SECOND QUARTERLY REPORT:
SOLAR CELL PERFORMANCE
MATHEMATICAL MODEL

Prepared for the
California Institute of Technology
Jet Propulsion Laboratory

CASE FILE COPY

By
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December 15, 1969

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ABSTRACT

Results of the second three months of an effort to prepare a computer program for silicon solar cell performance in the space environment are reported. The previously completed mathematical model has been reviewed in the light of current research activity and improvements to the model are suggested. Numerical and graphic results of computations are provided.
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INTRODUCTION

This is the second quarterly report on a one year program to provide computational methods for prediction of solar cell performance in a natural radiation environment. It covers work performed during the period 1 September 1969, through 30 November 1969. The model for the theoretical calculations of solar cell performance under space conditions, prepared under contract 952246 with the Jet Propulsion Laboratory, is the basis of this work. During the first quarter of this program, that model was refined and computer routines written to permit the examination of changes in cell parameters due to various environmental factors.

Section I presents a brief summary of the status of the computer program. Sections II and III describe the actual work done in expanding the original mathematical model for greater versatility in manipulation. Numerical values for electrical parameters are exhibited for different conditions of cell thickness, junction depth, temperature, and radiation fluence. Finally, conclusions are drawn from this work, and suggestions are made for future investigations in Section IV.
I. STATUS OF COMPUTER PROGRAM

Efforts during the past three months have been concentrated on preparing a workable computer program for solar cell performance, beginning with the routines assembled so far. A workable program now exists using a simplified version of the model. This program, in Fortran IV, has been run on an IBM 360/65 in order to see if it can generate the appropriate response of cell performance to variations in a number of parameters. These responses are presented in later portions of this report; they appear to us to be correct, within the limitations of the program.

The earlier computer routine for deriving the minority carrier concentration across the cell, and from this the photovoltaic current output, has been rewritten once more. This routine performs its function by means of the variable mesh interval technique described in the last Quarterly Report. The routine also includes consideration of the surface layer. To include this portion of the calculation exactly, an electric field $E$ should be added to the basic continuity equation. The field is caused by the large gradient of diffused atoms which results from the manufacturing process used in forming solar cell junctions. Work is currently in progress to incorporate the field terms into the program. Because of the complexity of this function, which requires generating terms for the complimentary error function, its inclusion in the program caused the computer to overflow. The portion of the program which deals with this function therefore, must be simplified and debugged to prevent this overflow.

The formula, presented in the last report, to generate minority carrier concentration $n$ is:

$$n_{k+1} = n_k + \frac{h_k + h_k^2 qE/kT}{h_k^2 + (1 - h_k qE/kT) + h_k^2 (1 - h_k qE/kT)}$$

$$- n_k \left[ h_{k+1} \left( 1 - h_k qE/kT - 2 h_k^2/L_k^2 \right) \right]$$

$$+ \left( 2 h_k^2 h_{k+1} C_{k+1}/D \right)$$

(1)
where \( h \) is the width of the interval, the subscript \( k \) identifies the interval and the other symbols have the meanings shown in Appendix A. For the base region we are assuming the \( E \) vanishes; this would not be true for drift field solar cells.

The effects of the coverslide in reducing and moderating radiation fluences have been included in a shielding routine for the program. This calculation is in accordance with the revised mathematics for electron shielding, described in the last report. Briefly, the damage coefficient is reduced by a factor depending on the areal mass (grams/cm\(^2\)) of the coverslide. The reduction is given by:

\[
K(E, \rho_d) = K(E,0) \exp \left( -10 \frac{\rho_d}{E^{1.5}} \right)
\]

where \( \rho_d \) is the areal mass and \( E \) is the initial electron energy. The factor was derived for space fluxes; the reduction of laboratory beams by a coverslide would be somewhat different.
II. ANALYSIS OF MATHEMATICAL MODEL

A. Computational Techniques

The difference equation for the evaluation of the continuity equation, which was derived in the First Quarterly Report was expanded so that the effects of the n-layer, temperature, resistivity, and light intensity could be easily included. Figure 1 shows the flow chart for the expanded program.

Using the values given by Tada (1) for $\tau_p$ and $D_p$ in the n region, $L_{p_{no}} = 1 \times 10^{-4}$ cm. In the base region, $L_{n_{no}}$ is assumed to be $1.5 \times 10^{-2}$ cm. The concentration of minority carriers was then plotted as a function of distance, x, in the cell, considering a 14 mil cell with a 0.5 micron n surface region, and a base resistivity of 10 ohm-cm. Two hundred increments were taken on each side of the junction. The results of this calculation are shown in Figure 2.

The current contribution from each side of the junction is the product of $q$, $D$, and the slope of the concentration in the vicinity of the junction. Therefore the photovoltaic current density is

$$J = q D_n \left| \frac{n_1}{h_1} \right| p + q D_p \left| \frac{n_{-1}}{h_{-1}} \right| n$$

The negative subscripts indicate that the interval is taken to the left (surface region side) of the junction. For the case under consideration, this method of computation shows $J_p = 33.853$ mA/cm$^2$ and $J_n = 4.276$ mA/cm$^2$, therefore $J = 38.129$ mA/cm$^2$. 

4.
Fig. 1. Flow chart for computer program.
Figure 2. Minority carrier concentrations in solar cell with AMO sunlight.
B. Temperature Effects

Both theoretical and experimental data show that \( I_L \) changes very slowly with temperature. Waddel (2) gives a temperature coefficient of 0.0758 mA/°C for a 2cm\(^2\) solar cell, while Reynard (3) shows approximately 0.05%/°C. The major contribution to the variation of the current with temperature comes from the change in \( I_o \), the diode saturation current. The variation is a fairly complex function, since it depends on the ideal diode saturation current, which varies as \( T^{2.5} e^{-E_s/kT} \), and on the generation-recombination current in the space charge region, which is approximately proportional to \( e^{-E_s/2kT} \) at high injection levels. In silicon, the latter dependence will generally dominate. In addition, \( I_o \) is also a function of the light intensity. The effect of temperature on this function has not yet been fully defined.

If the effect of the light on \( I_o \) is considered to be constant with temperature, \( I-V \) curves can be plotted from various temperatures, as shown in Figure 3. \( I_o \) can be calculated for a given cell from the standard solar cell equation:

\[
I = I_L - I_o (e^V + IR/V_o - 1)
\]

Using \( I_L = 38.129 \) mA, \( V_{oc} = 0.520 \) V, and \( V_o = 43 \) mV, the value of \( I_o \) at 300°K is found to be 0.213 μA at AM0 sunlight illumination. The curves in Figure 3 are plotted assuming that \( I_o \) increases 0.05%/°C, \( R \) is negligible, and \( I_o \) varies as \( e^{-E_s/2kT} \). From the curves, the temperature coefficient of \( V_{oc} \) is found to be approximately 0.56%/°C. Reynard's experiments (3) show temperature coefficients between 0.4 and 0.6%/°C.

From the computations used for the graphs the maximum power \( P_{max} \) can be calculated. Figure 4 shows a plot of \( P_{max} \) vs. \( T \).
Figure 3. I-V curves as a function of temperature

Figure 4. Temperature dependence of maximum output power
C. Effects of Cell Thickness and Junction Depth

One of the major advantages of the mathematical model of the solar cell under consideration is that it greatly facilitates examining the effects of changes in the physical parameters. It was therefore a simple matter to study the effects of cell thickness $t$, and junction depth $d$, on the distribution of carrier concentration, from which the current density and maximum power are readily calculable. Figure 5. shows the carrier distribution for a 7 mil cell with 0.2 $\mu$ and 1.0 $\mu$ junction depths and for a 14 mil cell with 0.2 $\mu$ and 1.0 $\mu$ junction depths. Only two curves are shown for the n-layer, since it is obvious that the surface region will not be affected by the overall cell thickness.

From Table 1, it can be seen that for solar cells thicker than the diffusion length of minority carriers in the base, $J_{sc}$, $V_{oc}$, and $P_m$ increase only slightly with cell thickness. The cell parameters depend more strongly on the junction depth.

It is interesting to note, that while the overall current density decreases with increasing junction depth, the current contribution from the n side is increasing from 2.42 mA/cm$^2$ to 5.49 mA/cm$^2$. For the reason discussed in Section III, $J_n$ is much less sensitive to radiation damage than $J_p$. Therefore, increasing the depth of diffusion of a solar cell will cause the current degradation due to irradiation to be slowed down.
carrier concentration (carriers/cm$^3$)

Figure 5. Minority carrier concentrations in cells with different junction depths and thicknesses.
Table 1. Dependence of solar cell parameters on cell thickness $t$ and junction depth $d$, as computed from the mathematical model for an n/p solar cell in AM0 sunlight.

<table>
<thead>
<tr>
<th>$d$ (mil)</th>
<th>7</th>
<th>10</th>
<th>14</th>
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<tr>
<td>0.2</td>
<td>40.222</td>
<td>40.855</td>
<td>41.168</td>
</tr>
<tr>
<td>0.5</td>
<td>37.171</td>
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<tr>
<td>1.0</td>
<td>32.279</td>
<td>33.908</td>
<td>34.235</td>
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$J_{sc}$ in mA/cm²

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<th>14</th>
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<td>0.2</td>
<td>0.44963</td>
<td>0.45047</td>
<td>0.45079</td>
</tr>
<tr>
<td>0.5</td>
<td>0.44569</td>
<td>0.44651</td>
<td>0.44696</td>
</tr>
<tr>
<td>1.0</td>
<td>0.43863</td>
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$V_{oc}$ in Volts

<table>
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<th>10</th>
<th>14</th>
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<td>0.5</td>
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<tr>
<td>1.0</td>
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<td>10.005</td>
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$P_m$ in mW
III. RADIATION DAMAGE

A. Electron Irradiation

The damage coefficient for electron irradiation may be calculated from Equation 2 by using the values given previously (4) for \( K(E,0) \). Figure 6 shows the degradation of \( J \) with electron fluence for varying thickness of coverslide and electron energy of 0.8 MeV. The variation of the current density from both n- and p- sides was considered; however, it was found that for the maximum fluence considered ( \( \Phi = 10^{16} \text{ e/cm}^2 \), no coverslide ) \( J_n \) decreased only 0.0147\%, and could therefore be considered constant for all cases.

In an experiment performed by Reynard, (5) \( I_{sc} \) was reduced to 75.3\% of its initial value by exposure to a fluence of \( 4.5 \times 10^{16} \text{ electrons/cm}^2 \) of 0.8 MeV electrons on uncovered cells. The current reduction predicted by the model for the same conditions is 77.1\%. However, using a coverslide of 0.112 g/cm\(^2\) thickness (.020 mils of Corning 7940 Fused Silica) Reynard found almost no difference in the amount of degradation. He further found that after the units were allowed to anneal for one month, the devices with the coverslides recovered a greater amount than did the uncovered cells. The indication therefore, is that the coverslide and/or adhesive also degraded. After annealing, Reynard's values for \( \frac{I_{sc}}{I_{sco}} \) are 79.0\% for the uncovered units, and 82.0\% for the covered devices. Both Reynard (6) and Morgan (7) indicate that the coverslide material used in the experiment will darken approximately 2 - 3\%. The theoretical value of \( \frac{I_{sc}}{I_{sco}} \) for \( \Phi = 4.5 \times 10^{16} \) and \( pd = 0.112 \) is 87.7\% if the coverslide darkening is taken into account, the value is further reduced to 85.1 - 85.9\%. This is in reasonably good agreement with the experimental value, considering that the damage to the adhesive is unknown.
Fig. 6. Effect of coverslide thickness on degradation of $J_L$.
If a coverslide and adhesive are used which have negligible degradation, Figure 7 may be used to determine the minimum coverslide thickness necessary to insure that there will not be more than a designated reduction in output current after a fixed fluence. For instance, if the requirements of the experiment are such that not more than a 10\% reduction in current can be tolerated after a fluence of $10^{16}$ electrons/cm$^2$ of 0.8 MeV electrons the minimum coverslide thickness is 0.203 gm/cm$^2$. If the coverslide material is fused silica, with a density of 2.20 gm/cm$^2$, the thickness will be $\frac{0.203}{2.20} = 0.0923$ cm, or approximately 36 mils. Sets of curves for electrons of other energies can readily be constructed.

B Proton Irradiation

In order to get a more accurate prediction of proton damage than can be obtained by using an "effective" damage coefficient, the model allows the use of the best estimate for the damage coefficient at each incremental step within the cell. This is particularly important for low energy protons, whose range $R$ is less than the thickness of the cell, since in such a case the energy and the flux are changing rapidly with distance.

The expanded program also allows inclusion of the current contribution from the surface region. This region is much less sensitive to radiation damage than the base region, since it is thinner, the energy of the protons is higher in that region (making $K$ smaller), and the diffusion length is much lower before irradiation. The initial contribution from the n-side is 11\% of the total current; however, after irradiation with $9_p = 5 \times 10^{14}$/cm$^2$ of 270 keV protons, current from the surface region represents 56\%. The effect of including the surface is to decrease the rate at which $J_L$ falls off with $9_p$. This addition though, is still not nearly sufficient to account for the difference shown in Figure 11 of the First Quarterly Report between the model and the experimental values of Statler and Curtin.$(8)$ Additional work which may help reconcile the model with the experimental values is underway.
Figure 7. Effect of coverslide on current degradation
IV. CONCLUSIONS

A. General Summary

The work performed during this quarter was primarily concerned with perfecting the number of routines available for the program. In particular, the program was adjusted so that the parameters associated with the n surface layer could be included. In addition, it was made capable of handling variations in temperature, resistivity, light intensity, junction depth, cell thickness, coverslide thickness, and radiation. The radiation variables which can be used are: particle type (electron or proton), energy and fluence.

Values were taken for most of the above variables in order to check out the program. Many of the results are given in the body of the report. In addition, smaller routines were written which would generate I-V curves (under varying conditions of $I_{sc}$, $V_{oc}$ and $T$) and calculate the maximum output power for those curves.

The results of the computations are in keeping with the experimental evidence. Summarizing the results given in the report, we find:

1. The carrier concentration distribution can be readily plotted for the entire cell.
2. $V_{oc}$ decreases with increasing temperature, primarily due to the increase in $I_0$.
3. $P_{max}$ decreases with increasing temperature.
4. $I_{sc}$, $V_{oc}$, and $P_{max}$ decrease with increasing junction depth.
5. $J_{sc}$, $V_{oc}$, and $P_{max}$ increase very slowly with increasing cell thickness (for thicknesses greater than the diffusion length of minority carriers in the base region).
6. The surface region is essentially unaffected by electron irradiation.
7. The degradation of solar cells due to electron irradiation can be predicted as a function of electron energy, fluence, and coverslide thickness.

8. The degradation of solar cells due to proton irradiation can be predicted as a function of proton energy and fluence.

B. Future Work

Future work will include further analysis of solar cell performance in light of current theory and observation, expansion of the program to include the field due to the non-uniform impurity distributions in the surface region, preparation and simplification of the completed computer program, debugging and checkout, and evaluation of the program.

The analysis of solar cell performance at present is concentrated on the problem of non-uniform damage. There is still disagreement between the experimental data and the model as far as the variation of $I_L$ with low energy proton fluence is concerned. Investigations into the physics of the cell and refinements in the model are continuing, so that these differences may be reconciled. Effects such as the impurity gradient field, junction depth, minority carrier life times under high injection conditions, etc. will be examined.

The completed computer program shall be capable of predicting the power degradation of solar cells in a combined environment of low and high energy protons and electrons and provide the necessary framework to optimize the solar cell coverglass combination for a particular radiation environment with respect to cell base resistivity, cell thickness, coverslide thickness, and cell substrate type and thickness. The program is to be presented in operable form, listed in Fortran IV, suitable for input to an IBM 360/65.
Standard modular debugging will be employed for program check-out. It will be tested and evaluated against published experimental results and continually updated throughout the course of this contract as additional experimental data are analyzed.

C. **New Technology**

After a diligent review of the work performed under this contract, it was determined that no new innovation, discovery, improvement or invention was developed.
APPENDIX A
Definitions of Symbols

\[ D'_n = \text{diffusion coefficient of electrons} \]
\[ D_P = \text{diffusion coefficient of holes} \]
\[ E = \text{Electric field} \]
\[ E_g = \text{energy gap} = 1.12 \text{ eV in Si} \]
\[ I_L = \text{light generated current} \]
\[ I_o = \text{diode saturation current} \]
\[ I_{sc} = \text{short circuit current} \]
\[ I_{sco} = \text{short circuit current in undamaged solar cell} \]
\[ J = \text{current density} \]
\[ J_L = \text{light generated current density} \]
\[ J_n = \text{current density generated in surface (n) region} \]
\[ J_P = \text{current density generated in base (p) region} \]
\[ J_{sc} = \text{short circuit current density} \]
\[ k = \text{Boltzmann's constant} = 8.617 \text{ eV/}^{0}\text{K} \]
\[ K = \text{damage coefficient} \]
\[ L_{no} = \text{diffusion length of electrons in p region before radiation damage} \]
\[ L_P = \text{diffusion length of holes in n region} \]
\[ L_{po} = \text{diffusion length of holes in n region before radiation damage} \]
\[ n = \text{carrier concentration} \]
\[ P_{\text{max}} = \text{maximum output power} \]
\[ q = \text{electronic charge} = 1.602 \times 10^{-19} \text{ coulombs} \]
\[ T = \text{absolute temperature} \]
\[ V_{oc} = \text{open circuit voltage} \]
\[ \phi_e = \text{fluence of electrons} \]
\[ \phi_P = \text{fluence of protons} \]
\[ \tau_p = \text{lifetime of holes in n material} \]
REFERENCES


