

CONSTANT OF STEP-BY-STEP IONIZATION OF ATOMS

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Tom 187, No.4, 787-790,
Izdatel'stvo "NAUKA", 1969

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SUMMARY

The object of this work is to establish the constant of step-by-step ionization in a large discharge tube by way of its dependence on temperature and density of electrons and properties of different atomic gases. The ratio of such constants is established for two such gases. The earlier calculations by Bates et al are proved to be incorrect.

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1. A step-by-step ionization in a discharge tube takes place in the case when the departure time of an excited atom on the tube walls is much greater than its destruction time under the effect of electron impact, i. e., the step-by-step-ionization is realized at sufficiently large transverse dimensions of the discharge tube and high electron densities. In the process of step-by-step ionization the excited atom undergoes a whole sequence of excited states, whereupon the transitions between these states are conditioned by atom's collisions with electrons. Under the considered conditions, that is, for large dimensions of the discharge tube, the ionization constant is independent of tube's dimensions, and since the distribution function of free electrons by velocities is Maxwellian (high electron density), this quantity depends only on temperature of electrons, on their density and on the properties of ionized atoms. The object of the present work is the finding of such a dependence.

2. We shall establish the relationship between the constants of step-by-step ionization in two different atomic gases. At the same time, we shall consider that the temperature of electrons is much smaller than the excitation energy of atoms. The density of free electrons, formed as a result of step by-step ionization per unit of time is:

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$$dN_e / dt = N_e \sum_n k_n N_n,$$

where N_e is the density of electrons; N_n is the density of atoms in the state n ; k_n is the constant of these atoms' direct ionization. The constant of step-by-step ionization k_{ion} is introduced on the basis of the relation

$$dN_e / dt = k_{ion} N_a N_e,$$

where N_a is the density of atoms in the ground state. Hence,

$$k_{io} = \sum_n k_n \frac{N_n}{N_a}. \quad (1)$$

When obtaining the relations between constants of step-by-step ionization in different gases, we shall make use of the fact that the properties of strongly excited atoms do not depend on the kind of atom, for they are determined by the Coulomb interaction of the electron with the charge of atom's residue. This is why the constants of direct ionization k_n of the excited atom are identical for atoms of arbitrary kind. Besides, the ratio of densities of atoms $N_n^{(1)}/N_n^{(2)}$, which are in identical strongly excited states in two different gases for identical densities and temperatures of electrons, is independent of the number of this state. Denoting $N_n^{(1)}/N_n^{(2)} = s$ and taking into account that at low temperatures of electrons the quantity k_{ion} is determined by the destruction of strongly excited atoms, we shall obtain for the ratio of constants of step-by-step ionization of different kinds of atoms with identical density and temperature of electrons:

$$k_{ion}^{(1)} / k_{ion}^{(2)} = s N_a^{(2)} / N_a^{(1)}.$$

Here, indices 1 and 2 are related to atoms of the respective kind.

For the determination of the proportionality factor s , which is independent of the density of electrons, we shall consider the case of high electron density, when the collision frequencies of electron with excited atoms substantially exceed the emission frequencies of the excited atom. In this case the density of excited atoms is determined by the Boltzmann law:

$$N_n = \frac{g_n}{g_a} N_a \exp\left(\frac{I_n - I}{T}\right).$$

where T is the temperature of electrons, $g_n = g_i g_e n^2$; g_i is the statistical weight of the ion; $g_e = 2$; g_a is the statistical weight of the atom in ground state;

I, I_n are respectively the ionization potentials in ground and excited state. Making use of this, we find the proportionality factor, which leads to the following relation between the constants of step-by-step ionization for atoms of different kind at identical temperature and density of electrons:

$$k_{\text{ion}}^{(1)} \frac{g_{1a}}{g_{1i}} \exp\left(\frac{I_1}{T}\right) = k_{\text{ion}}^{(2)} \frac{g_{2a}}{g_{2i}} \exp\left(\frac{I_2}{T}\right), \quad (2)$$

Here g_{1a}, g_{2a}, I_1, I_2 are respectively the statistical weight and the ionization potential of the corresponding atom in ground state; g_{1i}, g_{2i} is the statistical weight of the ion.

3. Let us consider with more detail the case of high density of electrons, when the radiation of excited atoms does not play any part and the density of excited atoms responds to the Boltzmann law:

$$N_n = N_a g_n / g_a \exp(-\epsilon_n / T),$$

where ϵ_n is the energy of excitation of the corresponding level, $g_n = g_e g_i n^2$ is its statistical weight. Then the ionization constant is

$$k_{\text{ion}} = \frac{1}{N_a} \sum_n k_n N_n = \frac{g_i}{g_a} \int n^2 dn k_n \exp\left(-\frac{\epsilon_n}{T}\right).$$

Inasmuch as at inelastic collisions with an excited atom the transferred energies are of the order of the temperature of electrons, the basic contribution to this integral is made by excited states with ionization energy of the order of the temperature of electrons $me^4 / h^2 n^2 \sim T$. Taking this into account, we obtain

$$k_{\text{ion}} \sim \frac{g_i}{g_a} n^3 \langle v\sigma \rangle \exp\left(-\frac{I}{T}\right) \sim \frac{g_i}{g_a} \frac{me^{10}}{h^3 T^3} \exp\left(-\frac{I}{T}\right),$$

where the characteristic velocity of collision of electrons with the atom, i. e. $v \sim \sqrt{T/m}$, the ionization cross-section $\sigma \sim e^4 / T^2$ for an excited atom with binding energy $me^4 / h^2 n^2 \sim T$ coincides by order of magnitude with the exchange cross section of two free electrons with kinetic energies of the order of the temperature of electrons with energy of the order of the temperature T . Thus, in the given case we shall obtain for the constant of step-by-step ionization

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of the atom by electron impact:

$$k_{\text{ion}} = A \frac{g_i}{g_a} \frac{m e^{10}}{h^3 T^3} \exp\left(-\frac{I}{T}\right), \quad (3)$$

where the coefficient A is independent of the temperature of electrons and of the kind of atoms.

T A B L E 1

PLASMA Temperature of electrons 1000°K	Hydrogen				Alkaline metal				Hydrogen-like			
	1	3	16	32	1	2	4	8	4	8	16	32
A	0,005	0,011	0,069	0,025	0,051	0,036	0,022	0,031	0,16	0,045	0,021	0,066

Plotted in the above Table 1 are the values of the coefficient A obtained by way of processing the calculations by Bates, Kingston and McWhirter [1] for the constant of step-by-step ionization. At the same time, the ionization constant, computed in this work for high densities of electrons, was substituted in the right-hand part of formula (3) and from the relation thus obtained we found the coefficient A. According to the obtained result, the coefficient A must be identical in all cases. At the same time the obtained dependence is valid within the limits of low temperature $T / I \ll 1$, when free electrons are forming as a result of ionization of strongly excited atoms. The fact that the processing of calculations does not lead to identical values of A should be ascribed to the shortcomings of the calculations by Bates, Kingston and McWhirter [1]. Indeed, what have been utilized in that work for cross sections of inelastic transitions between the state of atoms under the action of the electron impact is in fact the empirical formulas of Grizinskiy, whose precision is quite limited. The error in the cross-section accrues during the computation of the ionization constant, so that the error in this constant may be greater than the error in the cross-section of inelastic transition, and, roughly speaking, by as many factors as there are transitions performed by a valence electron prior to ionization.

4. We shall make use of the information that can be extracted from the process of triple recombination of electrons and ions within the limits, when the density of electrons is high, so that the radiation plays no part. The considered process proceeds according to the scheme $2e + X^+ \rightarrow e + X$, and it is opposite in detail to the process of interest to us, that is, the step-by-step ionization.

According to investigations of the three-piece recombination mechanism [2 - 6], the recombining electron spends the basic recombination time at levels with ionization energy of the order of the temperature. This agrees well with our own assertion, whereby the step-by-step ionization is determined mainly by the disintegration of excited atoms with an ionization potential of the order of the temperature.

In case of equilibrium between the processes of triple recombination and step-by-step ionization, the relationship between the constants (k_{ion}) of step-by-step ionization and those (α_{rec}) of triple recombination has the form

$$k_{ion} N_e N_a = N_e N_i \alpha_{rec},$$

where N_a is the density of atoms in ground state, N_i is the density of ions. At the same time, the recombination coefficient's dependence on density and temperature of electrons has the form [2 - 6]:

$$\alpha_{rec} = b N_e e^{10} / \sqrt{mT}^{3/2}, \quad (4)$$

where b is the proportionality constant. Utilizing the Sach formula

$$\frac{N_e N_i}{N_a} = \frac{g_e g_i}{g_a} \left(\frac{mT}{2\pi h^2} \right)^{3/2} \exp \left(- \frac{I}{T} \right),$$

we again obtain formula (3) for the ionization constant:

$$k_{ion} = A \frac{g_i}{g_a} \frac{m e^{10}}{h^3 T^3} \exp \left(- \frac{I}{T} \right); \quad A = \frac{2b}{(2\pi)^{3/2}} (g_e = 2) \quad (5)$$

The values of coefficient A, obtained from the processing of results of Bates, Kingston and McWhirter calculation for the recombination constant, are compiled in Table 2:

T A B L E 2

Temperature of electrons 1000°K	0.25	0.5	1.0	2.0	2.0	4
b	4.1	2.6	2.4	19	0.97*	0.8**
A	0.52	0.33	0.1	0.21	0.12	0.1

* Hydrogen plasma; ** Alkaline metal plasma

T A B L E 3

Source	[3]	[4]	[5]	[6]
b	3.1	5.9	3.2A	4.2
A	0.38	0.73	0.39A	0.52

Given in Table 3 are the values of the coefficient in formula (4), which were found in the works [3 - 6]. In the Hinnov & Hirschberg work [3] the Thomson theory was applied for excitation cross section of the atom by electron impact when obtaining formula (4); in the Makin and Keck work [4], this formula was obtained on the basis of the variation method. In the Gurevich and Pitayevskiy's work [5] the triple recombination of the electron with a multi-charge ion with great charge $Z \gg 1$ was investigated. In this case formula (4) contains still one more multiplier, which for $Z \gg 1$ is $\Lambda = \ln Z$. For $Z \sim 1$ the quantity Λ is ~ 1 . Finally, in the D'Angelo work [6], formula (4) was obtained on the basis of Thomson theory for a triple capture. As may be seen, the values of the proportionality factor b , found in the above-indicated works, agree well among themselves, particularly if we take into account the roughness of models used in these works. The value of the coefficient A in Table 3 was found on the basis of formula (5). As may be seen, the results presented in Table 3 are in contradiction with the Bates, Kingston and Whirter data [1] (see Tables 1 and 2). This questions the reliability of the indicated calculations, which is linked with the utilization in them of Grizinskiy's formulas for the transition cross-sections.

5. In conclusion let us compare the constant of step-by-step and direct ionization in case of low temperatures and high densities of electrons. The atom's ionization cross-section by electron impact near threshold is practically determined by the linear dependence on the difference of electron energy and the ionization potential:

$$\sigma_{\text{ion}} = \sigma_0 (\epsilon/I - 1),$$

where σ_0 is of the order of atomic magnitude. Hence we find for the constant of direct ionization, averaged by the Maxwellian distribution of electrons:

$$k_{\text{dir}} \approx \sqrt{8T / \pi m \sigma_0} \exp(-I / T).$$

Comparing this with the step-by-step ionization constant (5), we find

$$k_{\text{dir}} / k_{\text{step}} \sim (h^2 T / m e^4)^{1/2} \ll 1.$$

Therefore, at high densities of electrons and low temperatures, there materializes a step-by-step mechanism of free electron formation. Note that the distribution function of electrons at the tail usually decreases more sharply than the Maxwellian, i.e. the direct ionization constant is still less than what we obtained.

*** E N D ***

References follow...

R E F E R E N C E S

1. D.R. BATES, A.E. KINGSTON, R.W.P. McWHIRTER,
Proc. Roy. Soc. A., 267, 297, 1962;
270, 155 1962.
2. B.M. SMIRNOV, - Atomnye stolknoveniya i elementarnye protsessy
v plazme.
Atomic Collisions and Elementary Processes in
Plasma, M., 1968
3. E. HINNOV, J.G. HIRSCHBERG,
Phys. Rev., 125, 795, 1962.
4. B. MAKIN, J.C. KECK,
Phys. Rev., Let., 11, 281, 1963.
5. A.B. GUREVICH, L.P. PITAEVSKIY,
ZhETF, 46, 1281, 1964.
6. N. D'ANGELO, - Phys. Rev., 140A, 1488, 1965.

Received on 27 Nov.1968.

CONTRACT NO.NAS-5-12487
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Translated by
Andre L. Brichant
26 August, 1969.