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GRAVITATIONAL EXPERIMENTS FOR A PROPOSED VENUS ORBITER

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GREENBELT, MARYLAND

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ABSTRACT

A Venus orbiter offers a unique opportunity to determine the oblateness and other anomalous gravitational properties of that cloud shrouded, moonless planet. While the Venus mass (or μ constant) should also be improved from the period and semi-major axis data on the orbiter, the most important new knowledge will come from tests of the anomalous properties of the gravitational field. These tests should be sensitive enough to detect low-order gravitational anomalies on Venus of about the same magnitude as those for the Earth. Reasonable lower bounds will be forthcoming from the long-term tracking of the Venus orbiter, on the nonequilibrium stress which the planet's interior is capable of supporting.

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INTRODUCTION

Recently, Goddard Space Flight Center has proposed developing a planetary explorer spacecraft to study Venus close up and for an extended period from Venus orbit.[1] Such an orbiter offers a unique opportunity to determine some of the anomalous gravitational properties of that planet.

VENUS DATA AND ORBITER ANALYSIS

The latest radar probe data on the rotation of Venus[2] makes it clear that the rotational oblateness of the planet (in hydrostatic equilibrium) will be insignificant compared to what we might call the anomalous gravitational oblateness (related to stress-supported density anomalies in the planet).

Venus apparently rotates once in 245 Earth days. The flattening of the figure of a rotating fluid planet is proportional to the square of the rotation rate.[3] Thus, since the mass and dimensions of the Earth and Venus are roughly the same, we would expect the flattening to be about $1/(245)^2$ that of the Earth. The gravitational oblateness (J_2), is also proportional to the square of the rotation rate (to first order) if the rotation is strong enough. A more detailed calculation shows the hydrostatic J_2 of Venus to be about $1/(236)^2$ that of the Earth. This is about 400 times smaller than what Venus could support if it had approximately the same interior strength as the Earth.[3] This stress-supported value is about 8×10^{-6}. It should be noted that there is a clear presumption here that the gravitational oblateness of Venus will not be related to the equatorial plane defined by its rotation. In fact, J. D. Anderson of the Jet Propulsion Laboratory (JPL) reports a value of $(J_2)_{Venus} = (-5 \pm 10) \times 10^{-6}$ from the analysis of range rate tracking data on the Mariner 5 flyby.[4] The negative sign indicates a possible equatorial mass deficiency or a slight football shape to Venus, emphasizing the nonrotational character of the oblateness. What this means is that all five gravity coefficients of second degree (C_{20}, C_{21}, S_{21}, C_{22}, S_{22}) will probably be of equal significance for the planet; the equatorial ellipticity (C_{22}, S_{22}) perhaps outstanding since the rotation of Venus may be in a resonance lock with the Earth.[5] The determination of C_{21} and S_{21} is equivalent to locating the gravitational north pole. The problem of discriminating these five equally significant second-degree effects from observations of a single orbiter will be a formidable one. Certainly the data on the previous Venus flyby's (Mariner 2 and 5) will be useful to combine with the orbiter data to reduce the high correlations which may be expected between these effects. In addition, the planned, lower altitude, dragged orbit will supply new information useful for this discrimination.

To get some idea of what kind of anomalous gravity results can be expected from the orbiter, in terms of precision, it seems reasonable to use J. D. Anderson's analysis of the Mariner 5 data as a guide. Using essentially the same deep space two-way Doppler tracking on the flyby that will be available for the orbiter, J. D. Anderson reports a precision of 10×10^{-6} for $(J_2)_{Venus}$. This value was achieved from a numerical analysis of continuous tracking of the probe to a minimum altitude of about 3500 km and beyond to occultation. The probe was within measurable influence of the anomalous field for only a few hours. For a conservative calculation, we may assume that one half of each orbit will be similarly observed with the same J_2 recovery precision as for Mariner 5. The orbiter's period is planned to be about 6 hours. The tracking lifetime will be about 110 days.[1] Thus, there will be about 440 orbits observed. The $\sqrt{n}$ law, then, promises a precision of $10 \times 10^{-6}/(440)^{1/2} = 0.5 \times 10^{-6}$ for $(J_2)_{Venus}$, or to about one part in ten if $(J_2)_{Venus} = 8 \times 10^{-6}$.

However, additional information will be available on long-term changes of the angular orbit elements to supplement these "half-revolution" or short-period analyses. For a six-hour equatorial Venus orbiter with periapsis-apoapsis altitudes of 1000-20,000 km, the secular rotations of the line of apsides and the orbit plane due to a J_2 of 8×10^{-6} in 110 days will amount to about 1/2 and 1 degree, respectively. The closest possible orbiter (semi-major axis = 1 Venus radius) would show changes of about 8-1/2 and 17 degrees in these elements in 110 days. According to J. D. Anderson,[4] the planet-centered coordinates of Mariner 5 during encounter were known to within about 20 km. The average distance of the drag-free orbiter from the center of Venus will be about 2.74 Venus radii or $2.74 \times 6.06 \times 10^3 = 16{,}000$ km. Thus, planet-centered angular elements for the orbiter might be determinable to $20 \times 57.3/16{,}000 = .07$ degrees. Again, the $\sqrt{n}$ law with 440 orbit updates should give the 110-day drift of these angular elements to a precision of $.07/(440)^{1/2} = .003$ degrees.

Thus, this long-term measurement could determine $(J_2)_{Venus}$ to a precision better than one part in a hundred or to an uncertainty in J_2 of less than 0.1×10^{-6}. This contrasts quite favorably with the accuracy of $(J_2)_{earth}$, which is known now, from combined satellite solutions, to within 0.05×10^{-6}. It is clear that long-term observation of the orbiter will establish quite sharp lower bounds on the nonequilibrium interior stress which Venus is capable of supporting. It is not so clear as to how accurately the 3 principal moments of inertia (derivable from the mass, rotation J_2 and J_{22}) of the planet can be determined. This determination would set just as interesting upper bounds on the supportable stress, at least through second order in the planet's figure. The problem here stems from the anticipated difficulty of separting the five second-degree gravity effects.

HIGHER DEGREE GRAVITY EFFECTS

The fact that the surface of Venus is at a temperature near the melting point of lead and the mass is close to that of the Earth's, argues strongly that the planet exists throughout at closer to hydrostatic equilibrium than the Earth.[6] Thus, by analogy with results on Earth satellites, it is very unlikely that any anomalous effect of very short period (from high-degree gravity harmonics) will be observed. This is true even with continuous tracking from Earth near the 1000 km periapsis.

Nevertheless, on the off chance that large short wavelength anomalies do exist (from mass concentrations or discontinuities near the surface, for example) periapsis should be placed so as to be under continuous observation from Earth for as long a time as possible. In fact, observation of both periapsis and apoapsis should be insured since these points are also sensitive to the determination of the principal oblate parameters of the planet from long-term observation.

RESONANT ORBITS

In the sense of being planet-stationary, all Venus orbits are nearly resonant since the rotation is so slow. However, the "energy resonances" (i.e., those of the semi-major axis) which are the useful ones for geodesy with Earth satellites (on longitude gravity terms) will not be the strong ones on the Venus orbiter. Rather, the m daily terms (m = longitude frequency) in the potential function will exhibit resonance like behavior with small divisors, and long-period and amplified effects, merely because the Venus day is 245 Earth days.

For example, all longitude gravity harmonics with frequency or order $m = 2$ (i.e., C_{22}, S_{22}) would have effects of period = 245/2 = 123 days. Equally significant because of the likely arbitrariness of the gravitational equator, the C_{21} and S_{21} terms will have effects of a 245-day period which will aid in their discrimination from the other second-degree effects.

Typical low frequency m daily position perturbations on Earth satellite orbits of about 2 Earth radii are of the order of 100 meters.[7] Since these effects are inversely proportional to the period, if the Venus mass anomalies are analogous to the Earth's, these same m daily effects for Venus should be about $245 \times 100 = 25$ km. This again is at the discrimination level for the deep space Doppler tracking of Mariner 5.[4] In terms of velocity measurements, for a sinusoidal variation of 25 km over 123 days, the position rate amplitude is 1.5 cm/sec. Therefore, considering the $\sqrt{n}$ law again, the combined effects of the lowest order longitude harmonics of Venus should be readily detectable from long-term position changes over the tracking lifetime of the orbiter.

However, to detect deviations of this long period (123 and 245 Earth days), the semi-yearly (Venus year = 225 Earth days) sun perturbations which will be strong on the orbiter, will have to be carefully removed from the tracking data.

VENUS MASS

Tracking of the orbiter over many revolutions should certainly aid in refinement of the Venus mass which is now known to perhaps 5 significant figures from the major deviations of the Mariner 2 and 5 flyby's. How much improvement to the Venus mass the orbiter will make is not clear, however, because it will, on the average, be much closer to Venus than the flyby trajectories were. Thus the mass determined from the orbiter may be subject to greater uncertainty than the flyby's from errors in the location of the spacecraft, the Venus center of mass, possible atmospheric drag, and the uncertainty of the higher degree harmonics.

For an Earth analogy, the greatest precision in the Earth's mass has been achieved as a byproduct from the Ranger moon impact trajectories (similar but even more sensitive to mass than flyby's), and not from analysis of the thousands of Earth satellites including the moon.

SUMMARY

A Venus orbiter of semi-major axis between 1 and 3 planet radii, under nearly continuous tracking observation from Earth over a period of about 100 days, should detect the expected gravitational oblateness of Venus to about one part in a hundred.

Additionally, it may be expected that the Venus mass will be improved somewhat (perhaps to the sixth significant figure) from the drag-free orbiter observations, principally of the period.

At levels of tracking accuracy similar to those achieved in Mariner 2 and 5 (i.e., planet-centered position uncertainties of 25 km and range rate errors of 1 to 2 cm/sec.), the expected long-period effects of low-order longitude gravity harmonics should also be detectable.

Under the assumption of nearly continuous orbiter tracking information over a 100-day arc, it should be possible, though perhaps with some high correlations, to discriminate all of the second-degree gravity harmonics of Venus to an accuracy of something less than one part in a hundred. The accuracy should still be fine enough to draw significant conclusions about the internal stress support capabilities of Venus (both upper and lower bounds), at least to second degree in the planet's figure.

To insure maximum coverage of periapsis, where the greatest sensitivity to any short-period gravitational anomalies will occur, periapsis should be placed on the Earth's side of the Venus orbit. In fact, because of its value in discriminating secular gravity effects, the entire line of apsides should be under continuous observation for as long as possible. This will probably mean an initial placement of this line close to the Venus-Earth line at insertion but ahead of it in the direction of the earth's motion.

Atmospheric drag on the orbiter will tend to hide the long-period longitude harmonic effects since these are strongest along track. It will also affect the rotation of the line of apsides, important in the oblateness (J_2) discrimination. For this reason, since a drag-decay experiment is planned, at least the first two months of the orbiter's tracking lifetime should be drag-free. This would avoid the uncertainties inherent in removing this effect in gravitational studies.

Since the strength of this gravity experiment depends critically on utilizing as many orbits as possible (to overcome the relatively weak tracking information), as much time as possible should be devoted to the tracking task, at least to the extent of well defining the line of apsides and the nodes of each orbit.

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