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GEOS SATELLITE TRACKING CORRECTIONS FOR REFRACTION IN THE TROPOSPHERE

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GSFC

GODDARD SPACE FLIGHT CENTER
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ABSTRACT

Different agencies using the tracking data from the GEOS geodetic tracking systems employ different mathematical models of the atmosphere or different approximations to the same model. Consequently, different refraction corrections are applied by different users of the same data. This paper compares the magnitude, at different elevation angles, of some of the different tropospheric refraction correction formulations utilized by Goddard Space Flight Center, Wallops Island, Smithsonian Astrophysical Observatory, Army Map Service, Naval Air Systems Command, Central Radio Propagation Laboratory, and Eastern Test Range for refraction corrections to the laser, GRARR, SECOR, TRANET, and C-band radar measurements of range, range rate, and elevation angle of earth satellites. At a 15-degree elevation angle and for a nominal surface refractivity of $N_s = 313 \times 10^{-6}$, the maximum differences between the above formulations are about 2.3 meters in range, 1.4 cm/sec in range rate, and 31 seconds of arc in elevation angle.

CONTENTS

ILLUSTRATIONS

TABLES

TABLES (Cont)

APPENDIXES

GEOS SATELLITE TRACKING CORRECTIONS FOR REFRACTION IN THE TROPOSPHERE

INTRODUCTION

The refraction correction formulas for correcting the elevation angle (E), range (R), and range rate ($\dot{R}$) tracking data from the GEOS geodetic tracking systems are not standardized. Different refraction correction formulas are applied by different users of the same data. The different formulas arise from the use of different mathematical models of the atmosphere or from different approximations to the same model.

This document presents some of the different sets of tropospheric refraction correction formulas now being used on GEOS data, and compares the magnitude of the corrections to E, R, and $\dot{R}$ among the different formulations at different elevation angles.

Five sets of refraction correction equations for E, R, and $\dot{R}$ are being used by the Goddard Space Flight Center (GSFC) on GEOS data. These are found in a refraction study done for GSFC in 1964 by Dr. J. J. Freeman and in four GSFC orbit determination programs (ODP's). The four ODP's are the operational orbit differential correction (DC) program, the NONAME definitive orbit program, the geodetic data analysis program (GDAP), and the network analysis program (NAP). These refraction formulas for E, R, and $\dot{R}$ are compared with each other in this document. The formulas for refraction correction in E and R are also compared with the Army Map Service (AMS) and Engineer Topographic Laboratory (ETL) SECOR R refraction correction formulas, with the Wallops Island C-band radar E and R refraction correction formulas, and with the GSFC and Smithsonian Astrophysical Observatory (SAO) optical laser R refraction correction formulas. The formulas for $\dot{R}$ are compared with the Applied Physics Laboratory (APL) Hopfield and Naval Weapons Laboratory (NWL) Astro range rate refraction correction formulas.

All the E and R refraction correction formulas are also compared with the Central Radio Propagation Laboratory (CRPL) tables of the reference exponential troposphere (Reference 1) and with the results of ray tracings through the same CRPL reference exponential troposphere using the Eastern Test Range (ETR) REEK program. In addition, the $\dot{R}$ refraction correction formulas are compared with values of $\frac{d(\Delta R)}{dt} = \dot{E}\,\frac{d(\Delta R)}{dE}$, where $\frac{d(\Delta R)}{dE}$ is the rate of change of REEK ray-trace range error (ΔR) with respect to E obtained by differentiating the polynomial fitted to the curve of ΔR vs E, and $\dot{E}$ is the time rate of change E.

In Appendix A, the tropospheric refraction correction equations are first given as they appear in the referenced source documents and then are converted to a common notation and functional form to simplify the comparisons of these equations.

TROPOSPHERIC REFRACTION CORRECTION FORMULAS FOR E, R, AND $\overset{o}{R}$

The E, R, and $\overset{o}{R}$ tropospheric refraction correction formulas used with GEOS data are listed in Tables 1, 2, and 3, respectively. All formulas listed in these tables employ nominal or first-order correction functions for E, R, and $\overset{o}{R}$, defined as:

$$\Delta E_{to} = N_s \operatorname{ctn} E$$

$$\Delta R_{to} = H N_s \csc E$$

$$\Delta \overset{o}{R}_{to} = H N_s \overset{o}{E} \operatorname{ctn} E \csc E$$

where N_s = refractivity at station (nominally $N_s = 0.000313$)

H = scale height of exponential model of troposphere from the expression $N(h) = N_s e^{-h/H}$ (nominally, if $N_s = 0.000313$ at $h = 0$, then from the CRPL model, $H = 6951.25$ meters)

E = measured elevation angle

$\overset{o}{E}$ = time rate of change of E. In these comparisons $\overset{o}{E}$ is calculated from an overhead pass for a satellite at a height of $(4/3) \times 10^6$ meters (see Appendix B)

At low-elevation angles the nominal correction functions all approach infinity, whereas the true E, R, and $\overset{o}{R}$ tropospheric radio refraction corrections at the horizon ($E = 0^\circ$) are all finite with values of about 0.7°, 100 meters, and 200 cm/sec, respectively. Therefore, some of the formulations employ restraining functions which, when multiplied with the nominal correction functions, produce new correction functions to yield finite refraction corrections closer to the true values at low-elevation angles.

The limiting values for the E, R, and $\overset{o}{R}$ tropospheric refraction corrections obtained from the formulas in Tables 1, 2, and 3 and from the CRPL tables are given in Table 4 for the horizon case ($E = 0^\circ$) and for the zenith case ($E = 90^\circ$). For the zenith case, the E tropospheric radio refraction correction vanishes. At the satellite point of closest approach (PCA) to the station, the $\overset{o}{R}$ tropospheric radio

Table 1

Elevation Angle Refraction Correction Equations

Refraction Formulation	Appendix A Equation	Elevation Angle Refraction Correction ΔE ΔE (radians) = Obs - Corr.
GSFC DC	A1	ΔE_{to}
GSFC Freeman	A4	ΔE_{to}
GSFC NONAME	A7	$\Delta E_{to}\left(\frac{1}{0.93 + 0.0164 \operatorname{ctn} E}\right)$
GSFC GDAP	A10a	$\Delta E_{to}\left(\frac{2}{1 + \sqrt{1 + 0.0045154 \csc^2 E}}\right)$
GSFC NAP-1	A10b	$\Delta E_{to}\left[\frac{0.000350}{N_s}\left(\frac{2}{1 + \sqrt{1 + 0.004 \csc^2 E}}\right)\right]$
Wallops C-band	A17	ΔE_{to}

where $\Delta E_{to} = N_s \operatorname{ctn} E$ = first order correction

Table 2

Range Refraction Correction Equations

Refraction Formulation	Appendix A Equation	Range Refraction Correction ΔR ΔR (meters) = Obs - Corr.
GSFC DC	A2a	$\Delta R_{to}\left[\frac{8750}{H}\left(\frac{1}{\sqrt{1 + 0.000772\, \mathrm{ctn}^2 E}}\right)\right]$
GSFC DC	A2b	$\Delta R_{to}\ \frac{8750}{H}$
GSFC Freeman	A5	$\Delta R_{to}\left(1 - \frac{H}{R_s}\, \mathrm{ctn}^2 E\right)$
GSFC NONAME	A8	$\Delta R_{to}\left[\frac{8432.336}{H}\left(\frac{1}{1 + 0.026\, \csc E}\right)\right]$
GSFC GDAP	A11a	$\Delta R_{to}\left[\frac{7200}{H}\left(\frac{2}{1 + \sqrt{1 + 0.0045154\, \csc^2 E}}\right)\right]$
GSFC NAP-1	A11b	$\Delta R_{to}\left[\frac{2.7432}{HN_s}\left(\frac{2}{1 + \sqrt{1 + 0.004\, \csc^2 E}}\right)\right]$
GSFC & SAO lasers	A13a	$\Delta R_{to}\left(\frac{2.1}{HN_s}\right)$
SAO lasers (after May 1968)	A13b	$\Delta R_{to}\left[\frac{2.238 + 533.5\, N_s}{HN_s(1 + 10^{-3}\cos E \csc^2 E)}\right]$
AMS SECOR	A14	$\Delta R_{to}\left[\frac{2.7}{HN_s}\ \left(\frac{1}{1 + 0.0236\, \mathrm{ctn} E}\right)\right]$
Wallops C-band	A18	$\Delta R_{to}\left(\frac{7600}{H}\right)$

where $\Delta R_{to} = HN_s \csc E = H\Delta E_{to} \sec E$ = first order correction

Table 3
Range Rate Refraction Correction Equation

Refraction Formulation	Appendix A Equation	Range Rate Refraction Correction, $\Delta \dot{R}$ $\Delta \dot{R}$ (meters/sec) = Obs - Corr.
GSFC DC	A3a $(E \leq 10^{\circ})$	$\Delta \dot{R}_{to} \left[\frac{8743.25}{H} \frac{1}{(1 + 0.000772\ ctn^2 E)^{3/2}} \right]$
GSFC DC	A3b $(E > 10^{\circ})$	$\Delta \dot{R}_{to} \left(\frac{8750}{H} \right)$
GSFC Freeman	A6	$\Delta \dot{R}_{to} \left[1 + \frac{H}{R_s} (1 - 3\ csc^2 E) \right]$
GSFC NONAME	A9	$\Delta \dot{R}_{to} \left[\frac{8432.336}{H} \frac{1}{(1 + 0.026\ cscE)^2} \right]$
GSFC GDAP	A12a	$\Delta \dot{R}_{to} \left[\frac{7200}{H} \frac{2}{\sqrt{1 + 0.0045154\ csc^2 E} + (1 + 0.0045154\ csc^2 E)} \right]$
GSFC NAP-1	A12b	$\Delta \dot{R}_{to} \left[\frac{2.7432}{HN_s} \frac{2}{\sqrt{1 + 0.004\ csc^2 E} + (1 + 0.004\ csc^2 E)} \right]$
APL TRANET	A15	$\Delta \dot{R}_{to} \frac{R_s}{H} \sin^2 \Gamma \left[f(E) \right]$
NWL TRANET	A16	$\Delta \dot{R}_{to} \frac{2.3}{HN_s}$

In equation A15,

$$f(E) = 1 + \left(\frac{2R_s}{H_t^2} \sin E \right) \left[\sqrt{H_t^2 + 2R_s H_t + R_s^2 \sin^2 E} - R_s \sin E + (R_s + H_t) \ln \frac{R_s (1 + \sin E)}{(R_s + H_t) + \sqrt{H_t^2 + 2R_s H_t + R_s^2 \sin^2 E}} \right]$$

where $\Delta \dot{R}_{to} = -HN_s \dot{E}\ cscE\ ctnE = -\Delta R_{to} \dot{E}\ ctnE = -H \Delta E_{to} \dot{E}\ cscE$ = first order correction refraction correction vanishes. For the overhead pass assumed here, PCA occurs at zenith and all the E and $\dot{R}$ refraction corrections calculated from the formulas listed here vanish as they should at $E = 90^{\circ}$.

Table 4
Limiting Values for E, R, and $\dot{R}$ Refraction Corrections
(Using $N_s = 0.000313$, H = 6951.25 Unless Built-In)

Refraction Formula	Horizon Case ($E = 0^{\circ}$)			Zenith Case ($E = 90^{\circ}$)*
	ΔE (degrees)	ΔR (meters)	$\Delta \dot{R}$ (cm/sec)	ΔR (meters)
DC	∞	98.6	331.1	2.74
Freeman	∞	∞	∞	2.18
NONAME	1.09	101.5	364.7	2.57
GDAP	0.53	67.1	93.2	2.25
NAP-1	0.63	86.8	128.1.	2.74
Laser	—	∞	—	2.10
SECOR	—	114.4	—	2.70
C-band	∞	∞	—	2.22
Hopfield	—	—	186.5	—
Astro	—	—	∞	—
CRPL	0.72	104.2	—	—
REEK	0.57	77.1	—	2.18

In the Freeman correction formulas for R and $\dot{R}$, the restraining function does not prevent the correction from becoming infinite at the horizon ($E = 0^{\circ}$). However, it does serve to improve the nominal correction values for higher elevation angles ($E > 8^{\circ}$), as will be seen later. Dr. Freeman claims validity for his equations only for $E > 30^{\circ}$.

GRAPHS OF TROPOSPHERIC REFRACTION CORRECTIONS FOR E, R, AND $\dot{R}$

In Figures 1, 2, 3, and 4, the E, R, and $\dot{R}$ refraction corrections from the different formulas and from the REEK ray traces and CRPL tables are plotted against elevation angle. In all cases the tracking station is assumed to be at sea level. For the radio frequency systems, a refractivity of $N_s = 0.000313$ and a compatible scale height of H = 6951 meters from the CRPL tables are assumed if not otherwise specified. The value $N_s = 0.000313$ is the long-term average value at the earth's surface for the United States (Reference 13). For comparison of the optical

*For the zenith case ($E = 90^{\circ}$), all formulas for ΔE and $\Delta \dot{R}$ yield a zero correction.

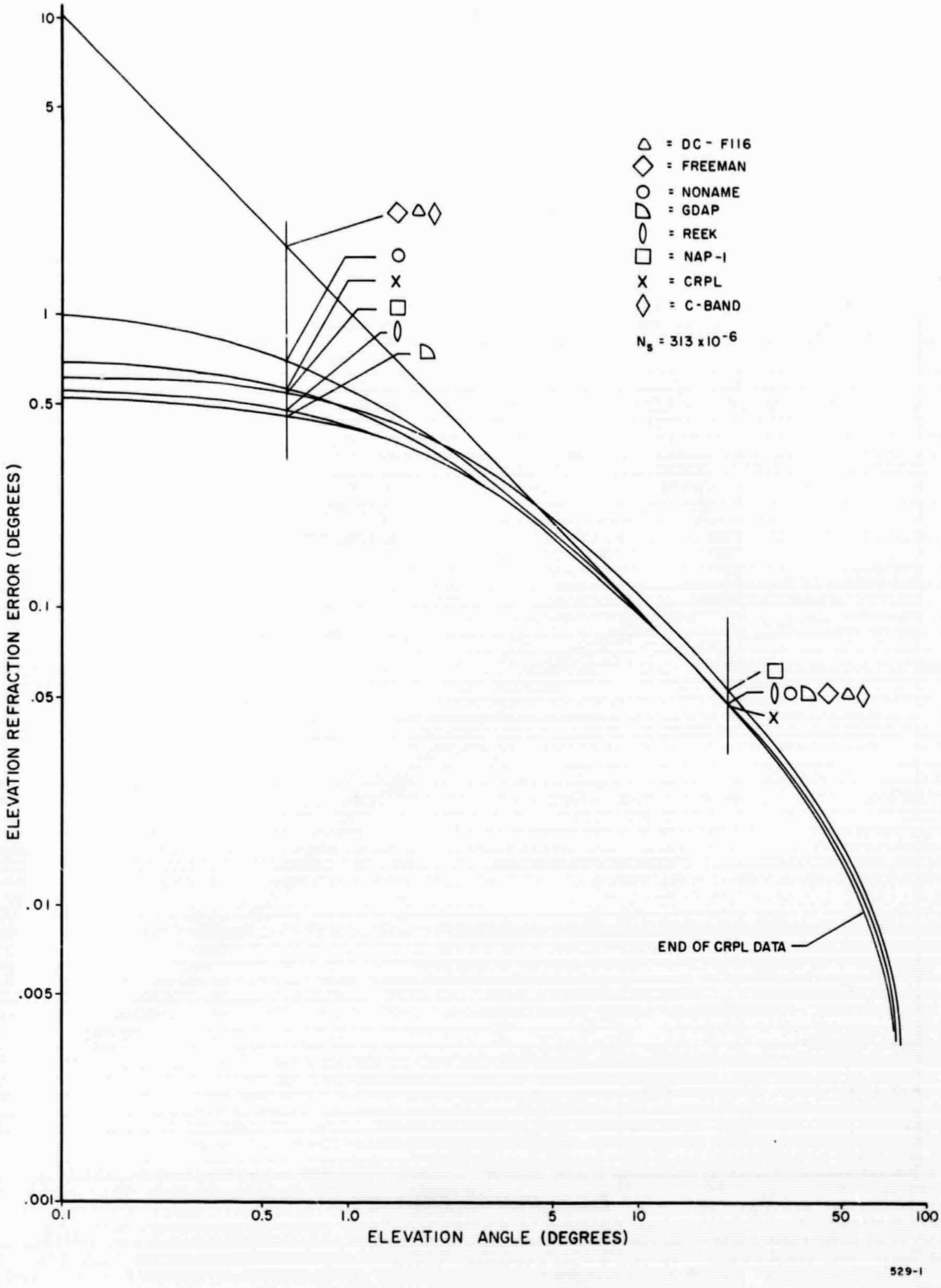

Figure 1. Elevation Error Due to Tropospheric Refraction

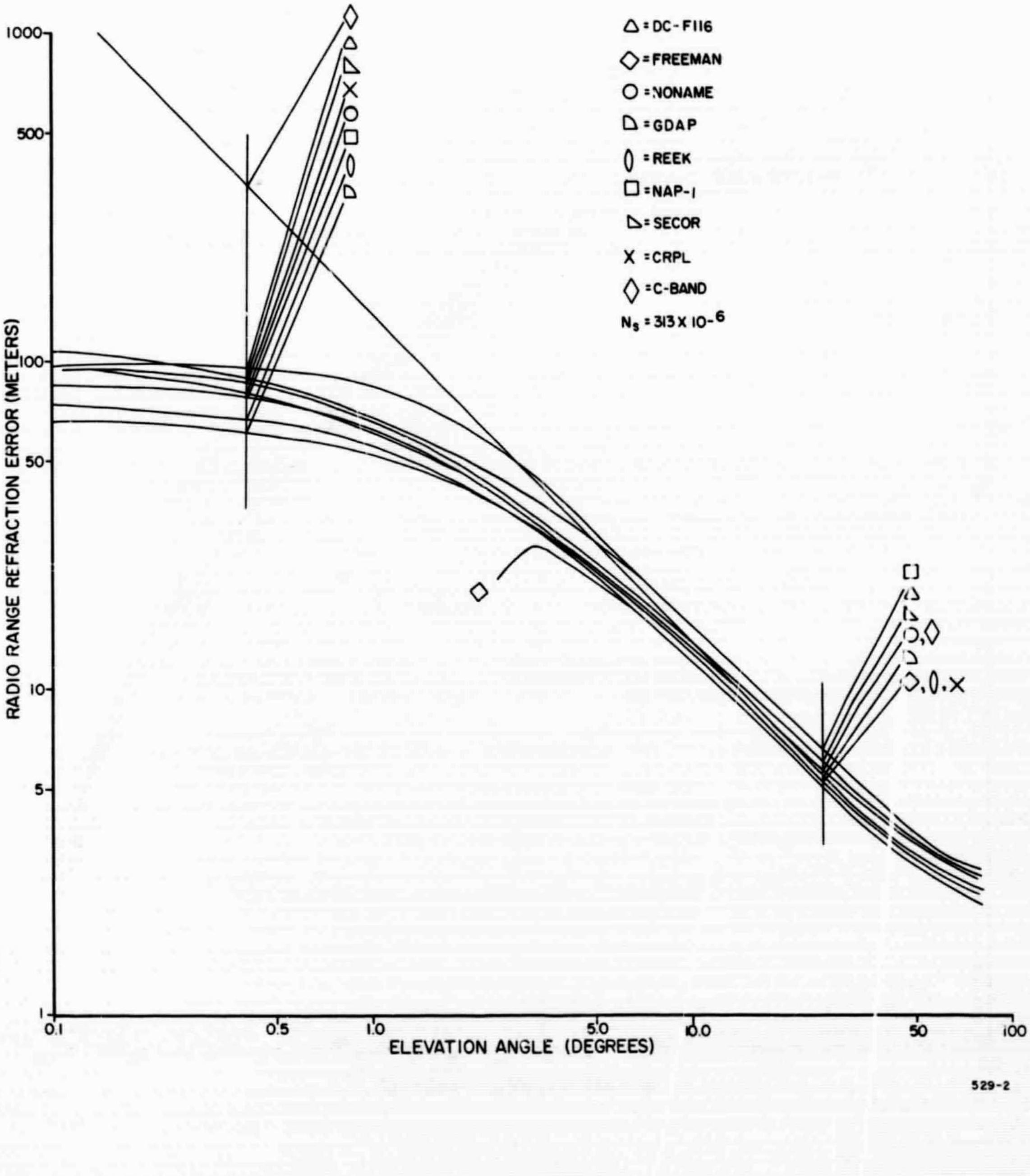

Figure 2. Range Error Due to Tropospheric Refraction

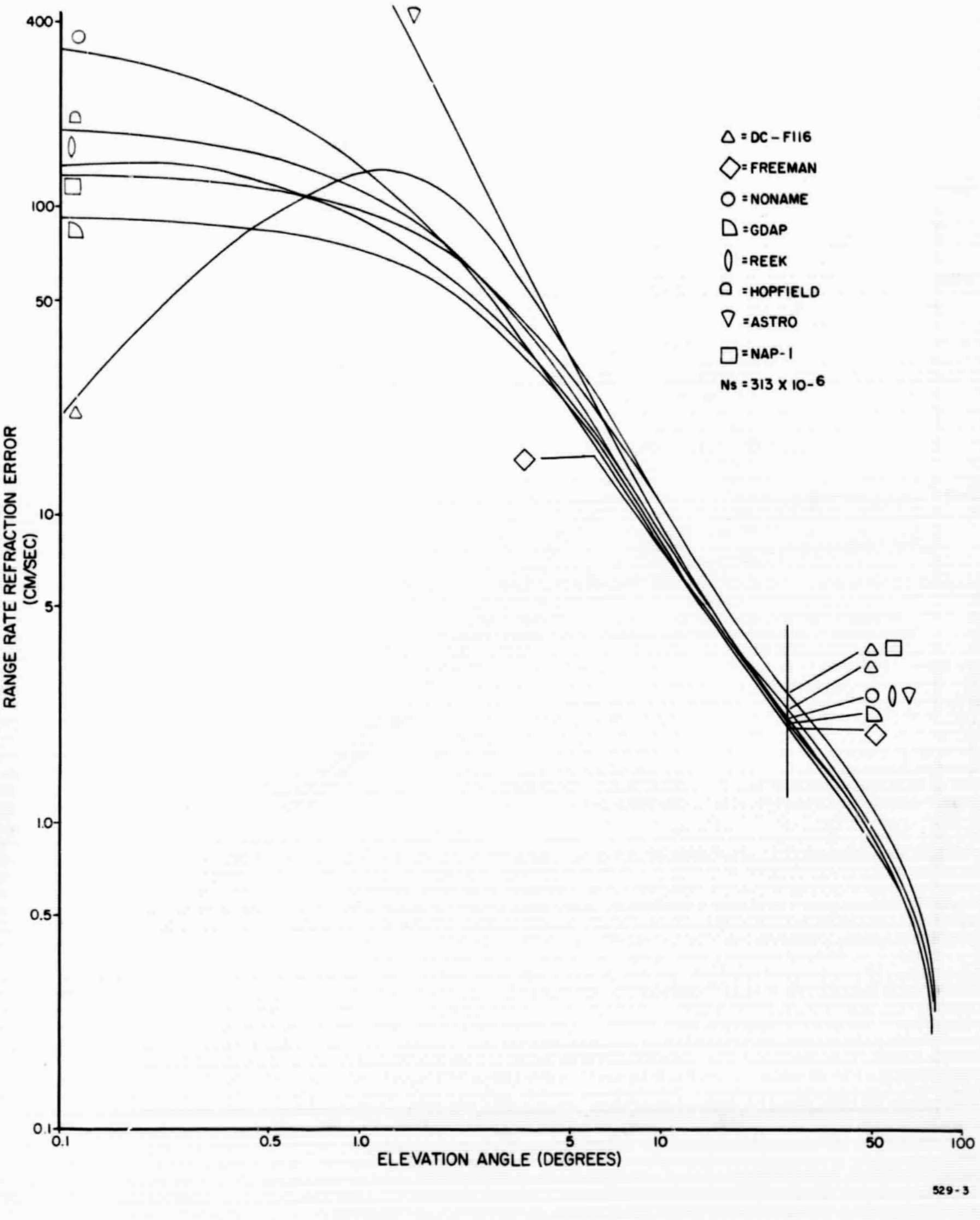

Figure 3. Range Rate Error Due to Tropospheric Refraction

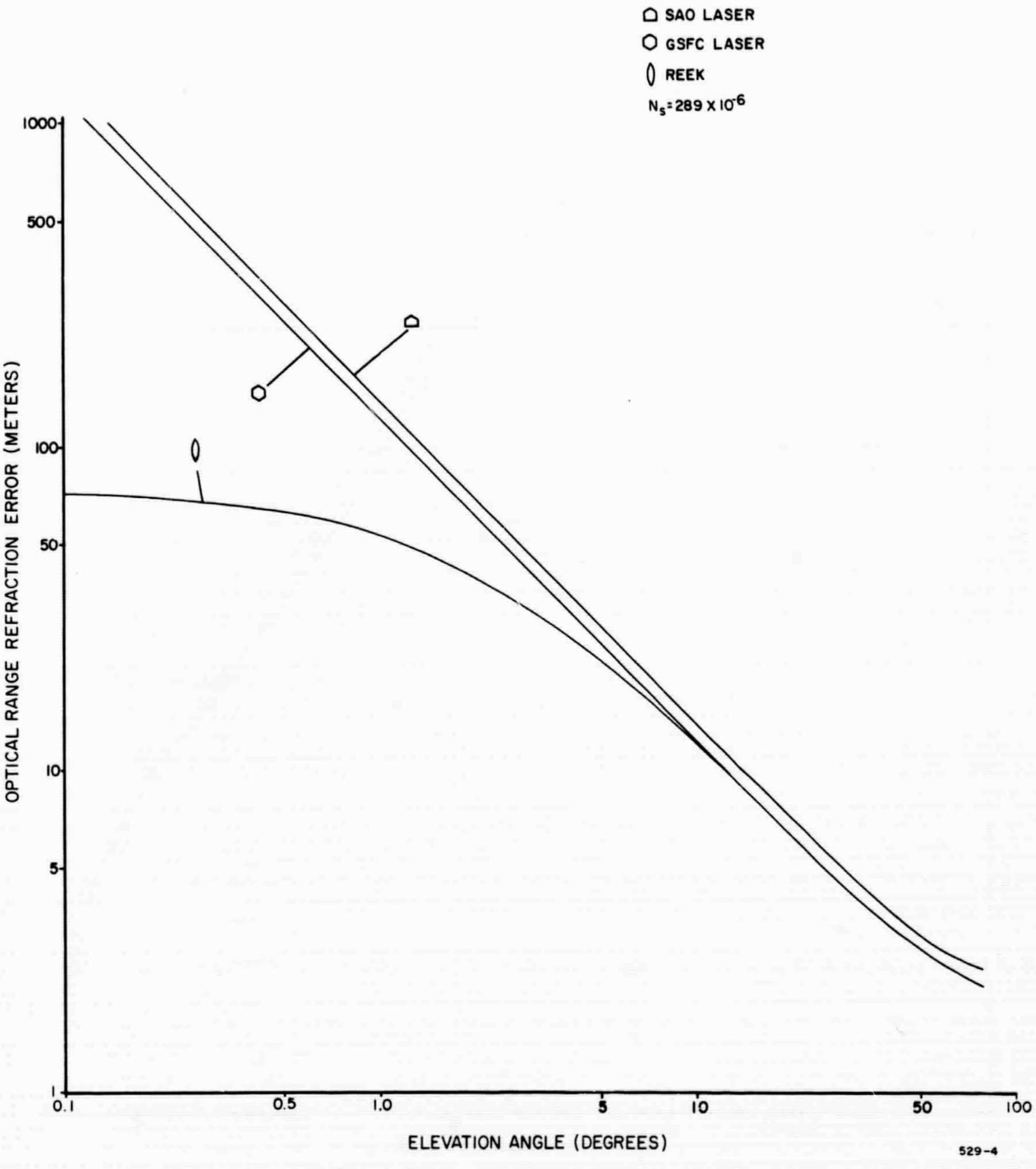

Figure 4. Range Error Due to Tropospheric Refraction, Laser Systems

frequency laser formula, an $N_s = 0.000289$ and a compatible scale height of $H = 7367$ meters are used in REEK, since they are closer to the average values for the laser optical frequency.

In Figures 5, 6, and 7, the E, R, and $\overset{o}{R}$ refraction corrections from the different formulas are plotted against N_s for several different elevation angles to show the amount of variation which can arise from different values of N_s. For each value of N_s in the Freeman formulas for ΔR and $\Delta\overset{o}{R}$, a compatible value of H is determined from the CRPL tables. A nominal H is built into all the other formulas for ΔR and $\Delta\overset{o}{R}$.

In Figures 8, 9, and 10, the restraining functions which modify the nominal refraction corrections for E, R, and $\overset{o}{R}$ are plotted against elevation angle to show more clearly the essential differences between the various refraction correction formulas.

DEVIATION AMONG REFRACTION CORRECTIONS FOR CONSTANT N_s

For elevation angles above about 10^o, where most satellite tracking is done, the refraction error curves in Figures 1, 2, 3, and 4 deviate from each other much less than at lower elevation angles. Also, the magnitude of the corrections is much less at the higher elevation angles. The deviations among correction formulas are shown more clearly in the restraining function plots of Figures 8, 9, and 10. For comparison purposes, an effective restraining function for each REEK ray trace is obtained by dividing the complete REEK correction at each E by the appropriate nominal correction at that E.

The maximum deviations among the corrections in Figures 1, 2, and 3 are given in Table 5 for several angles.

Table 5
Maximum Deviation Among Refraction Corrections

E (degrees)	ΔE (arc sec)	ΔR (meters)	$\Delta\overset{o}{R}$ (cm/sec)
2.5	450.4	33.2	75.2
4.0	162.0	12.2	33.8
10.0	46.1	3.5	2.8
15.0	28.5	2.3	1.4
40.0	9.1	1.0	0.4

It is evident that some of the spread among the computed refraction corrections could be reduced by a different choice of built-in constants in the formulas.

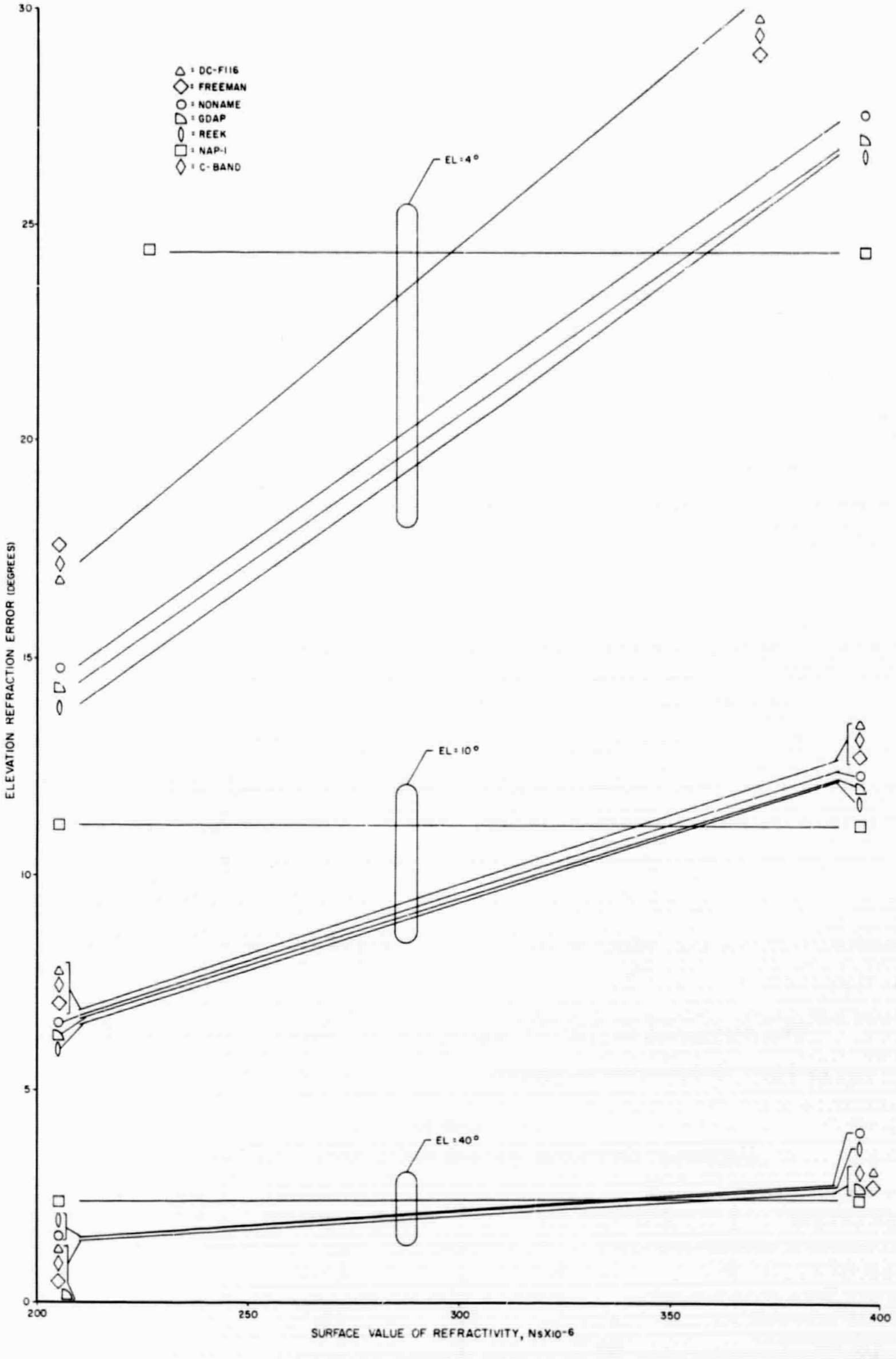

Figure 5. Elevation Angle Error versus Surface Value of Refractivity

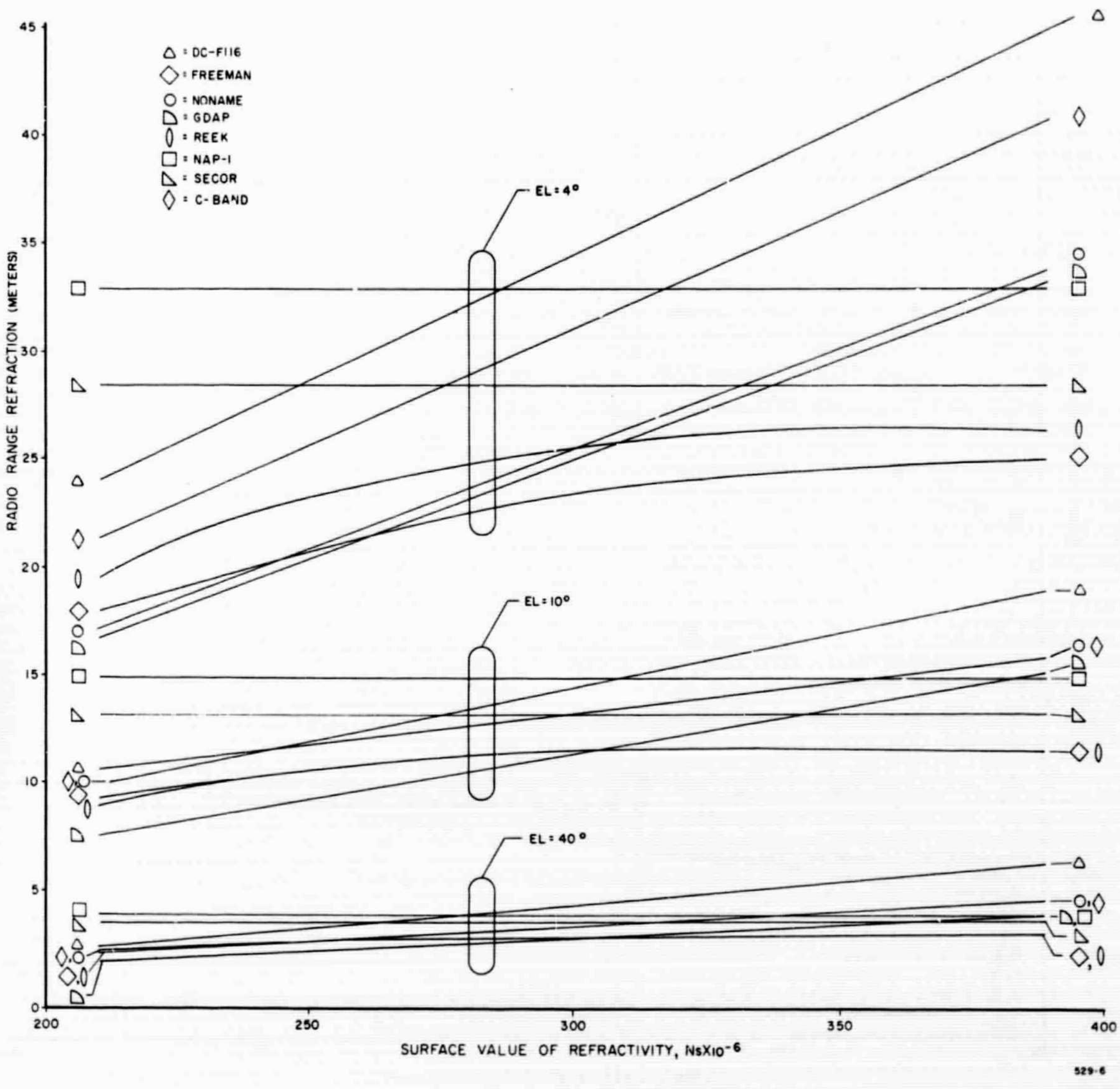

Figure 6. Range Error versus Surface Value of Refractivity

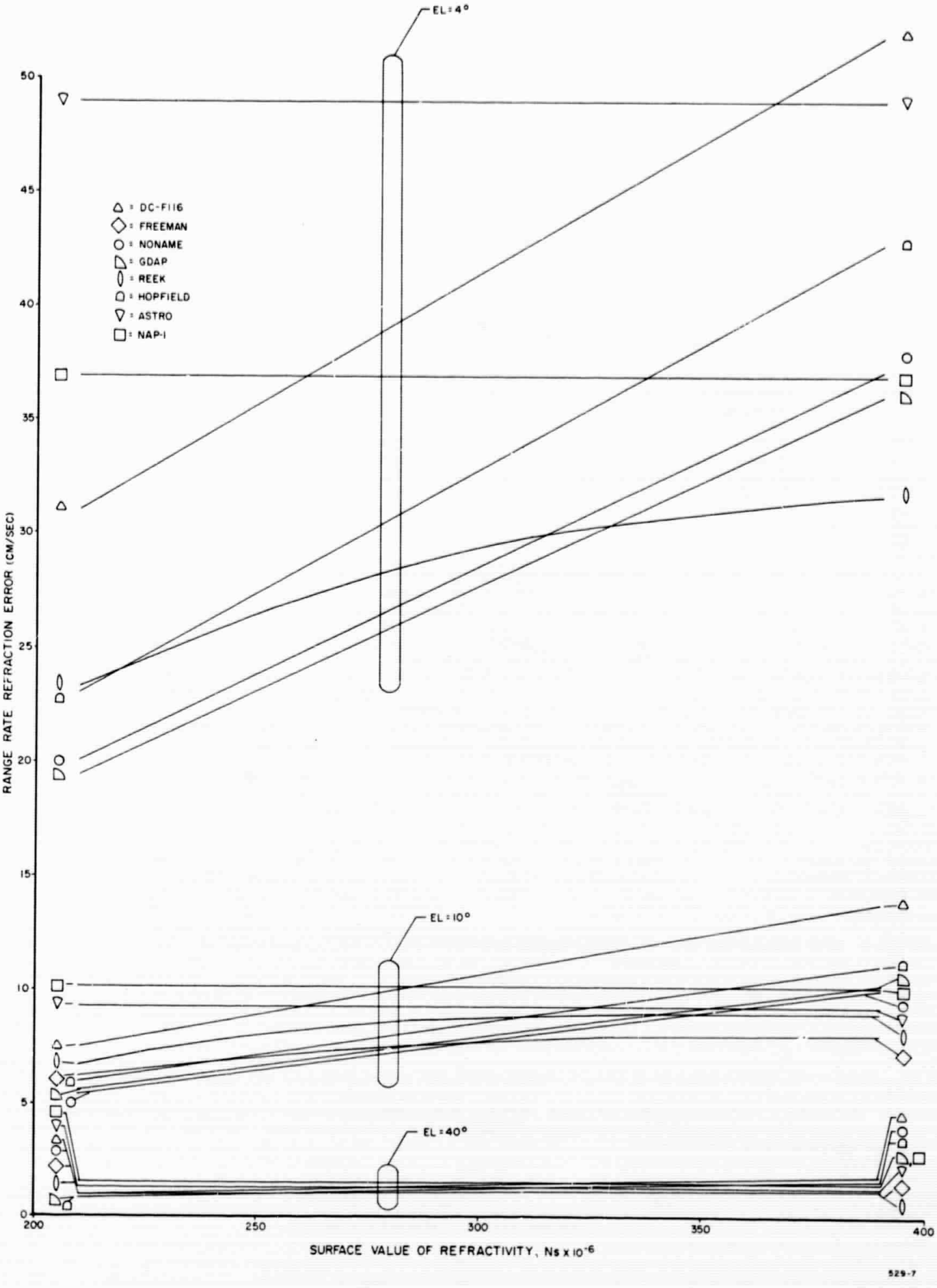

Figure 7. Range Rate Error versus Surface Value of Refractivity

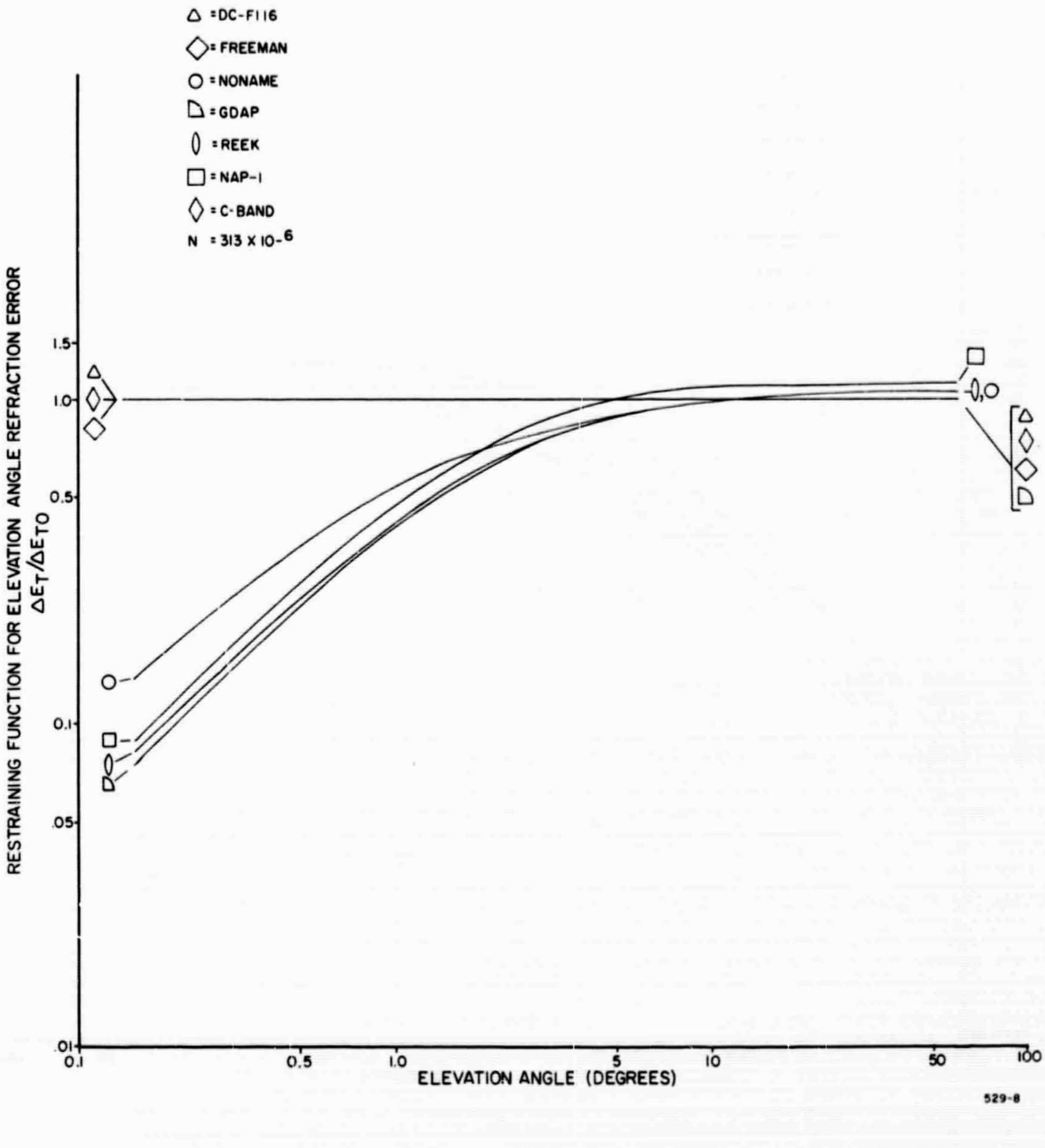

Figure 8. Ratio of ΔE_t to ΔE_{to} for Elevation Tropospheric Error

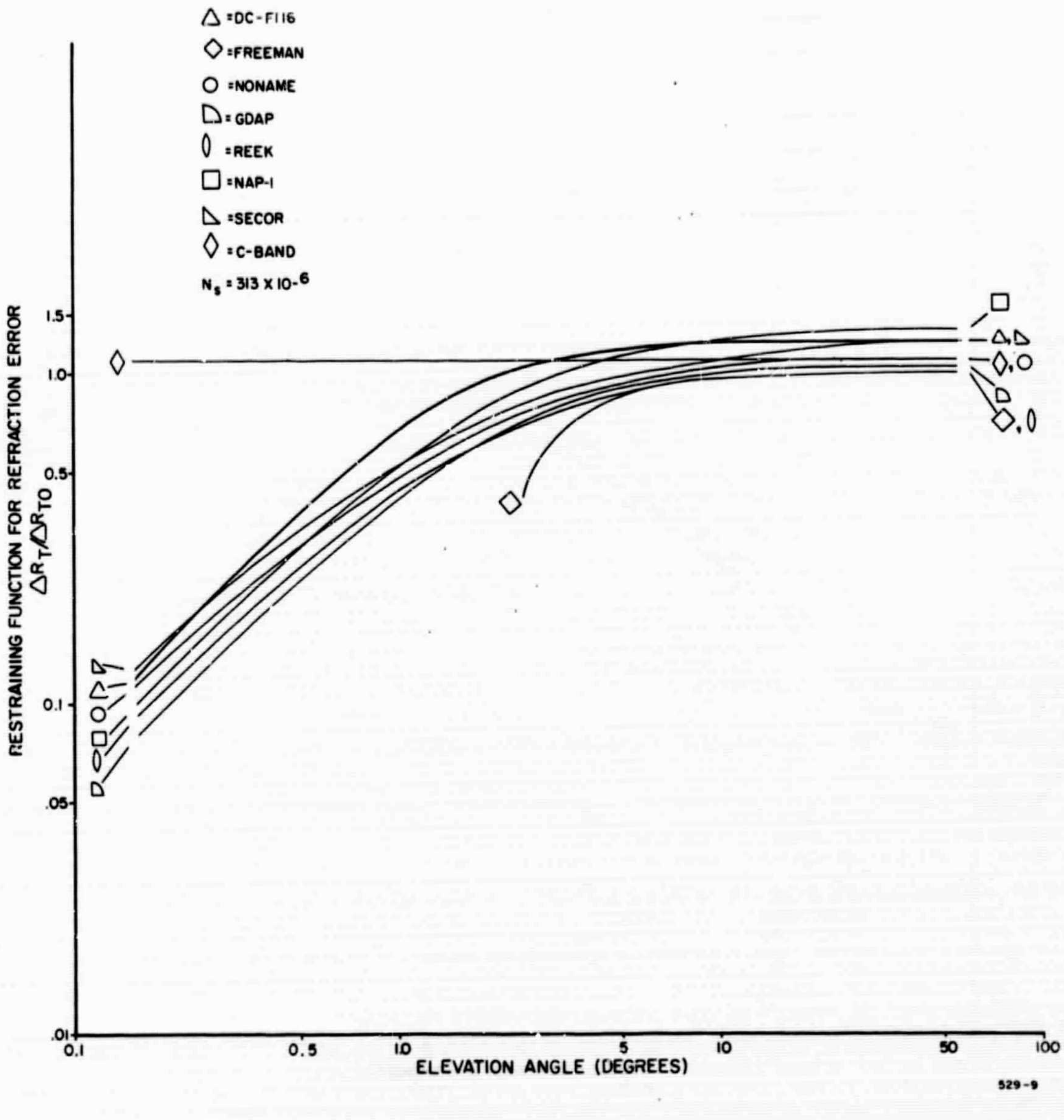

Figure 9. Ratio of ΔR_t to ΔR_{to} for Range Tropospheric Error

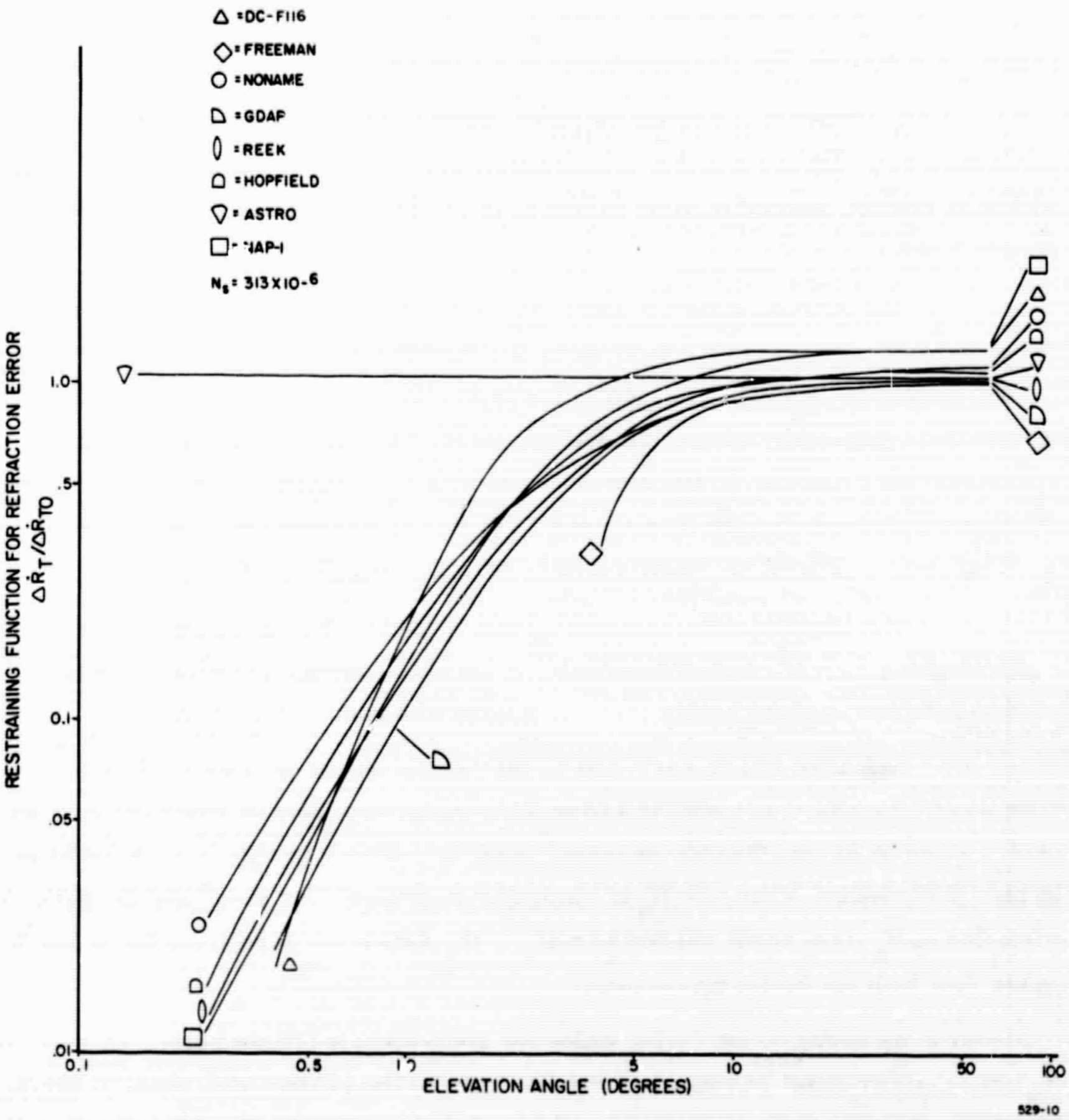

Figure 10. Ratio of $\Delta \overset{o}{R}_t$ to $\Delta \overset{o}{R}_{to}$ for Range Rate Tropospheric Error

VARIATION IN REFRACTION CORRECTIONS WITH CHANGE IN N_s

In Table 1, all the refraction correction formulas for E are linear functions of N_s for a given E, except the NAP-1 formula which uses a constant built-in value of $N_s = 0.000350$.

Since the NAP-1 formula is otherwise practically the same as the GDAP formula, the NAP-1 formula yields the same correction for E as the GDAP formula near $N_s = 0.000350$, but diverges for other values of N_s.

In Tables 2 and 3, the DC, NONAME, GDAP, and C-band refraction correction formulas for R and $\overset{o}{R}$ are all linear functions of N_s for constant E. These formulas contain nominal built-in values of H = 8750, 8432.336, 7200, and 7600 meters, respectively.

The NAP-1, laser, SECOR, and Astro refraction correction formulas for R or $\overset{o}{R}$ all contain nominal built-in values for the product HN_s. These are 2.7432 meters for NAP-1, 2.1 meters for laser, 2.7 meters for SECOR, and 2.3 meters for Astro. Therefore, the NAP-1, laser, SECOR, and Astro refraction correction formulas for R and $\overset{o}{R}$ do not vary with N_s. Again, since the NAP-1 and GDAP formulas are practically the same, the NAP-1 formula yields the same corrections for R and $\overset{o}{R}$ as the GDAP formula near $N_s = \frac{2.7432}{7200} = 0.000381$.

The Hopfield refraction correction formula for $\overset{o}{R}$ is a linear function of N_s for a given E, but uses a tropospheric scale height associated with a quadratic model of the troposphere rather than the H used with exponential models.

As shown in Figures 6 and 7, the refraction corrections for R and $\overset{o}{R}$ from the REEK ray-trace and the Freeman formulas are the only nonlinear functions of N_s, since these are the only functions which introduce a change in H, as indicated in the CRPL tables, whenever N_s is changed. In general, for $E > 8^o$ and for normal variation in N_s from about 253 to 377×10^{-6}, the Freeman corrections for R and $\overset{o}{R}$ agree best with the REEK corrections.

As shown in the CRPL tables for some sample tropospheres, changes in N_s are partially offset by opposite changes in H, so the product HN_s tends to remain constant. The product HN_s is contained in all the formulas for R or $\overset{o}{R}$ refraction corrections. However, those formulas which allow N_s to change, but use a built-in value of H, introduce more error than the formulas which use a built-in constant value for the product HN_s. For example, from reference (7) the variation in N_s is about $\pm 60 \times 10^{-6}$ about a mean value of 313×10^{-6}, a variation of about $\pm 20\%$.

From the CRPL model this leads to a variation in HN_s of less than 8%, as shown in Table 6.

Table 6
HN_s Variations from CRPL Tables

N_s (10^{-6})	$\%\Delta(N_s)$	H (meters)	HN_s (meters)	$\%\Delta(HN_s)$
252.9	-19.2%	7920.85	2.00318	-7.9%
313.0	0	6951.25	2.17574	0
377.2	20.5%	5772.81	2.17750	+0.1%

Thus, if the center nominal constant value for HN_s is used in the refraction correction formulas for R and $\dot{R}$, normal tropospheric variations may lead to an error of -7.9% for low values of N_s, and negligible error for high values of N_s. Correcting for N_s alone without a corresponding change in H leads to overcorrection errors up to 20% for high values of N_s and up to 11% for low values of N_s.

RELATIONSHIPS BETWEEN REFRACTION CORRECTIONS FOR E, R, AND $\dot{R}$

NOMINAL CORRECTIONS

Several relationships connecting the nominal refraction correction formulas for E, R, and $\dot{R}$ are apparent by inspection of these formulas. They are:

$$\Delta E_{to} = N_s \operatorname{ctn}E = \frac{\Delta R_{to}}{H} \cos E$$

$$\Delta R_{to} = HN_s \csc E = H \Delta E_{to} \sec E$$

$$\Delta \dot{R}_{to} = -HN_s \dot{E} \csc E \operatorname{ctn} E = -\Delta R_{to} \dot{E} \operatorname{ctn} E = -H \Delta E_{to} \dot{E} \csc E$$

$$= \frac{d\,(\Delta R_{to})}{dt} = \dot{E} \frac{d\,(\Delta R_{to})}{dE}$$

ΔE AND ΔR RELATIONSHIP

It is of interest to determine whether relationships similar to the above also hold for the complete GEOS tropospheric refraction correction formulas for E, R, and $\dot{R}$ in Tables 1, 2, and 3, including those formulas with restraining functions.

For the GDAP and NAP-1 GEOS tropospheric refraction correction formulas in Tables 1 and 2, a similar relationship is found to exist; i.e.,

$$\Delta E = \frac{\Delta R}{H} \cos E \quad \text{holds for GDAP and NAP-1}$$

However, this relationship does not hold for the DC, Freeman, and NONAME formulas. The DC and Freeman formulas for ΔE are merely the nominal correction formulas ΔE_{to}, whereas the DC and Freeman formulas for ΔR represent a higher-order approximation to the true values. It is probably necessary that the level of approximation in the ΔE and ΔR formulas be the same if the above relationship is to hold. The NONAME formulas for ΔE and ΔR may also represent slightly different levels of approximation.

ΔR AND $\Delta \overset{o}{R}$ RELATIONSHIP

It can easily be shown that, for all the GEOS tropospheric refraction correction formulas in Tables 2 and 3, the range rate refraction correction formula for $\Delta \overset{o}{R}$ is simply the time derivative of the formula for ΔR.

Thus,

$$\Delta \overset{o}{R} = \frac{d\,(\Delta R)}{dt} = \overset{o}{E}\,\frac{d\,(\Delta R)}{dE}$$

holds for the DC, Freeman, NONAME, GDAP, and NAP-1 formulas. This is not generally stated in the source documents.

The GEOS range rate measurements are all basically derived by determining the number of doppler cycles in a short time interval. The number of doppler cycles in this time interval corresponds to the range shift in spacecraft position relative to the station during the same time interval, where range shift is measured in wavelengths of the transmitter frequency. The measured range shift includes the differential effect of the atmosphere on the doppler cycles at the beginning and end points in spacecraft position corresponding to the beginning and end points of the short-measurement time interval. Therefore, it is physically reasonable that the range rate refraction correction should be expressed as a differential in range refraction corrections per corresponding time differential or as a derivative, with respect to time, of the range refraction correction formula.

CONCLUSIONS

The purpose of this study was to compare the different tropospheric refraction correction techniques used with GEOS E, R, and $\overset{o}{R}$ data to determine whether significant deviations exist between these techniques. If deviations exist, it is difficult to know which technique gives the best results, since no perfect model of the troposphere is available. However, REEK ray-trace results and the CRPL tabulated results were included as references for comparison of the results from the various formulas, and should indicate which formulas are best.

The following conclusions summarize the results of this study.

1. The GEOS tropospheric refraction correction formulas and the REEK ray-trace results for E, R, and $\overset{o}{R}$ are in poor agreement for $E < 10^{o}$.

2. These formulas and ray-trace results for $E > 10^{o}$, where most satellite tracking is done, differ in E, R, and $\overset{o}{R}$ by insignificant amounts in comparison to most operational requirements. However, for the GEOS Observation Systems Intercomparison Investigation (GOSII), these differences (see page 11) are equivalent in size to observed differences in system biases for some of the best electronic geodetic tracking systems. Therefore, the systematic differences in refraction correction techniques are affecting the GOSII results.

3. The refraction correction formulas for R and $\overset{o}{R}$ contain the product HN_s. For best results, whenever N_s is changed, the value of H should also be changed, using the CRPL model. Otherwise, using HN_s = constant is preferable to using a measured N_s and a constant H.

4. For all the GEOS tropospheric refraction correction formulas examined in this report, the range rate refraction correction for $\Delta \overset{o}{R}$ is simply the time derivative of the formula for ΔR. In the source documents, the $\Delta \overset{o}{R}$ and ΔR formulas are derived independently.

5. The Minitrack Interferometer direction cosine tropospheric refraction correction formulas, which are documented in Reference 2, are the proper nominal corrections for angle trackers which determine the direction of the incident ray. However, for interferometer systems, the refraction error is much smaller, and it is better to make no refraction correction at all to the dual ray interferometer data than to use the single ray angle tracker refraction correction (see Appendixes A and C).

APPENDIX A
REFRACTION CORRECTION FORMULATIONS

APPENDIX A

REFRACTION CORRECTION FORMULATIONS

Different groups using data from the GEOS geodetic tracking systems employ different refraction correction equations. This appendix lists some of the different refraction correction equations, and describes how they are being used. For comparison of the equations, the original forms and notations are converted into a common form and notation.

A.1 GSFC ORBIT DIFFERENTIAL CORRECTION PROGRAM (DC)

The GSFC Orbit Differential Correction Program (DC) was developed by the Advanced Orbit Programming Branch at Goddard. It is used for operational orbit updating and contains optional refraction correction formulas for interferometer direction cosines and for range and range rate. It is documented in reference 2, program no. F116.

A.1.1 DC MINITRACK DIRECTION COSINE CORRECTION OR ELEVATION ANGLE CORRECTION

The direction cosine corrections are:

$$\ell_c = \frac{\ell}{q} \text{ and } m_c = \frac{m}{q} \qquad \text{equation (2), F116}$$

where

ℓ, m = measured direction cosines

ℓ_c, m_c = refraction corrected direction cosines

$$q = \frac{1 + N_o}{1 + A_s - N_1(h, t)/h}$$

$A_s = N_s = (\mu - 1)$ = tropospheric refractivity at the station; a constant for a given station for a given month. For example, a typical $N_s = 0.000313$.

$1 + N_o = 1$

Keeping the tropospheric part

$$\ell_c = \ell(1 + A_s) \qquad\qquad m_c = m\,(1 + A_s)$$

or

$$\Delta \ell = \ell - \ell_c = -A_s \ell \qquad\qquad \Delta m = m - m_c = -A_s m$$

are the changes in observed direction cosines due to refraction in the DC program.

As shown below, these equations are equivalent to a single refraction correction equation for elevation angle.

By definition

$$\ell = \cos E \sin A \qquad\qquad m = \cos E \cos A$$

where E = apparent elevation angle of incident ray
A = apparent azimuth angle of incident ray

Refraction causes small changes, ΔE, in elevation angle, but in azimuth, $\Delta A = 0$

therefore, taking the derivatives

$$\Delta \ell = -\Delta E \sin E \sin A \qquad\qquad \Delta m = -\Delta E \sin E \cos A,$$

$$\Delta E = -\frac{\Delta \ell}{\sin E \sin A} = \frac{\ell A_s}{\sin E \sin A} \qquad\qquad \Delta E = -\frac{\Delta m}{\sin E \cos A} = \frac{m A_s}{\sin E \cos A}$$

$$= A_s \operatorname{ctn} E; \qquad\qquad = A_s \operatorname{ctn} E$$

$$\Delta E = A_s \operatorname{ctn} E = N_s \operatorname{ctn} E = \Delta E_{to} \text{ (radians)} \tag{A1}$$

This equation is the nominal refraction correction for angle trackers measuring the elevation angle of the incident ray and is treated as such for the comparisons in this paper. For further discussion of interferometer refraction, see Appendix C.

A.1.2 DC RANGE CORRECTION

From equation (12), F116

$$\Delta R_1 = R - R_c = C_1 A_s \csc\phi_1 + B_s N_1 (h, t) \csc\phi_2 \qquad E \leq 10^o$$

From equation (14), F116

$$\Delta R_2 = R - R_c = C_1 A_s \csc E + B_s N_1 (h, t) \csc E \qquad E > 10^o$$

where R = measured range

R_c = refraction corrected range

For the troposphere alone

$$\Delta R_1 = C_1 A_s \csc\phi_1 \qquad E \leq 10^o$$

where $\phi_1 = \cos^{-1} (\cos\phi_{1m} \cos E)$

$$\phi_{1m} = \tan^{-1} T_{1m}$$

$$T_{1m} = 0.027759$$

$$\tan\phi_{1m} = 0.027759 = \frac{\sin\phi_{1m}}{\cos\phi_{1m}}$$

$$\cos\phi_{1m} = \frac{\sin\phi_{1m}}{\tan\phi_{1m}} \cong 0.999614 \quad \text{from trigonometric tables}$$

$$\cos\phi_1 = 0.999614 \cos E$$

$$\csc\phi_1 = \frac{1}{(1 - \cos^2\phi_1)^{1/2}} = \frac{1}{\left[1 - (0.999614 \cos E)^2\right]^{1/2}}$$

$$C_1 = 8750 \text{ meters}$$

$$\Delta R_1 = \frac{8750\ A_s}{\left(1 - 0.999614^2 \cos^2 E\right)^{1/2}} = \frac{8750\ N_s}{\left(1 - 0.999228 \cos^2 E\right)^{1/2}} \qquad \text{(A2a)}$$

$$= (HN_s \csc E) \left[\frac{8750}{H} \frac{\sin E}{\left(1 - 0.999228^2 \cos^2 E\right)^{1/2}}\right]$$

$$= \Delta R_o \left[\frac{8750}{H} \frac{1}{\left(1 - 0.000772\ \operatorname{ctn}^2 E\right)^{1/2}}\right] \text{(meters)} \qquad E \leq 10^o$$

$$R_2 = C_1\ A_s \csc E = 8750\ N_s \csc E \qquad \text{(A2b)}$$

$$= (HN_s \csc E)\left(\frac{8750}{H}\right) = \Delta R_o \left(\frac{8750}{H}\right) \cdot \text{(meters)} \qquad E > 10^o$$

A.1.3 DC RANGE RATE CORRECTION

From equation (13) F116, for $E \leq 10^o$

$$\Delta \dot{R}_1 = \dot{R} - \dot{R}_{c1} = \Bigg\{ B_s \left[\underline{\dot{r}}(t) \cdot \underline{r}^*(t)\right] \frac{dN_1(h,\ t)}{dh} \csc \phi_2$$

$$-C_1 A_s \csc^2 \phi_1 \operatorname{ctn} \phi_1 \cos \phi_{1m} \dot{E} \sin E$$

$$-B_s N_1\ (h,\ t) \csc^2 \phi_2 \operatorname{ctn} \phi_2 \left(\frac{R_s}{R_s + h_{ms}(t)}\right) \dot{E} \sin E \Bigg\}$$

From equation (15) F116, for $E > 10^o$

$$\Delta \dot{R} = \dot{R} - \dot{R}_{c2} = \Bigg\{ \left[B_s \underline{\dot{r}}(t) \cdot \underline{r}^*(t)\right] \frac{dN_1\ (h,\ t)}{dh}$$

$$- \left[C_1 A_s + B_s N_1\ (h, t)\right] \dot{E} \operatorname{ctn} E \Bigg\} \csc E$$

where $\overset{o}{R}$ = measured range rate

$\overset{o}{R}_c$ = refraction corrected range rate

The tropospheric part is

$$\Delta \overset{o}{R}_1 = -C_1 A_s \csc^2 \phi_1 \, \mathrm{ctn}\phi_1 \cos\phi_{1m} \overset{o}{E} \sin E \qquad E \leq 10^o$$

$$\Delta \overset{o}{R}_1 = -8750 A_s (.999614) \overset{o}{E} \sin E \csc^3 \phi_1 \cos\phi_1 \qquad E \leq 10^o \quad \text{(A3a)}$$

$$= \frac{-8750 N_s (.999614)^2 \overset{o}{E} \sin E \cos E}{\left[1 - (0.999614 \cos E)^2\right]^{3/2}}$$

$$= (-HN_s \overset{o}{E} \, \mathrm{ctn}E \csc E) \left[\frac{8750}{H} (.999614)^2 \frac{1}{(1 + 0.000772 \, \mathrm{ctn}^2 E)^{3/2}}\right]$$

$$= \Delta \overset{o}{R}_{to} \left[\frac{8743.25}{H} \frac{1}{(1 + 0.000772 \, \mathrm{ctn}^2 E)^{3/2}}\right] \text{(meters/sec)}$$

$$\Delta \overset{o}{R}_2 = -C_1 A_s \overset{o}{E} \, \mathrm{ctn}E \csc E \qquad E > 10^o$$

$$\Delta \overset{o}{R}_2 = -8750 N_s \overset{o}{E} \, \mathrm{ctn}E \csc E \qquad E > 10^o \quad \text{(A3b)}$$

$$= (-HN_s \overset{o}{E} \, \mathrm{ctn}E \csc E)\left(\frac{8750}{H}\right) = \overset{o}{R}_{to} \left(\frac{8750}{H}\right) \text{(meters/sec)}$$

A.2 FREEMAN RANGE CORRECTION

The following formulas were formulated by J. J. Freeman Associates, Inc., under Contract NAS5-9782 for the Operations Evaluation Branch (OEB) at Goddard and are documented in Reference 5. The purpose of the contract was to develop better refraction correction equations for use in the calibration of angle, range, and range rate systems. These formulas are being used by the OEB for Goddard Range and Range Rate System (GRARR) quality assurance, GEOS-GRARR data validation, and some GEOS data intercomparisons.

A.2.1 FREEMAN ELEVATION ANGLE CORRECTION

The Freeman elevation angle correction (flat-earth approximation) is

$$\Delta E = E - E_c = \text{ctn}E \left[N_s \left(1 - \frac{H}{h} \right) - \frac{1}{h} \int_o^h N_i \, dh \right] \qquad \text{(A13) ref. 5}$$

where E = observed elevation angle

E_c = refraction corrected elevation angle

H/h is neglected since the scale height $H \sim 7,225$ meters and target height for GEOS, $h > 10^6$ meters $\therefore \frac{H}{h} \sim .007 \ll 1$

The tropospheric formula is

$$\Delta E = N_s \text{ctn}E = \Delta E_{to} \text{ (radians)} \qquad \text{(A4)}$$

A.2.2 FREEMAN RANGE CORRECTION

The Freeman range correction (quasi-flat-earth approximation) is

$$\Delta R = R - R_c = \frac{1}{\cos\alpha} \left[\int_o^h |N| \, dh - \frac{\tan^2\alpha}{R_s} \int_o^h |N| h' \, dh' \right] \qquad \text{(A25) ref. 5}$$

where R = electromagnetically determined range

R_c = true refraction corrected range (meters)

h = satellite height $\sim 10^3$ km for GEOS

h' = distance from earth surface to ray path

R_s = distance from earth center to surface

$\alpha = (\pi/2 - E)$ = apparent zenith angle

N = (absolute value of tropospheric or ionospheric refractivity = $|\mu - 1|$)

$$\Delta R = \frac{1}{\sin E} \left[\int_o^h |N| \, dh - \frac{\cot^2 E}{R_s} \int_o^h |N| h \, dh \right]$$

In the previous equation, if $|N|$ assumes the value

$$N = N_s e^{-h/H}$$

N_s = surface refractivity

H = scale height $\sim$ 7 km

$$\therefore \text{ at } h' = h, \ |N| = N_s e^{\frac{-1000}{7}} \sim 0,$$

then the tropospheric formula is

$$\Delta R = N_s H \left(1 - H\frac{ctn^2 E}{R_s}\right) cscE \tag{A5}$$

$$= \Delta R_o \left[1 - \frac{H}{R_s} \ ctn^2 E\right]$$

A.2.3 FREEMAN RANGE RATE CORRECTION

From equation (A51), reference 5

$$\Delta \overset{o}{R} = N_p \left\{ V_R + V_\theta \ \tan \alpha \left(1 - \frac{h}{R_s \cos}\right)\right\} -$$

$$\frac{V_\theta \tan \alpha}{R\cos} \int_o^h |N| \left[1 + \frac{h'}{R_s}\left(1 - \frac{3}{\cos^2 \alpha}\right)\right] dh'$$

where $V_\theta = R\overset{o}{E}$

As h' approaches the GEOS satellite height of approximately 10^3 km, the tropospheric refractivity at the target, N_p, approaches zero. Therefore the range rate correction equation reduces to

$$\Delta \overset{o}{R} = -\frac{\overset{o}{E} \cos E}{\sin^2 E}\left[\int_o^h |N| dh' + \frac{1 - 3/\sin^2 E}{R_s} \int_o^h |N| h' dh'\right]$$

In the previous equation, if $|N|$ assumes the value

$$|N| = N_s e^{-h'/H}$$

then the tropospheric formula is

$$\Delta \overset{o}{R} = -HN_s \frac{\overset{o}{E} \cos E}{\sin^2 E} \left[1 + \left(1 - \frac{3}{\sin^2 E} \right) \frac{H}{R_s} \right] \qquad \text{(A6)}$$

$$= -HN_s \overset{o}{E} \text{ctn} E \csc E \left[1 + \frac{H}{R_s} (1 - 3 \csc^2 E) \right]$$

$$= \Delta R_{to} \left[1 + \frac{H}{R_s} (1 - 3 \csc^2 E) \right]$$

A.3 GSFC NONAME PROGRAM

The NONAME program was jointly developed by the Mission Trajectory Determination Branch (MTDB) at GSFC and the Wolf Research and Development Company under NASA contract NAS 5-9756-710. It was designed to produce accurate satellite orbits from geodetic quality data. The MTDB and OEB groups at GSFC have used this program for both long-arc and short-arc GEOS data intercomparisons. The program contains refraction corrections for elevation angle, range, and range rate. The current corrections are somewhat more sophisticated than those used in earlier versions of this program as reported in reference 24.

The following formulas were obtained from Reference 19.

$$\Delta E = \frac{N_s}{(0.0164 + 0.93 \tan E)} = N_s \text{ ctn} E \left[\frac{1}{(0.0164 \text{ ctn} E + 0.93)} \right] \qquad \text{(A7)}$$

$$= \Delta E_o \frac{1}{(0.93 + 0.0164 \text{ ctn} E)}$$

$$\Delta R = \frac{+8432.336 N_s}{(0.026 + \sin E)} = HN_s \csc E \left[\frac{8432.336}{H (1 + 0.026 \csc E)} \right] \qquad \text{(A8)}$$

$$= \Delta R_{to} \left(\frac{8432.336}{H} \right) \frac{1}{1 + 0.026 \csc E}$$

$$\Delta \overset{o}{R} = \frac{8432.336\ N_s \overset{o}{E}\ \cos E}{(0.026 + \sin E)^2} \tag{A9}$$

$$= HN_s\ \overset{o}{E}\ \operatorname{ctn} E\ \csc E \left[\left(\frac{8432.336}{H} \right) \frac{\sin^2 E}{(0.026 + \sin E)^2} \right]$$

$$= \Delta \overset{o}{R}_{to} \left[\frac{8432.336}{H} \ \frac{1}{(1 + 0.026 \csc E)^2} \right]$$

A.4 GSFC GDAP AND NAP-1 PROGRAMS

The Geodetic Data Adjustment Program (GDAP) was developed for the OEB at GSFC by DBA Systems, Inc., under contract NAS 5-9860. It is a short-arc orbit determination program with error model regression capability and has been used on the CDC-3200 computer for GEOS data intercomparisons over data spans of about 1/4 orbit. The Network Adjustment Program (NAP-1) was later developed for the Mission and Systems Analysis Branch (MSAB) at GSFC by the same company under contract NAS 5-10588. NAP-1 extends the GDAP capability to long arcs and to operation on the IBM 360 computers at GSFC. Both GDAP and NAP-1 contain refraction correction equations for elevation angle, range, and range rate. The functional forms of the refraction corrections are the same in both programs, but since different values of the constants were chosen, both sets of equations are given here.

A.4.1 ELEVATION ANGLE REFRACTION CORRECTION

The GDAP, elevation angle refraction correction as given in reference 27, equation (13.1), is

$$\Delta Z = \frac{\alpha_1 \sin Z_o}{\cos Z_o + \sqrt{\cos^2 Z_o + \ell^2}}$$

where $\Delta Z = Z_o - Z_c = -(E - E_c) = -\Delta E$

$Z_o = (\pi/2) - E$ = observed zenith angle

$Z_c = (\pi/2) - E_c$ = refraction corrected zenith angle

$\alpha_1 = 2a = 2N_s$

$r_o = R_s$ = earth radius at station

$a = n_o - 1 = N_s$ = refractivity at station

$\ell^2 = 4H/R_s = \frac{4(7200)}{6378166} = 00.0045154$

H = tropospheric scale height (nominal 7200 meters assumed in GDAP)

R_s = radius of earth at observer (6,378,166 meters at sea level)

$$\Delta E = \frac{2N_s \csc E}{\sin E + \sqrt{\sin^2 E + 4H/R_s}} = (N_s \operatorname{ctn} E)\left(\frac{2}{1 + \sqrt{(1 + 4H/R_s \csc^2 E)}}\right) \qquad \text{(A10a)}$$

$$= \Delta E_{to}\left(\frac{2}{1 + \sqrt{1 + 0.00045154 \csc^2 E}}\right) \text{ (radians) GDAP}$$

The NAP-1 elevation angle refraction correction (given in a private communication) is

$$\Delta E = \frac{\alpha_1 \csc E}{\sin E + \sqrt{\ell^2 + \sin^2 E}}$$

where $\alpha_1 = 0.0007$

$\ell^2 = 4H/R_s = 0.004$

$$\Delta E = \frac{0.0007 \operatorname{ctn} E}{1 + \sqrt{1 + 0.004 \csc^2 E}} = (N_s \operatorname{ctn} E)\left[\frac{0.00035}{N_s} \frac{2}{1 + \sqrt{1 + 0.004 \csc^2 E}}\right] \qquad \text{(A10b)}$$

$$= \Delta E_{to}\left[\frac{0.00035}{N_s}\left(\frac{2}{1 + \sqrt{1 + 0.004 \csc^2 E}}\right)\right] \text{ (radians) NAP-1}$$

A.4.2 RANGE REFRACTION CORRECTION

The GDAP range retraction correction, from reference 4, appendix C, section 1a, is

$$\Delta R_t = \frac{2(n_o - 1)\, H_o}{\sin E + \sqrt{\sin^2 E + \frac{4H_o}{R_o}}}$$

where $R_t = \Delta R$ = (observed range - corrected range)

$n_o = 1 + N_s$ ground index of refraction

$H_o = H$ = scale height (nominal 7200 meters assumed in GDAP)

$R_o = R_s$ = radius of earth at observer (6,378,166 meters at sea level)

E = elevation angle

$$\Delta R = \frac{2N_s H}{\sin E + \sqrt{\sin^2 E + \frac{4H}{R_s}}} = (HN_s \csc E)\left[\frac{(7200)}{H} \frac{2}{1 + \sqrt{1 + \frac{(4)\ 7200}{R_s} \csc^2 E}}\right]$$

$$= \Delta R_{to} \left[\left(\frac{7200}{H}\right) \frac{2}{1 + \sqrt{1 + \frac{28800}{R_s} \csc^2 E}}\right]$$

In GDAP, R_s and N_s are calculated for each pass. For these comparisons, a sea level R_s of 6,378,166 meters and an N_s of 313 (10^{-6}) are assumed.

$$\Delta R = \Delta R_{to} \left[\left(\frac{7200}{H}\right) \frac{2}{1 + \sqrt{1 + 0.0045154 \csc^2 E}}\right] \text{(meters)} \quad \text{GDAP} \qquad \text{(A11a)}$$

The NAP-1 range refraction correction is

$$\Delta R = \frac{\alpha_2}{\sin E + \sqrt{\ell^2 + \sin^2 E}}$$

where $\alpha_2 = 5.4864$ meters

$$\ell^2 = 4H/R_s = 0.004$$

$$\Delta R = \frac{5.4864 \csc E}{1 + \sqrt{1 + 0.004 \csc^2 E}} = (HN_s \csc E) \frac{5.4864}{HN_s\left(1 + \sqrt{1 + 0.004 \csc^2 E}\right)} \quad \text{(A11b)}$$

$$= \Delta R_{to}\left[\frac{2.7432}{HN_s} \frac{2}{1 + \sqrt{1 + 0.004 \csc^2 E}}\right] \text{(meters)} \quad \text{NAP-1}$$

Instead of assuming a nominal H = 7200 meters and calculating N_s for each pass as in GDAP, NAP-1 assumes a nominal $HN_s = 2.7432$ and a nominal $HN_s = 2.7432$ and a nominal $4H/R_s = 0.004$.

A.4.3 RANGE RATE REFRACTION CORRECTION

The GDAP range rate correction, from reference 4, appendix C. section 2a, is

$$\Delta \overset{o}{R} = \frac{-\Delta R_T \overset{o}{E} \csc E}{\sqrt{\sin^2 E + 4H_o/R_o}} = \frac{-2N_s H}{\left(\sin E + \sqrt{\sin^2 E + 4H/R_s}\right)} \frac{\overset{o}{E} \cos E}{\sqrt{\sin^2 E + 4HR_s}} \quad \text{(A12a)}$$

$$= \frac{-2\,(HN_s \overset{o}{E} \operatorname{ctn} E \csc E)\,7200}{H\left(1 + \sqrt{1 + 0.0045154 \csc^2 E}\right)\sqrt{1 + 0.0045154 \csc^2 E}}$$

$$= -\Delta \overset{o}{R}_{to}\left[\left(\frac{7200}{H}\right)\frac{2}{\left(1 + \sqrt{1 + 0.0045154 \csc^2 E}\right)\sqrt{1 + 0.0045154 \csc^2 E}}\right]\left(\frac{m}{sec}\right)$$

GDAP

The NAP-1 range rate refraction correction is

$$\Delta \overset{o}{R} = \frac{-\Delta R \overset{o}{E} \cos E}{\sqrt{\ell^2 + \sin^2 E}} = \frac{-5.4864 \csc E}{\left(1 + \sqrt{1 + 0.004 \csc^2 E}\right)} \frac{\overset{o}{E} \csc E}{\sqrt{\ell^2 + \sin^2 E}} \qquad \text{(A12b)}$$

$$= - \frac{5.4864 \, (HN_s \overset{o}{E} \operatorname{ctn} E \csc E)}{HN_s \left(1 + \sqrt{1 + 0.004 \csc^2 E}\right) \sqrt{1 + 0.004 \csc^2 E}}$$

$$= -\Delta \overset{o}{R}_{to} \left[\left(\frac{2.7432}{HN_s} \right) \frac{2}{\left(1 + \sqrt{1 + 0.004 \csc^2 E}\right) \sqrt{1 + 0.004 \csc^2 E}} \right] \left(\frac{m}{sec} \right) \quad \text{NAP-1}$$

A.5 GSFC and SAO LASER RANGE REFRACTION CORRECTION

The GSFC and Smithsonian Astrophysical Observatory (SAO) lasers employ pulsed ruby rods operated at an optical frequency with a wavelength of 6943 angstroms. At this frequency tropospheric refractivity is primarily a function of pressure and temperature with very little dependence on relative humidity. So far, laser data have been acquired only at high elevation angles ($E > 15^o$), due to transmitter power and reflector limitations. Consequently, both GSFC and SAO use a nominal laser range refraction correction. From references 17, 18, and 28 this is

$$\Delta R = R - R_c = 2.1 \csc E = (HN_s \csc E) \left(\frac{2.1}{HN_s} \right) \qquad \text{(A13a)}$$

$$= \Delta R_{to} \left(\frac{2.1}{HN_s} \right) \quad \text{(meters)}$$

and SAO changed in May 1968 to

$$\Delta R = \frac{2.238 + 0.0414 \, PT^{-1} - 0.238 \, h_s}{\sin E + 10^{-3} \operatorname{ctn} E} = \frac{2.238 + 533.5 \, N_s}{\sin E + 10^{-3} \operatorname{ctn} E} \qquad \text{(A13b)}$$

where P = station pressure in millibars

T = station temperature in degrees Kelvin

$N_s = 77.6 \frac{P}{T} \times 10^{-6} = 0.000289$ for nominal P and T

h_s = station height above mean sea level, assumed to be zero.

A.6 AMS AND ETL SECOR RANGE REFRACTION CORRECTION

The Army Map Service (AMS) tropospheric range refraction correction for SECOR was obtained from reference 20, page 37.

$$\Delta R = \frac{K_1 (1 - e^{-ZR})}{\sin E + K_2 \cos E}$$

where

E = elevation angle

K_1 = 2.7 meters = zenith refraction value

K_2 = 0.0236 = "control constant"

$Z = \frac{1}{H} = \frac{1}{7000}$ meters

R = slant range (meters) $> 10^6$ meters for GEOS

$e^{-ZR} \sim e^{-143} \sim 0$ for GEOS. This term is needed when tracking targets within the troposphere such as aircraft. For any satellite tracks it may be ignored.

$$\Delta R = \frac{2.7 \csc E}{1 + 0.0236 \operatorname{ctn} E} = \frac{2.7}{HN_s} \frac{(HN_s \csc E)}{1 + 0.023 \operatorname{ctn} E} \qquad \text{(A14)}$$

$$= \Delta R_{to} \frac{2.7}{HN_s} \left(\frac{1}{1 + 0.023 \operatorname{ctn} E} \right) \quad \text{(meters)}$$

The Army Engineer Topographic Laboratory (ETL) Tropospheric range refraction correction for SECOR is also given in reference 20, page 49. This is the same as that given above for GDAP in equation (A11a).

A.7 APL TRANET RANGE RATE REFRACTION CORRECTION

The Applied Physics Laboratory (APL) tropospheric range rate refraction correction for TRANET was obtained from reference 8, equations 8 and 9.

$$\Delta \overset{o}{R} = -N_s R_s \overset{o}{E} \left[f(E) \right] = -(HN_s \overset{o}{E} \operatorname{ctn} E \csc E) \frac{R_s}{H} \sin^2 E \left[f(E) \right] \qquad \text{(A15)}$$

$$= - \Delta \overset{o}{R}_{to} \frac{R_s}{H} \sin^2 E \left[f(E) \right]$$

where $$f(E) = 1 + \left(\frac{2R_s}{H_t^2}\sin E\right)\left[A - R_s \sin E + (R_s + H_t)\ell n \frac{R_s(1+\sin E)}{(R_s + H_t) + A}\right]$$

$$A = \sqrt{R_s^2 \sin^2 E + 2R_s H_t + H_t^2}$$

H_t = 23,000 meters = best fitting equivalent thickness of troposphere above station

R_s = 6,378,166 meters in this comparison

This formula was derived from a quadratic rather than an exponential fit to the tropospheric refractivity profile. Consequently it looks quite different than some of the other range rate refraction formulas.

More recently, in reference 25, the same author has further developed and refined this technique for range rate refraction correction by using two separate quartic fits to the wet and dry components of the troposphere.

A.8 NWL TRANET RANGE RATE REFRACTION CORRECTION

The Naval Weapons Laboratory (NWL) tropospheric range rate refraction correction for TRANET is given in equation (A6) of reference 21 and in reference 26 describing the NWL ASTRO computer program.

$$\Delta f = -\frac{fs}{c}\frac{\sin Z}{\cos^2 Z}\overset{o}{Z}\left[h(\bar{n}-1)\right] = -\frac{fs}{c}\Delta\overset{o}{R}$$

where Δf = refraction correction to Doppler frequency

$\Delta\overset{o}{R}$ = range rate refraction correction

Z = zenith angle = $(\pi/2 - E)$

f_s = satellite transmitter frequency

$\bar{n}$ = mean tropospheric refractive index between station and satellite

h = height of satellite above the station

$h(\bar{n} - 1)$ = 2.3 meters for a station at mean sea level

$$\Delta \overset{o}{R} = - \frac{\cos E}{\sin^2 E} \overset{o}{E} \left[h(\bar{n} - 1) \right] = -2.3 \overset{o}{E} \operatorname{ctn} E \csc E = -\Delta \overset{o}{R}_{to} \left(\frac{2.3}{HN_s} \right) \left(\frac{\text{meters}}{\text{sec}} \right) \quad \text{(A16)}$$

A.9 WALLOPS STATION C-BAND PROGRAM

The Wallops Station C-band program is documented in reference 29.

A.9.1 WALLOPS C-BAND ELEVATION CORRECTION

$$\Delta E = N_s \operatorname{ctn} E = E_{to} \quad \text{(A17)}$$

A.9.2 WALLOPS C-BAND RANGE CORRECTION

$$\Delta R = 7600\, N_s \csc E \quad \text{(A18)}$$

$$= \Delta R_{to} \left[\frac{7600}{H} \right]$$

APPENDIX B

ELEVATION RATE DETERMINATION

APPENDIX B

ELEVATION RATE DETERMINATION

The required expression for elevation rate is derived from the geometry shown in Figure B-1.

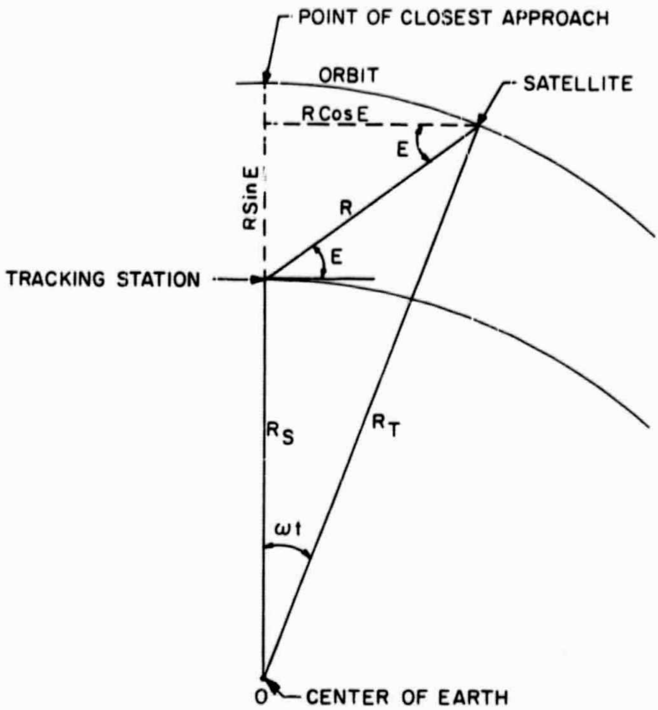

Figure B-1. Plane containing satellite, tracking station, and center of earth. (A circular overhead orbit around a spherical stationary earth is assumed.)

$$\sin\omega t = R\cos E/R_T;\ \cos E = R_T \sin\omega t/R \qquad \text{(B1)}$$

$$\cos\omega t = (R_s + R\sin E)/R_T;\ \sin E = (R_T \cos\omega t - R_s)/R \qquad \text{(B2)}$$

$$R = (R_s^2 + R_T^2 - 2R_T R_s \cos\omega t)^{1/2} \qquad \text{(B3)}$$

substitute (B3) into (B1).

$$\cos E = R_T \sin \omega t \; (R_s^2 + R_T^2 - 2R_T R_s \cos \omega t)^{-1/2}$$

then by differentation,

$$-\sin E \frac{dE}{dt} = \omega R_T R^{-1} \cos \omega t - \omega R_T R^{-3} (R_T R_s \sin^2 \omega t) \qquad \text{(B4)}$$

substitute (B2) into (B4)

$$-R^{-1}(R_T \cos \omega t - R_s) \frac{dE}{dt} = \omega R_T R^{-1}(R^2 \cos \omega t - R_T R_s \sin^2 \omega t)/R^2$$

$$\frac{dE}{dt} = -\omega R_T (R_T^2 \cos \omega t + R_s^2 \cos \omega t - 2R_T R_s \cos^2 \omega t - R_T R_s + R_T R_s \cos^2 \omega t)/R^2 (R_T \cos \omega t - R_s)$$

$$\frac{dE}{dt} = -\omega R_T (R_T - R_s \cos \omega t)\,(R_T \cos \omega t - R_s)/R^2 (R_T \cos \omega t - R_s$$

$$\frac{dE}{dt} = -\omega R_T (R_T - R_s \cos \omega t) \,/\, (R_T^2 + R_s^2 - 2R_T R_s \cos \omega t) \qquad \text{(B5)}$$

R_T = distance from the center of earth to the satellite (7,711,499 meters)

R_s = distance from the center of earth to the station (6,378,166 meters)

R = distance from the station to the satellite

ω = angular velocity of the satellite = $2\pi/T$

T = period of the satellite = $T_c(R_T/R_s)^{3/2}$ page 96 of reference 22

T_c = 84.347 minutes, the period of a circular orbit at the earth's surface

APPENDIX C

MINITRACK INTERFEROMETER TROPOSPHERIC REFRACTION CORRECTION

The Minitrack interferometer measures the phase difference between the signals received by two ground antennas spaced a known distance apart (see Figure C-1). If the propagation time of a wavefront from the satellite to antenna A_1 is t_1, and to antenna A_2 is t_2, then the interferometer measures the time difference, $\Delta t = (t_2 - t_1)$, between these signals.

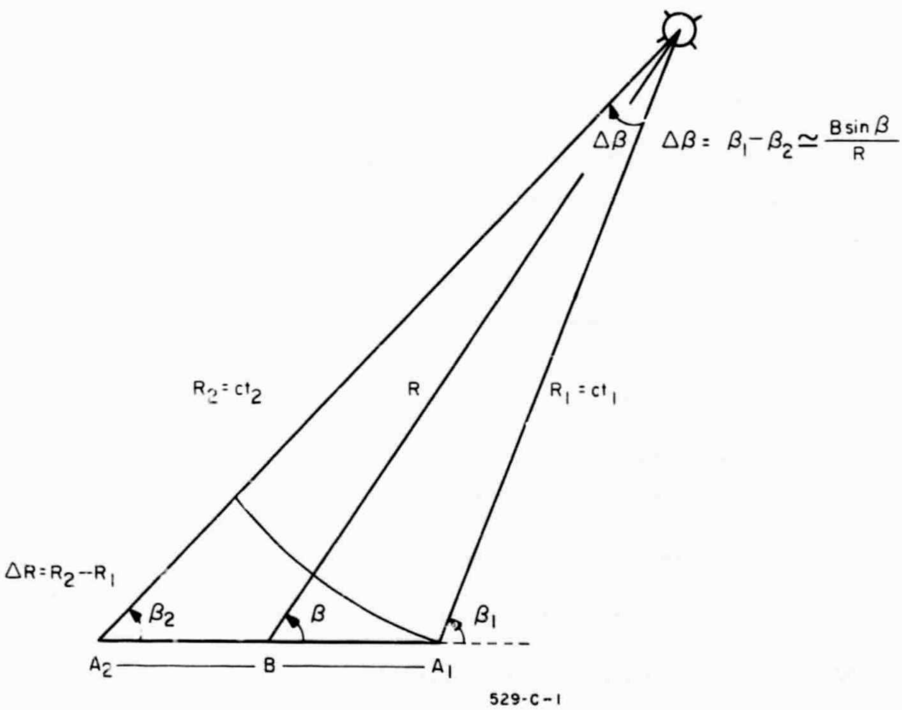

Figure C-1. Interferometer Geometry (No Atmosphere)

The effect of the troposphere is to bend the two ray paths and retard the signals along these ray paths from the satellite to the two antennas. Thus, the propagation time to antenna A_1 increases from t_1 to $t_1 + \Delta t_1$, and the propagation time to antenna A_2 increases from t_2 to $t_2 + \Delta t_2$, causing the measured time difference to increase from $(t_2 - t_1)$ to $(t_2 + \Delta t_2) - (t_1 + \Delta t_1)$, a net increase of $\delta(\Delta t) = (\Delta t_2 - \Delta t_1)$.

In other words, the difference in retardation along each ray path due to the atmosphere causes a slight increase of $\delta(\Delta t)$ in the measured Δt.

If the two rays were parallel, as from a distant radio star, and if the atmosphere consisted of horizontally homogeneous plane parallel layers, the retardation along the two rays would be identical, so the $\Delta t_2 = \Delta t_1$, and $\delta(\Delta t) = 0$. In this case, there is no change in the measured Δt due to the introduction of an atmosphere; and, since the true angle to the star is also unchanged, there is no need to make a refraction correction to the derived interferometer angle. It is primarily because the two ray paths to a near-earth satellite are not quite parallel and because the atmosphere is not horizontally but spherically stratified that a differential $\delta(\Delta t)$ is introduced.

For the Minitrack interferometers, the baselines are horizontal at the midpoint and the longest baseline, B, is less than 125 meters; whereas the range, R, to a near-earth satellite is at least 150 kilometers. Thus $B \ll R$. Under this condition, and assuming that no atmosphere is present, the interferometer direction cosine can be expressed as follows:

$$m_o = \cos\beta = \frac{c\Delta t}{B} = \frac{\Delta R}{B}$$

where: m_o = direction cosine for no atmosphere

β = interferometer angle between direction line to satellite and baseline at baseline center.

c = velocity of light in vacuum

B = length of baseline in meters

$\Delta t = t_2 - t_1$ = difference in propagation time from satellite to antennas A_2 and A_1 as seen by interferometer.

$\Delta R = R_2 - R_1$ = difference in propagation distance (meters) from satellite to antennas A_2 and A_1 (derived from $c\Delta t$)

Also, from Figure C-1, the change in β, $\Delta\beta$, is

$$\Delta\beta = \beta_1 - \beta_2 \simeq \frac{B\sin\beta}{R} \qquad \text{for } B \ll R.$$

When the troposphere is introduced, the measured propagation time difference, Δt, is increased by the differential retardation $\delta(\Delta t) = \Delta t_2 - \Delta t_1$, so that with no correction for refraction, the measurement is interpreted as:

$$m = \cos\beta = \frac{[\Delta t + \delta(\Delta t)]\ c}{B} = \frac{\Delta R + \delta(\Delta R)}{B} = m_o + \Delta m$$

The change in the derived direction cosine due to the troposphere is therefore:

$$\Delta m = m - m_o = \frac{\delta(\Delta R)}{B} \tag{C1}$$

where: $\delta(\Delta R)$ = differential change due to the troposphere in the derived range difference along the two ray paths.

From Table 2 on page 4, the nominal derived increase in a single path range observation due to the tropospheric retardation is given as

$$\Delta R_{to} = HN_s \csc E$$

where: H = tropospheric scale height ($\sim 7 \times 10^3$ meters)

N_s = refractivity at station ($\sim 3 \times 10^{-4}$)

E = elevation angle of satellite

Thus, for the interferometer, the effect of the troposphere is to increase the derived range difference between the two ray paths by:

$$\delta(\Delta R) = (\Delta R_{to})_2 - (\Delta R_{to})_1 = HN_s (\csc E_2 - \csc E_1)$$

$$\simeq \frac{\partial(\Delta R_{to})}{\partial E} \cdot \Delta E = (-HN_s \csc E \ \mathrm{ctn} E)\Delta E$$

where: $(\Delta R_{to})_2$ = predicted increase in range observation along ray path to antenna A_2 due to tropospheric retardation.

$(\Delta R_{to})_1$ = same for the ray path to antenna A_1.

$\frac{\partial(\Delta R_{to})}{\partial E} = -HN_s \csc E \ \mathrm{ctn} E$ = rate of change of predicted tropospheric increase in derived range with a change in elevation angle, E.

$\Delta E = E_2 - E_1$ = small change in elevation angle between the ray paths to the two antennas due to the geometrical separation of the antennas. The small correction to ΔE, due to the different changes in E_1

and E_2 caused by the troposphere, is ignored here. For $E \geq 10^o$, the error in ΔE due to this effect is

$$\delta(\Delta E) = \frac{\partial(\Delta E)}{\partial E} \Delta E = \frac{\partial(N_s \operatorname{ctn} E)}{\partial E} \Delta E = -N_s \csc^2 E \Delta E, \text{ which is} \leq .01 \Delta E.$$

The difference in elevation angle, ΔE, for the two ray paths to the two antennas is related to the difference in interferometer angle, $\Delta\beta$, for the two ray paths to the two antennas as shown in Figure C-2.

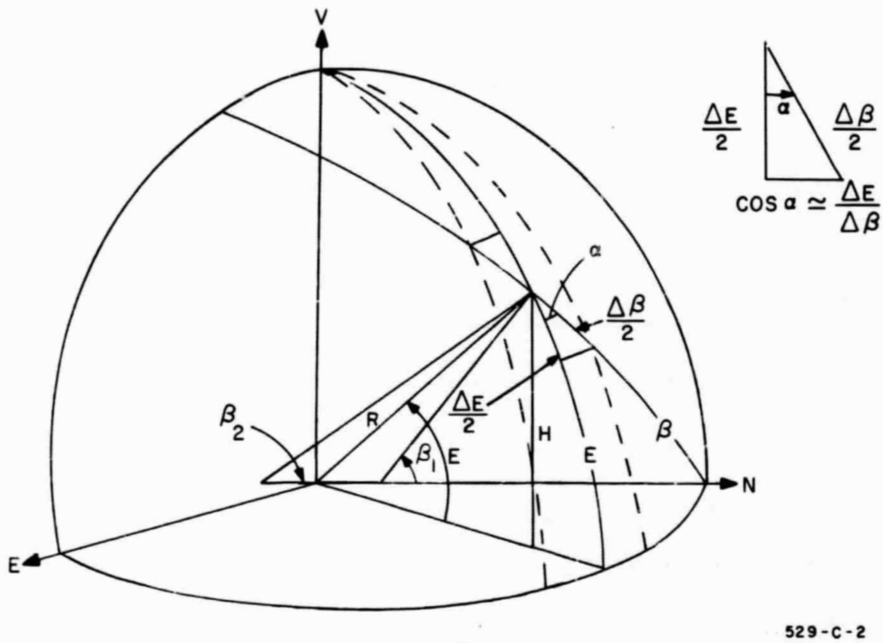

Figure C-2. Relation Between ΔE and $\Delta\beta$

At the satellite, the angle $\Delta\beta$ is defined as the angle subtended by the baseline. Therefore, the projection of the baseline on the great circle trace of the interferometer angle β produces the short arc $\Delta\beta$ at the satellite. The ΔE corresponding to this $\Delta\beta$ is also shown in Figure C-2. The arcs $\frac{\Delta\beta}{2}$ and $\frac{\Delta E}{2}$ form two sides of a small right spherical triangle with included angle α. Since the spherical triangle is small, it may be replaced by its projection on the tangent plane as shown in Figure C-2.

$$\text{Then } \Delta E = \Delta\beta \cos\alpha = \frac{B \sin\beta}{R} \cos\alpha. \tag{C2}$$

To determine $\cos\alpha$, return to the large right spherical triangle with sides E, β, and A. From spherical trigonometry,

$$\cos\alpha = \tan E \; \mathrm{ctn}\beta \tag{C3}$$

Now returning to equation (C1)

$$\Delta m = \frac{\delta(\Delta R)}{B} = \frac{1}{B}\,(-HN_s \csc E\; \mathrm{ctn} E)\Delta E \tag{C4}$$

Using equation (C2)

$$\Delta m = \frac{1}{B}\,(-HN_s \csc E\; \mathrm{ctn} E)\,\frac{B\sin\beta}{R}\,\cos\alpha$$

Using equation (C3)

$$\Delta m = \frac{1}{B}\,(-HN_s \csc E\; \mathrm{ctn} E)\,\frac{B\sin\beta}{R}\,(\tan E\; \mathrm{ctn}\beta)$$

$$= \frac{(-HN_s \csc E)}{R}\cos\beta = -\frac{\Delta R_{to}}{R}\cos\beta = -\frac{\Delta R_{to}}{R}\,m$$

or $$\frac{\Delta m}{m} = -\frac{\Delta R_{to}}{R} \tag{C5}$$

For the flat-earth approximation (see Figure C-2)

$$R = H_t \csc E$$

Then $$\frac{\Delta m}{m} = -\frac{\Delta R_{to}}{R} = -\frac{HN_s \csc E}{H_t \csc E} = -\frac{H}{H_t}\,N_s \tag{C6}$$

Or since $m = \cos\beta$

$$\Delta m = \Delta(\cos\beta) = -\sin\beta\Delta\beta$$

$$\Delta\beta = -\frac{\Delta m}{\sin\beta} = -\frac{\Delta m}{m}\,\mathrm{ctn}\beta = \frac{H}{H_t}\,N_s\,\mathrm{ctn}\beta \tag{C7}$$

For nominal values of scale height, $H = 7 \times 10^3$ meters, and of GEOS satellite height, $H_t = 10^6$ meters, $\frac{H}{H_t} = \frac{7 \times 10^3}{10^6} = 0.007$.

This result (C7) differs considerably from that given in reference 2, where: $m_o = \frac{m}{q} = m\,(1 + A_s)$

$$\Delta m = m - m_o = -A_s m$$

$$\frac{\Delta m}{m} = -A_s = -N_s \tag{C8}$$

or $$\Delta\beta = -\frac{\Delta m}{m}\,\mathrm{ctn}\beta = N_s\,\mathrm{ctn}\beta \tag{C9}$$

The Minitrack interferometer correction for the troposphere derived in (C7) says that the correction from reference 2 should be reduced by a factor of about 0.007 for the GEOS satellite.

On the other hand, (C7) is equivalent to that derived for the Minitrack interferometer aircraft calibrations in reference 30, where

$$m_o = m\bar{\mu} = m\,(1 + \bar{N})$$

$$\frac{m - m_o}{m} = \frac{\Delta m}{m} = -\,\bar{N} \tag{C10}$$

or
$$\Delta\beta = -\frac{\Delta m}{m}\,\text{ctn}\beta = \bar{N}\,\text{ctn}\beta \tag{C11}$$

Using an exponential model for the troposphere, it can be shown that

$$\bar{N} = \frac{HN_s}{H_t}$$

Assume $N(h) = N_s\, e^{-h/H}$

where it is implicit that h is height above the tracking site, not above sea level, and N_s is the refractivity at the site.

Then
$$\bar{N} = \frac{\int_o^{H_t} N(h)\, dh}{\int_o^{H_t} dh} = \frac{N_s}{H_t}\int_o^{H_t} e^{-h/H}\, dh = \frac{HN_s}{H_t}\left[1 - e^{-H_t/H}\right]$$

where $e^{-H_t/H} = e^{-1000/7} \longrightarrow 0$

$$\therefore\ \bar{N} = \frac{HN_s}{H_t}$$

and (C7) and (C11) are equivalent for satellite heights.

To show that (C7) is equivalent to (C11) for aircraft heights, it is better to rederive (C7) in terms of height above sea level instead of height above the site.

Then, instead of $\Delta R_{to} = HN_s \csc E$,

$$\Delta R_{to} = \int_{R_s}^{R_t} N(h)\, dr = \int_{H_s}^{H_t} N_o e^{-h/H}\, dh \csc E =$$

$$H \csc E \left[N_o e^{-H_s/H} - N_o e^{-H_t/H} \right] = H \csc E \left[N_s - N_t \right]$$

where: N_o = refractivity at sea level

N_s = refractivity at site

N_t = refractivity at aircraft

Then $$\frac{\Delta m}{m} = -\frac{\Delta R_{to}}{R} = -\frac{H(N_s - N_t) \csc E}{(H_t - H_s) \csc E} = -\frac{H(N_s - N_t)}{H_t - H_s} \tag{C12}$$

$$\Delta\beta = -\frac{\Delta m}{m} \operatorname{ctn}\beta = \frac{H(N_s - N_t)}{H_t - H_s} \operatorname{ctn}\beta \tag{C13}$$

For this relation to be equivalent to (C11) for aircraft heights, it must be shown that $\overline{N} = \dfrac{H(N_s - N_t)}{H_t - H_s}$

$$\overline{N} = \frac{\int_{H_s}^{H_t} N(h)\, dh}{\int_{H_s}^{H_t} dh} = \frac{\int_{H_s}^{H_t} N_o e^{-h/H} dh}{H_t - H_s} = \frac{H(N_o e^{-H_s/H} - N_o e^{-H_t/H})}{H_t - H_s}$$

$$= \frac{H(N_s - N_t)}{H_t - H_s}$$ so (C7) and (C11) are also equivalent for aircraft heights.

The advantage of this new derivation of (C7) or (C11) is that more precise expressions for the single path range (or range rate) refraction correction than those used here are available and can be used for a more precise interferometer refraction correction at low elevation angles, subject to the conditions that $B \ll R$ and the interferometer antennas are at the same height.

As an example, a more precise expression of $\delta(\Delta R)$ is available from the range rate refraction correction equation for GDAP from Table 3 on page 5:

$$\delta(\Delta R) = \frac{\partial \Delta R}{\partial E} \Delta E = \frac{d(\Delta R)}{dt} \Delta t =$$

$$(\Delta \overset{o}{R}_{to})\Delta t \left[\frac{7200}{H} \quad \frac{2}{\sqrt{1 + 0.0045154 \csc^2 E} + (1 + 0.0045154 \csc^2 E)} \right] =$$

$$\Delta \overset{o}{R}_{to} \Delta t \, f(E) = (-HN_s \csc E \ \text{ctn} E) \Delta E \, f(E)$$

Returning to (C1)

$$\Delta m = \frac{1}{B} \ \delta(\Delta R) = \frac{1}{B} \ (-HN_s \csc E \ \text{ctn} E) \Delta E \, f(E) \tag{C14}$$

Except for the factor f(E), (C14) is equivalent to (C4).

It follows that

$$\frac{\Delta m}{m} = -\frac{\Delta R_{to}}{R} \ f(E) \tag{C15}$$

and $$\Delta \beta = -\frac{\Delta m}{m} \ \text{ctn}\beta = \frac{\Delta R_{to}}{R} \ f(E) \ \text{ctn}\beta =$$

$$\frac{H(N_s - N_t)}{H_t - H_s} \ f(E) \ \text{ctn}\beta =$$

$$\overline{N} \ f(E) \ \text{ctn}\beta \tag{C16}$$

A similar derivation to that used to obtain (C15) and (C16) for the troposphere should apply to the Minitrack refraction correction for the ionosphere. This will be investigated at a later time.

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