magnitudes has, in the past, been predicated on the assumption that all sources have the same value of the parameter C. The models obtained by this method have values of the density parameter and the time since the beginning of expansion which were not consistent with the values of these parameters obtained by independent methods based on observation and models of the universe obtained by other means. Generally speaking, the density parameter was too high and the time since the beginning of expansion too short when compared with the independently determined values.

When the effects of evolution and selection in the data were accounted for by an independent method for determining the parameter C (this method allows C to vary from source to source), more acceptable models for the universe were found. It was concluded that the assumption that all sources in the data have the same value of the parameter C should not be used when analyzing observational data with the red-shift—magnitude relation. However, if the parameter C can be determined independently for each source, the red-shift—magnitude relation can be used for the analysis of observational data.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., July 10, 1969,
188-41-01-01-23.

APPENDIX A

SYMBOLS

A_g	age of Galaxy
$C = 5 \log_{10} \frac{c}{H_o} + M - 5$	
c	speed of light in a vacuum
H_o	Hubble parameter
i	refers to members of group, $i = 1, \ldots, n$
K	red-shift correction
k	curvature constant
M	absolute magnitude
$\dot{M}$	rate of change of absolute magnitude
ΔM	incremental absolute magnitude due to evolution
m	apparent magnitude
m_c	corrected observed apparent magnitude
m_{cal}	calculated apparent magnitude
m_{11}	calculated apparent magnitude for $\sigma_o = q_o = 1.0$
Δm	incremental apparent magnitude
$\overline{m}$	mean apparent magnitude
$\overline{m}_k$	mean apparent magnitude of kth group
$\overline{m}_{k_c}$	mean corrected apparent magnitude of kth group

APPENDIX A

n	number of sources in group
q_o	deceleration parameter
R	scale factor that describes geometric history of universe
R_o	present value of scale factor
$S(\omega)$	function that depends on curvature of space
T	time since beginning of expansion
t	time
t_o	present time
z	red shift
Λ	cosmical constant, $3 H_o^2(\sigma_o - q_o)$
σ^2	variance
σ_o	density parameter

APPENDIX B

OBSERVATIONAL DATA USED IN THE STUDY

The data used in the present paper with the red-shift—magnitude relation are the corrected apparent magnitudes and red shifts of galaxies that are members of clusters. Two principal groups of data fall into this category: data for 18 clusters reported by Humason, Mayall, and Sandage (ref. 8) and data for eight clusters observed by Baum. The red shifts and apparent magnitudes of the clusters of galaxies reported by Humason, Mayall, and Sandage (ref. 8) are as follows:

Cluster	Red shift, z	Apparent magnitude, m
Virgo	0.004	9.16
0316 + 4121	.018	12.51
1257 + 2812	.022	12.84
1603 + 1755	.036	14.12
2308 + 0720	.043	14.78
2322 + 1425	.044	15.04
1145 + 5559	.052	15.71
0106 - 1536	.053	15.21
1024 + 1039	.065	15.88
1239 + 1852	.072	15.22
1520 + 2754	.072	15.93
0705 + 3506	.078	16.26
1431 + 3146	.131	17.31
1055 + 5702	.134	17.31
0025 + 2223	.159	17.39
0138 + 1840	.173	17.16
0925 + 2044	.192	17.54
0855 + 0321	.202	17.84

The apparent magnitudes are photographic magnitudes and have been corrected for the aperture effect and the K correction by the methods given in reference 8. These investigators give no uncertainties for these magnitudes; however, De Vaucouleurs (ref. 12) indicates that good photographic magnitudes carry an error from ±0.1 to ±0.2. It was assumed that these magnitudes carried a similar error. A survey of reports on red-shift measurements indicates that for a given observing system the uncertainty is in the fourth decimal place. However, when the red shift for a given source is determined by more than one observing system, a comparison of the results indicates an uncertainty in the third decimal place. This uncertainty would seem to make the red shift of the Virgo cluster suspect since $z = 0.004$. However, the red-shift uncertainty mentioned applies to single spectrograms and the red shift of the Virgo cluster given is the mean for 73 members of the Virgo cluster; therefore, the red shift is probably accurate to three decimal places.

Baum has never published a definitive paper on the magnitudes of the eight clusters he observed. In reference 13, Baum gives the red shifts he measured and the band passes of the photometer. Baum's data used in this study are those published by McVittie (ref. 2, p. 190). The red shifts and apparent magnitudes for the eight clusters of galaxies observed by Baum are as follows:

Cluster	Red shift, z	Apparent magnitude, m
Virgo	0.004	9.2
1257 + 2812	.022	12.8
1520 + 2754	.072	15.6
1055 + 5702	.134	16.9
0925 + 2044	.192	17.4
0024 + 1654	.290	18.7
1448 + 2617	.350	18.5
1410 + 5224	.460	19.3

Baum's magnitudes were obtained by use of an eight-color photoelectric photometer and no red-shift correction is needed. Again, no uncertainties are given but De Vaucouleurs (ref. 12) gives the uncertainty for good photoelectric magnitudes as ±0.02 to ±0.03. Inasmuch as these uncertainties compare very well with those given for quasi-stellar objects, they were assumed to be applicable to Baum's data. The red-shift uncertainty discussion for the data of Humason, Mayall, and Sandage (ref. 8) applies to the data of Baum presented in reference 2.

The smaller errors that appear to apply to Baum's magnitudes make it the better of the two sets of data; however, there are too few points to make an adequate analysis. The data of Humason, Mayall, and Sandage, although more extensive, cut off at a red shift that is too low for an adequate calculation of a model for the universe.

These two sets of data when combined would constitute a more adequate set of data than either alone. The two sets of data were combined for this study as follows. The mean difference in apparent magnitude for the five sources common to each list was determined. Baum's data were then made fainter by the mean difference which was $0^m.18$.

For common sources the average was taken; otherwise, the data of Humason, Mayall, and Sandage or modified Baum data were used. The combined data are presented in table I. Inasmuch as the method reduces Baum's data to those of reference 8, it was assumed that all data now represented corrected apparent photographic magnitudes and that the uncertainty was $0.^{m}10$ to $0.^{m}20$.

In addition to the data for the red shift and apparent magnitude, an estimate of the age of the Galaxy and an estimate of the present density of the universe were also used. The age of the Galaxy can be determined from nuclear cosmochronology as was done by Fowler (ref. 14) and Hoyle (ref. 15) or from the age of the old population II stars in globular clusters. Fowler and Hoyle estimate the age of the Galaxy to be from $1.5 + 0.5 \times 10^{10}$ years to $1.5 - 0.3 \times 10^{10}$ years, an age that agrees well with the age of the population II stars in globular clusters. An age of the Galaxy of 1.5×10^{10} years has been used in previous cosmological studies (for example, ref. 16) and this value was adopted for this study. The present density of the universe is based on matter that shows luminous and dynamical properties. Abell (ref. 17) estimates the density on this basis and found that the density could range from 10^{-29} g-cm^{-3} to 10^{-31} g-cm^{-3} with the most probable value being 10^{-30} g-cm^{-3}. The corresponding σ_0 interval is $0.0045 \leqq \sigma_0 \leqq 0.45$. The value of σ_0 corresponding to most probable density of Abell is 0.045. The methods used to estimate density do not rigorously account for intergalactic matter. There are two sets of observations in which intergalactic matter may have been detected. One observation by Koehler and Robinson (ref. 18) of absorption in the 21-cm line of hydrogen was interpreted as indicating an intergalactic medium with a density approximately 7×10^{-31} g-cm^{-3}. The second observation by Henry and coinvestigators (ref. 19) of radiation in the 40 Å to 60 Å band was interpreted as indicating a hot intergalactic plasma with a density of about 10^{-5} proton-cm^{-3}. If Koehler and Robinson are correct concerning intergalactic density, the value of σ_0 would not appreciably change. However, if Henry and his coworkers are correct about the intergalactic medium being a dense (10^{-5} proton-cm^{-3}) hot plasma, then σ_0 should be multiplied by 16. However, Henry and his coworkers have based these results on very limited observational data and until these results are confirmed, a value of σ_0 in the interval $0.45 \leqq \sigma_0 \leqq 0.0045$ is acceptable. This interval is centered on Abell's most probable value of the density.

The idea that the universe was once in a highly condensed state which took the form of a primeval fireball has been espoused by many investigators. (See ref. 20 for a brief review of the idea.) One of the more interesting results to come from the primeval fireball idea is that a relic of the fireball survives as universal thermal radiation of about 5° K to 6° K (ref. 21). A thermal radiation of about 3° K was detected by Penzias and Wilson (ref. 22) and has since been found to be isotropically distributed in the universe. The observational confirmation of the universal thermal radiation means that any model which is proposed for the observed universe must be such that this radiation can be explained by one or more features of the model. Such a feature would be an extremely small radius before the start of expansion.

APPENDIX C

A POSSIBLE METHOD FOR DETERMINATION OF THE PARAMETER C

The determination of the parameter C for sources in any group of data permits both evolution and selection to be taken into account. The following method, used to obtain values of C in table III, depends on two factors: a good estimate of the upper and lower bounds for σ_o and q_o and a knowledge of the uncertainty in the measurement of apparent magnitude. The first step is to select a source between $z = 0.0133$ and $z = 0.02$ and determine a value of C from the equation

$$C_1 = m - 5 \log_{10} z \tag{C1}$$

In this red-shift interval, σ_o and q_o being different from 1 do not introduce significant differences in the value of C. Let $\sigma_{o_l}, \sigma_{o_u}$ and q_{o_l}, q_{o_u} be the lower and upper bounds for σ_o and q_o. Use the value of C determined from equation (C1) (that is, C_1) and σ_{o_l}, q_{o_l} in equation (7) with $\dot{M} = 0$ to compute an apparent magnitude m_l for each source in the data group. Repeat the procedure for σ_{o_u}, q_{o_u} to obtain m_u. Next, evaluate the equation

$$m_{11} = 5 \log_{10} z + C_1 \tag{C2}$$

for each red shift and compute

$$m_{l_1} = m_{11} - \epsilon \tag{C3}$$

and

$$m_{u_1} = m_{11} + \epsilon$$

where ϵ is the uncertainty in the magnitude measurement. Then, determine the red shift at which $m_{l_1} = m_l$ and $m_{u_1} = m_u$. Call these red shifts $\tilde{z}_l$ and $\tilde{z}_u$. The relationship of the quantities used to determine C is shown in figure 6. The following set of inequalities is used to estimate C:

For	and	the parameter C is
$z \leqq \tilde{z}_l$ $z \leqq \tilde{z}_u$	$(m_{l_1} + n\epsilon) \leqq m \leqq (m_{u_1} + n\epsilon)$	$C_1 + n\epsilon$
$z \leqq \tilde{z}_l$ $z > \tilde{z}_u$	$(m_{l_1} + n\epsilon) \leqq m \leqq (m_u + n\epsilon)$	$C_1 + n\epsilon$
$z > \tilde{z}_l$ $z \leqq \tilde{z}_u$	$(m_l + n\epsilon) \leqq m \leqq (m_{u_1} + n\epsilon)$	$C_1 + n\epsilon$
$z > \tilde{z}_l$ $z > \tilde{z}_u$	$(m_l + n\epsilon) \leqq m \leqq (m_u + n\epsilon)$	$C_1 + n\epsilon$

APPENDIX C

In the foregoing inequalities, n is zero or a positive or negative even integer. Positive n represents decreasing brightness and negative n represents increasing brightness. The process for determining the possible values of C is to start with $n = 0$ and decrease n until all values of m are greater than $m_u + n\epsilon$. The process is repeated for increasing n until all values of m are less than $m_l + n\epsilon$. If during this process m is equal to one of the boundary values when z is less than one of the critical values, that galaxy has two values of C. For example, if $n = 0$ and $m = m_{l1}$ and because $m_{l1} = m_u - 2\epsilon$, the galaxy can have values of C equal to C_1 and $C_1 - 2\epsilon$. Because for $z > \tilde{z}_u$ and $z > \tilde{z}_l$ the difference $(m_u + n\epsilon) - (m_l + n\epsilon)$ increases rapidly and if ϵ is small, the distant sources can have a great many possible values of C. This large number of possible values of C can lead to a huge computing program of model determination because all possible combinations of C must be considered. Both selection and evolution operate to make the distant galaxies brighter than the nearer ones. Therefore, the computing time can be reduced if only those combinations are considered where C for sources that have a red shift greater than one or both of the critical values of red shift is less than or equal to C for sources where red shift is less than the critical values.

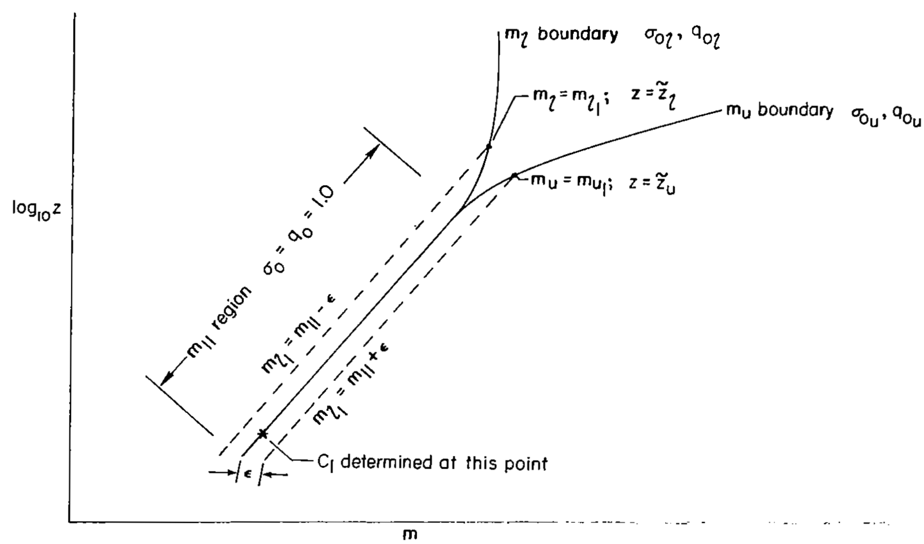

Figure 6.- Relationship of quantities used in the determination of the parameter C.

This method for determining possible values of the parameter C is easily programed for digital computers and was used to determine the values of C in table III. The list of values of the parameter C given in table III is only one of many possible combinations. Once C has been determined, it can be used directly in equation (7) with $\Delta M = 0$ or it can be used to generate Δm by subtracting $5 \log_{10} z + C$ from the observed magnitude for use in equation (10) as was done in this report. Because some combinations of C can produce a model with negative density, the model determination program should be instructed to report only those combinations of C that give positive values of σ_0.

REFERENCES

1. Sherman, Windsor L.: Zero-Density Cosmological Models and Their Applicability to the Observed Universe. NASA TN D-2601, 1965.
2. McVittie, G. C.: General Relativity and Cosmology. Univ. of Illinois Press, 1965.
3. Solheim, J. E.: Relativistic World Models and Redshift-Magnitude Observations. Mon. Notic. Roy. Astron. Soc., vol. 133, no. 3, 1966, pp. 321-338.
4. Sherman, Windsor L.; and Wallace, Sylvia A.: Relativistic Models of the Universe With Pressure Equal to Zero and Time-Dependent Uniformity. NASA TR R-282, 1968.
5. Allen, C. W.: Astrophysical Quantities. Second ed., Athlone Press (London), 1963.
6. Tinsley, Beatrice M.: Evolution of the Stars and Gas in Galaxies. Astrophys. J., vol. 151, no. 2, pt. 1, Feb. 1968, pp. 547-565.
7. Scott, Elizabeth L.: The Brightest Galaxy in a Cluster as a Distance Indicator. Astron. J., vol. 62, no. 8, Oct. 1957, pp. 248-265.
8. Humason, M. L.; Mayall, N. U.; and Sandage, A. R.: Redshifts and Magnitudes of Extragalactic Nebulae. Astron. J., vol. 61, no. 3, Apr. 1956, pp. 97-162.
9. Sherman, Windsor L.: Uniform, Zero Pressure Relativistic Models of the Universe and Observational Data. Astron. J. (Abstr.), vol. 71, no. 6, Aug. 1966, pp. 397-398.
10. Chiu, Hong-Yee: A Cosmological Model for Our Universe. I. Ann. Phys. (N.Y.), vol. 43, no. 1, June 2, 1967, pp. 1-41.
11. Dicke, R. H.: The Age of the Galaxy From the Decay of Uranium. Astrophys. J., vol. 155, no. 1, pt. 1, Jan. 1969, pp. 123-134.
12. De Vaucouleurs, G.: Classification of Galaxies by Form, Luminosity, and Color. Problems of Extra-Galactic Research, G. C. McVittie, ed., Macmillan Co., c.1962, pp. 3-21.
13. Baum, W. A.: Photoelectric Magnitudes and Red-Shifts. Problems of Extra-Galactic Research, G. C. McVittie, ed., The Macmillan Co., c.1962, pp. 390-400.
14. Fowler, William A.: Nuclear Astrophysics. Amer. Phil. Soc., 1967.
15. Hoyle, F.: The Age of the Galaxy. Proc. Roy. Soc. (London), ser. A, vol. 260, no. 1301, Feb. 21, 1961, pp. 201-204.
16. Omer, Guy C., Jr.; and Vanyo, James P.: Closed Model Universes With a Cosmological Constant. Astrophys. J., vol. 145, no. 2, Aug. 1966, pp. 555-559.

17. Abell, G. O.: Clustering of Galaxies. Annual Review of Astronomy and Astrophysics, Vol. 3, Leo Goldberg, ed., Annu. Rev., Inc., 1965, pp. 1-22.

18. Koehler, J. A.; and Robinson, B. J.:, Intergalactic Atomic Neutral Hydrogen. I. Astrophys. J., vol. 146, no. 2, Nov. 1966, pp. 488-503.

19. Henry, R. C.; Fritz, G.; Meekins, J. F.; Friedman, H.; and Byram, E. T.: Possible Detection of a Dense Intergalactic Plasma. Astrophys. J. (Lett. Ed.), vol. 153, no. 1, pt. 2, July 1968, pp. L11-L18.

20. Harrison, Edward R.: The Early Universe. Phys. Today, vol. 21, no. 6, June 1968, pp. 31-39.

21. Gamow, G.: The Physics of the Expanding Universe. Vistas in Astronomy, Vol. 2, Arthur Beer, ed., Pergamon Press, 1956, pp. 1726-1732.

22. Penzias, A. A.; and Wilson, R. W.: A Measurement of Excess Antenna Temperature at 4080 Mc/s. Astrophys. J. (Lett. Ed.), vol. 142, no. 1, July 1, 1965, pp. 419-421.

NASA-Langley, 1969 —— 30 L-5648