ANALYSIS OF THE GAMMA-RAY-BACKSCATTER TECHNIQUE FOR DIRECT MEASUREMENT OF THE DENSITY OF THE MARTIAN ATMOSPHERE

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SUMMARY

Studies have been conducted of a technique that uses the backscattering of gamma radiation emitted by a radioisotope source to measure the ambient density of the Martian atmosphere during entry of a spacecraft into the atmosphere. The results show the feasibility of the measurement up to an altitude of about 100 000 feet (30.5 km) in the least dense of the Voyager-Mars (VM) model atmospheres. No probes or special windows through the spacecraft skin or heat shield are required because of the penetrating ability of the radiation. A source strength at Mars of 20 curies (1 curie = 3.7 × 10^10 disintegrations per second) of gamma radiation is required. The measurement is essentially unaffected by the composition of the atmosphere, by the shock layer, and by motions of the vehicle. An engineering-model density sensor, which has been fabricated and tested, has a mass of 5 pounds (2.3 kg), a volume of 90 inches^3 (1475 cm^3), and requires 5 watts of power for operation. Further miniaturization of the device appears practical. Sterilization of the sensor is not expected to present major problems.

INTRODUCTION

Density is a quantity of prime importance to be measured during the probing of planetary atmospheres. The atmospheric density profile is not only of considerable scientific value but is also an important engineering measurement needed for optimum aerodynamic design of future spacecraft. In general, information on the atmospheric density from a planet's surface to as high in its atmosphere as possible is desired. However, the mission profile usually presents limitations. For a large part of the proposed Martian atmosphere entry trajectories, the instrument package will be located behind an ablative heat shield and the entry capsule engulfed in a high-temperature flow field. Under these conditions, measurement techniques which avoid ports, probes, or special thin windows that might compromise the integrity of the heat shield or structure are desirable. In addition, the shock layer and vehicle motions should have a minimum effect on the measurement.

A technique employing backscattering of gamma radiation emitted by a radioisotope source to measure density has been shown to be effective under these conditions. This
concept is shown schematically in figure 1. It consists of a source of gamma radiation, a detector, and shielding material to eliminate direct transmission between the two. Because of their penetration ability, gamma rays can propagate out through the vehicle structure and heat shield without severe attenuation. A fraction of the emerging gamma-ray flux is backscattered by the atmosphere into the detector. The backscatter interaction, Compton scattering of gamma rays by atomic electrons, is linearly dependent upon the atmospheric density in the scattering region. By proper location of sensor components and collimation of the source emission and detector field of view, the effective scattering volume or measurement region can be located in the undisturbed ambient gas beyond the shock wave. Calibration accounts for any attenuation of the gamma-ray beam in the heat shield; attenuation in the shock layer is generally very small and can be neglected. The density profile is then obtained by correlating the measured density with spacecraft altitude during entry.

For measurement of Martian atmospheric density, the potential advantages of the gamma-ray-backscatter technique are

(1) The measurement is a direct, linear function of the ambient atmospheric density

(2) No probes or special windows through the vehicle skin or heat shield are required because of the penetrating ability of the radiation

(3) Atmospheric composition measurements are not required for accurate density measurements

(4) The shock layer and vehicle motions have a negligible effect upon the measurement.

In general, the gamma-ray-backscatter technique is applicable to other planetary atmospheres; however, this report will discuss the results of NASA in-house and contractor studies of the feasibility of using the technique to measure the ambient density of the Martian atmosphere during entry.

SYMBOLS

A effective atomic weight of the atmosphere

B background counting rate

BD direct-transmission background counting rate
BN  natural-radiation background counting rate
BS  skin-scatter background counting rate

\[ f_1, f_2, f_3, f_4 \]  factors of K which express differences in the various sensor-response parameters between the time of sensor calibration and the time of measurement during atmospheric entry (See appendix.)

G  gross or total counting rate
K  proportionality constant relating backscattered counting rate and atmospheric density

N  Avogadro's number
R  counting rate per unit density measured during sensor calibration
S  backscattered counting rate, or signal
t_B  time for background measurement
t_G  time for gross or total counting-rate measurement

\[ X = \frac{G}{K} \]
\[ Y = \frac{B}{K} \]

Z  effective atomic number of the atmosphere

\( \rho \)  density of the scattering medium (atmosphere)
\( \sigma \)  standard deviation
\( \sigma_s \)  total atmospheric Compton scattering cross section
\( \sigma_{s,e} \)  Compton scattering cross section per electron (a function of the gamma-ray energy)
BACKGROUND

Gamma-Ray Scattering

A discussion of the several ways in which a gamma ray can interact with matter can be found in the literature. (See, for example, ref. 1.) Only four interactions are significant for gamma-ray energies in the range between 10 keV and 10 MeV. These interactions are illustrated in figure 2, which is a plot of the relative probability of occurrence of each interaction as a function of gamma-ray energy, for nitrogen (N2) or carbon dioxide (CO2) at standard temperature and pressure. (Curves for N2 are presented in ref. 2 from cross-section data tabulated in ref. 3. Similar calculations for CO2 and other gases from ref. 3 data have been found to yield similar results.) The photoelectric effect is an ionization process in which the gamma-ray energy is completely transferred to an electron bound to the nucleus of an atom. In the same general energy region is Rayleigh, or coherent, scattering. This interaction involves small-angle scattering of the gamma ray from tightly bound atomic electrons and has a low probability of occurrence. At higher energies pair production becomes important. In this phenomenon, the gamma ray interacts with the intense field surrounding the nucleus and is converted into a positron-electron pair. A minimum gamma-ray energy of 1.02 MeV, twice the rest-mass energy of an electron, is required. Figure 2 also illustrates Compton, or incoherent scattering, in which the gamma ray acts like a particle and undergoes a collision with an electron. The electron absorbs some of the gamma-ray energy and is freed from the atom. The scattered gamma-ray has an energy which depends upon its original energy, the energy absorbed by the electron and the scattering angle. As can be seen in figure 2, over much of the energy range shown, Compton interactions have the highest probability of occurrence. Also, of the four mechanisms involved, only Compton scattering can produce a large-angle or backscattered gamma ray which will reach the detector.

Concept-Feasibility Flight Tests

Two flight tests have been conducted in the earth's atmosphere to examine the feasibility of using backscattered gamma rays to measure atmospheric density. The launch vehicle for these tests was the Nike-Apache sounding rocket. The sensor payload, shown schematically in figure 3, included a source of approximately 30 curies of the isotope cerium-144 (1 curie = 3.7 \times 10^{10} disintegrations per second) and a sodium iodide scintillation detector.

Test data for one of the flights are shown in figure 4. The counting rate and density are presented as a function of altitude. For comparison, density determined from Arcasonde measurements at low altitudes as well as density data from reference 4 at higher altitudes are shown. The point scatter at high altitudes occurs because of the
statistical uncertainty inherent in counting random nuclear events, and also because a large background counting rate had to be subtracted from each data point.

The flight-test results demonstrated the feasibility of using backscattered gamma rays to measure atmospheric density but indicated that for measurement of a thin atmosphere such as that of Mars, improvements in signal-to-noise ratio were necessary. As a result, investigations were initiated to reduce background noise and to develop a high-purity radioisotope source. A study of the sterilization resistance of a miniaturized gamma-ray-backscatter density sensor was also initiated. Additional details of the feasibility flight tests can be found in reference 5.

RESPONSE OF MARTIAN ATMOSPHERE SENSOR

Signal Response

In studying the feasibility of using backscattered gamma rays to measure the density of the Martian atmosphere, the parameters which affect sensor response must be examined. For a typical measurement geometry such as that shown in figure 1, the backscattered counting rate, or signal, $S$ is a function of the gamma-ray flux emerging from the source, gamma-ray energy, attenuation in the gamma-ray path, Compton scattering cross section in the atmosphere, detector area and efficiency, and the geometry parameters, such as the source-detector separation and the collimation angles. For fixed source, detector, and geometry, all these parameters are constant with two exceptions, the attenuation in the shock layer and the atmospheric Compton scattering cross section $\delta_s$. As discussed in the appendix, for the Martian atmosphere the attenuation in the shock layer, although not constant, is expected to be very small and can be neglected. The total atmospheric Compton scattering cross section is given by

$$\sigma_s = \sigma_s, e N \rho \frac{Z}{A}$$

where $\delta_s, e$ and $N$ are constant. Thus, the backscattered counting rate, or signal, can be represented by

$$S = K \rho \frac{Z}{A}$$

where $K$ is a proportionality constant which includes the source, detector, and geometry parameters previously mentioned.

This expression shows that the response of the device is a direct function of atmospheric density. The only other atmospheric parameter which affects the signal is the ratio of atomic number to atomic weight $Z/A$. This ratio is equal to 0.5 for all gases in the Voyager-Mars model atmospheres, VM-1 through VM-10 (table 1 from ref. 6) with the exception of argon which has a $Z/A$ ratio of approximately 0.45. This value represents
a 3.5 percent uncertainty in density measurement for the maximum argon concentration postulated (VM-6 model atmosphere). Thus, atmospheric composition measurements are not required for accurate density measurements.

Background Noise

During descent of the sensor through the Martian atmosphere, background radiation will contribute to the total counting rate. It is important to reduce background noise for maximum measurement sensitivity and to precisely measure background counting rate for maximum measurement accuracy. The background counting rate will be measured just prior to atmospheric entry and subtracted from the total counting rate so that the backscattered counting rate, or signal, can be determined. Since the signal increases with increasing density and the background noise will be approximately constant, the signal-to-noise ratio will continually improve during entry. Three factors which contribute to the background noise are

(1) Natural-background radiation. In reference 7 it was estimated that cosmic rays and other sources of natural-background radiation would contribute on the order of 50 counts per second to the total counting rate for a typical detection system and measurement geometry.

(2) Direct-transmission noise. Direct transmission of gamma rays between source and detector can be reduced by shielding; however, high-energy radiation is difficult to shield. In the flight payloads described earlier, a thickness of 15 inches (38 cm) of lead was required to shield the detector from a small amount of 2-MeV radiation from the cerium-144 source. If the source had only emitted the primary energy, 134 keV, less than 0.4 inch (1 cm) of shielding would have been required. Since weight is of prime importance on a Mars mission, the radiation source must have no high-energy gamma-ray component which would require extensive shielding. Additional discussion of radiation source requirements can be found later in this paper.

(3) Skin-scatter noise. The background counting rate which was measured on the Nike-Apache flight tests was about an order of magnitude greater than had been predicted for cosmic radiation and other natural sources. As discussed in reference 8, the major part of this high background noise level was found to be caused by multiple scattering of gamma rays within the vehicle skin and into the detector. Results of in-house studies and those reported in reference 8 show several potential methods of minimizing skin-scatter noise. It can be reduced considerably by choosing a geometry that requires the gamma ray to undergo a third scatter before being detected. Skin-scatter noise has also been found to be a strong function of the source-detector separation distance. Small increases in this separation will significantly reduce the noise without appreciably affecting the backscattered signal. (See ref. 9.)
Measurement Range

For a typical geometry, such as that shown in figure 1, sensor response to atmospheric density can be estimated by examining the probability functions that determine the path of any gamma ray emitted by the source, scattered by the atmosphere toward the detector, and detected. The product of these probabilities integrated over the total volume of interaction represents the response of the device. This approach is further discussed in reference 8.

For a typical measurement of Martian atmospheric density, sensor response was estimated by using the techniques of reference 8, the geometry of figure 1, and the following conditions:

(1) Source-detector separation, 1.5 feet (0.5 m)
(2) Source energy, 100 keV
(3) Attenuation of gamma-ray beam in skin, 10 percent; this is what might be expected in a low-density silicone-elastomer heat-shield material of the type being considered for Mars entry missions
(4) Detector, 3-inch (7.6-cm) diameter, 100-percent efficient for 100-keV gamma radiation
(5) Density varied with altitude for the extremes of Mars model atmospheres, VM-8 and VM-9, as shown in figure 5
(6) Source strength at Mars of 20 gamma curies (a gamma curie is equivalent to $3.7 \times 10^{10}$ gamma rays per second)
(7) The capsule heat shield was discarded at an altitude of 25 000 feet (7.6 km) for both VM-8 and VM-9

For these conditions, sensor response in counts per second was found to be

$$S = (7.0 \times 10^8)\rho$$

before heat-shield ejection, and

$$S = (7.6 \times 10^8)\rho$$

after heat-shield ejection, where $\rho$ is in units of g/cm$^3$.

By using the density-altitude data of figure 5 and 1-second counting intervals, sensor response is shown as a function of altitude for VM-8 and VM-9 in figure 6. For VM-8, the device begins to respond to density at an altitude of about 150 000 feet (45.8 km) with a counting rate of a few counts per second increasing to 10 000 counts per second at the Martian surface. In the much more dense VM-9 atmosphere, the sensor begins to respond
to density at an altitude of 400 000 feet (122 km) with counting rate increasing to 20 000 counts per second at the planet's surface.

If necessary, to avoid interference with the landed spacecraft experiments, the gamma-ray backscatter sensor could be ejected from the spacecraft when the heat shield is discarded, or a shielding mechanism could be actuated to "turn off" the source.

Measurement Accuracy

In the appendix, an expression is derived for the relative standard deviation or uncertainty in density in terms of the total or gross counting rate $G$, background counting rate $B$, net counting rate, or signal, $S$, and proportionality constant $K$ as

$$\frac{\sigma(\rho)}{\rho} = \left[ \frac{\sigma^2(G)}{S^2} + \frac{\sigma^2(B)}{S^2} + \frac{\sigma^2(K)}{K^2} \left( \frac{G^2 + B^2}{S^2} \right) \right]^{1/2}$$

The various factors which influence the standard deviations of $G$, $B$, and $K$ are also discussed in the appendix.

The precision of a measurement of Martian atmospheric density can be estimated by using this expression and the following assumptions:

1. Signal varies with altitude as shown in figure 6 and discussed in the preceding section.

2. The total background counting rate is 100 counts per second. Fifty counts per second are due to skin scattering and the remainder comes from such sources as natural background and direct transmission. The background reduces to 50 counts per second after heat-shield ejection, as shown in figure 6.

3. The systematic error $\frac{\sigma(K)}{K}$ is 0.03 (3 percent). A quantitative analysis of all factors which affect $\frac{\sigma(K)}{K}$ is beyond the scope of this report. However, based on a study of the feasibility flight-test results, an estimate of 3 percent systematic error appears reasonable.

The resultant 1-sigma uncertainty in density measurement is listed in table 2 and plotted in figure 7. The results show for VM-8 a 100 percent uncertainty at 135 000 feet (41.2 km), 10 percent at 90 000 feet (27.5 km), and 3 percent at the Martian surface. For VM-9 comparable results are 100 percent at 370 000 feet (112.8 km), 10 percent at 235 000 feet (71.7 km), and 3 percent at the surface.
SYSTEM REQUIREMENTS

Source

A primary characteristic of the gamma-ray source is the energy spectrum of the emitted gamma rays. Energy selection for a backscatter sensor is complicated by having several conflicting criteria. As described in reference 2, these criteria include awareness that in order to use the technique, the gamma rays must penetrate the heat shield and return after having lost some energy in scattering. Maximum penetration ability implies the need for a high energy. However, the cross section for scattering at large angles is a maximum at low energy. A low gamma-ray energy is also desirable so that weight required to shield the detector from the source does not become impractical as previously discussed. Evaluation of these conflicting criteria can best be accomplished by a trade-off exercise. For a measurement of the Martian atmosphere, an optimum range of source energy is 50 keV to about 150 keV. (See ref. 10.)

The most ideal source would be monoenergetic; thus all the gamma rays would be emitted at the energy of interest, and a narrow energy discrimination window could be established to increase the signal-to-noise ratio. The radioisotope comes closest to being the ideal monoenergetic source. The inherent high reliability and low weight compared with an X-ray tube make the radioisotope most suitable for the Mars application.

Desirable source characteristics for the measurement of Martian atmospheric density are summarized as follows:

1. Gamma-ray-emitting radioisotope
2. Primary gamma-ray energy between 50 and 150 keV
3. No high-energy gamma ray or beta particle which would require excessive shielding mass to minimize direct transmission noise
4. Source half-life, minimum of 150 to 200 days
5. High specific activity (curies per gram of source material)

Under interagency agreement with Langley, the Oak Ridge National Laboratory of the Atomic Energy Commission has identified gadolinium-153 as the most suitable radioisotope source for the Martian atmospheric measurement. (See ref. 11.) $^{153}$Gd has a half-life of 241 days and emits gamma rays with an energy of about 100 keV and an abundance of 46 percent. It can be produced by two methods. The first is to irradiate a target enriched with $^{152}$Gd in a nuclear reactor. $^{153}$Gd is produced by the reaction

$$^{152}\text{Gd} + \text{neutron} \rightarrow ^{153}\text{Gd} + \text{gamma ray}$$
Material produced by this method has a low specific activity and high cost because of the requirement for a highly enriched target. Also, terbium impurities (\(^{160}\text{Tb}\) emits high-energy gamma rays) were found in irradiated samples of gadolinium. Terbium-gadolinium chemical separations are difficult, and it is doubtful that a high-purity source could be fabricated.

The second technique is to irradiate a natural europium target, containing about 50 percent \(^{151}\text{Eu}\), in a reactor. The reaction sequence for production of \(^{153}\text{Gd}\) is:

\[
\begin{align*}
^{151}\text{Eu} + \text{neutron} & \rightarrow \text{gamma ray} + ^{152}\text{Eu} \\
& \beta \text{ decay} \\
^{152}\text{Gd} + \text{neutron} & \rightarrow ^{153}\text{Gd} + \text{gamma ray}
\end{align*}
\]

The isotope produced by this method has a high specific activity and can be readily purified to less than 10 parts per million of contamination. Also, because the target for irradiation does not require enrichment, the isotope can be produced fairly inexpensively. The production sequence for obtaining the desired quantities of gadolinium is now being optimized. (See ref. 11.)

For a specific mission, the required source strength depends upon a number of parameters, including the desired source output at time of use, source self-absorption, source half-life, and abundance of the gamma ray of interest. For example, to obtain an output of 20 gamma curies during descent through the Martian atmosphere, it would be necessary to initially produce 150 to 200 curies of \(^{153}\text{Gd}\).

Detection System

In order to minimize the required source strength, the detection system must be designed for maximum gamma detection efficiency. This requirement dictates the need for a scintillation-crystal—photomultiplier-tube detector. The detector area should be maximized consistent with such factors as the increased shielding weight and the increased susceptibility of large crystals to physical damage from shock and vibration.

The detector output can be processed by using fairly standard techniques to produce a signal compatible with the telemetry system. Either analog or digital techniques can be used.

Regardless of the processing network employed, some form of automatic gain control (AGC) will be necessary to compensate for the poor gain stability of a scintillation detector with temperature. A standard AGC technique in nuclear systems is to use the counting rate derived from a small, long half-life gamma or alpha radiation source placed near the crystal to control the gain by changing the photomultiplier tube voltage.

Physical Characteristics

Based on conceptual design studies (ref. 7) updated to take into account the latest knowledge of the Martian atmosphere and recent Mars mission studies, an
engineering-model density sensor has been fabricated for the purpose of sterilization testing. The device has a mass of 5 pounds (2.3 kg), a volume of 90 inches$^3$ (1475 cm$^3$), and requires 5 watts of power for operation. These figures include the radioisotope source, all necessary shielding, scintillation detector, signal conditioning electronics, and mounting hardware. Further miniaturization of the device appears practical.

**STERILIZATION**

The requirement for the sensor to withstand and be unaffected by sterilization presents constraints in the selection of materials and components. Studies have shown that components of the type required can be found which have been qualified in the environment specified by reference 12. The major area of uncertainty has been the availability of suitable photomultiplier tubes. However, development programs being conducted by phototube manufacturers indicate that suitable devices can be obtained. (See, for example, ref. 13.) Testing of the engineering-model density sensor mentioned above is planned to verify the resistance of the device to the sterilization environment.

**CALIBRATION**

Calibration is necessary to establish sensor response as a function of atmospheric density. This must be accomplished in a vacuum chamber where gas density can be carefully controlled and varied. Pressure and temperature measurements are made to monitor gas density, and counting-rate measurements are made to monitor sensor output.

The calibration should take place in as large a vacuum chamber as possible to minimize scattering of gamma rays from the chamber walls and into the detector. In the feasibility flight-test program, calibration tests were conducted in the largest available chamber, the 60-foot vacuum sphere at the Langley Research Center. Even so, it was necessary to perform the tests with an 0.5-curie source, since wall scattering would have exceeded the counting-rate capability of the sensor with the 30-curie flight source. The resultant calibration data were then scaled by the ratio of the source strengths, 30 curies/0.5 curie.

As discussed in the appendix, this scaling technique requires that the ratio of the gamma-ray outputs of the two sources be precisely known. To accomplish this, both calibration and flight sources are designed to have the same self-absorption and contain radioactive material from the same production batch. Then, to verify that the sources are constructed identically, direct-transmission measurements of the gamma-ray output of each source are made.
RADIATION SAFETY

Handling and use of radioisotope sources of the type and size required for this sensor requires certain precautions to satisfy applicable regulations and minimize radiation exposure to personnel. Acceptable procedures for the safe handling of a large gamma-ray source developed and exercised during the feasibility flight-test program are directly applicable to a planetary mission. In addition, the engineering-model sensor mentioned earlier was designed with a remotely actuated shield mechanism to cover the source during prelaunch operations so that the radiation dose to individuals working in the vicinity of the sensor is below the limit permitted in an unrestricted area of 2 millirem per hour, 0.5 rem per year.

The source-shield assembly will also incorporate sufficient thermal protection to protect the structural integrity of the source in case of a fireball explosion during launch operations.

CONCLUDING REMARKS

Studies have been conducted of the technique employing backscattering of gamma radiation emitted from a radioisotope source to directly measure the ambient density of the Martian atmosphere during entry of a spacecraft into the atmosphere. The measurement is essentially unaffected by the composition of the atmosphere, by the shock layer, and by motions of the vehicle. The results show the feasibility of the measurement during high-speed entry through the heat shield as well as after heat-shield ejection. For a typical configuration, the estimated 1-sigma uncertainty in density measurement is ±15 percent at 100 000 feet (30.5 km) altitude in a VM-8 (least dense) model atmosphere, improving to ±3 percent at the Martian surface. These estimates were made for a source strength of 20 curies (1 curie = $3.7 \times 10^{10}$ disintegrations per second) of gamma radiation at time of measurement. This would require production of 150 to 200 curies of gadolinium-153, the most suitable source for the Martian atmospheric measurement.

Techniques have been developed for producing this isotope with the proper purity. An engineering-model density sensor, which has been fabricated and tested, has a mass of 5 pounds (2.3 kg), has a volume of 90 inches$^3$ (1475 cm$^3$), and requires 5 watts of power for operation. Further miniaturization of the device appears practical. Tests are planned to verify the sterilization resistance of the engineering-model sensor.

Langley Research Center,
National Aeronautics and Space Administration,
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APPENDIX

DERIVATION OF DENSITY-ERROR EXPRESSION

An expression for the uncertainty in density measurement as a function of total and background counting rates and of systematic error is derived.

By assuming that the backscattered counting rate, or signal, is directly proportional to atmospheric density, \( S = K \rho \) or

\[
\rho = \frac{S}{K}
\]  

(A1)

The signal is obtained by subtracting the background counting rate from the total or gross counting rate; that is,

\[
S = G - B
\]  

(A2)

thus

\[
\rho = \frac{1}{K} (G - B) = \frac{G}{K} - \frac{B}{K}
\]  

(A3)

If it is assumed that \( X = \frac{G}{K} \) and \( Y = \frac{B}{K} \)

then

\[
\rho = X - Y
\]  

(A4)

The standard deviation of \( \rho \), \( \sigma(\rho) \), in terms of the standard deviations of \( X \) and \( Y \) is

\[
\sigma(\rho) = \left[ \sigma^2(X) + \sigma^2(Y) \right]^{1/2}
\]  

(A5)

From the definitions of \( X \) and \( Y \), their standard deviations can be given in terms of the standard deviations of \( K \), \( G \), and \( B \); that is,

\[
\sigma(X) = X \left[ \left( \frac{\sigma(K)}{K} \right)^2 + \left( \frac{\sigma(G)}{G} \right)^2 \right]^{1/2}
\]  

(A6)

similarly,

\[
\sigma(Y) = Y \left[ \left( \frac{\sigma(K)}{K} \right)^2 + \left( \frac{\sigma(B)}{B} \right)^2 \right]^{1/2}
\]  

(A7)

Substitution of equations (A6) and (A7) into equation (A5) gives

\[
\sigma(\rho) = \left[ \frac{\sigma^2(G)}{K^2} + \frac{G^2 \sigma^2(K)}{K^4} + \frac{\sigma^2(B)}{K^2} + \frac{B^2 \sigma^2(K)}{K^4} \right]^{1/2}
\]  

(A8)
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Division of equation (A8) by \( \rho \) yields the relative standard deviation or uncertainty in density

\[
\frac{\sigma(\rho)}{\rho} = \frac{1}{\rho} \left[ \frac{\sigma^2(G)}{K^2} + \frac{G^2 \sigma^2(K)}{K^4} + \frac{\sigma^2(B)}{K^2} + \frac{B^2 \sigma^2(K)}{K^4} \right]^{1/2}
\]  

(A9)

Substitution of equation (A1) into equation (A9) gives

\[
\frac{\sigma(\rho)}{\rho} = \frac{K}{S} \left[ \frac{\sigma^2(G)}{K^2} + \frac{G^2 \sigma^2(K)}{K^4} + \frac{\sigma^2(B)}{K^2} + \frac{B^2 \sigma^2(K)}{K^4} \right]^{1/2}
\]

or

\[
\frac{\sigma(\rho)}{\rho} = \left[ \frac{\sigma^2(G)}{S^2} + \frac{\sigma^2(B)}{S^2} + \frac{\sigma^2(K)}{S^2} \left( \frac{G^2 + B^2}{S^2} \right) \right]^{1/2}
\]  

(A10)

Equation (A10) expresses the uncertainty in density in terms of the standard deviations of gross and background counting rates with respect to the signal and the standard deviation of the proportionality constant \( K \) modified by the term \((G^2 + B^2)/S^2\).

The standard deviation of the gross or total counting rate is given by

\[
\sigma(G) = \left( \frac{G}{t_G} \right)^{1/2}
\]

(A11)

where \( t_G \) is the measurement time. This time is generally established by a trade-off between long time for minimum uncertainty in \( G \) (small \( \sigma(G) \)) and short time for minimum density change with altitude during the measurement period.

The background counting rate can be assumed to consist of several factors:

\[
B = B_D + B_S + B_N
\]

The standard deviation of \( B \) is given by

\[
\sigma(B) = \left( \frac{B}{t_B} \right)^{1/2}
\]

(A12)

where \( t_B \) is the time for background measurement. It is expected that ample measurement time will be available prior to atmospheric entry to determine the background counting rate with sufficient precision so that the uncertainty \( \sigma(B) \) will be negligible.

The proportionality constant \( K \) can be stated in terms of a number of factors as

\[
K = f_1 f_2 f_3 f_4 R
\]

The calibration factor \( R \) is the counting rate per unit density of the sensor which is measured during sphere test calibration. The factors \( f_1, f_2, f_3, \) and \( f_4 \) express...
APPENDIX

differences in the various parameters affecting sensor response between the time of measurement during atmospheric entry and the time of sensor calibration. For example,

\[ f_1 = \text{the source emission rate at the measurement time compared with the emission rate during calibration} \]

\[ f_2 = \text{the amount of attenuation in the gamma-ray path at the measurement time compared with the attenuation during calibration} \]

\[ f_3 = \text{detection-system response at measurement time compared with the response during calibration} \]

\[ f_4 = \text{the atmospheric scattering efficiency, or cross section per unit density at measurement time, compared with the scattering efficiency during calibration}. \]

The relative standard deviation of \( K \) is then given by

\[
\frac{\sigma(K)}{K} = \left[ \frac{\sigma^2(f_1)}{f_1^2} + \frac{\sigma^2(f_2)}{f_2^2} + \frac{\sigma^2(f_3)}{f_3^2} + \frac{\sigma^2(f_4)}{f_4^2} + \frac{\sigma^2(R)}{R^2} \right]^{1/2} \tag{A13}
\]

The factor \( f_1 \) is a function of the initial source strength, the half-lives of the primary and any contaminant gamma-ray energies, and the source self-absorption. If the source to be used for flight can be used for calibration, then only the half-lives of the gamma-ray energies would need to be known to minimize \( \sigma(f_1) \). The half-lives can be determined by taking a small sample of the main source and monitoring its output with a stable detection system during the flight mission. However, if different sources must be used for calibration and measurement, then the ratio of the gamma-ray outputs of the two sources must be precisely known. This ratio is determined from mass analysis of the flight and calibration sources which have been fabricated to have the same self-absorption characteristics and to contain radioactive material from the same production batch. In addition, direct-transmission counting-rate measurements of the gamma-ray output of each source must be made.

The uncertainty in the amount of attenuation in the gamma-ray path \( f_2 \) is primarily a function of the shock layer formed outside of the spacecraft during entry. Even under extreme conditions of a thick, high-density shock layer, the attenuation in the shock layer is estimated to be less than 0.1 percent for a 100-keV gamma ray. Therefore, the attenuation in the shock layer can be neglected.

The factor \( f_3 \) is a function of the system parameters, gain, detector efficiency, and so on. Changes in these parameters such as gain shifts due to temperature or aging of the scintillation crystal will increase the uncertainty. However, as discussed in the text, the signal conditioning system will incorporate automatic gain control (AGC) circuitry which will compensate for changes of this type. Therefore, the uncertainty \( \sigma(f_3) \) should be very small.
APPENDIX

The factor $f_4$ is a function of the atmospheric scattering efficiency or cross section. As discussed in the text, this parameter varies only slightly with atmospheric composition; therefore, the uncertainty $\sigma(f_4)$ should be very small even for atmospheres containing appreciable argon.

The parameter $R$, the counting rate per unit density, is determined by calibrating the sensor in a large vacuum sphere. The calibration experiment must be carefully designed to minimize extraneous effects such as scattering from the walls of the sphere so that $R$ can be precisely determined.
REFERENCES


### TABLE 1 - MARS MODEL ATMOSPHERES

From reference 6

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Dimension</th>
<th>VM-1</th>
<th>VM-2</th>
<th>VM-3</th>
<th>VM-4</th>
<th>VM-5</th>
<th>VM-6</th>
<th>VM-7</th>
<th>VM-8</th>
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<td>( P_o )</td>
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<td></td>
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<td>20.9</td>
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<td>(^\circ)R/1000 ft</td>
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TABLE 2.- CALCULATED 1-SIGMA UNCERTAINTY IN GAMMA-RAY-BACKSCATTER MEASUREMENT OF ATMOSPHERIC DENSITY FOR MARS MODEL ATMOSPHERES VM-8 AND VM-9

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<tr>
<th>Altitude (ft)</th>
<th>Uncertainty in density, percent</th>
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<td></td>
<td>VM-8</td>
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<td>15 000</td>
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<td>400 000</td>
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Figure 1.- Schematic drawing of gamma-ray-backscatter configuration for in-flight measurement of atmospheric density.
Figure 2.- Relative probability of occurrence of principal interactions of gamma rays with matter for energies from 10 keV to 10 MeV.

Relative probability of occurrence (for $N_2$ or $CO_2$)

- A - Photoelectric effect
- B - Rayleigh (coherent) scattering
- C - Compton (incoherent) scattering
- D - Pair production

Gamma-ray energy, keV

0 10 10^2 10^3 10^4
Figure 3.- Payload of Nike-Apache backscatter density sensor.
Figure 4.- Detected counting rate (background subtracted) and density as a function of altitude for an earth-atmosphere flight test (data from ref. 5).
Figure 5.- Density as a function of altitude for extremes of Mars model atmospheres, VM-8 and VM-9.
Figure 6. Calculated sensor response or signal $S$ as a function of altitude for VM-8 and VM-9 atmospheres. Assumed background $B$ is also shown. Increase in $S$ and decrease in $B$ at 25,000 ft is due to heat-shield ejection.
Figure 7.- Calculated 1-sigma uncertainty in density measurement as a function of altitude for extremes of model atmospheres, VM-8 and VM-9.
"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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