A MODEL ATMOSPHERE FOR EARTH RESOURCES APPLICATIONS

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ABSTRACT

A computer subprogram set is described which permits the use of radiosonde data to provide model atmosphere data for earth resources applications.
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SUMMARY

All earth resources remote-sensing techniques are affected by the atmosphere lying between the target and the sensor. The computer program presented in this report offers a method of numerical use of radiosonde data so that atmospheric effects may be assessed and possibly removed from the signal.

INTRODUCTION

The objectives of the NASA Earth Resources Program are to determine the performance capabilities of various sensors, to discover signature criteria of resources, and to develop new sensors and systems that will eventually enable management of earth resources. To accomplish these objectives, certain absolutes which may be used to evaluate sensing systems and techniques must be established. The laboratory usually offers the best testing environment, but the type of target, the conditions of the path of the signal, and other testing parameters are limited. In general, the laboratory is so restrictive that a successful laboratory test of a remote sensor is necessary but not sufficient to ensure proper operation of the sensor in an application. Therefore, much of the testing is performed in the same environment in which the instrument is expected to operate. Testing under such conditions requires that the data concerning the environment between the instrument platform (e.g., an aircraft or a spacecraft) and the target be as accurate as possible. Thus, determination of the "ground truth" and description of the state of the atmosphere in the path of the electromagnetic signal are necessary.

Remote-sensing techniques are affected by the atmosphere lying between the target and the sensor. The amount of noise introduced into the signal by the interaction between the atmosphere and the signal depends upon the type of sensor, the wavelength employed, and the meteorological conditions prevailing at the time of the experiment. Since the NASA Earth Resources Program remote-sensing effort is in a developmental stage, the effects of this interaction are presently being determined, and hopefully, the model atmosphere for earth resources applications, presented in this paper, will facilitate analyses of such effects.

The computer subprogram set presented in this paper offers a self-consistent method for numerically calculating the state of the atmosphere based on radiosonde
data given in terms of significant levels of pressure, temperature, and temperature-dewpoint depression. After data from the radiosonde closest to an aircraft or spacecraft remote-sensing target have been obtained and after these data have been inserted into the computer subprogram set, a programmer has almost any desirable atmospheric parameter available for use in his computer programs. In particular, the subprogram set described in this paper makes available all the necessary quantities for calculation of infrared and microwave absorption or refraction, or both. However, no attempt has been made in this paper to include atmospheric absorption calculations in the model atmosphere; only the basic atmospheric data necessary for the previously mentioned calculations are provided.

The model atmosphere was written in the FORTRAN V computer language for the Univac 1108 computer. However, the program is also compatible with Control Data Corporation and IBM FORTRAN IV compilers. Copies of the computer cards are available upon request from David E. Pitts, TF8, Manned Spacecraft Center, Houston, Texas 77058.

**SYMBOLS**

- $C_{\text{sound}}$: computer symbol ANS(4), speed of sound, m/sec
- $d$: computer symbol DUM/D1, increment of the slant path from $r$ to $r'$, cm
- $e$: computer symbol ANS(19), $E(X)$, water-vapor pressure, mbar
- $e_s$: computer symbol ANS(20), $E(X)$, saturation water-vapor pressure, mbar
- $f_w$: computer symbol $F(P, X)$, correction for the departure of the air and water-vapor mixture, from ideal-gas law
- $g$: computer symbol ANS(5), acceleration caused by gravity, $f(Z)$, cm/sec$^2$
- $g_0$: surface gravity, $g$ at $R_e$, cm/sec$^2$
- $H$: computer symbol $H(I)$, geopotential altitude, m
- $H_a$: computer symbol $H_A, HLOW$, geopotential altitude at A, where $H_a < H_b$, m
- $H_b$: computer symbol $H_B$, geopotential altitude at B, m
- $H_p$: computer symbol ANS(15), pressure scale height, km
- $H_\rho$: computer symbol ANS(16), density scale height, km
\( \frac{M_i}{m} \) mass percentage of the \( i \)th constituent

\( m \) computer symbol ANS(7), molecular weight of the atmosphere, g/g-mole

\( m_b \) molecular weight at \( H_b \), g/g-mole

\( m_d \) computer symbol XMO, molecular weight of the dry atmosphere, g/g-mole

\( m_0 \) computer symbol XMO, molecular weight at the surface, g/g-mole

\( m_w \) molecular weight of water, g/g-mole

\( n' \) computer symbol XN2, refractive index at \( r' + (1/2)\Delta Z \)

\( n'' \) computer symbol XN1, refractive index at \( r'' + (1/2)\Delta Z \)

\( n_{STP} \) computer symbol ANS(17), refractive index of air at STP

\( n(Z) \) computer symbol ANS(18), refractive index of air as a function of \( \lambda, T, \) and \( P \)

\( P \) computer symbol ANS(1), atmospheric pressure, mbar

\( P_a \) computer symbol PLOW, atmospheric pressure at \( H_a \), mbar

\( P_b \) computer symbol PHIGH, atmospheric pressure at \( H_b \), mbar

\( q \) computer symbol ANS(13), specific humidity, g/kg

\( q_s \) computer symbol ANS(14), specific humidity at saturation, g/kg

\( R \) computer symbol RO, universal gas constant, \( 8.31432 \times 10^7 \) ergs/(mole °K)

\( R_e \) computer symbol RE, mean radius of the earth, 6371.299 km

\( \text{Rel} \) computer symbol ANS(12), relative humidity, percent

\( R_X \) computer symbol XS-XL, \( X \)-component of \( (\frac{r_i^{sp} - r_l^i}{r_i^{sp} - r_l^i}) \), km

\( R_Y \) computer symbol YS-YL, \( Y \)-component of \( (\frac{r_i^{sp} - r_l^i}{r_i^{sp} - r_l^i}) \), km

\( R_Z \) computer symbol HS-HL, \( Z \)-component of \( (\frac{r_i^{sp} - r_l^i}{r_i^{sp} - r_l^i}) \), km
\( r \)  computer symbol \( \text{ANS}(10) \), mixing ratio of the water in the atmosphere, g/kg

\( r' \)  computer symbol \( S_2 \), distance to shell \( Z + \Delta Z \) on the refracted path, km

\( r'' \)  computer symbol \( S_1 \), distance to shell \( Z \) on the refracted path, km

\( r'_1 \)  distance from the center of the earth to a target, km

\( r_s \)  computer symbol \( \text{ANS}(11) \), mixing ratio required for the saturation of water in the atmosphere, g/kg

\( r_{sp} \)  distance from the center of the earth to a spacecraft, km

\( S \)  computer symbol \( S \), Sutherland's constant, 110.4 °K

\( s \)  distance

\( T \)  computer symbol \( \text{ANS}(2) \), kinetic atmospheric temperature, °K

\( T^* \)  computer symbol \( \text{ANS}(6) \), virtual temperature, °K

\( T_a \)  computer symbol \( T(\ ) \), temperature at \( H_a \), °K

\( T_{a}^* \)  computer symbol \( \text{TVLOW} \), virtual temperature at \( H_a \), °K

\( T_b \)  computer symbol \( \text{ANS}(2) \), temperature at \( H_b \), °K

\( T_{b}^* \)  computer symbol \( \text{TVHIGH} \), virtual temperature at \( H_b \), °K

\( T_d \)  dewpoint temperature, °K

\( T_{d,a} \)  computer symbol \( \text{TD}(\ ) \), dewpoint temperature at \( H_a \), °K

\( T_{d,b} \)  computer symbol \( \text{ANS}(9) \), dewpoint temperature at \( H_b \), °K

\( T_m \)  molecular scale temperature, °K

\( \text{TO} S \)  computer symbol \( \text{TOS} \), angle between \( r'_1 \) and \( r_{sp}' \), rad

\( t \)  time

\( VV \)  identifier of the significant-level data set of radiosonde code

\( Z \)  computer symbol \( Z \), geometric altitude, km
$Z_1$ computer symbol ZL, altitude of a target above the earth, km

$Z_{sp}$ computer symbol ZS, altitude of a spacecraft above the earth, km

$\beta$ computer symbol BETA, $1.458 \times 10^{-6}$ kg/(sec °K m)

$\gamma$ ratio of specific heats

$\zeta$ computer symbol PHI, angle from the zenith down to the tangent to the path at the target, rad

$\zeta''$ computer symbol C(3), distance upward from a local station to a spacecraft, rad

$\eta''$ computer symbol C(2), distance eastward from a local station to a spacecraft, rad

$\theta_1$ computer symbol THETAL, target longitude, input card, deg (internally, rad)

$\theta_{sp}$ computer symbol THETAS, spacecraft longitude, input card, deg (internally, rad)

$\lambda$ computer symbol XLAMDA, wavelength, microns

$\mu$ computer symbol ANS(8), coefficient of viscosity, kg/(msec)

$\xi$ computer symbol SUM1, dummy variable, rad

$\xi''$ computer symbol C(1), distance southward from a local station to a spacecraft, rad

$\rho$ computer symbol ANS(3), atmospheric density, g/cm$^3$

$\rho_d$ density of dry air, g/cm$^3$

$\rho_w$ density of water vapor, g/cm$^3$

$\phi'$ computer symbol PHIPR, angle between $r'$ and the path of the ray after refraction, rad

$\phi''$ computer symbol PHI, angle between $r''$ and d, rad

$\phi_1$ computer symbol PHIL, target latitude, input card, deg (internally, rad)
MODEL ATMOSPHERES

Model atmospheres for earth resources applications may be described as one of three types: preflight, flight, and postflight. Preflight model atmospheres include those which have been developed from aerospace flight-support models (refs. 1 and 2) and statistical models of cloud cover over the earth (ref. 3). The last of these indicates the probability of success on spacecraft- or aircraft-borne photographic missions for earth resources applications.

Flight model atmospheres are calculated from sounding-type remote-sensing devices aboard spacecraft or aircraft. Flight model atmospheres are not presently well developed, but when they are well developed, they will represent the ultimate in knowledge of the "air truth" until special-purpose instruments that will perform atmospheric noise extraction in real time are developed.

Postflight model atmospheres are based upon standard meteorological soundings and are used to assist in the development of flight model atmospheres. These postflight model atmospheres may be described as predictive and nonpredictive.

Predictive postflight model atmospheres use equations of motion, thermodynamics, and continuity and standard meteorological soundings to predict (in time and space) the state of the atmosphere near the target for a remote sensor mounted on an instrument platform. This type of model atmosphere is not presently well developed. Nonpredictive postflight model atmospheres offer a self-consistent method of calculating a model atmosphere at the position of a radiosonde which may be located near the experiment platform. The subprogram model atmosphere set discussed in this paper has the capability of performing either as a nonpredictive postflight model atmosphere or as a preflight model atmosphere, depending on the form of the input data.

EQUATIONS FOR THE MODEL ATMOSPHERE

The model atmosphere may generally be considered to be in a state of quasi-static equilibrium. That is, when the equations of motion, thermodynamics, and continuity are scaled and when closed sets are found, the large-scale (i.e., the first order) vertical-component solution will show that, except near clouds with high-velocity updrafts, the hydrostatic equation

\[
\frac{\partial P}{\partial Z} = -\rho g
\]  

(1)
applies well. In equation (1), $P$ is atmospheric pressure, $Z$ is geometric altitude, $ho$ is atmospheric density, and $g$ is the acceleration caused by gravity. At pressures and temperatures experienced in the atmosphere of the earth, the ideal-gas law is usually accurate to within 1 percent. The equation of state

$$
\rho = \frac{P_m}{RT}
$$

(2)

is a form of the ideal-gas law, where $m$ is the molecular weight of the atmosphere, $R$ is the universal gas constant, and $T$ is the kinetic atmospheric temperature.

With certain reasonable and valid assumptions, the proper combination of the hydrostatic equation (eq. (1)) and the ideal-gas law (eq. (2)) results in equations (3) and (4), which are derived in detail in reference 4. If $\partial T^*/\partial H \neq 0$, where $T^*$ is virtual temperature and $H$ is the geopotential altitude, then

$$
P_b = P_a \left( \frac{T_b^*}{T_a^*} \right)^{g_o m_d/[R(\partial T^*/\partial H)]}
$$

(3)

and if $\partial T^*/\partial H = 0$, then

$$
P_b = P_a \exp \left[ -\frac{g_o m_d(H_b - H_a)}{RT_a^*} \right]
$$

(4)

In equations (3) and (4), $P_b$ is the atmospheric pressure at $H_b$, $P_a$ is the atmospheric pressure at $H_a$, $T_b^*$ is the virtual temperature at $H_b$, $T_a^*$ is the virtual temperature at $H_a$, $g_o$ is the surface gravity, $m_d$ is the molecular weight of the dry atmosphere, $H_a$ is the geopotential altitude at $A$, and $H_b$ is the geopotential altitude at $B$. In the upper atmosphere, a fictitious temperature designated as molecular scale temperature $T_m$ is defined in order to include variations in molecular weight (caused by molecular dissociation) and temperature in one variable.

$$
T_m = T \frac{m_o}{m}
$$

(5)
where \( m_D \) is the molecular weight at the surface. Similarly, in the lower atmosphere, a quantity designated as virtual temperature \( T^* \) is defined in order to include variations in molecular weight (caused by water vapor) and temperature in one variable.

\[
T^* = T \frac{m_d}{m} \quad (6)
\]

Therefore, \( T^* \) and \( T_m \) may be used interchangeably in equations (3) and (4); this fact enables the use of equations (3) and (4), which were derived for planetary atmospheres in reference 4.

As shown in appendix A, the proper combination of the equation of the state of dry air, the equation of the state of moist air, and equation (6) gives the exact expression of \( T^* \) as a function of temperature, pressure, and water-vapor pressure.

\[
T^* = \frac{T}{1 - 0.37803 \frac{f_w e}{P}} \quad (7)
\]

where \( f_w \) is the correction factor for the departure of the air and water-vapor mixture (from the ideal-gas law) and \( e \) is water-vapor pressure. Equations (3) and (4), which are the fundamental equations of subroutine MODATM calculations, are used in different forms to find the altitude of the significant levels and to find the pressure at a level between significant levels.

**Subroutine MODATM**

When atmospheric data at a particular altitude are desired, either geometric altitude is used as the calling variable, or pressure is used as the calling variable and a corresponding geometric altitude is calculated by using equations (3) and (4). Geopotential altitude \( H \) is calculated by

\[
H = \frac{Z (R_e)}{R_e + Z} \quad (8)
\]

where \( R_e \) is the mean radius of the earth. Geopotential altitude is then used to calculate temperature, virtual temperature, and molecular weight.
Temperature is calculated by

\[ T_b = T_a + \frac{\partial T}{\partial H} (H_b - H_a) \]  

(9)

where \( T_b \) is the temperature at \( H_b \), and \( T_a \) is the temperature at \( H_a \). Virtual temperature is calculated by

\[ T_b^* = T_a^* + \frac{\partial T^*}{\partial H} (H_b - H_a) \]  

(10)

Molecular weight is calculated by

\[ m_b = \frac{m_a T_a}{T_b^*} \]  

(11)

where \( m_b \) is the molecular weight at \( H_b \).

When \( P \) and \( T^* \) are known, a form of the equation of state (eq. (2))

\[ \rho = \frac{P m_d}{R T^*} \]  

(12)

is used to calculate density. Then, additional quantities related to altitude, pressure, density, molecular weight, temperature, and virtual temperature are calculated. The equations for the speed of sound \( C_{\text{sound}} \), acceleration of gravity \( g \), coefficient of viscosity \( \mu \), saturation mixing ratio \( r_s \), saturation specific humidity \( q_s \), pressure scale height \( H_p \), and density scale height \( H_\rho \) are as follows:

\[ C_{\text{sound}} = \sqrt{\gamma \frac{RT^*}{m_d}} \]  

(13)

\[ g = g_0 \left( \frac{R_e}{R_e + Z} \right)^2 \]  

(14)
\[ \mu = \frac{\beta T^{3/2}}{T + S} \]  

(15)

\[ r_s = \frac{0.62197 f_w e_s}{(P - f_w e_s)} \]  

(16)

\[ q_s = \frac{0.62197 f_w e_s}{(P - 0.37803 f_w e_s)} \]  

(17)

\[ H_p = \frac{RT^*}{m_d g} \]  

(18)

\[ H_p = \frac{1}{1 + \frac{1}{T^*} \left( \frac{\partial T^*}{\partial Z} \right)} \]  

(19)

where \( \gamma \) is the ratio of specific heats, \( \beta \) is \( 1.458 \times 10^{-6} \), \( S \) is Sutherland's constant, and \( e_s \) is the saturation water-vapor pressure. Equations (13), (15), (18), and (19) are derived in reference 1, equation (14) is derived in reference 4, and equations (16) and (17) are derived in reference 5. The \( f_w \)-factor is calculated by a function subprogram simulating tables 89 and 90 given in reference 6.

For calculations of variables describing the amount of water vapor in the atmosphere, dewpoint temperature \( T_d \) is calculated as follows:

\[ T_{d, b} = T_{d, a} + \frac{\partial T_d}{\partial H} (H_b - H_a) \]  

(20)

where \( T_{d, b} \) is the dewpoint temperature at \( H_b \), and \( T_{d, a} \) is the dewpoint temperature at \( H_a \). The equilibrium vapor pressure over a plane surface of water (ref. 6) is then calculated.
The formula for the vapor pressure over ice (ref. 6) may also be used.

\[
e = 1013.246 \times 10^{-6} \left[ \frac{1}{1.0 + \left( \frac{273.16}{T_d} \right)} \right] - 5.0208 \log_{10} \left( \frac{273.16}{T_d} \right) - 1.3818 \times 10^{-7} \left( 11.344 \left[ 1.0 - \left( \frac{T_d}{273.16} \right) \right]^{-0.01} + 8.31328 \times 10^{-3} \right) + 3.4624 \left[ 1.0 + \left( \frac{373.16}{T_d} \right) \right]^{-1.0}
\]

(21)

The choice of the temperature ranges during which each of the previously mentioned equations for \( e \) is used is determined by the programmer (function \( E(X) \)). As presently set up, only equation (21) is used. Equations (21) and (22) are used for calculating \( e_s \) by using \( T \) in place of \( T_d \).

With the previously discussed basic quantities available, the remaining atmospheric quantities may be calculated. The equations for the mixing ratio \( r \), relative humidity \( \text{Rel} \), specific humidity \( q \), refractive index \( n_{\text{STP}} \) (in wavelength), and refractive index \( n(Z) \) (in \( P, T, \) and wavelength) are as follows (ref. 5):

\[
r = \frac{0.62197f_w e}{(P - f_w e)}
\]

(23)

\[
\text{Rel} = \frac{r}{r_s} \times 100
\]

(24)

\[
q = \frac{0.62197f_w e}{(P - 0.83603f_w e)}
\]

(25)
For the infrared region (ref. 7)

\[
n_{STP} = 1 + 10^{-8} \left( 6432.8 + \frac{2949810.0}{146 - \frac{1}{\lambda^2}} + \frac{25540}{41 - \frac{1}{\lambda^2}} \right) \tag{26}
\]

and

\[
n(Z) = 1 + \left( n_{STP} - 1 \right) \left( \frac{1 + \frac{288.15}{273.16}}{1 + \frac{P}{273.16}} \right) \frac{P}{1013.25} \tag{27}
\]

where \( \lambda \) is wavelength. If the wavelength is in the microwave region (\( \lambda > 12500 \) microns, i.e., \( \lambda > 1.25 \) centimeters), then

\[
n(Z) = 1.0 + \left[ 1.0 \times 10^{-6} \left( 77.6 \frac{P}{T} \right) \right] + 373000.0 \frac{e}{T^2} \tag{28}
\]

as shown in reference 8.

The input variables of MODATM are included in the calling argument, and all output variables (i.e., the variables calculated by equations (3) to (28)) are stored in a "common block" in the array ANS. Detailed instructions on the use of subroutine MODATM are included in comment cards. For data-card information, see the discussion on subroutine INPUT in this report.

Subroutine INPUT

The purpose of subroutine INPUT is to read the input data cards necessary to set up the significant levels of various atmospheric parameters (i.e., altitude, pressure, temperature, and dewpoint temperature) for subroutine MODATM. Subroutine INPUT is initiated by MODATM whenever pressure (i.e., \( ANS(1) \)) is set equal to a number which is less than zero, and because of this fact, many sets of radiosonde data may be used successively, but not concurrently.

The input data may be of the form given in the significant levels (i.e., \( VV \)) of pressure, temperature, and temperature-dewpoint depression for a radiosonde. Table I shows an example of radiosonde data and the key to the radiosonde code. Table II gives the input data cards for the example shown in table I.

Subroutine INPUT is also constructed to accept input data other than radiosonde code \( VV \). If the first data card encountered is blank, then each of the next data cards
will be read in unencoded form (i.e., as altitude, temperature, and relative humidity). An example of the input data cards necessary to set up the 15° N annual model (ref. 2) is included in table III.

Levels of possible condensation are indicated by the word "condensation" in the print-out of the significant levels. This occurrence is determined by \( T - T_d < 2^\circ K \) at 1500 meters and \( T - T_d < 8^\circ K \) at 9000 meters, which is expressed by the approximate expression

\[
T - T_d < 1.0 + 0.000777H \text{ (meters)}
\]  

Subroutine REFRAC

Subroutine REFRAC is included to assist in making refracted path calculations throughout the atmosphere. The basic equations are developed (ref. 9) from Snell's law

\[
n' \sin \phi' = n'' \sin \psi
\]  

and from the law of sines

\[
\frac{\sin \phi''}{r'} = \frac{\sin \psi}{r``}
\]  

as shown in figure 1. In equations (30) and (31), \( n' \) is the refractive index at \( r' + (1/2)\Delta Z \), \( \phi' \) is the angle between \( r' \) and the path of the ray after refraction, \( n'' \) is the refractive index at \( r`` + (1/2)\Delta Z \), \( \psi \) is the angle between \( r' \) and \( d \), \( \phi'' \) is the angle between \( r`` \) and \( d \), \( r' \) is the distance to shell \( Z + \Delta Z \) on the refracted path, and \( r`` \) is the distance to shell \( Z \) on the refracted path.

The combination of equations (30) and (31) gives

\[
\phi' = \sin^{-1}\left(\frac{n''r'' \sin \phi''}{n'r'}\right)
\]  

and

\[
\psi = \sin^{-1}\left(\frac{r'' \sin \phi''}{r'}\right)
\]
Thus, by using known values for \( r'', r', \lambda, \) and \( \phi'' \) and by initiating MODATM to obtain values for \( n'' \) and \( n' \), the angles \( \phi' \) and \( \psi \) are calculated. If a continuous path is desired, \( \phi'' \) should be set equal to \( \phi' \), and \( r'' \) and \( r' \) should be incremented. Then, subroutine REFRAC should be called again.

Slant-path calculations are also made available by using the law of sines to calculate the increment \( d \) of the slant path from \( r \) to \( r' \) as follows:

\[
d = \frac{r'' \sin(\phi'' - \psi)}{\sin \psi}
\]  

Since subroutine MODATM is called by subroutine REFRAC and since subroutine MODATM is called last for the altitude corresponding to the middle of \( d \), the array ANS may be used externally to calculate the amount of water vapor or the total atmospheric mass that was traversed over distance \( d \). For the initial calculation at the target point, the angle \( \zeta \) (i.e., \( \phi'' \)) is needed; therefore, subroutine PATH is provided to calculate \( \zeta \) for the programmer.

**Subroutine PATH**

The principal purpose of subroutine PATH is to calculate the angle \( \zeta \); however, while calculating \( \zeta \), it is also convenient to calculate the columnar mass and the precipitable water vapor along this path. These three quantities are stored in the array ANS. If subroutine PATH is called prior to the calling of subroutine MODATM, ANS(1) will be set equal to -1.0, and subroutine MODATM will be called such that subroutine INPUT is activated, eliminating the future need to call subroutine INPUT externally. Subroutine PATH is thus programmed to be called only once for each radiosonde sounding.

The initial guess at \( \zeta \) is calculated by finding \( (r_{sp} - r_{1}) \), the vector from the target (1) to the spacecraft (sp), as shown in figure 2 and as developed in reference 10. The components of \( (r_{sp} - r_{1}) \) are

\[
R_X = (R_e + Z_{sp}) \cos \theta_{sp} \cos \phi_{sp} - (R_e + Z_1) \cos \theta_1 \cos \phi_1
\]

\[
R_Y = (R_e + Z_{sp}) \sin \theta_{sp} \cos \phi_{sp} - (R_e + Z_1) \sin \theta_1 \cos \phi_1
\]

and

\[
R_Z = (R_e + Z_{sp}) \sin \phi_{sp} - (R_e + Z_1) \sin \phi_1
\]
where $\theta_{sp}$ is the longitude of the spacecraft, $\theta_1$ is the longitude of the target, $\phi_1$ is the latitude of the target, $\phi_{sp}$ is the latitude of the spacecraft, $Z_{sp}$ is the altitude of a spacecraft above the earth, and $Z_1$ is the altitude of the target above the earth.

The components $(R_X, R_Y, R_Z)$ are found by coordinate transformation in the coordinate system of the target to be $\xi''$, $\eta''$, and $\zeta''$, which are the respective distances southward, eastward, and upward from a local station to the target.

\[
\begin{align*}
\begin{bmatrix}
\xi'' \\
\eta'' \\
\zeta''
\end{bmatrix}
&= 
\begin{bmatrix}
\sin \phi_1 \cos \theta_1 & \sin \phi_1 \sin \theta_1 & -\cos \phi_1 \\
-\sin \theta_1 & \cos \theta_1 & 0 \\
\cos \phi_1 \cos \theta_1 & \cos \phi_1 \sin \theta_1 & \sin \phi_1
\end{bmatrix}
\begin{bmatrix}
R_X \\
R_Y \\
R_Z
\end{bmatrix}
\end{align*}
\] (38)

The unrefracted zenith angle

\[\zeta = \tan^{-1}\left[\frac{\sqrt{(\xi'')^2 + (\eta'')^2}}{\zeta''}\right]\] (39)

can then be found. Next, the angle $TOS$ between $r_1'$ and $r_{sp}'$ is calculated by using the definition of the dot product

\[TOS = \cos^{-1}\left(\frac{\overrightarrow{r_1'} \cdot \overrightarrow{r_{sp}'}}{||\overrightarrow{r_{sp}'||} \cdot ||\overrightarrow{r_1'||}}\right)\] (40)

so that the best refracted path from the target to the spacecraft (fig. 1) may be found by iteration.

Iteration of paths from the equations developed in the description of subroutine REFRAC is used to find $\phi'$ and $\psi$ for each level, and since

\[\Delta \xi = \phi'' - \psi\] (41)

integration proceeds until

\[\Sigma \Delta Z = Z_{sp} - Z_1\] (42)
Then, $\sum \Delta \xi$ is compared to TOS for the purpose of iterating on $\zeta$ as follows

$$\zeta(t + \Delta t) = \zeta(t) - \frac{(\sum \Delta \xi - TOS)}{2}$$

(43)

until $|\sum \Delta \xi - TOS| \leq 0.0001$ radian (0.0057°). This procedure yields an accuracy on $\zeta$ of approximately $3 \times 10^{-3}$ radian (0.17°). The quantities columnar mass and precipitable centimeters of water along this refracted path are calculated, respectively, in the following equations.

$$\int_{r_1}^{r_{sp}} \rho \, ds \approx \Sigma \rho \cdot d$$

(44)

and

$$\int_{r_1}^{r_{sp}} q \rho \, ds \approx \Sigma q \rho d$$

(45)

The increments on $\Delta Z$ are made to be multiples of 10 smaller than $Z_{sp} - Z_1$, such that

$$Z_{sp} - Z_1 = \Delta Z \cdot i$$

(46)

where $i$ is 10, 100, 1000, et cetera and $\Delta Z \leq 0.2$ kilometer.

**Subroutine ATMOS3**

The subroutine ATMOS3 reproduces the U.S. Standard Atmosphere, 1962 (ref. 1). Subroutine ATMOS3 is called with geometric altitude from which geopotential altitude is calculated. The equations which are subsequently used for ATMOS3 are many of those developed for subroutine MODATM. Equations (3) to (5) and (8) to (15) are common to both subroutines. The main difference between subroutines ATMOS3 and MODATM is that in subroutine ATMOS3, all the significant levels are included in a data statement so that no data cards are necessary, and the output variables are more limited; that is, only the first eight variables in array ANS are available. These variables are pressure, temperature, density, speed of sound, acceleration of gravity, molecular scale temperature, molecular weight, and coefficient of
viscosity. The main purpose for including subroutine ATMS3 is that if atmospheric data above the maximum-altitude radiosonde data are required of subroutine MODATM, then ATMS3 is automatically called. The main impact subroutine ATMS3 has on analyses is that if the maximum usable radiosonde altitude is <10 kilometers, significant water vapor will be ignored since the subroutine ATMS3 includes no water vapor. Instructions on the use of subroutine ATMS3 are included in comment cards in the subprogram. The computer print-out, including all subroutines, is shown in appendix B.

CONCLUDING REMARKS

It is hoped that this nonpredictive model atmosphere for earth resources applications will fill the need for atmospheric data until predictive postflight or flight models can be developed.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, November 15, 1969
160-75-03-00-72
REFERENCES


May 10 1969 0000Z

Table I. - Lake Charles, Louisiana, Radiosonde and Code

| TT 60004 72240 99016 23266 01008 00146 21467 00512 85517 |
| 08463 35017 70118 04273 32033 50577 13571 29543 40743 26569 |
| 27572 30946 38567 27590 20217 519// 15400 589// 10850 673// |
| 88999 |
| 66280 27595Ø |

| VV 6000/ 72240 00016 23266 11970 18068 22831 06662 33813 |
| 11075 44609 02171 55400 26569 66290 40166 77243 461// 88227 |
| 451// 99193 535// 11100 673// 31313 25069 451// '';/''/Ø |

| QQ 60000 72240 90012 01008 35512 35007 90346 36009 36013 34524 |
| 90789 33530 34031 33031 91246 31535 32539 31534 9205// 29044 |
| 27582 9302/ 27588 27595Ø |

2nd Trans

| WW 6000/ 72240 70866 661// 50071 633// 30391 551// 20653 |
| 497/ 10115 411// 07358 403// |
| 88950 681// '''''''' |
| 77999Ø |

| YY 6000/ 72240 11950 681// 22920 657// 33600 665// 44230 |
| 511// 55100 411// 66070 403//Ø |

| LL 60000 72240 XMTDØ |

---

a. The significant level code is VV. For VV, the code is itipp TT'T'dd where
   ii = identifier of a set of data; the two characters are identical (e.g., 00, 11, 22, 33).
   ppp = pressure in mbar except the 4th character from the right is suppressed (e.g., 970 = 970 mbar, and 016 = 1016 mbar).
   TTT = temperature, + if last digit is even, and - if last digit is odd.
   dd = dewpoint temperature. If 00-49, multiply by 0.1 for °C; 50 = 5.0 °C; 51-55, not used; 56-99, subtract 50 for °C.
   That is, 02 = 0.2, 56 = 6.0, 60 = 10."

Slashes indicate no data.
TABLE II. - INPUT DATA CARDS FOR LAKE CHARLES, LOUISIANA, RADIOSONDE DATA

<table>
<thead>
<tr>
<th>STATEMENT NUMBER</th>
<th>CONTINUATION</th>
<th>FORTRAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOCATION</td>
<td>OPERATION</td>
<td>VARIABLE FIELD</td>
</tr>
<tr>
<td>0.1.6</td>
<td>2.3</td>
<td>2.6.6.</td>
</tr>
<tr>
<td>9.7.0</td>
<td>1.8</td>
<td>0.6.8.</td>
</tr>
<tr>
<td>8.3.1</td>
<td>0.6</td>
<td>6.6.2.</td>
</tr>
<tr>
<td>8.1.3</td>
<td>1.1</td>
<td>0.7.5.</td>
</tr>
<tr>
<td>6.0.9</td>
<td>0.2</td>
<td>1.7.1.</td>
</tr>
<tr>
<td>4.0.0</td>
<td>2.6</td>
<td>5.6.9.</td>
</tr>
<tr>
<td>2.9.0</td>
<td>4.0</td>
<td>1.6.6.</td>
</tr>
<tr>
<td>2.4.3</td>
<td>4.6</td>
<td>1.0.0.</td>
</tr>
<tr>
<td>2.2.7</td>
<td>4.5</td>
<td>1.0.0.</td>
</tr>
<tr>
<td>1.9.3</td>
<td>5.3</td>
<td>5.0.0.</td>
</tr>
<tr>
<td>1.0.0</td>
<td>6.7</td>
<td>3.0.0.</td>
</tr>
<tr>
<td>-1...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE III. - INPUT DATA CARD FORMAT FOR 15° N ANNUAL MODEL ATMOSPHERE

<table>
<thead>
<tr>
<th>STATEMENT NUMBER</th>
<th>LOCATION</th>
<th>OPERATION</th>
<th>VARIABLE FIELD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000E+01</td>
<td>0.13250E+03</td>
<td>2.9, 6.5, 7.5</td>
</tr>
<tr>
<td>1</td>
<td>0.000E+01</td>
<td>0.39000E+02</td>
<td>2.9, 6.5, 7.5</td>
</tr>
<tr>
<td>2</td>
<td>0.000E+01</td>
<td>0.43000E+02</td>
<td>2.8, 6.5, 7.5</td>
</tr>
<tr>
<td>3</td>
<td>2.500E+01</td>
<td>0.37800E+02</td>
<td>2.8, 6.1, 7.5</td>
</tr>
<tr>
<td>4</td>
<td>5.000E+01</td>
<td>0.37500E+02</td>
<td>2.8, 9.5, 3.5</td>
</tr>
<tr>
<td>5</td>
<td>0.000E+01</td>
<td>0.3632300E+02</td>
<td>2.7, 9.0, 3.5</td>
</tr>
<tr>
<td>6</td>
<td>0.000E+01</td>
<td>0.3491100E+02</td>
<td>2.6, 5.0, 3.5</td>
</tr>
<tr>
<td>7</td>
<td>0.000E+01</td>
<td>0.3376400E+02</td>
<td>2.5, 1.0, 3.0</td>
</tr>
<tr>
<td>8</td>
<td>0.000E+01</td>
<td>0.4284300E+02</td>
<td>2.3, 7.0, 2.0</td>
</tr>
<tr>
<td>9</td>
<td>1.000E+00</td>
<td>0.000E+00</td>
<td>0.0, 0.5</td>
</tr>
</tbody>
</table>

Blank Card
Figure 1. - Refraction-path geometry through a spherically symmetric atmosphere.
Figure 2. - Resultant vector from the target to the spacecraft in fixed-earth center coordinates.
APPENDIX A

DERIVATION OF VIRTUAL TEMPERATURE $T^*$

The equations of the state of dry air

$$\rho_d = \frac{(P - f_w e)m_d}{RT}$$  \hspace{1cm} (A1)

defined by water vapor

$$\rho_w = \frac{f_w e m_w}{RT}$$  \hspace{1cm} (A2)

and of wet air

$$\rho = \rho_d + \rho_w = \frac{f_w e m_w}{RT} + \frac{(P - f_w e)m_d}{RT}$$  \hspace{1cm} (A3)

can be used with the mass percentage formula for molecular weight

$$m = \frac{100}{\sum \frac{M_i}{m_i}} = \frac{\rho}{\frac{\rho_w}{m_w} + \frac{\rho_d}{m_d}}$$  \hspace{1cm} (A4)

to give a formula for the relationship of temperature, molecular weight, pressure, and
water-vapor pressure

$$m = \frac{f_w e m_w + (P - f_w e)m_d}{RT}$$  \hspace{1cm} (A5)$$
Equation (A5), when simplified, becomes

\[ m = m_d \left[ \frac{P + f_w e \left( \frac{m_w}{m_d} - 1 \right)}{P} \right] = m_d \left( 1 - 0.37803 \frac{f_w e}{P} \right) \]  
(A6)

By employing the definition of \( T^* \)

\[ T^* = \frac{m_d T}{m} \]  
(A7)

and by using equation (A6), the exact expression for \( T^* \) may be found in terms of \( T, e, \) and \( P \)

\[ T^* = \frac{T}{\left( 1 - 0.37803 \frac{f_w e}{P} \right)} \]  
(A8)
APPENDIX B
SUBROUTINES

00101 10 SUBROUTINE HOUATM (Z,PP,TEST,XALAMU)
00103 20 DIMENSION H(25),P(25),T(25),TD(25),ANS(35),TV(25)
00104 30 COMMON ANS
00105 40 DATA RO/8.31432E+07/,XMO/28.9664/,BETA/1.458E-06/,S/110.4/,RE/6.37
00105 50 1.299E+03/,G/980/665/,CONN/3.41631947E-02/
00105 60 C
00105 70 C**************************************************************************************************************************
00105 80 C
00105 90 C Z IS IN KM, PP IS IN MB
00105 100 C ANS IS OUTPUT VARIABLES
00105 110 C XALAMU IS THE WAVELENGTH IN MICRONS FOR WHICH YOU ARE CALCULATING
00105 120 C ATOMIC REFRACTION
00105 130 C IF TEST .EQ. PRES THEN PRESSURE IS USED AS HEIGHT INDICATOR
00105 140 C IF TEST .NE. PRES THEN GEOMETRIC ALTITUDE (KM) IS HEIGHT INDICATOR
00105 150 C YOU MUST SET ANS(I)=1.0 BEFORE ENTERING THE SUBROUTINE THE FIRST TIME
00105 160 C RU IS THE UNIVERSAL GAS CONSTANT BASED ON THE CARBON 12 ATOMIC WEIGHT
00105 170 C SCALE IN ERGS/(DEG KELVIN*GM-MOLE)
00105 180 C XMO IS MOLECULAR WEIGHT OF AIR CALCULATED FROM THE COMPOSITION OF DRY
00105 190 C AIR USING THE CARBON 12 ATOMIC WEIGHT SCALE, FOUND IN THE U.S.
00105 200 C STANDARD ATMOSPHERE 1962, PAGE 9, GIVEN IN GM/TM-MOLE
00105 210 C BETA IS A CONSTANT USED IN SUTHERLAND'S VISCOSITY EQUATION, GIVEN IN
00105 220 C KG/SEC-M^2(1/2)
00105 230 C S IS SUTHERLAND'S CONSTANT IN DEG. KELVIN
00105 240 C RL = THE MEAN RADIUS OF THE EARTH IN METERS AS GIVEN BY THE SMITHSONIAN
00105 250 C METEOROLOGICAL TABLES, SIXTH EDITION, PUBLICATION 4014, R. J.
00105 260 C LIST, 1966
00105 270 C G IS ACCELERATION OF GRAVITY AT O EQUIPOENTIAL SURFACE LEVEL GIVEN IN
00105 280 C CM/SEC^2
00105 290 C CONN IS A CONSTANT GIVEN AS -M*G/RO WHERE M IS MASS AND G AND RO ARE AS ABOVE
00105 300 C
00105 310 C**************************************************************************************************************************
00105 320 C
C THE FOLLOWING IS AN EXAMPLE OF A CALLING PROGRAM FOR MODATM AND PATH

DIMENSION ANS(35)
COMMON ANS
X Lamda=+6
ZS=20.0
PHI S=30.0
THETA S=90.0
ZL=0.0
PHI L=30.0
THETAL=90.0
CALL PATH (X Lamda,ZS,PHI S,THETA S,ZL,PHI L,THETAL)
WRITE (6,3) (ANS(K),K=Z1,Z3)
3 FORMAT (1X:1//,1X,1P3E14.4)
TEST=4HPRES
DO 1 I=1,20
Z=I
PP=1000.0-Z*50.
CALL MODATM (Z,PP,TEST,X Lamda)
WRITE (6,2) TEST,Z,ANS(N),N=1,24
2 FORMAT (1X,A4,4X,1P12E9.3,/,1P13E9.3,/) CALL EXIT
END

CT=288.15/273.16+1.0
CT IS 1.0 + RATIO OF SURFACE TEMPERATURE TO ICE TEMPERATURE
IF(ANS(1),GE,0.0) GO TO 15
ANS(1)=0.0
CALL INPUT (PIT,TD,M,H,TVM)
15 IF (TEST EQ 4HPRES) GO TO 7
HA= RE*Z/(RE+Z)*1000.0
HA IS GEOPOTENTIAL ALTITUDE IN METERS
DO 11 I=1,M
11 CONTINUE
9 CALL ATMOS3(Z)
ANS(9)=0.0
GO TO 52
13 I=I-1
DH=H(I+1)-H(I)
D=(TV(I+1)-TV(I))/DH
<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>( W = (T(i+1) - T(i)) / \Delta H )</td>
</tr>
<tr>
<td>77</td>
<td>( D = (T(i+1) - T(i)) / \Delta H )</td>
</tr>
<tr>
<td>78</td>
<td>( \Delta H = H(i) - H(i-1) )</td>
</tr>
<tr>
<td>C</td>
<td>( <em>H</em> ) is in meters</td>
</tr>
<tr>
<td>C</td>
<td>( <em>Z</em> ) is in kilometers</td>
</tr>
<tr>
<td>C</td>
<td>( <em>P</em> ) is pressure</td>
</tr>
<tr>
<td>C</td>
<td>Pressure is in millibars</td>
</tr>
<tr>
<td>C</td>
<td>( <em>T</em> ) is temperature</td>
</tr>
<tr>
<td>C</td>
<td>Temperature is in degrees Kelvin</td>
</tr>
<tr>
<td>C</td>
<td>( <em>d</em> ) is density</td>
</tr>
<tr>
<td>C</td>
<td>Density is in grams per cubic centimeter</td>
</tr>
<tr>
<td>C</td>
<td>( <em>s</em> ) is speed of sound</td>
</tr>
<tr>
<td>C</td>
<td>Speed of sound is in meters per second</td>
</tr>
<tr>
<td>C</td>
<td>( <em>a</em> ) is acceleration of gravity</td>
</tr>
<tr>
<td>C</td>
<td>Acceleration of gravity is in centimeters per second squared</td>
</tr>
<tr>
<td>C</td>
<td>( <em>v</em> ) is virtual temperature</td>
</tr>
<tr>
<td>C</td>
<td>Temperature is in degrees Kelvin</td>
</tr>
<tr>
<td>C</td>
<td>( <em>m</em> ) is molecular weight</td>
</tr>
<tr>
<td>C</td>
<td>( <em>\mu</em> ) is coefficient of viscosity</td>
</tr>
<tr>
<td>C</td>
<td>Viscosity is in kilograms per hour per meter</td>
</tr>
<tr>
<td>C</td>
<td>( <em>d</em> ) is dew point temperature</td>
</tr>
<tr>
<td>C</td>
<td>Temperature is in degrees Kelvin</td>
</tr>
<tr>
<td>C</td>
<td>( <em>r</em> ) is mixing ratio R</td>
</tr>
<tr>
<td>C</td>
<td>Mixing ratio is in parts per thousand, i.e., ( \frac{1000}{X} ) grams per kilogram</td>
</tr>
<tr>
<td>C</td>
<td>( <em>s</em> ) is saturation mixing ratio RS</td>
</tr>
<tr>
<td>C</td>
<td>Saturation mixing ratio is in parts per thousand, i.e., ( \frac{1000}{X} ) grams per kilogram</td>
</tr>
<tr>
<td>C</td>
<td>( <em>h</em> ) is relative humidity</td>
</tr>
</tbody>
</table>
C RELATIVE HUMIDITY IS IN PERCENT (0/0)
C Ans(13) is specific humidity
C Ans(14) is saturation specific humidity
C Ans(15) is pressure scale height
C Ans(16) is density scale height
C Ans(17) is refractive index developed by edlen in terms of wavelength alone
C Ans(18) is refractive index developed by penndorf in terms of wavelength, temperature, and pressure
C Ans(19) is water vapor pressure in mb
C Ans(20) is the saturation water vapor pressure in mb
C Ans(21) is the zenith angle from groundstation in radians
C Ans(22) = the total gm/cm**2 or columnar mass along the slant path
C Ans(23) = total gm/cm**2 of water vapor along the slant path, it is equivalent to precipitable cm of water
C Ans(24) = total path length in cm
C Ans(21) thru Ans(24) are calculated in subroutine path
C Ans(2) = T(I1) - W*DH
C Ans(6) = TV(I1) - W*DH
C Ans(19) = TD(I1) - W*DH
C Ans(1) = PRES(P(1), D(1), V(1), Ans(6), DH)
C Go to 14
00159 160* 12 I=11
00160 161* ANS(1)=P(1)
00160 162* ANS(2)=T(1)
00160 163* ANS(3)=TV(1)
00160 164* ANS(4)=T0(1)
00161 165* 14 ANS(5)=G*RL/(RE+2)**2
00162 166* ANS(6)=XMO/(R0*ANS(6))/1000*0
00162 167* ANS(7)=XMO*ANS(2)/ANS(6)
00162 168* ANS(8)=.Objects(ANS(2))**3/(ANS(2)**5)
00162 169* ANS(9)=E(ANS(2))
00162 170* ANS(10)=RANS(19),ANS(1),ANS(2)
00162 171* ANS(11)=RANS(20),ANS(1),ANS(2)
00162 172* ANS(12)=ANS(10)/ANS(11)*100*0
00162 173* ANS(13)=Q(ANS(1),ANS(2))
00162 174* ANS(14)=Q(ANS(1),ANS(2))
00162 175* ANS(15)=R*ANS(6)/(XMO*ANS(5))**1*GE*.05
00162 176* ANS(16)=ANS(15)/(1*U*RO/(XMO*ANS(5))**1*OE*01)
00162 177* IF (X/LAMDA*0E**-12500*00) GO TO 20
00162 178* C THIS MEANS IF X/LAMDA 15*GE* *25 CM USE MICROWAVE REFRACTIVITY
00162 179* ANS(17)=U*0E-09*(1432*8*294980+1/146+1*(X/LAMDA**2)+25540+/
00162 180* 10*(1-1*(X/LAMDA**2))
00162 181* ANS(18)=1*U*ANS(17)**1*90*(CT/(1*O+ANS(2)/273.16))**ANS(1)/1013.25
00162 182* GO TO 31
00162 183* 30 ANS(18)=1+1*U*OE-09*(77.6*ANS(1)/(ANS(2)+373000)*0*ANS(19)/(ANS(2)**1
00162 184* 1) +2)
00162 185* ANS(17)=ANS(18)
00162 186* 31 RETURN
00162 187* 7 DD 16 I=1,M
00162 188* 19* C PRESSURE
00162 189* 191* I=1
00162 190* IF (PP-P(1)) 16*41,17
00162 191* 16 CONTINUE
00162 192* HA=0*0
00162 193* 195* DHA=100*0
00162 194* 196* 51 DO 48 I=1,11
00162 195* 197* HA=HA+DHA
00162 196* 198* CALL ATMOS3(HA)
00162 199* 199* IF (ANS(1)*LE*U=0) GO TO 42
00162 200* 200* IF (ANS(1)*LT*PP) GO TO 49
00162 201* 201* 48 CONTINUE
00162 202* 202* 42 DO 10 I=2,35
00162 203* 203* 24) GO TO 10
00162 204* 204* ANS(1)=0*0
00162 205* 205* 10 CONTINUE
00245  206e  ANS(17)=I+0
00246  207e  ANS(18)=I+0
00247  208e  Z=HA
00250  209e  RETURN
00251  210e  41 Z=HA(I)*RE/(1000+U*(RE-M(I)/1000+U))
00252  211e  GO TO 12
00253  212e  49 IF (ABS(ANS(I))=PP) *LE* (+001*PP)) GO TO 50
00255  213e  MA=MA+DMA
00256  214e  DMA=DMA/10+0
00257  215e  GO TO 51
00260  216e  50 Z=HA
00261  217e  GO TO 9
00262  218e  17 I=I-1
00263  219e  U=TV(I+1)-TV(I)
00264  220e  1F(U) 20,21,20
00267  221e  20 D=CONN*ALOG(P(I+1)/P(I))*ALOG(TV(I+1)/TV(I))
00270  222e  ANS(6)*TV(I)*PP/P(I)**(D/CONN)
00271  223e  MA=MA(I+1)+ANS(6)-TV(I)/D
00272  224e  GO TO 22
00273  225e  C MA IS IN METERS
00273  226e  21 MA=MA(I+1)+TV(I)*ALOG(P/P(I))/CONN
00274  227e  22 Z=HA*RE/(1000+U*(RE-HA/1000+U))
00275  228e  GO TO 23
00276  229e  END

END OF UNIVAC 1108 FORTRAN IV COMPILATION, 0 #DIAGNOSTIC #ESSAGE(S)
C  ANS(5)=980.665 FOR ACC OF GRAVITY IN CM/(SEC**2)
C  ANS(6) IS THE RATIO OF MOLECULAR SCALE TEMPERATURE
C  ANS(7) IS THE MOLECULAR WEIGHT
C  ANS(8) IS THE RATIO OF COEF OF VISCOSITY (NU/MUSL)
C  ANS(9)=1.8946-9.5 TO COEF IN KM/HR-SEC
C  W IS THE VERTICAL KINETIC TEMPERATURE GRADIENT
C  THIS RADII RE* IS CHOSEN TO AGREE WITH THE U.S STANDARD AT 90 KM, BUT IT
C  ALSO IS A BEST FIT TO ALL LEVELS BELOW 90 KM, ABOVE 90 KM THE LEVELS
C  THAT ARE BREAK POINTS WERE CALCULATED FROM GEOMETRIC TO GEOP USING *RE*
C
Z=Z*1000*0

IF (Z-7000000*0) 10,50,50
CONTINUE

HA=RE*Z/(RE*2)

DO 1 M=1,23
I=M

IF (H(I)-HA) 1,2,3
CONTINUE

GO TO 50

GO TO 5

I=I+1

D=(T(I)+1-T(I))/H(I+1)-H(I)
W=(A(I)+1)-A(I)/H(I+1)-H(I)

GO TO 4

GO TO 5

GO TO 5

IF (Z<=000000*0) 7,7,7

IF (Z<=000000*0) 8,6,6

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5

GO TO 5
00167  86* \[ \text{ANS}(8) = \frac{(T(2)+S)}{(\text{ANS}(2) \cdot T(2)+S)} \cdot \text{SQR}(\text{ANS}(2)+3) \]
00170  87* IF (1) 12,13,14
00173  88* 12 \text{COHN} = \text{ALOG}(P(1+1)/P(1))/\text{ALOG}(T(1+1)/T(1))
00174  89* \text{ANS}(1) = P(1)/P(2) \cdot (\text{ANS}(6) \cdot T(2)/T(1)) \cdot (\text{COHN}^2)
00175  90* GO TO 19
00176  91* 13 \text{COHN} = \text{ALOG}(P(1+1)/P(1))/(H(1+1)-H(1)) \cdot T(1)
00177  92* \text{ANS}(1) = P(1)/P(2) \cdot \text{EXP}(\text{COHN}^2 \cdot (H(1)-H(1))/\text{ANS}(6) \cdot T(2))
00200  93* 14 \text{ANS}(3) = \text{ANS}(1)/\text{ANS}(6)
00201  94* \text{ANS}(1) = \text{ANS}(1) + 0.132 \cdot 5 \cdot 0.3
00202  95* \text{ANS}(2) = \text{ANS}(2) - 0.132 \cdot 5
00203  96* \text{ANS}(3) = \text{ANS}(3) - 0.132 \cdot 5
00204  97* \text{ANS}(4) = \text{ANS}(4) - 0.132 \cdot 5
00205  98* \text{ANS}(5) = \text{ANS}(5) - 0.132 \cdot 5
00206  99* \text{ANS}(6) = \text{ANS}(6) - 0.132 \cdot 5
00207 100* \text{ANS}(8) = \text{ANS}(8) - 0.132 \cdot 5
00210 101* Z = Z / 1000 + 0
00211 102* GO TO 53
00212 103* 50 DO 51,1,8
00215 104* 51 \text{ANS}(1) = 0.0
00217 105* 53 RETURN
00220 106* END

END OF UNIVAC 1108 FORTRAN V COMPILATION.
DIAGNOSTIC MESSAGE(S)
**SUBROUTINE INPUT (P,T,TD,H,TV,H)**

**DIMENSION P(1),T(1),TD(1),H(1),TV(1)**

**C** THIS INPUT SUBROUTINE IS SET UP TO TAKE STANDARD PRINTOUT OF CODE WV,

1. **C** (IE SIGNIFICANT LEVELS OF A RADIOSONDE) AND SET ALTITUDES, VIRTUAL TEMPERATURES, AND AMBIENT TEMPERATURES OR IF A BLANK CARD

2. **C** PRECEDES THE DATA THE INPUT DATA IS OF THE FORM HEIGHT, PRESSURE,

3. **C** TEMPERATURE, AND RELATIVE HUMIDITY

**C**

**COND=SH**

**NSAT=SH**

**ON=SH**

**M=O**

**H(1)=O,0**

**WRITE (6,25)**

**25 FORMAT (1X,4X,'EARTH RESOURCES MODEL ATMOSPHERE, 1969,**

**29X,'THE SIGNIFICANT LEVELS FOR THE MODEL ATMOSPHERE ARE AS FOLLOWS:',//,**

**34X,'ALT=10X,PRES=10X,TEMP,9X,TD=11X**

**3 X,'TV=',//34X,'(M)=10X,(MB)=10X, (K)-10X,(K)-10X,(K)=/**

**C**

**C** THIS SECTION INPUTS CODED DATA

**DO I=1,25**

**READ (5,3) P(I),T(I),TD(I)**

**C** THIS IS THE FORMAT FOR READING RADIOSONDE DATA

**FORMAT(1X,F3.0,1X,F3.0,F2.0)**

**C ALTITUDE IN METERS**

**C PRESSURE IN MB**

**C T AND TD IN DEG KELVIN**

**IF (P(I) .LE. 0.0 AND T(I) .LE. 0.0 AND TD(I) .LE. 0.0) GO TO 1**

**IF (P(I) .LE. 0.0) GO TO 2**

**M=M+1**

**IF (T(I) .LE. LT .1000.0) P(I)=P(I)+1000.0**

**IF (AMOD(T(I),2.0)+GT.0.0) T(I)=T(I)**

**T(I)=T(I)+1**

**IF (TD(I) .LE. 50.0 AND TD(I) .LE. 55.0) WRITE (6,4)**

**4 FORMAT (9X,'INVALID TD INPUT DATA')**
00145  45*  IF (TU(I) .GE. 50.0 .AND. TD(I) .LE. 99.0) TU(I)=TU(I)=50.0
00147  46*  IF (TD(I) .LE. 0.01) TU(I)=T(I)+273.16
00151  47*  TD(I)=T(I)-TD(I)
00152  48*  T(I)=T(I)+273.16
00153  49*  TD(I)=TD(I)+273.16
00154  50*  1 CONTINUE
00156  51*  GO TO 2
00156  52*  C
00156  53*  C
00156  54*  C
00156  55*  C
00156  56*  C
00156  57*  11 M=0
00156  58*  DO 12 I=1,25
00160  59*  C  THIS IS THE FORMAT FOR READING SIGNIFICANT LEVELS IN NON-CODED FORM
00163  60*  READ (5,13) M(I),P(I),T(I),TD(I)
00171  61*  13 FORMAT (E9.3,E12.6,F7.2,F3.0)
00171  62*  C  TD(I) HERE, IS RELATIVE HUMIDITY, UNTIL A TD(I) IS FOUND BY ITERATION
00172  63*  DELT=100.0
00173  64*  GUESS=50.0
00174  65*  Ri=R(E(T(I)),P(I),,T(I))
00175  66*  992 DO .990 L=1,11
00200  67*  GUESS=GUESS+DELT
00201  68*  REL=R(E(GUESS),P(I),GUESS)*100.0/R1
00202  69*  Q=REL-TD(I)
00203  70*  IF (Q) 990,991,995
00206  71*  990 CONTINUE
00210  72*  CALL EXIT
00211  73*  995 GUESS=GUESS-DELT
00212  74*  DELT=DELT/10.0
00213  75*  991 IF (ABS(Q)+GT.01) GO TO 992
00215  76*  TD(I)=GUESS
00216  77*  IF (P(I),LE.0.0) GO TO 2
00220  78*  M=M+1
00221  79*  12 CONTINUE
00221  80*  C
00221  81*  C
00221  82*  C
00223  83*  2 DO 5 I=1,M
00226  84*  IF(TD(I) .LE. 0.01) GO TO 7
00230  85*  TV(I)=T(I)/(1.0-1.0-37803*E(TD(I))*F(P(I),T(I))/P(I))
00231  86*  GO TO 5
00232  87*  7 TV(I)=T(I)
00233  88*  5 CONTINUE
DO 26 I = M, 25
H(I) = H(M)
P(I) = P(M)
DO 6 I = 1, M

IF (ABS(T(I) - T(I)) + GT = 1.0 + H(I) * Q00777) GO TO 27
CONDE = 5H
NSATI = 5H
QN = 5H
WRITE (5, 29) H(I), P(I), T(I), TD(I), TV(I), CONDE, NSATI, QN
FORMAT (26X, 1P2E13.3, 3UP3F13.2, 1X, 3A5)

IF (CONDE = EQ + 5H) GO TO 6
CONDE = 5H
NSATI = 5H
QN = 5H
CONTINUE
WRITE (5, 86)
FORMAT (/ /)
RETURN
END

END OF UNIVAC 1108 FORTRAN V COMPILATION. 0 DIAGNOSTIC MESSAGE(S)
SUBROUTINE REFHAC (Z1, Z2, XALAMDA, PHI, PHIPR, PSI, SLANT)

DIMENSION ANS(35)

COMMON ANS

DATA R6/6371.297/

C

C IN ORDER TO CALCULATE A CONTINUOUS PATH YOU MUST EXTERNALLY SET PHI=PHIPR

C Z1, Z2, PHI, AND XALAMDA ARE INPUT VARIABLES

C Z1 AND Z2 ARE IN KM AND XALAMDA IS IN MICRONS

C PHIPR, PSI, AND SLANT ARE OUTPUT VARIABLES

C PHI, PHIPR, AND PSI ARE IN RADIANS AND SLANT IS IN CM

C IF YOU WANT AMOUNT OF GM/CM2 (COLUMNAR MASS) OF ATMOSPHERE FROM Z1 TO Z2

C USE ANS(3)*SLANT. GM/CM2 OF WATER IS ANS(3)*SLANT=ANS(3)/1000,0.

C SINCE ALL ANS ARRAY IS IN COMMON, YOU CAN DO THIS EXTERNALLY.

C

C**************************************************************

S1=RE+Z1

S2=RE+Z2

DELT=(Z2-Z1)/Z+U

CALL HODATM(ZZ+DELT,PP,4HALTI,4XALAMDA)

D2=ANS(3)

XN2=ANS(18)

CALL HODATM(Z1+DELT,PP,4HALTI,4XALAMDA)

DI=ANS(3)

XN1=ANS(18)

PSI=ISISINV(S1*4SIN(PHI1)/S2)

PHIPR=ISISINV(S1*SIN(PHI)*XN1/(S2*XN2))

SLANT=S1*4SIN(PHI1)/SIN(PSI)+1.0*U

RETURN

END
SUBROUTINE PATH (X, LAMBDA, ZS, PHIS, THETAS, ZL, PHIL, ITHETAL)

DIMENSION ANS(35), A(3,3), B(3), C(13)

COMMON ANS

DATA PI/3.14159265/; CON/1.0174532925/; RE/6371.299/

C

C QUANTITIES ENDING IN S ARE FOR THE SATELLITE
C QUANTITIES ENDING IN L ARE FOR THE GROUND LOCAL

C -Q1- AND -Q2- ARE DUMMY VARIABLES
C -XS, YS, AND HS- ARE THE RECTANGULAR COORDINATES OF THE SPACECRAFT
C -AL, YL, AND HL- ARE THE RECTANGULAR COORDINATES OF THE GROUND LOCAL
C THE ANGLE ABD IS THE ANGLE BETWEEN THE SUBSATELLITE POINT AND TARGET
C ANGLE ABD IS FOUND BY USING THE DOT PRODUCT AND TAKING THE INVERSE COS
C *UO92833 RADIANS IS THE TOTAL REFRACTION ON A PASS THRU U.S. STANDARD
C *SUM IS THE TOTAL ANGLE CHANGE DURING REFRACTION
C *SUM1 IS THE SUM OF ALL DELTA Ai CALCULATED BY LAW OF SINES
C *SUM2 IS PRECIPITABLE CM OF WATER OR GM/CM**2 OF WATER VAPOR
C *SUM3 IS THE TOTAL COLUMNAR MASS IN THE SLANT PATH
C *SUM4 IS THE TOTAL SLANT PATH IN CM
C PHIL IS IN RADIANS
C ANS(121) IS THE ZENITH ANGLE FROM GROUNDSTATION IN RADIANS
C ANS(22) = THE TOTAL GM/CM**2 OR COLUMNAR MASS ALONG THE SLANT PATH
C ANS(23) = TOTAL GM/CM**2 OF WATER VAPOR ALONG THE SLANT PATH. IT IS
C EQUIVALENT TO PRECIPITABLE CM OF WATER
C ANS(24) = TOTAL PATH LENGTH IN CM
C C
C PHIL=PHIL*CON
C THETAS=THETAS*1.-CON
C PHIL=PHIL*CON
C THETAL=THETAL*1.-CON
C DDEL =ABS(ZL-ZS)
C DO 80 I=1,32000
C DDEL=DDEL/10.-0.
C L=1
C IF(DDEL*LE.2.*0) GO TO 81
C DO CONTINUE
C CALL EXIT
00130 45  81 IF (L.E.1) DEL1=DEL1/10.0
00132 46  ANS(1)=1.0
00133 47  Q1=RE+ZS
00134 48  Q2=COS(PHIS)
00135 49  XS=Q1*COS(THETAS)*Q2
00136 50  YS=Q1*SIN(THETAS)*Q2
00137 51  HS=Q1*SIN(PHIS)
00138 52  Q2=COS(PHIL)
00139 53  Q1=RE+ZL
00140 54  XL=Q1*COS(THETAL)*Q2
00141 55  YL=Q1*SIN(THETAL)*Q2
00142 56  HL=Q1*SIN(PHIL)
00143 57  ABD=COSINV((XS*XL)+(YS*YL)+(HS*HL))/(SQRT(XS**2+YS**2+HS**2))
00144 58  1/SQRT(XL**2+YL**2+HL**2))
00145 59  DO 3 I=1,3
00146 60  3 C(I)=0.0
00147 61  C FROM HERE TO STATEMENT 4 FINDS THE VECTOR (C) FROM THE TARGET TO THE
00148 62  SATELLITE
00149 63  A(I,1)=SIN(PHIL)*COS(THETAL)
00150 64  A(I,2)=-SIN(THETAL)
00151 65  A(I,3)=COS(PHIL)*COS(THETAL)
00152 66  A(2,1)=SIN(PHIL)*SIN(THETAL)
00153 67  A(2,2)=CO5(THETAL)
00154 68  A(2,3)=SIN(THETAL)
00155 69  A(1,1)=SIN(PHIL)
00156 70  A(1,2)=0.0
00157 71  A(1,3)=SIN(THIL)
00158 72  B(I)=XS*XL
00159 73  B(I)=YS*YL
00160 74  B(I)=HS*HL
00161 75  DO 4 I=1,3
00162 76  4 C(I)=A(I,M)*B(M)+C(I)
00163 77  PHIL=PHIL/CON
00164 78  PHIL=PHIL/CON
00165 79  THETAL=THETAL/-CON
00166 80  PHIS=PHIS/CON
00167 81  THEtas=THEThAS/-CON
00168 82  PHI=ATAN2(SQRT(C(1)**2+C(2)**2),C(3))
00169 83  IF (PHI.GT.017) PHI=PHI+0092833
00170 84  IF (PHI/CON.GT.90.0) WRITE (6,88)
00171 85  88 FORMAT (/<<<IX,190.0 DEG,1X,IT IS HIGHLY PROBABLE THAT THE AICRAFT OR SPACE
00172 86  2CRAPT CANNOT SEE THE TARGET,\///)
00173 87  89 CALL MODA1M (ZL+DEL1*5,PP,*4HALTI,IXLAMDA)
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00214     89*   PHI=INT=PHI
00215     90*   Z1=ZL
00216     91*   D1=ANS(3)
00217     92*   WATER1=ANS(13)
00220     93*   XNI=ANS(18)
00221     94*   SUM=0+0
00222     95*   SUM1=0+0
00223     96*   SUM2=0+0
00224     97*   SUM3=0+0
00225     98*   SUM4=0+0
00226     99*   DO 1 I=1,32000
00231    100*   Z2=Z1+DELT
00232    101*   S1=RE+Z1
00233    102*   S2=RE+Z2
00234    103*   CALL MODATM (Z2+DELT*5,PP,4HALTI,XLAMDA)
00235    104*   D2=ANS(3)
00236    105*   WATER2=ANS(13)
00237    106*   XN2=ANS(18)
00240    107*   PS1=SININV(S1*SIN(PHI)/S2)
00241    108*   PHI=PR=SININV(S1*SIN(PHI)*XN1/(S2*XN2))
00242    109*   DUM=D1*S1*SIN(PHI-P51)/SIN(P51)*1+0E+05
00243    110*   SUM1=SUM1+PHI-P51
00244    111*   SUM2=SUM2+WATER1+DUM/1000+0
00245    112*   SUM3=SUM3+DUM
00246    113*   SUM4=SUM4+DUM/D1
00247    114*   IF (Z2+GE+Z5) GO TO 82
00251    115*   SUM=SUM+ABS(PHIPR-P51)
00252    116*   PHI=PHIPR
00253    117*   Z1=Z2
00254    118*   D1=D2
00255    119*   WATER1=WATER2
00256    120*   1 XNI=XN2
00260    121*   CALL EXIT
00261    122*     82 CONTINUE
00262    123*   Q=SUM1-ABD
00263    124*   PHI=PHINT-Q/2+0
00264    125*   IF (ABS(Q)+GE+0001) GO TO 89
00266    126*   ANS(21)=PHI
00267    127*   ANS(22)=SUM3
00270    128*   ANS(23)=SUM2
00271    129*   ANS(24)=SUM4
00272    130*   IF (PHI/CONST.LE.+90.+0) GO TO 83
00274    131*   WRITE (6,B7)
00276    132*     87 FORMAT (1X)//,1X,THE ANGLE FROM ZENITH IS GREATER THAN 90.+0)
FUNCTION COSINV(A)
2. C THIS FUNCTION CALCULATES THE INVERSE COSINE OF A.
3. COSINV=ATAN2(SQR(T(1-Q*A**2))*A)
4. RETURN
5. END

END OF UNIVAC 1108 FORTRAN V COMPILATION, 0 DIAGNOSTIC MESSAGE(S)

FUNCTION SININV(A)
2. C THIS FUNCTION CALCULATES THE INVERSE SINE OF A.
3. SININV=ATAN2(A*(SQR(T(1-Q*A**2))))
4. RETURN
5. END

END OF UNIVAC 1108 FORTRAN V COMPILATION, 0 DIAGNOSTIC MESSAGE(S)

FUNCTION Q(P,T)
2. C Q = SPECIFIC HUMIDITY WITH UNITS OF GM/KG
3. C SPECIFIC HUMIDITY=GM OF WATER VAPOR / (KG OF AIR INCLUDING WATER VAPOR)
4. X=EIT
5. Q=Q*621979X/(P=0.37803X)*1000+0
6. RETURN
7. END

END OF UNIVAC 1108 FORTRAN V COMPILATION, 0 DIAGNOSTIC MESSAGE(S)
FUNCTION ALTITU (TVHIGH,TVLOW,PHIGH,PLOW,HLOW)

GIVEN THE TEMPERATURE AND PRESSURE AT EACH OF 2 POINTS AND THE ALTITUDE OF
THE LOWER POINT, THIS FUNCTION CALCULATES THE ALTITUDE OF THE HIGHER POINT.

ALTITU IS IN METERS, CONN IS A CONSTANT = -M*G/R

DATA CONN=-3.41*31977L*02/
D=TVHIGH-TVLOW
IF(D) 2,3,2
D=CONN/(ALOG(PHIGH/PLOW)*ALOG(TVHIGH/TVLOW))
ALTITU =HLOW+(TVHIGH-TVLOW)/D
GO TO 6
3 ALTITU =HLOW+TVLOW*ALOG(PHIGH/PLOW)/CONN
6 RETURN
END
FUNCTION PRES(PLOW,D,TVLOW,TVHIGH,DH)
DATA CONN=-3.4163197E-02/
C
C THIS PROGRAM CALCULATES PRESSURE -PRES- AT SOME POINT -DH- ABOVE A
C POINT IN THE ATMOSPHERE HAVING PRESSURE -PLOW- WHERE -D- IS THE
C TEMPERATURE GRADIENT AND -TVHIGH- AND -TVLOW- ARE CORRESPONDING
C TEMPERATURES -CONN- IS CONSTANT = -M*G/R
C
C IF(D) 2,3,2
C PRES=PLOW*(TVHIGH/TVLOW)**(CONN/D)
C GO TO 4
C PRES=PLOW*EXP(-CONN*DH/TVLOW)
C RETURN
C END

END OF UNIVAC 1108 FORTRAN V COMPILATION.  O *DIAGNOSTIC* MESSAGE(S)
C FORMULA FOR VAPOR PRESSURE OVER WATER

\[ E = 1013.246 \times 10^{-2} \times 10^{-4} \times (1 - 10^4 TS/X)^5 \times 1.28185 \times 10^4 \times \log_{10}(TS/X) - 1.3816 \]

\[ E = 0.7 \times (10 - 10^{0.45} \times (1 - 10^4 TS/X)^2 + 10) - 8.1328 \times 10^{-2} \times (10^{-1} \times 10^{-2} (-3.59414) - 1) \]

GO TO 5

E = 0 * 0

RETURN

FUNCTION R(S,P,X)

C THIS ROUTINE CALCULATES THE MIXING RATIO (GM OF H2O/KG OF DRY AIR)

C BASED ON X WHICH IS TEMPERATURE IN DEG KELVIN

C R(S,P,X) = 0/00 (IE PARTS PER THOUSAND)

C S IS VAPOR PRESSURE OF WATER

C P IS TOTAL ATMOSPHERIC PRESSURE IN MB

C IF (S) 7,6,7

CONTINUE

R = 1.8016 \times 5 \times (P - S \times F(P,X)) \times 1000,0

C R IS IN GM/KG

RETURN

R = 0 * 0

RETURN

END

END OF UNIVAC 1108 FORTRAN V COMPILATION.  O *DIAGNOSTIC* MESSAGE(S)
FUNCTION F(P,A)

DIMENSION TE(12), PE(11), U(12,11)

DATA ((U(I,J)), I=1,12; J=1,12) / 1,1,2,3,4,5,6,12,18,30,42,53,65/

DATA (TE(I)), I=1,12) / 15,23,31,40,49,58,67,76,85,94,103,112/

DO 10 I=1,12
  E=TE(I)
  DO 10 J=1,12
    F=E*U(J,12-I)
  10 CONTINUE

DO 10 I=1,12
  DO 10 J=1,12
    F=F*U(1,12-J)
  10 CONTINUE

END