TECHNIQUES FOR SELECTION AND ANALYSIS OF PARACHUTE DEPLOYMENT SYSTEMS

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16. Abstract

Parachute deployment systems based on forced ejection, extraction using a drogue parachute, and extraction using a rocket motor were studied. Equations which approximate the linear dynamics of deployment prior to inflation were derived. An expression for approximating the suspension-line tension was developed. A theoretical model describing the resistance of the packed parachute to unfurling was formulated. In a general manner, the problems associated with each of the basic types of deployment techniques were identified and compared.
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SUMMARY

A study has been conducted to develop techniques for selection and analysis of parachute deployment systems. Particular emphasis was directed to parachute deployment from vehicles having low ballistic coefficients. Deployment systems based on forced ejection, extraction using a drogue parachute, and extraction using a rocket motor were considered. General equations which approximate the linear dynamics of deployment prior to inflation were derived. An expression for approximating the suspension-line tension was developed. A theoretical model describing the resistance of the packed parachute to unfurling was developed, and equations were derived by which parameters appearing in this model can be determined from a ground test. An expression for the reaction load generated by a forced ejection device was derived. A method for estimating the ejection velocity in a flight environment, based on the ejection velocity determined in a ground test, was formulated. Methods of approximating weights of the alternate deployment systems were developed. In a general manner, the problems associated with each of the basic types of deployment techniques were identified and compared.

INTRODUCTION

The successful operation of any parachute depends on satisfactory performance of the deployment system. Deployment of a parachute is defined herein as the process of extending the parachute canopy and suspension lines to a position at which satisfactory inflation can occur. Usually, the design of the deployment system is based on experience and simple approximations. This method of determining the deployment system requirements is quite satisfactory for most applications. However, recent interest in aerodynamic decelerator systems for planetary entry vehicles has pointed out the need for an optimum deployment technique and a more detailed analysis of the deployment process.

The dynamics of parachute deployment has received only limited study. The classic empirical treatment given in the "Parachute Handbooks" (refs. 1 and 2) is usually sufficient for design purposes but is also evidence of the lack of rigorous analytical description of the deployment process. Reference 3, similar in some respects to the present paper, is probably the first successful attempt at a preliminary description of the deployment process.
The present paper provides an extension of the analysis given in reference 3. The equation for the motion of the parachute bag relative to a vehicle is derived. A mathematical technique for determining the resistance of the parachute to unfurling is outlined. These expressions can be used to calculate a time history of the deployment motion in order to evaluate the overall success of the deployment and to determine the vehicle trajectory with greater accuracy. An expression is derived for calculating the tensile loading in the unfurling parachute. The relative merits of deployment systems based on either forced ejection, extraction by a drogue parachute, or extraction by a rocket are discussed on the basis of the present analysis and weight approximations.

SYMBOLS

A  reference area, meter$^2$
C  effective porosity, grams/newton-second
CD  drag coefficient, $\frac{F_D}{q_\infty A}$
E  energy, joules
F  force, newtons
g  acceleration due to gravity, meters/second$^2$
Is  specific impulse of propellant, seconds
IT  total impulse of extractor rocket, newton-seconds
K  parameter characterizing pressure component of unfurling resistance, seconds/meter
L  stroke of mortar, meters
m  mass, kilograms
m'  linear mass density of parachute, kilograms/meter
dm  mass of differential element of unfurling parachute, kilograms
Δm  mass of a portion of unfurled parachute, kilograms (see sketch 1)
\( p \) pressure, newtons/meter\(^2\)

\( q \) dynamic pressure, newtons/meter\(^2\)

\( R \) gas constant, newton-meters/kilogram-degree Kelvin

\( T \) suspension-line tension, newtons

\( t \) time, seconds

\( V \) volume, meter\(^3\)

\( v \) velocity, meters/second

\( \Delta v \) ejection velocity, meters/second

\( W \) weight, newtons

\( x \) deployed distance, meters (see fig. 1)

\( \gamma \) vehicle flight-path angle, degrees (see fig. 2)

\( \zeta \) propellant mass fraction

\( \eta \) ratio of drag coefficient of parachute bag in vehicle wake to that in free stream

\( \theta \) temperature of gas, degrees Kelvin

\( \xi \) ratio of drag coefficient of drogue parachute or extractor rocket in vehicle wake to that in free stream

\( \rho \) atmospheric density, kilograms/meter\(^3\)

Subscripts:

\( B \) mouth of parachute bag

\( b \) parachute deployment bag
D  drag

d  drogue parachute

e  combination of parachute deployment bag, its contents, and drogue parachute or extractor rocket

F  flight environment

f  furled portion of parachute remaining in bag

fr  friction

G  ground test

i  behind unfurling layer of parachute

M  mortar

p  main parachute

R  reaction on vehicle

r  rocket

re  unfurling resistance

T  rocket thrust

t  mortar tube

u  unfurled portion of parachute

V  suspension-line attachment point at vehicle

v  vehicle

o  conditions just prior to parachute ejection

4
free-stream conditions

Dots over symbols denote time derivatives.

DESCRIPTION OF VEHICLE-PARACHUTE SYSTEM

The geometry of an arbitrary vehicle, alternate parachute deployment systems, and a partially deployed parachute is shown in figure 1. Prior to deployment, the parachute is tightly packed in the parachute deployment bag which is stowed in the vehicle. The purpose of the deployment bag is to reduce snatch loads and to prevent premature inflation. At deployment, the bag containing the parachute either is forcefully ejected from the vehicle by a mortar, a bellows, a catapult, or another similar device, or is extracted from the vehicle by a drogue parachute or an extractor rocket. As the bag moves away from the vehicle, the parachute is unfurled from the parachute bag by the suspension lines which are attached to the vehicle. The deployment process is considered to be complete, for the purpose of this analysis, when the parachute is entirely unfurled. This motion can be approximated by using a linear analysis if both the angular rates of the vehicle and the total time required to complete the deployment process are small.

A diagram showing the forces which act on the vehicle, the partially unfurled parachute, and the partially filled deployment bag is given in figure 2. These forces affect the vehicle deceleration rate, the tensile forces in the parachute, and the deployment rate.

ANALYSIS

In the analysis to follow, the equations of motion for the parachute deployment bag relative to the vehicle are presented. Also, an equation is derived which expresses the tension in the partially unfurled parachute in terms of the physical characteristics of the system and of the deployment rate which can be determined by a numerical solution of the fundamental equations of motion. In addition to these equations, a technique is given by which the resistance of the packed parachute to unfurling can be approximated.

Equations of Motion

The derivation of the fundamental equations of motion can be simplified considerably by making the following assumptions:

(1) The motion is linear.
(2) The deployment system is inelastic.
(3) The partially unfurled parachute is in tension during the deployment process.
(4) The aerodynamic drag of the partially unfurled parachute is neglected.

(5) The deployment rate is much less than the velocity of the vehicle.

Under these assumptions, the equations of motion for the parachute deployment bag relative to the vehicle are derived in appendix A and can be expressed as

\[
\ddot{x} = \frac{F_T + \left[\xi(CDA)_d \text{ or } \eta(CDA)_b\right]q_{\infty}}{m_e} - \left[\frac{(CDA)_v q_{\infty}}{m_v + m_u} + \frac{m'}{m_v + m_u} \dot{x}^2 + \left(\frac{1}{m_v + m_u} + \frac{1}{m_e}\right)F_{re}\right]
\]  

(1)

and

\[
\dot{v}_v = -\left[\frac{(CDA)_v q_{\infty}}{m_v + m_u} + \frac{m'}{m_v + m_u} \dot{x}^2 + \left(\frac{1}{m_v + m_u}\right)F_{re} + g \sin \gamma\right]
\]  

(2)

where

\[\ddot{x}\] deployment acceleration

\[F_T\] thrust of extractor rocket

\[\eta, \xi\] wake parameters

\[CDA\] drag area

\[q_{\infty}\] free-stream dynamic pressure

\[m\] mass

\[m'\] linear mass density of parachute

\[\dot{x}\] deployment rate

\[F_{re}\] resistance of packed parachute to unfurling

\[g \sin \gamma\] component of acceleration of gravity along vehicle axis

and subscripts

\[b\] parachute bag
d

droge parachute

r

extractor rocket

v

vehicle

u

unfurled portion of parachute

e

combination of parachute bag, its contents, and a drogue parachute or an extractor rocket

Suspension-Line Tension

An expression for the tension in the parachute suspension lines can be obtained by using equation (2) and the equation of motion for the vehicle. Considering the forces shown in figure 2, the equation of motion for the vehicle is

\[
\dot{v}_V = - \left[ \frac{T_V + (C_A)_{v \infty}}{m_v} + g \sin \gamma \right]
\]  

where \( T_V \) is the tension in the suspension lines at the vehicle attachment point. Combining equations (2) and (3) gives

\[
T_V = m'x^2 + F_{re} + m_u(\dot{v}_V + g \sin \gamma)
\]  

The tension \( T \) in the partially unfurled parachute at any arbitrary position (see sketch 1) can be determined by considering the equation of motion of the portion of the unfurled parachute between the point of interest and the vehicle attachment point. Since

Sketch 1.
the acceleration of this portion of the unfurled parachute is equal to the acceleration of
the vehicle, the equation of motion is
\[
\dot{v}_V = \frac{T_V - T}{\Delta m} - g \sin \gamma
\]  
(5)
where \( T \) is the suspension-line tension at an arbitrary position along the unfurled parachute and \( \Delta m \) is the mass of the portion of the unfurled parachute between the point of
interest and the vehicle attachment point. Substituting equation (4) into equation (5) yields
\[
T = m'x^2 + F_{re} + (m_u - \Delta m)(\dot{v}_V + g \sin \gamma)
\]  
(6)
which expresses the variation of the tension throughout the unfurled portion of the
parachute.

Unfurling Resistance

In order to define completely the motion, the resistance of the parachute to unfurling
must be expressed in terms of empirically determined constants and characteristics of
the motion. The force of resistance to unfurling \( F_{re} \) is composed of frictional and
pressure forces and can be expressed by
\[
F_{re} = F_{fr} + (p_{\infty} - p_l)A_b
\]  
(7)
where
\( F_{fr} \) friction between unfurling parachute and internal walls of parachute bag
(fig. 3)
\( p_{\infty} \) free-stream static pressure
\( p_l \) pressure in low pressure region generated behind unfurling layer of material
\( A_b \) cross-sectional area of parachute deployment bag

In an actual deployment process, both the frictional and the pressure forces oscillate about some average value due to the mechanism of unfolding. The overall deployment motion can be approximated by using values of the frictional and pressure forces averaged over one unfolding cycle.

A simple approximation to the internal pressure averaged over one cycle is derived
in appendix B. By using this approximation, the resistance can be expressed as
\[
F_{re} = F_{fr} + p_{\infty}A_b \frac{Kx}{1 + Kx}
\]  
(8)
where $K$ depends on the packing configuration and other construction details and must, consequently, be determined experimentally.

A technique for determining the friction and pressure parameters $F_{fr}$ and $K$ from a ground test of the deployment system is discussed in appendix C. In order to obtain a unique solution for these parameters, it is necessary to assume that they are constant over an arbitrary interval of the deployment sequence. In appendix C, two convenient intervals are chosen. The first interval is the unfurling of the suspension lines and the second interval is the unfurling of the parachute canopy.

**SELECTION OF PARACHUTE DEPLOYMENT SYSTEMS**

Selection of a parachute deployment system can be based on varied criteria. For most applications, reliability is the primary consideration. However, for specialized applications, such as entry vehicles, design limitations can be quite restrictive. For example, the structural loads generated by the deployment system, the deployment time, and the system weight may be subject to specified limitations. Therefore, selection and analysis of the parachute deployment system are related and cannot be separated. In this section, representative types of deployment systems are discussed with particular emphasis on the problems associated with parachute deployment from an entry vehicle having a low ballistic coefficient.

**Forced Ejection Systems**

Forced ejection is a frequently used parachute deployment technique. The mortar, catapult, and pressure bellows are examples of mechanisms designed to produce a forced ejection of the packed parachute. The reliability inherent in the simplicity of this technique is quite attractive. However, deployment systems based on this method have excessive weight and high reaction loads. The problem of large reaction loads generated by forced ejection parachute deployment is particularly severe on vehicles having low ballistic coefficients, since the required ejection velocity is relatively high.

The basic design specification for a forced-ejection-type deployment system is, therefore, the required ejection velocity, that is, the minimum velocity for which the suspension lines remain in tension throughout the deployment process. Extremely high opening loads can result when the suspension lines become slack prior to inflation. The variation in suspension-line tension can be determined from equation (4) and the results of a simultaneous numerical integration of equations (1) and (2). A complete description of the vehicle, the vehicle wake, the parachute, and the unfurling resistance is required. The integration is performed for a range of ejection velocities until the minimum velocity for which the suspension-line tension, as computed from equation (4), is non-negative throughout the deployment sequence.
The weights of forced-ejection-type deployment systems can vary significantly from one design to another. Any general expression relating the mortar weight to the parachute weight and the required ejection velocity would be subject to considerable question. During the Planetary Entry Parachute Program (ref. 4), considerable experience was gained in the design of mortars for ejecting parachutes weighing 175 to 325 newtons at velocities between 35 and 40 meters per second. Within this range, the mortar weight is essentially independent of the design ejection velocity, primarily because the mortar-tube wall thickness is determined by minimum gage limitations rather than by the hoop stress resulting from internal pressure. However, the mortar weight was found to depend significantly on the mortar-tube volume. This volume is directly related to the parachute weight for a given packing density. The weights of mortars designed during the Planetary Entry Parachute Program are as follows:

<table>
<thead>
<tr>
<th>Parachute weight, newtons</th>
<th>Ejection velocity, meters/second</th>
<th>Mortar weight, newtons</th>
</tr>
</thead>
<tbody>
<tr>
<td>178</td>
<td>38</td>
<td>81</td>
</tr>
<tr>
<td>289</td>
<td>34</td>
<td>111</td>
</tr>
<tr>
<td>334</td>
<td>40</td>
<td>105</td>
</tr>
</tbody>
</table>

Relatively high reaction loads are generated by forced ejection devices since a large amount of energy must be transferred to the parachute pack over a small displacement. The total vehicle (including the structure, subsystems, and instrumentation) must be designed to withstand this high loading. Therefore, the peak reaction load should be reduced as much as possible. It is easily shown that for a specified ejection velocity and a given mortar stroke, constant pressure in the mortar tube results in the minimum peak reaction load. For this reason, constant internal pressure is an objective of mortar system design. The erodible-orifice-type nozzle is an example of a technique by which approximately constant internal pressures have been obtained. The theoretical minimum peak reaction load is given by

$$ F_R = \frac{1}{L} \left[ \frac{1}{2} (m_P) G (\Delta v)^2 \right] $$

(9)

where

- $F_R$ reaction load
- $L$ mortar stroke
- $m_P$ parachute mass
\( \Delta v \) ejection velocity

subscript \( G \) vertical ground test

If the internal force is not a function of the surrounding environment, the reaction load in flight is the same as in ground tests. Equation (9) is plotted in figure 4 for a range of mortar strokes, parachute weights, and ejection velocities.

The ejection velocity obtained in flight is not, in general, equal to the ejection velocity measured in ground tests. An expression relating these two velocity increments is derived in appendix D and is as follows:

\[
(\Delta v)_F = \left( \left( \frac{W_p}{W_v} \right)^G \left[ 1 + \frac{(W_p)_F}{W_v} \right] \left( \Delta v \right)_G^2 + \frac{2[(p_\infty)_G - (p_\infty)_F]A_M L}{(m_p)_G} \right)^{1/2}
\]  

(10)

where the subscript \( F \) refers to flight conditions, \( W \) is weight, and \( A_M \) is the cross-sectional area of the mortar. The derivation of this expression assumes that the mortar-tube pressure displacement history in flight is approximately the same as that in ground tests. Equation (10) indicates that the flight ejection velocity is, in general, larger than the ejection velocity determined in ground tests.

The overall efficiency of the mortar system can be improved by reducing the unfurling resistance. The friction component of this force can be minimized by lining the deployment bag with a material having a low coefficient of friction. This alteration also reduces friction heating which can be a significant problem when high ejection velocities are required. The pressure force can essentially be eliminated by appropriate vent holes in the bag. Both of these modifications reduce the unfurling resistance and thereby the required ejection velocity is minimized.

Another less severe problem associated with a mortar deployment is possible penetration of the parachute canopy by the parachute deployment bag after inflation. This problem can be alleviated by attaching the bag to the parachute apex. However, for some configurations, the attached bag can severely hamper the inflation process. An alternate solution would be to attach a small drogue parachute to the bag. This drogue parachute must have sufficient drag area to insure separation from the vehicle in the wake of the inflated main parachute.

**Drogue Parachute System**

Parachute deployment using a drogue or pilot parachute has numerous advantages. The system is quite flexible since the parachute extraction force is applied continuously over the entire deployment sequence. Compared with a forced ejection system, the
The drogue system is usually lighter in weight and produces smaller reaction loads. These reductions are due primarily to the reduced energy requirements of the drogue parachute mortar. However, the drogue parachute must produce drag while in the vehicle wake. For a vehicle having a low ballistic coefficient, the wake can substantially reduce the drag produced by the drogue. In addition, a drogue system is more complex than a forced ejection system because the drogue parachute itself must be deployed.

In order to analyze and select an appropriate drogue parachute system, equations (1) and (2) must simultaneously be numerically integrated for a variety of drogue-parachute sizes and locations until all constraints are satisfied. In general, the limiting constraint for a drogue system is the maximum allowed deployment time. The absolute minimum drogue-parachute size is determined by the initial location of the drogue, the ballistic coefficient of the vehicle, the characteristics of the vehicle wake, and the weight of the main parachute. If maximum deployment time is unconstrained, an extremely lightweight system can be designed which requires a long extraction time but maintains tension in the suspension lines throughout the deployment process. In order to complete the analysis, the deployment of the drogue parachute must also be studied by using techniques discussed in the previous section.

The weight of the drogue parachute deployment system includes the weight of the drogue parachute and the weight of the forced ejection system for the drogue. The approximate weight of the drogue parachute can be estimated from the following empirical relationship (ref. 5):

\[ W_d = 0.12(C_d A)_d + (0.28 \times 10^{-3})q_{\infty}(C_d A)_d^{3/2} \]  

(11)

where \( W_d \) is the weight of the drogue parachute in newtons. The first term accounts for the canopy weight and the second term accounts for the suspension-line weight. Typical weights of the drogue parachute based on equation (11) are given in figure 5.

The design of a parachute deployment bag for any extractor system differs from that for a forced ejection system. The extraction force must be transferred to the portion of the parachute remaining in the bag. Therefore, the bag must have high frictional resistance or some type of stowage loops to transfer the extraction force. For this reason, a certain amount of frictional energy dissipation and heating cannot be avoided.

The drogue parachute system must also be sized to separate from the vehicle after the deployment sequence. The drogue parachute must perform this separation function in the wake of the inflated main parachute. Since this wake can cause large reductions in the drag produced by the drogue parachute, separation considerations may limit the minimum size of the drogue.
Rocket Extraction System

A rocket extraction system for parachute deployment has all the advantages of a drogue parachute system. Due to continuous application of the extraction force, the system is flexible. Theoretically, it is lightweight. In addition to these advantages, the rocket extraction system produces very light, if any, reaction loads, is a single-stage system, and is only slightly dependent on the characteristics of the vehicle wake.

The weight of an extractor rocket is given by

\[ W_r = \frac{I_T}{\xi I_s} \]  

(12)

where

\( W_r \) initial weight of extractor rocket
\( I_T \) total impulse of rocket
\( I_s \) specific impulse of propellant
\( \xi \) propellant mass fraction

Equation (12) is plotted in figure 6 for representative extractor rockets.

Discussion

A general rule for selection of a parachute deployment system cannot be formulated, since design criteria vary considerably for various applications. Therefore, selection must be based on a detailed analysis of each available option under the specific constraints and objectives associated with the particular application. The techniques presented in this report provide methods for performing this analysis.

Several basic generalities can be stated. For vehicles having high ballistic coefficients, required ejection velocities are relatively low. Consequently, an appropriate forced ejection system would generate relatively light reaction loads. Since, of the alternatives, the forced ejection system is the simplest, it would probably be more desirable in this situation.

For vehicles having low ballistic coefficients, required ejection velocities can be relatively high. Therefore, if weight is critical and if large reaction loads cannot be tolerated, some type of extractor system would be preferable. In general, the performances of the drogue parachute system and the rocket extractor system are comparable.
However, the rocket extractor system produces a smaller reaction load and is the lighter weight system for short deployment times. If long deployment times can be tolerated, the drogue parachute system is, in theory, the lighter weight system.

The previously discussed techniques were used to analyze the dynamics of parachute deployment on the second flight test of the Planetary Entry Parachute Program (refs. 4 and 6). Equations (1), (2), and (4) were solved numerically by using a fourth-order Runge-Kutta integration routine (Langley computer program E1547) on a digital computer for appropriate values of the parameters and coefficients. Figure 7 shows time histories of deployed distance, deployment velocity, and suspension-line tension obtained from the solution. Flight data obtained from references 4 and 6 are also shown in the figure. As can be seen, the numerical solution closely approximates the flight data.

CONCLUDING REMARKS

An analysis has been made of the dynamics of parachute deployment prior to inflation. The deployment system was assumed to be inelastic and only linear motion was considered. The equations of motion were derived for analysis of parachute deployment by either forced ejection, extraction by a drogue parachute, or extraction by a rocket. The characteristics of each of these systems were studied. A numerical solution to the equations of motion was found to closely approximate the deployment dynamics of forced ejection parachute deployment behind a vehicle having a low ballistic coefficient in a low-dynamic-pressure environment.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., December 1, 1969.
APPENDIX A

DERIVATION OF THE EQUATIONS OF MOTION

The equations for the motion of the parachute deployment bag relative to the vehicle are derived in this appendix. In the derivation, the following basic assumptions are made:

(1) The motion is linear.
(2) The parachute is inelastic.

The fundamental mass particles of the assumed system and the forces internal to this system of particles are identified in the following sketch:

Sketch 2.

The equation of motion for the combined vehicle and the partially unfurled parachute is

\[
\dot{v}_v = -\left[\frac{(F_D)_V + (F_D)_u + T_B}{m_v + m_u} + g \sin \gamma\right]
\]  \hspace{1cm} (A1)

where

\[
\begin{align*}
\dot{v} & \quad \text{inertial acceleration} \\
F_D & \quad \text{aerodynamic drag force}
\end{align*}
\]
APPENDIX A

\( T_B \) suspension-line tension at parachute bag mouth

\( m \) mass

\( g \) acceleration due to gravity

\( \gamma \) vehicle flight-path angle

and subscripts

\( v \) vehicle

\( u \) partially unfurled parachute

The discrete change in the momentum of an identified mass particle of the unfurling parachute can be represented by

\[
(T_B - F_{re})dt = dm(v_v - v_b)
\]  \hspace{1cm} (A2)

where \( F_{re} \) is the resistance of the parachute to unfurling and \( dm \) is the mass of the identified mass particle. This equation can be rewritten

\[
T_B - F_{re} = m'\dot{x}^2
\]  \hspace{1cm} (A3)

where \( m' \) is the linear mass density of the parachute and \( \dot{x} \) is the deployment rate.

The equation of motion for the parachute bag, its instantaneous contents, and the drogue parachute (or extractor rocket) is

\[
\ddot{x} - \dot{v}_v = \frac{[F_T + (F_D)_{d (or r)} + (F_D)_{b} - F_{re}]}{m_e} + g \sin \gamma
\]  \hspace{1cm} (A4)

where

\[
m_e = m_f + m_b + m_d (or r)
\]  \hspace{1cm} (A5)

The subscript \( f \) refers to the furled parachute in the bag, the subscript \( d (or r) \) refers to the drogue parachute (or extractor rocket), and \( F_T \) is the thrust of the extractor rocket.
APPENDIX A

The relative acceleration is obtained by using equations (A1), (A3), and (A4) and can be expressed as

\[
\ddot{x} = \frac{F_T + (F_D)_d (\text{or } r) + (F_D)_b}{m_e} - \left[ \frac{(F_D)_v + (F_D)_u}{m_v + m_u} \dot{x}^2 + \left( \frac{1}{m_v + m_u} + \frac{1}{m_e} \right) F_{re} \right]
\]

(A6)

The aerodynamic drag of the partially unfurled parachute is much less than the aerodynamic drag of the vehicle and can be neglected. The aerodynamic drag of the vehicle is given by

\[
(F_D)_v = (C_D A_v) q_\infty
\]

(A7)

where \( C_D \) is the drag coefficient, \( A \) is the reference area, and \( q_\infty \) is the free-stream dynamic pressure given by

\[
q_\infty = \frac{1}{2} \rho_\infty v_v^2
\]

(A8)

with \( \rho_\infty \) being the free-stream density.

The drag of the parachute deployment bag in the wake of the vehicle can be expressed as

\[
(F_D)_b = \eta (C_D A)_b q_\infty
\]

(A9)

where \( \eta \), the experimentally determined ratio of the drag coefficient of the bag in the vehicle wake to the drag coefficient of the bag in the free stream, is a function of the body geometry, Mach number, Reynolds number, and separation distance. The deployment velocity reduces the effective dynamic pressure a small amount; however, this effect is usually negligible.

The aerodynamic drag of a drogue parachute (or an extractor rocket) is given by

\[
(F_D)_d (\text{or } r) = \xi (C_D A)_d (\text{or } r) q_\infty
\]

(A10)

where \( \xi \) is the ratio of the drag coefficient of the drogue parachute (or extractor rocket) in the vehicle wake to the corresponding drag coefficient in the free stream.

Substituting equations (A7), (A9), and (A10) into equation (A6) gives the following basic equation for the motion of the parachute bag relative to the vehicle:
\[ \ddot{x} = \frac{F_T}{m_e} + \left[ \xi(CDA)d \text{ or } r \right] + \eta(CDA)_b q_\infty - \left[ \frac{(CDA)_v q_\infty}{m_v + m_u} + \frac{m'}{m_v + m_u} \dot{x}^2 + \left( \frac{1}{m_v + m_u} + \frac{1}{m_e} \right) \right] r_{re} \] 

where

\[ \dot{v}_v = -\left[ \frac{(CDA)_v q_\infty}{m_v + m_u} + \frac{m'}{m_v + m_u} \dot{x}^2 + \left( \frac{1}{m_v + m_u} \right) r_{re} + g \sin \gamma \right] \]
APPENDIX B

AN APPROXIMATION TO THE DEPLOYMENT BAG INTERNAL PRESSURE

The purpose of the following analysis is to express the static pressure behind an unfurling layer of the packed parachute as a function of the deployment rate, the ambient static pressure, and experimentally determined constants. In order to approximate the internal pressure, the unfurling parachute is assumed to be a porous piston being extracted from a rigid cylinder. The equation of state for the contained gas can be expressed as

\[ p_i = \frac{m_i}{V_i} R \theta_i \]  \hspace{1cm} (B1)

where \( p_i, m_i, R, \) and \( \theta_i \) are, respectively, the pressure, mass, gas constant, and temperature of the gas behind the unfurling layer of material.

If the expansion process is assumed to be isothermal, the time rate of change of the internal pressure is given by

\[ \dot{p}_i = \frac{1}{V_i} \left[ R \theta_i \dot{m}_i - A_b p_i \dot{x} \right] \]  \hspace{1cm} (B2)

where \( A_b \) is the cross-sectional area of the bag and \( \dot{x} \) is the deployment rate.

The effective mass diffusion resulting from the combined mass diffusion and bulk mass transport is assumed to be governed by an effective porosity. This relationship can be expressed as

\[ \dot{m}_i = CA_b (p_\infty - p_i) \]  \hspace{1cm} (B3)

where \( C \) is the effective porosity and \( p_\infty \) is the ambient static pressure.

Using equations (B1) to (B3) gives the internal pressure

\[ p_i = \frac{p_\infty - K \frac{V_i}{A_b} p_i}{1 + K \dot{x}} \]  \hspace{1cm} (B4)

where \( K \) is assumed constant over a complete cycle of the unfolding process.
APPENDIX B

The second term in the numerator in equation (B4) is negligible with respect to the free-stream static pressure when the pressure component of the unfurling resistance is significant. With this simplification, the expression for the internal pressure reduces to

\[
p_i = \frac{p_\infty}{1 + Kx}
\]  

(B5)
APPENDIX C

A METHOD FOR DETERMINING UNFURLING RESISTANCE PARAMETERS FROM A GROUND TEST

In this appendix, a technique is developed for determining the parachute deployment bag friction $F_{fr}$ and the pressure force parameter $K$ from a vertical ground test of the deployment system.

For a vertical ground test in which the vehicle acceleration $\ddot{v}_V$ is zero, equations (1) and (2) expressing the motion of the bag reduce to

$$\ddot{x} = \frac{F_T}{m_e} - \left\{ g + \frac{F_{re} + \frac{1}{2}\rho_{\infty} \dot{x}^2 \left[ (C_{DA})_b + (C_{DA})_r \right]}{m_e} \right\}$$

which can be solved for the unfurling resistance and written as

$$F_{re} = F_T - m_e(\ddot{x} + g) - \frac{1}{2}\rho_{\infty}\dot{x}^2 \left[ (C_{DA})_b + (C_{DA})_r \right]$$

(C2)

With equation (C2), the unfurling resistance can be calculated from the motion determined in a ground test. However, as given in equation (8), the unfurling resistance can be expressed as

$$F_{re} = F_{fr} + p_{\infty} A_b \frac{K\dot{x}}{1 + K\dot{x}}$$

(C3)

In order to uniquely determine the bag friction $F_{fr}$ and the pressure force parameter $K$, these parameters are assumed to be constant over some interval of the deployment process. Dividing the complete deployment process into the two fundamental intervals of suspension-line unfurling and canopy unfurling is recommended. By using equation (C2), the unfurling resistance is calculated at two displacements $x_1$ and $x_2$ in the selected interval. The two equations resulting from substitution of the calculated unfurling resistances and the corresponding deployment velocities into equation (C3) can be solved simultaneously for the resistance parameters. The result can be written as

$$K = -B - \left( p^2 - \frac{1}{x_1 x_2} \right)^{1/2}$$

(C4)
APPENDIX C

and

\[ F_{fr} = F_{re,1} - p_\infty A_b \frac{K \dot{x}_1}{1 + K \dot{x}_1} \]  \hspace{1cm} (C5)

where

\[ B = \frac{1}{2} \frac{(\dot{x}_1 + \dot{x}_2) - \frac{p_\infty A_b}{F_{re,1} - F_{re,2}}(\dot{x}_1 - \dot{x}_2)}{\dot{x}_1 \dot{x}_2} \]

The subscripts 1 and 2 denote correspondence to the displacements \(x_1\) and \(x_2\), respectively.
A METHOD FOR PREDICTING FLIGHT EJECTION VELOCITY FROM A GROUND TEST

The energy output of a mortar, with the dissipation due to wall friction neglected, can be expressed as

\[ E = \int_0^L (p_t - p_\infty)A_M \, dx \]  \hspace{1cm} (D1)

where

- **E** \quad energy transferred to ejected mass
- **p_t** \quad pressure in mortar tube
- **p_\infty** \quad ambient pressure
- **A_M** \quad cross-sectional area of mortar
- **L** \quad mortar stroke

If it is assumed that the displacement history of the mortar-tube pressure is not affected by small changes in the ejection velocity, the energy output of mortar in flight reduces to

\[ (E)_F = (E)_G + \left[ (p_\infty)_G - (p_\infty)_F \right]A_M L \]  \hspace{1cm} (D2)

where the subscripts **F** and **G** refer to conditions present in flight and during a ground test, respectively. A measure of the energy output of a mortar is the maximum kinetic energy of the exiting parachute pack in a ground test. This relationship can be expressed by

\[ (E)_G = \frac{1}{2}(m_p)_G(\Delta v)_G^2 \]  \hspace{1cm} (D3)

where **m_p** is the mass of the parachute pack, and **\Delta v** is the ejection velocity.

In flight, the energy is transferred to both the parachute pack and to the vehicle. The change in the total kinetic energy of this system can be written as

\[ (E)_F = \frac{1}{2}(m_p)_F(v_p)_F^2 - (v_F)_o^2 + \frac{1}{2} m_v(v_v)_o^2 - (v_v)_o^2 \]  \hspace{1cm} (D4)
where \( v \) is the absolute velocity, \( m \) is mass, and the subscripts \( p \), \( v \), and \( o \) refer, respectively, to the parachute pack, the vehicle, and conditions immediately before parachute ejection.

The momentum of the system of particles representing the vehicle and the parachute pack is approximately constant during parachute ejection since the ejection force is internal to the system and momentum losses due to aerodynamic and gravity forces during ejection of the parachute pack are negligible. Conservation of momentum can be expressed by

\[
\left[ m_v + (m_p)_F \right] (v_v)_o = m_v v_v + (m_p)_F v_p
\]

The approximate flight ejection velocity is found by solving equations (D2), (D3), and (D4) simultaneously. The result can be written

\[
(\Delta v)_F = \left[ \frac{(W_p)_G}{(W_p)_F} \right] \left[ 1 + \frac{(W_p)_F}{W_v} \right] \left\{ (\Delta v)_G^2 + \frac{2 [(p_\infty)_G - (p_\infty)_F] A_M L_0}{(m_p)_G} \right\}^{1/2}
\]
REFERENCES


Figure 1.- Sketch of deployment configuration.
Figure 2.- Forces affecting the dynamics of parachute deployment.
Figure 3.- Cutaway of the parachute bag showing the forces which resist parachute unfurling.
Figure 4.- Reaction load acting on the vehicle during forced ejection parachute deployment.
Figure 5.- Approximate weights of a drogue parachute.
Figure 6.- Effect of propellant mass fraction and specific impulse on the extractor rocket weight.
Figure 7.- Comparison of a numerical solution and the flight data for the deployment dynamics of the second flight test on the Planetary Entry Parachute program (refs. 4 and 6).
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—National Aeronautics and Space Act of 1958

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