The Determination of Stability Domains for Nonlinear Dynamical Systems

Present day guidance systems can be described for finite intervals of time by nonlinear, nonautonomous differential equations. Moreover, the control laws for these systems are, in some cases, generated by iterative procedures. Such a system is far more complicated than any system which has been successfully analyzed by current state-of-the-art stability analysis techniques other than simulation. Before an analysis of an actual guidance system can be undertaken, it is necessary to expand the knowledge of several fundamental aspects of the over-all problem of stability.

The concepts of Liapunov's direct method for the stability analysis of nonlinear dynamical systems has generated a continually expanding research program aimed at finding analytical stability analysis techniques applicable to highly complex physical systems. A research effort has been directed toward several aspects of the stability problem pertinent to the analysis of space vehicle guidance systems. This effort was focused on: (a) effective use of present techniques and the development of new techniques for determining the domain of stability (exactly or approximately) of high order nonlinear systems; (b) numerical means for implementing these techniques; (c) the relationship of Liapunov stability to finite time stability; (d) stability analysis of nonautonomous systems; (e) formulation of mathematical models of guidance schemes; and (f) analysis of a simplified time-dependent closed-loop guidance system. A numerical algorithm for determining an "optimal" quadratic estimate of the domain of attraction of an equilibrium solution of a quasi-linear differential equation was formulated and developed. The estimate was optimal in the sense of largest enclosed volume and was based upon the use of a quadratic form Liapunov function. The numerical experiments performed with this algorithm and the conclusions drawn from them are also described.