NEUTRONIC DESIGN OF A REACTOR CORE CONTAINING
HEAT PIPES FOR APPLICATION TO A NUCLEAR AIRPLANE

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SUMMARY

A successful design of a nuclear aircraft propulsion system must meet all specifications of performance and safety at a minimum weight. Heat pipes utilized for removing core heat may be able to reduce system weight by eliminating the need for heavy pumps and high pressure piping.

A contract was awarded to C. Silverstein for the study of heat pipe applications to nuclear aircraft propulsion systems (ref. 1). This study was limited to the thermal and mechanical design of heat pipes and did not include core criticality calculations.

Therefore, a study was conducted by Lewis Research Center concurrently to perform the neutronic calculation on the proposed heat pipe reactor design. The study conducted revealed that utilizing heat pipes for a nuclear airplane reactor application appeared promising when heat pipe performance was applied to the limit of heat pipe technology. The design parameters calculated are
Number of heat pipes ............... 2740/half core
Radial heat flux .................. $0.88 \times 10^6$ Btu/hr-ft$^2$
                               ($2.77 \times 10^6$ watts/meter$^2$)
Core diameter ..................... 45 inches (114 cm)
                               [0.075 in. (0.19 cm) fuel, 0.035 in. (0.089 cm) clad]
Core L/D .......................... 1.0
Heat pipe vapor temperature ...... $2100^\circ$ F (1422 K)
Fuel enrichment ................... 37.5 percent
Fuel loading ....................... 3770 kg
Clad temperature .................. $2300^\circ$ F (1530 K)
Clad stress ......................... 10 000 psi (6890 N/cm$^2$)
Core $K_{eff}$ ....................... 1.1
Power ............................. 300 MW

The active core diameter of 45 inches (114 cm) can be decreased by increasing the enrichment and reducing the fuel loading of the core. This would be desirable since control drum application appears difficult at 45 inch (114 cm) outside diameter due to the reduced boundary flux.

INTRODUCTION

Many studies have been conducted where heat pipes are applied to nuclear reactor cores or thermionic energy conversion devices. These studies indicate possible advantages in efficiency and design simplification. Weight reduction has also been accomplished through the elimination of pumps and high pressure piping.

On January 22, 1969 a contract was awarded to Mr. C. C. Silverstein for the study of heat pipe applications in nuclear aircraft propulsion systems. This study included a preliminary design evaluation of a heat transport system in which heat pipes were used for removing and transporting the reactor core heat to the aircraft engine air heaters. This study was limited to the thermal and mechanical design of heat pipes and did not include core criticality calculations.
Therefore, a reactor design study was conducted by Lewis Research Center concurrently with Mr. Silverstein. Its purpose was to provide reactor neutronic information which could be used, together with heat pipe information, to establish reactor size and weight and subsequent shielding size and weight. The neutronic calculations determine the core size and fuel enrichment for a core $K_{eff}$ of 1.1, given the heat pipe design characteristics. This report presents the results of this study.

**SYMBOLS**

- $A_0$: internal heat generation rate, Btu/hr-cu.in. (watt/cc.)
- $C_1, C_2$: integration constant
- $D$: core diameter, in. (cm)
- $eV$: neutron energy, electron volts
- $K$: thermal conductivity, Btu/hr-ft-$^0F$ (watts/meter-$K$)
- $n$: number of heat pipes
- $L$: heat pipe length, core length, in. (cm)
- $q$: heat flux, Btu/hr-ft$^2$ (watts/meter$^2$)
- $r$: radius, in. (cm)
- $S_r$: radial spacing, in. (cm)
- $t$: thickness, in. (cm)
- $t$: temperature, $^0F$ (K)
- $u$: neutron energy, lethargy

**Subscripts:**
- $A$: Axial
- $cell$: heat pipe fuel cell
- $cl$: clad
- $core$: active reactor core
A sketch of the proposed design is presented in figure 1. The core is cylindrical. It consists of heat pipe fuel elements surrounded by four inch (10.2 cm) thick side and end reflectors. The heat pipes start at the core midplane, exit through the top and bottom end reflectors and terminate in heat exchangers just outside the end reflectors.

The heat pipes consist of a fluid, wick and tube. The part of the heat pipe within the core is surrounded by an annulus of fuel which in turn is surrounded by cladding. This fueled portion covers about half of the active length of the heat pipe. The heat pipe fuel elements are clustered tightly such that nominally zero spacing occurs between their outer diameters within the active core region. With the fuel and cladding terminating at the end reflectors, there is space between the heat pipes in the heat exchangers. The heat pipes are supported by the end reflectors. These reflectors are in turn supported by a containment vessel.

The heat pipes and reactor core were designed so that a minimum amount of design feedback was involved. For example, the heat pipes were surrounded by the fuel rather than vice versa so that once the heat pipes were designed a change in fuel thickness did not necessitate recalculation of the heat pipe.

The initial heat pipe design proposed by Silverstein resulted in a much too large active core diameter. A second heat pipe design was satisfactory. These heat pipes are circular in cross section and consist of a liquid metal vapor space, a capillary wick, and a liquid annulus all enclosed in a tube (fig. 1). They are required to remove 300 MW of reactor thermal power at a maximum vapor temperature of 2100° F (1422 K).
The heat pipe design is reported in reference 1. A summary of the design is as follows:

**Materials**
- Heat pipe fluid - Sodium
- Wick and wall - TZM moly

**Heat pipe cross section**
- Circular consisting of inner vapor space of radius 0.25 inches (0.636 cm) surrounded by annular TZM wick 5 mils (0.013 cm) thick with 30 percent void and effective pore diameter of 2 microns. Wick is surrounded by 10 mils (0.025 cm) thick annulus filled with sodium (which also occupies wick void volume). The sodium annulus is surrounded by TZM containment wall with thickness of 33 mils (0.084 cm).

**Heat pipe length**
- 20 Inch (50.8 cm) evaporator length in each half of core

**Average temperatures**
- Heat pipe vapor - 2000° F (1367 K)

**Maximum axial heat flux**
- 307 kW/in.² (47.6 kW/cm²)

**Average axial heat flux**
- 279 kW/in.² (43.3 kW/cm²)

**Average heat transport**
- 54.9 kW

**rate/heat pipe**

Surrounding each heat pipe is a fuel annulus, clad on its outside diameter. Both the thickness of the fuel and its enrichment can be varied. For calculating the required fuel thickness, fuel enrichment, and core size the following reactor design criteria was established:
Core fuel ........................................... UN
Clad ................................................. TZM
Core $K_{\text{eff}}$ ..................................... 1.1
Core lifetime ...................................... 10 000 hr
Peak fuel burnup ................................. 5 percent
Fuel thickness ................................. 0.075 in. (0.19 cm)
Clad thickness ................................. 0.035 in. (0.089 cm)
Active core diameter ...................... 45 in. (114 cm) max
Side and end reflector ................. 4 in. (10.2 cm) - Moly
Radial peak to average power ratio .......... 1.1
Axial peak to average power ratio .......... 1.0

**DESIGN PROCEDURE**

Using the heat pipe design outlined above and the reactor specifications, the number of heat pipes, radial heat flux, core diameter, heat pipe fuel cell radial temperature gradient, fuel loading, and enrichment can be calculated. The following is a presentation of the calculations of these parameters.

**Number of Heat Pipes**

With the heat pipes removing core heat through both end reflectors, the reactor can be designed considering symmetry about its midplane. Therefore, 150 MW of heat must be removed from each half of the core.

The required number of heat pipes for one half core is expressed as follows:

$$n = \frac{150 \text{ MW} \times 4}{q_A \pi D_v^2}$$  \hspace{1cm} (1)
where $D_v$ is the vapor diameter of the heat pipe. For a vapor diameter of 0.5 inch (1.27 cm) and an average axial heat flux of 279 kW/in.², then 2740 heat pipes are required in each half the core.

Core Diameter

The active core diameter is dependent upon the number of heat pipes, fuel thickness and cladding thickness. Since the fuel and cladding thickness have been specified the core diameter has in effect been dictated by the 2740 required tubes.

Figure 4 presents the proposed design of the heat pipe fuel cell. The area of this hexagonal cell may be calculated as follows:

$$\text{A}_{\text{cell}} = \frac{\sqrt{3}}{2} \left( 2r_6 + S_r \right)^2$$

(2)

Multiplying equation (2) by the number of heat pipes gives the active core area if the scallops of the outer perimeter are neglected.

$$\text{A}_{\text{core}} = n \frac{\sqrt{3}}{2} \left( 2r_6 + S_r \right)^2$$

(3)

From equation (3) the diameter of the core is

$$D_{\text{core}} = \sqrt{\frac{4}{\pi} \text{A}_{\text{core}}}$$

$$\quad = \sqrt{\frac{4\sqrt{3}}{\pi} n \left( 2r_6 + S_r \right)^2}$$

(4)

or in terms of fuel thickness $t_f$ and cladding thickness $t_{cl}$, equation (4) becomes

$$D_{\text{core}} = \sqrt{\frac{2\sqrt{3}}{\pi} n \left( 2r_4 + 2t_f + 2t_{cl} + S_r \right)^2}$$

(5)
Finally, for an assumed $t\text{_{cl}}$ of 0.035 inch (0.089 cm), an assumed $t\text{_f}$ of 0.075 inch (0.19 cm), a $S\text{_r}$ of 0, and 2740 tubes, the core diameter is 45 inches (114 cm). This value meets the maximum core diameter design specification. A smaller core diameter could be obtained by reducing the clad and/or fuel thickness or by making the fuel cross sectional geometry hexagonal instead of circular.

**Radial Heat Flux**

The heat removed axially from the core enters the heat pipes radially through its walls. The radial heat flux is dependent upon the axial heat flux and the pipe's length and diameter or

$$q\text{_r} = \frac{q\text{_A} r_1^2}{2\pi r_1 L} = \frac{q\text{_A} r_1}{2L}$$

(6)

The reactor core requirement of a $L/D$ of 1 has set the heat pipe length for half a core at approximately 20 inches (50.8 cm). For a $q\text{_A}$ of 279 kW/in.$^2$ (43.3 kW/cm$^2$) of vapor cross section, the $q\text{_r}$ is $0.88\times10^6$ Btu/hr-ft$^2$ (2.77$\times10^6$ W/m$^2$). The radial heat flux directly affects both the maximum fuel temperature and thermal stresses of the fuel. The selection of UN as the fuel reduces this effect over fuels such as $UO_2$ due to UN's higher thermal conductivity.

**Heat Pipe Fuel Cell Radial Temperature Gradient**

The axial heat removed by the heat pipes is generated in the UN fuel surrounding the heat pipe tube wall. This internal heat generation and resultant radial flux results in a radial temperature gradient from the heat pipe liquid and wall interface through the tube wall, fuel, and to the clad wall interface around the fuel. No temperature gradient exists in the cladding since no heat is transferred.
The total temperature gradient from the fuel outer wall to the heat pipe inner wall is

$$\Delta t = t_5 - t_3 = \frac{A_0}{4} \left\{ \frac{(r_4^2 - r_3^2)}{K_f} + \frac{2r_5^2 \ln \frac{r_5}{r_4}}{K_{cl}} + \frac{2 \ln \frac{r_4}{r_3} (r_4^2 - r_3^2)}{K_{cl}} \right\}$$

(See Appendix A for derivation.)

A radial heat flux of 0.88 Btu/hr-ft$^2$ ($2.77 \times 10^6$ W/m$^2$) results in a $\Delta T$ of 200$^\circ$. For a maximum heat pipe vapor temperature of 2100$^\circ$ F (1422 K) the resultant clad temperature is 2300$^\circ$ F (1530 K).

For TZM at 2300$^\circ$ F (1530 K), the 10000 hour creep-rupture stress is about 10000 psi (6890 N/cm$^2$). Since the clad has no thermal gradients (no heat is lost radially outward) its stresses would be limited to that caused by fuel swelling, fuel growth, fission product gases, etc.

Fuel Loading

The 0.075 inch (0.19 cm) thick fuel surrounding each heat pipe also sets the total fuel loading of the core in kg of weight. With the reactor core required to operate for 10000 hours at 300,000 MW, the amount of fuel that will be used can be calculated as follows:

$$300 \times 10^6 \text{ Watts} \times 10000 \text{ hr} \times 3600 \text{ sec/hr} = 10.8 \times 10^{15} \text{ Watt-sec}$$

For $^{235}$U fuel the burn-up rate is $1.452 \times 10^{-11}$ gm $^{235}$U/Watt-sec of operation. The total amount of fuel used is

$$10.8 \times 10^{15} \text{ Watt-sec} \times 1.452 \times 10^{-11} \text{ gm } ^{235}\text{U/Watt-sec} = 157 \text{ kg}$$

From figure 7 a 0.075 inch (0.19 cm) thick fuel for 5480 tubes (total core) results in a core loading of 3770 kg. This loading would result in a peak burn-up of 4.2 percent which is less than the design goal of 5 percent.
Now all thermal and physical design variables have met the requirements of the section titled "Heat Pipe and Reactor Design Characteristics." The final design variable to satisfy is the enrichment for a core $K_{\text{eff}} = 1.1$.

**CORE CRITICALITY CALCULATIONS**

The code used in the neutronic analysis of this report is a two-dimensional discrete angular segmentation transport program referred to as TDSN (ref. 2). It is a numerical iterative finite difference method in which the continuous angular distribution of neutron velocities is represented by considering discrete angular directions. The output of the transport program includes the core multiplication factor, integrated power ratio and flux shapes for each energy group.

A required input to the TDSN transport program are the core cross sections. Multigroup cross sections used in this analysis were obtained from GAM II and GATHER-II programs (refs 3 and 4). These are then converted to macroscopic cross sections by a Lewis Research Center written code entitled MACROS.

For the calculations of this report the core was divided into one radial zone and one axial zone. Seven group microscopic cross sections were obtained from the GAM II and GATHER II programs over the energy ranges presented in table I. The macroscopic cross sections calculated by MACROS used the atom densities listed in figure 4. A $P_0$ approximation for neutron scattering and a $S_2$ discrete angle approximation were used in the neutronic calculations. The core $K_{\text{eff}}$ was calculated over a large range of enrichments. For each fuel enrichment the core atom densities were readjusted. The results of these calculations are presented in figure 3, core $K_{\text{eff}}$ as a function of Percent Fuel Enrichment. The required core $K_{\text{eff}}$ was achieved at a 37.5 percent fuel enrichment.

The normalized power distribution is plotted in figure 4. The plot shows that the radial peak to average power ratio is approximately 1.45.
compared to a desired 1.1. Since the fuel enrichment is low, however, fuel zoning could be applied to better meet this requirement. The low flux at the boundary also results in a low radial neutron leakage of 10 percent. This would negate the use of boundary control such as control drums. However, if the radial power distribution were flattened to nearer the 1.1 ratio the radial leakage would increase and boundary control may then be feasible.

**SUMMARY OF RESULTS**

The values of the design variables that meet the requirements set forth in the section titled 'Heat Pipe and Reactor Design Characteristics' section are

- **Number of heat pipes**: 2740/half core
- **Radial heat flux**: $0.88 \times 10^6$ Btu/hr-ft$^2$ ($2.77 \times 10^6$ watts/meter$^2$)
- **Core diameter**: 45 inches (114 cm)
  
  [0.075 in. (0.19 cm) fuel, 0.035 in. (0.089 cm) clad]
- **Core L/D**: 1.0
- **Heat pipe vapor temperature**: $2100^\circ$ F (1422 K)
- **Fuel enrichment**: 37.5 percent
- **Fuel loading**: 3770 kg
- **Clad temperature**: $2300^\circ$ F (1530 K)
- **Clad stress**: 10 000 psi (6890 N/cm$^2$)
- **Core $K_{eff}$**: 1.1
- **Power**: 300 MW

The heat pipe design may be compared with a liquid metal fast reactor design (ref. 5) and a water moderated thermal reactor design (ref. 6). The comparison is qualitative. The reactors were designed for the same power and life but for different burnups. The 45-inch (114-cm) diameter at a 4.2 percent burnup for the heat pipe core compares with a 30-inch
(76.2-cm) diameter fast reactor at a 12 percent burnup and 62-inch (157-cm) diameter thermal reactor at a 20 percent burnup. If the burn-up rate for the heat pipe design were increased to 10 percent, the core diameter reduces to 39 inches (99 cm). This may be possible since a fuel enrichment of 37.5 percent could be increased to maintain a core $K_{\text{eff}}$ of 1.1.

The scope of this study did not include the design of control rods or drums. The calculated boundary neutron flux and radial leakage, however, indicate that control drums would be difficult to apply. Therefore, a more optimized design should endeavor to flatten the radial power distribution, reduce the core diameter, or incorporate poison heat pipe tubes within the core.

CONCLUSIONS

The following conclusions are made based upon the preliminary analysis conducted in this report.

1. The concept of utilizing heat pipes for the removal of core heat in a nuclear aircraft 300 MW reactor appears promising when the heat pipe performance is pushed to the limit of present heat pipe technology.

2. The active core diameter of 45 inches (114 cm) (4.2 percent burn-up) compares with 30 inches (76.2 cm) for a liquid metal fast reactor (12 percent burnup) and 62 inches (157 cm) for a water moderated thermal reactor (20 percent burnup). A smaller core diameter of 39 inches (99 cm) is possible for the heat pipe core if a 10 percent burnup were assumed.

3. Control drum application for the current design will be difficult due to the reduced boundary flux and high energy level of the neutrons.

4. Further reduction in fuel loading and subsequent core diameter is possible by reducing the 0.075 inch (0.19 cm) fuel thickness and the 0.035 inch (0.089 cm) clad thickness.
APPENDIX A

For the case of heat production in a cylinder

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{dt}{dr} \right) + \frac{A_0}{K} = 0
\]

\[
d \left( r \frac{dt}{dr} \right) = -\frac{A_0 r}{K} \, dr
\]

\[
\frac{dt}{dr} = -\frac{A_0 r}{2K} + \frac{C_1}{r}
\]

The boundary conditions for equation (A1) are

\[
\frac{dt}{dr} = 0 \quad \text{at} \quad r = r_5
\]

\[
C_1 = \frac{A_0 r_5^2}{2K}
\]

\[
\frac{dt}{dr} = -\frac{A_0 r}{2K} + \frac{1}{2} \frac{A_0 r_5^2}{r} \frac{1}{2K}
\]

integrating equation (A2) gives

\[
t = \frac{A_0}{2K} \left\{ -\frac{r^2}{2} + r_5^2 \ln r \right\} + C_2
\]

The boundary conditions for equation (A3) are
Equation (A4) represents the temperature drop from the heat pipe tube-outer wall radially outward to the fuel outer wall. For the total temperature gradient from the fuel outer wall to the heat pipe inner wall (ref. \( r_5 \) and \( r_3 \) of fig. 2).

\[
Q = \frac{2\pi K_{cl} L}{\ln \frac{r_4}{r_3}} (t_4 - t_3) = A_0 \pi \left( r_4^2 - r_3^2 \right) L \\
\]

\[
t_4 - t_3 = \frac{A_0 \left( r_4^2 - r_3^2 \right) \ln \frac{r_4}{r_3}}{2K_{cl}} \\
\]

Substituting equation (A5) into equation (A4) and applying the boundary condition \( t = t_5 \) at \( r = r_5 \) gives
\[ t_5 - t_3 = \Delta T = \frac{A_0}{4} \left\{ \frac{(r_4^2 - r_3^2)}{K_f} \right\} + \frac{2r_5^2 \ln \frac{r_5}{r_4}}{K_{cl}} + \frac{2 \ln \frac{r_4}{r_5} (r_4^2 - r_3^2)}{K_{cl}} \] (A6)

REFERENCES


### TABLE I. - NEUTRON ENERGY GROUPS

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Figure 1: Proposed Reactor Design Containing Heat Pipes
Figure 2: Proposed Design of Heat Pipes in Reactor

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Figure 3: Core Key as a function of percent fuel enrichment