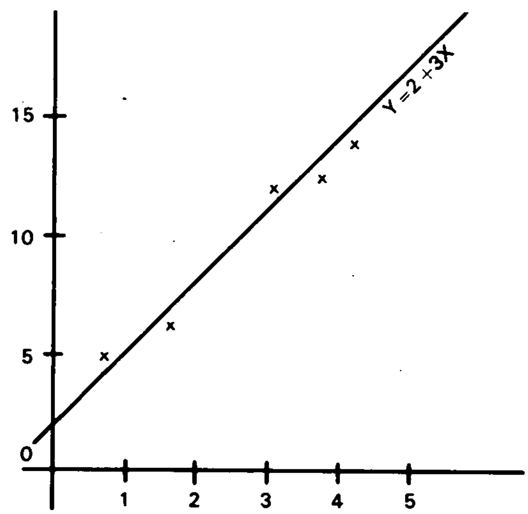
NASA TECH BRIEF

A Method for Rapidly Evaluating the Linearity of Calibration Data

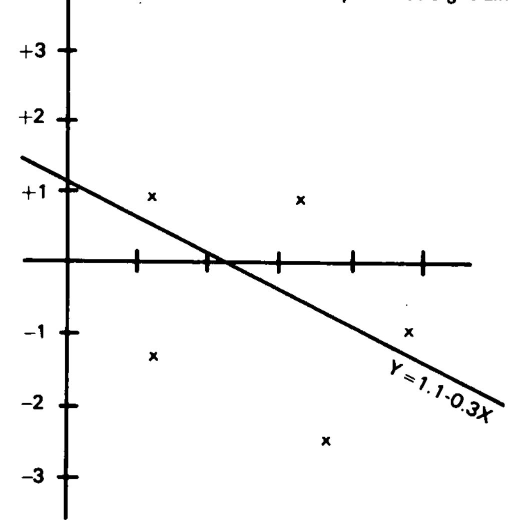
Full Scale Graph of Data Points and Reference Line



X_i	Y_i	Difference Between Y_i and Corresponding Point on Line
1	6	+1
2	7	−1
3	12	+1
4	12	−2
5	16	−1

Figure 1. Sample Data Set

Difference Plot and Least-Squares Straight Line Fit



X	Difference	Fitted Difference By Method of Least-Squares	Residual
1	+1	0.6	0.4
2	−1	0.1	−1.1
3	+1	−0.4	1.4
4	−2	−0.9	−1.1
5	−1	−0.4	0.4

Figure 2. Difference Plot

A simple technique is presented for determining whether or not a set of five data points lies within a specified close tolerance of a linear fit.

The following theorem, which justifies this technique, will be proved: Define two arbitrary constants, T and C. Select data points (X_i, Y_i) with $i = 1,2,3,4,5$ which have the property

$$X_{i+1} - X_i = C. \quad (1)$$

Let $f(X) = a_o + a_1 X$ be the least-squares linear fit to these data (regression of Y on X). If $g(X) = b_o + b_1 X$ is another line which has the property

$$|Y_i - g(X_i)| < T/1.6 \text{ for all } i, \quad (2)$$

then $$|Y_i - f(X_i)| < T \text{ for all } i. \quad (3)$$

Consider the following situation. A calibration check is run on amplifier modules with a linearity tolerance

(continued overleaf)

of 0.25% of the full-scale output. The input signal is increased in five equal steps. Full scale output is approximately 5 V (tolerance 0.0125 V). The expected results follow:

Coded input	*Nominal output*
0	0.00 V
1	1.25 V
2	2.50 V
3	3.75 V
4	5.00 V

A logical first step to verify acceptable data linearity is to plot the points and see if a line can be drawn to bring the points within tolerance. In this case, however, if a graph is plotted using a scale of 1 V : 2 in., the tolerance is only 0.025 in., less than 1/32 in. Thus, plotted on 8-1/2 × 11 in. graph paper, the tolerance in question is barely discernible. To avoid this problem of scale, a difference plot is used.

Figure 1 presents a sample data set (chosen deliberately to exaggerate certain aspects of difference plots) in tabular and graphic form, together with an approximately fit reference line: $Y = 2 + 3X$. Figure 2 shows the difference plot, a graph of the vertical separation of each point from the reference line. These differences may be plotted on a magnified scale, to allow easy display of discrepancies otherwise too small to see. The line plotted on Figure 2 is the least-squares best fit to the difference data

$$h(X) = 1.1 - 0.3X. \tag{4}$$

In general, the least-squares line h(X) fitted to a difference plot is related to the reference line g(X) used to determine the differences and to the least-squares line f(X) fitted to the raw data, by the relationship

$$f(X) = g(X) + h(X). \tag{5}$$

Thus, in this example, f(X) may be computed from the sum

$$\begin{aligned} g(X) &= 2.0 + 3.0X \\ h(X) &= \underline{1.1 - 0.3X} \\ f(X) &= 3.1 + 2.7X \,. \end{aligned}$$

Worst-case analysis was used to obtain the constant 1.6 which appears in equation 2. Let the tolerance on the y-distance be unity. Then the worst case of a line within tolerance exists when each of the five points is at the maximum distance of ± 1 from the reference line. There are 32 such cases, but, because of the condition of equation 1, imposed on the independent variable, certain of the 32 difference patterns possess the same maximum absolute distance of a point from the least-squares line fit. Identification of identical patterns of differences reduces to eleven the number of essentially different cases. Each of these cases is tabulated below, giving the absolute value of the residual for the farthest point from the least-squares line.

Difference pattern					*Maximum absolute residual*	
1	1	1	1	1	0.0	
1	1	1	1	-1	1.4	
1	1	1	-1	1	1.4	
1	1	1	-1	-1	0.8	
1	1	-1	1	1	1.6	greatest maximum residual
1	1	-1	1	-1	1.2	
1	1	-1	-1	1	1.2	
1	-1	1	1	-1	1.4	
1	-1	1	-1	1	1.2	
1	-1	-1	1	-1	1.4	
1	-1	-1	-1	1	1.2	

Since the greatest (absolute) y-distance of one of these points from a least-squares line is 1.6, the theorem, stated in equations 2 and 3, is proved. Note, however, that the theorem provides a sufficient, but not a necessary condition.

With this background, the amplifier module calibration data may be evaluated. The tolerance is adjusted to 0.0078 V (0.0125/1.6). A standard form is used to make difference plots of the calibration data.

If a line can be drawn on the difference plot which passes within 0.0078 V of each data point, the linearity requirement is satisfied. If no such line can be drawn, judgement is deferred, and the part is held for further investigation.

Notes:

1. This technique has been successfully applied to reduce delays in a large-scale testing program. About 95% of a large group of amplifier modules which were evaluated using this technique, were accepted without waiting for computer curve fits.
2. No additional documentation is available. Specific questions, however, may be directed to:
   Technology Utilization Officer
   Marshall Space Flight Center
   Huntsville, Alabama 35812
   Reference: B70-10085

Patent status:

No patent action is contemplated by NASA.

Source: Frances A. Norton Bari of
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