NASA
SPACE VEHICLE
DESIGN CRITERIA
(GUIDANCE AND CONTROL)

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SPACECRAFT GRAVITATIONAL TORQUES

MAY 1969

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

FOREWORD

NASA experience has indicated a need for uniform design criteria for space vehicles. Accordingly, criteria are being developed in the following areas of technology:

Environment
Structures
Guidance and Control
Chemical Propulsion

Individual components of this work will be issued as separate monographs as soon as they are completed. This document, Spacecraft Gravitational Torques, is one such monograph. A list of all monographs in this series issued prior to this one can be found on the last page of this document.

These monographs are to be regarded as guides to design and not as NASA requirements, except as may be specified in formal project specifications. It is expected, however, that the criteria sections of these documents, revised as experience may indicate to be desirable, eventually will be uniformly applied to the design of NASA space vehicles.

This monograph was prepared under the cognizance of the NASA Electronics Research Center. Principal contributors were Mark Harris and Robert Lyle of Exotech, Inc.

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Contributions in the form of design and development practices were also provided by many other engineers of NASA and the aerospace community.

Comments concerning the technical content of these monographs will be welcomed by the National Aeronautics and Space Administration, Office of Advanced Research and Technology (Code RVA), Washington, D.C. 20546.

May 1969

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CONTENTS

SPACECRAFT GRAVITATIONAL TORQUES

1. INTRODUCTION

In the design of spacecraft attitude control systems, all torques that tend to disturb the attitude of a spacecraft must be considered. One of these torques is the gravitational or gravity gradient torque which results from the variation in the gravitational force over the distributed mass of the spacecraft.

Determination of the gravitational torque requires knowledge of the gravitational field and the mass distribution properties of the spacecraft. This torque decreases with the cube of the orbital radius. At any orbital altitude the gravitational torque may be minimized by designing the spacecraft to be as nearly isoinertial, i.e., having equal principal moments of inertia, as practical. The gravitational disturbance torque is most likely to be a significant factor in the design of large spacecraft in low altitude orbits.

The gravitational disturbance torque is one of the factors that must be considered in the determination of spacecraft attitude motion, control actuator sizing, and expendable fuel requirements. Control or minimization of gravitational disturbance torques requires that attention be given to the arrangement of spacecraft equipment or that provision be made for adjustable balance masses. Rearrangement of the spacecraft is difficult and expensive after the configuration is established; therefore an accurate determination of the gravitational torque should be made during initial design and updated during development of the spacecraft.

Gravitational torques may be employed for spacecraft stabilization. When this is the design objective, mass properties are controlled to increase rather than decrease the differences between principal moments of inertia. Additional factors, such as variations in gravitational torque caused by orbit eccentricity and the oblateness of the Earth, become significant and must be considered. This monograph is primarily concerned with the disturbance torques that affect attitude control systems of vehicles stabilized by means other than gravity. The equations given may also be used to estimate first-order effects of the gravity field on gravity-stabilized vehicles.

The scope of this document has been limited to those gravitational torque effects that are independent of the spacecraft dynamics. Stability problems that involve the internal angular momentum, coupling between the libration and orbital periods of the spacecraft, and spacecraft flexibility are not covered.

2. STATE OF THE ART

2.1 General

Failure to consider the effects of gravitational disturbance torques during spacecraft design has caused mission degradation or failure in several instances. Where these torques have been considered, the correlation between calculated and observed behavior of the spacecraft has generally been quite good, except in instances where the inertial properties of the spacecraft were not completely known. The limiting factor in the assessment of gravitational disturbance torques is the difficulty of accurately determining the spacecraft inertial dyadic.

2.2 Historical Background[1]

The study of torques on a rigid body in a gravitational field is based on Newton's laws of motion and universal gravitation (1687). In 1749 the problem of an axially symmetric ellipsoid in an inverse square field was analyzed by d'Alembert (ref. 1) and Euler (ref. 2) in connection with the precession of the equinoxes produced by the torque on the Earth caused by the solar gravitational field. In 1754 d'Alembert expanded and generalized his work and this became a basis for the first treatment of the librations of the Moon (ref. 3). In 1764 Lagrange took up the problem of the librations of the Moon as the prize problem for the Royal Academy of Sciences and in 1780 published the first definitive treatment (ref. 4). The torque expression, essentially in its modern form, appears in the works of LaPlace (ref. 5) and Tisserand (ref. 6).

During the period after World War II, when the first artificial satellites were being designed and developed, the subject of gravitational torques was reexamined and their equations restated in modern matrix and vector notation. Expressions for the torque on a rigid asymmetrical body in an inverse square field are given by Roberson (refs. 7 and 8), Nidey (ref. 9), Hultquist (ref. 10), and Lur'e (ref. 11).

Because many of the satellites first placed in orbit were spin stabilized, the problem of predicting the motion of the spin axis due to gravitational disturbance torques has been extensively investigated (refs. 12, 13, 14, and 15). The major impetus for the detailed investigation of gravitational torques and their influence on the dynamics of an orbiting body was undoubtedly provided by continuing interest in the exploitation of this torque for passive stabilization of Earth-oriented satellites (sec. 2.3).

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[1] R. E. Roberson: "Dynamics and Control of Rotating Bodies" (to be published).

Digital computer methods are extensively used for the computation of gravitational torques as a part of multibody dynamic simulation. Programs capable of handling multiple-hinge, *n*-body problems exist in several forms (refs. 16 and 17), and more complex programs, which include the distributed characteristics of very long booms, have been developed (refs. 18 and 19).

2.3 Flight Experience

The earliest observations of the effects of gravitational torque on man made satellites related to the precession of the spin axis of spin-stabilized spacecraft. A detailed analysis of the various disturbance torques that acted on Sputnik 3 has shown that the gravitational torque was the major disturbance torque and exceeded the next largest disturbance, i.e., magnetic torque, by a factor of 6 (ref. 14). In this satellite one of the primary attitude sensors was a self-orienting magnetometer, and the low magnetic moment was presumably a requirement.

For the Explorer 11 the orientation of the angular momentum vector was determined from analysis of radio signals (refs. 14 and 20) and its motion checked against that calculated from consideration of the gravitational and magnetic disturbance torques acting on the satellite. Excellent agreement between the observed and calculated motion was obtained. The gravitational torque was slightly less than the magnetic torque.

In an early Agena flight, the gravitational torque was responsible for unexpected attitude behavior. When the Agena was actively stabilized with gas jets, the gravitational torque produced a negligible disturbance but when the spacecraft was coasting in a passive, spin-stabilized mode, its nonspherical mass distribution resulted in an unpredicted precession of the angular momentum vector. After the first observation of this phenomenon, a computer simulation that included the effects of gravitational torques was developed to predict the behavior of the Agena's spin axis. Subsequently, gravitational torques were advantageously used to orient similar spacecraft.

The first Canadian satellite, Alouette 1, was spin stabilized and employed four long antennas to study ionospheric phenomena (fig. 1). The long antenna booms caused a large inertia difference between the longitudinal and transverse axes of the satellite and, consequently, the gravitational torque caused a comparatively rapid precession of the spin axis (ref. 21). The motion of the Alouette 1 spin axis in terms of right ascension and declination as a function of time is shown in figure 2.

Gravitational torques were expected to be the dominant disturbance on the three Pegasus satellites, which deployed large wing panels for micrometeoroid detection. These satellites were neither actively nor passively controlled in attitude but accurate determination of the spacecraft orientation relative to a space-fixed coordinate system was a mission requirement.

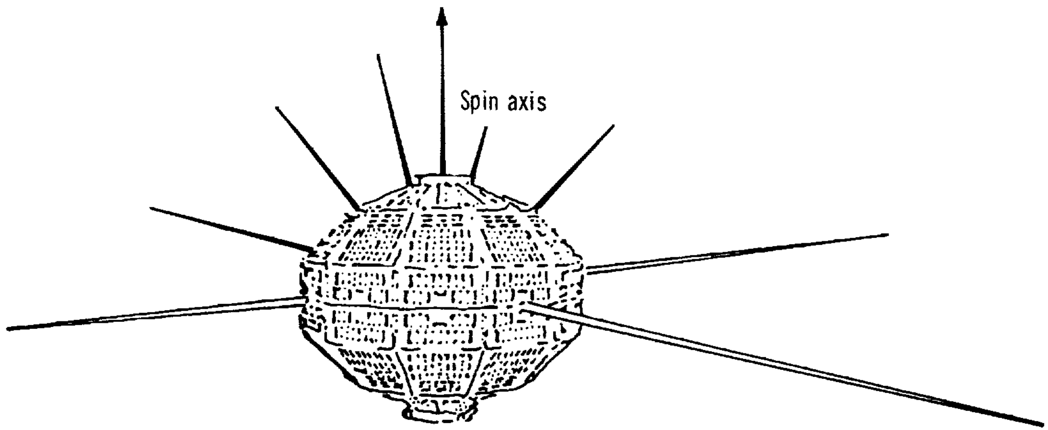


Figure 1.—Alouette 1.

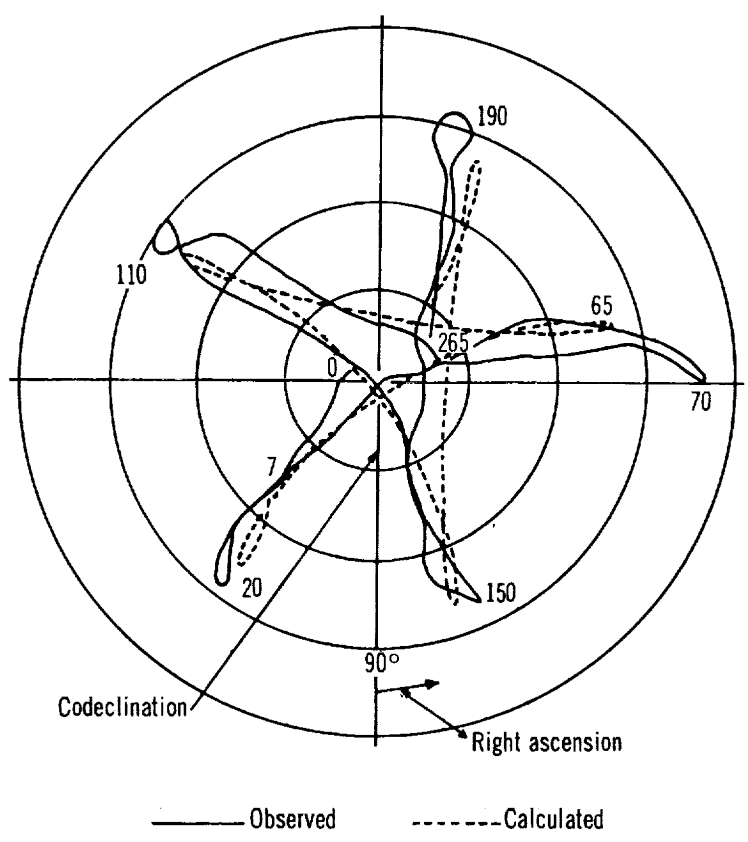


Figure 2.—Spin axis motion of Alouette 1 as a function of time in days (ref. 21).

The spacecraft contained a combination of solar sensors and Earth sensors to obtain the necessary attitude data (ref. 22). Although originally designed to have negligible angular momentum, all three satellites developed spin rates in the range of 7 to 10 deg/sec because of a venting problem. Considerable difficulty was encountered in obtaining agreement between the computed and observed attitude motions of these spacecraft. This flight experience exemplifies problems associated with the prediction of attitude motion when spacecraft dynamics, i.e., gyroscopic effects, are significant. Reference 23 shows that the observed motion can be attributed to gravitational torque provided that proper consideration is given to precession of the orbit plane.

The Lunar Orbiter furnishes an example of gravitational torque caused by a central body other than the Earth and, further, illustrates the importance of using the entire inertial dyadic in the computation of gravitational torques. The Lunar Orbiter employed active three-axis stabilization, and the spacecraft's geometric axes were not coincident with the principal inertia axes. In the lunar orbit the torque caused by the Moon's gravitational field was the only significant disturbance. Using the complete inertial dyadic, the average torque over an orbit was calculated as 9.75×10^{-6} N-m (Newton-meters). This value is about 40 percent larger than the value of 6.84×10^{-6} N-m which was computed on the basis of the moments along the spacecraft's geometric axes and neglecting product of inertial terms (ref. 24).

The effect of gravitational torque on mission requirements other than spacecraft orientation can also be important. The Radio Astronomy Explorer (RAE) uses four, 750-foot extension antennas for the dual purpose of obtaining gravitational stabilization and receiving radio signals at frequencies that are absorbed by the Earth's atmosphere (ref. 25). RAE's two pairs of V-shaped antennas attached to a comparatively small, rigid satellite body are neither parallel nor perpendicular to the local vertical. Consequently, each antenna is subjected to a gravitational torque that tends to bend it inward toward the local vertical as shown in figure 3. At the orbital altitude of 6000 kilometers, gravitational torque acting on the antennas causes the tips to deflect inward by 150 feet. During the design study, the possibility of using 1000-foot antennas was considered and rejected because, for antennas of this length, the deflection caused by gravitational torque is so large that the antenna pattern is adversely affected.

The RAE also furnished the impetus for the development of the equations and simulation of the dynamics of a flexible gravitationally stabilized body. Correlation between actual flight data and the results of the dynamics simulation (refs. 18, 19, and 26) has been excellent. An interesting feature of the RAE simulator system is the incorporation of a corrector module that adjusts parameters in the simulator to minimize deviations between measured and predicted spacecraft performance.

The determination of gravitational torques and the knowledge obtained from flight experience have been significantly advanced through the design, development, and flight of spacecraft in which gravitational torque is used to perform a desired function, e.g., stabilization of spacecraft attitude, unloading of momentum wheels, etc. The proceedings of the two recent symposia on gravitational stabilization (refs. 27 and 28) present an excellent summary of the state of the art and much information directly applicable to the determination of gravitational torque. A general survey of recent developments in the field together with a bibliography of more than 90 references is given in reference 29.

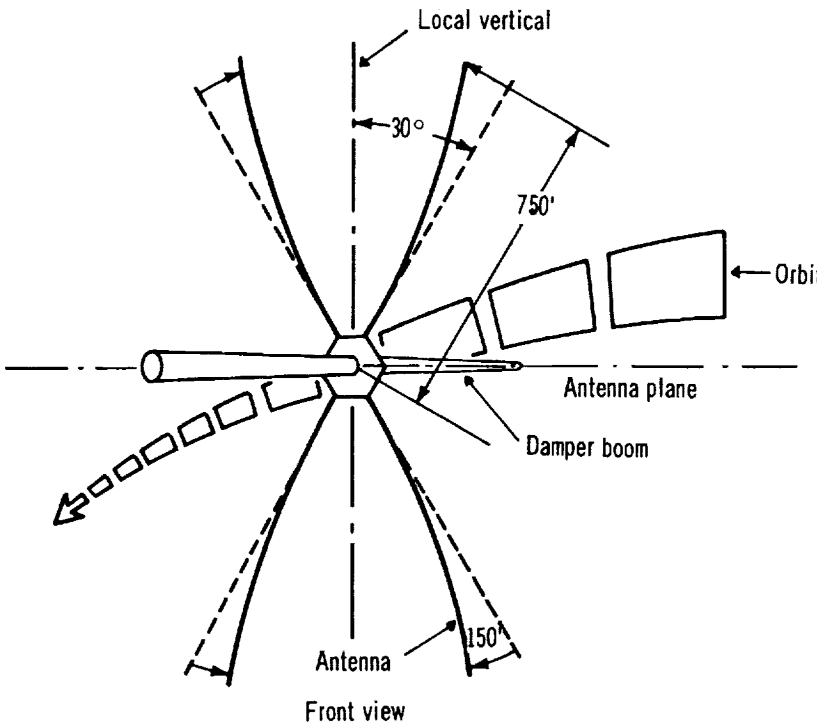


Figure 3.–Gravitational torque deflection of radio astronomy explorer antennas.

2.4 Gravitational Field of the Earth

The classical mathematical foundation of the theory of gravitational attraction was established during the 18th and 19th centuries. Todhunter (ref. 30) gives an excellent survey of early work in the field; Caputo (ref. 31) and Kaula (ref. 32) include both classical and modern developments. With the advent of close Earth satellites, there has been a considerable advance in the accurate determination of the Earth's gravitational field (refs. 33, 34, and 35). The current knowledge regarding the Earth's gravitational field has advanced far beyond the requirements for the determination of the gravitational torque acting on a spacecraft. In all known cases, adequate accuracy is obtained with the assumption of a central inverse square field, i.e.,

$$\mathbf{F} = -\frac{mk}{\mathrm{R}^2} R \tag{1}$$

where

$\mathbf{F}$ = vector force acting on a mass particle in the field, newtons
m = mass of particle, kilograms
k = gravitational constant of the attracting body
= 3.986032×10^{14} N-m^2/kg for Earth
R = distance from the particle to the mass center of the attracting body, meters
R = unit vector in the direction of the line joining the mass center of the attracting body and the particle, dimensionless

2.5 Gravitational Torque Equation

The vector torque equation for a rigid body is derived by determining the gravitational force $\mathbf{F}$ acting on each infinitesimal mass element dm, forming the vector cross product of this force with the vector to the body mass center, and integrating over all mass elements of the body (app. A). When a central inverse square field with gravitational constant k is assumed and the inertial dyadic $\mathbf{I}$ is used to describe the mass properties of the spacecraft, the simplified equation for gravitational torque, $\mathbf{L}_g$, in newton-meters (N-m), can be written as follows:

$$\mathbf{L}_g = \frac{3k}{\mathrm{R}^3}\left(R \times \mathbf{I} \cdot R\right) \tag{2}$$

If the orbit is eccentric, then (ref. 32)

$$\frac{k}{\mathrm{R}^3} = \omega_o^2 \left(\frac{1 + e \cos \omega_o t}{1 - e^2}\right)^3 \tag{3}$$

where

ω_o = mean orbital angular velocity, rad/sec
ω_o^2 = k/a^3 (rad/sec)^2
a = semimajor axis, meters
e = orbit eccentricity, dimensionless
t = time measured from perigee, seconds

If the satellite is in a circular orbit, the quantity k/R^3 is a constant equal to ω_o^2.

The simplifications used to develop equation (2) are (1) oblateness and higher order terms of the planetary gravitational field are ignored, (2) higher order inertial integrals of the orbiting body are ignored, and (3) the mass of the satellite is assumed to be negligible with respect to the mass of the Earth. DeBra (ref. 36) gives an expression for the gravitational torque that includes the oblateness term and neglects higher order inertial integrals. Meirovitch (ref. 37) has investigated the effect of the higher order inertial integrals for circular orbits about a spherically symmetric Earth.

Even for low altitude orbits the torque computed using equation (2) will provide adequate accuracy, i.e., ignoring oblateness and higher order terms will not change the result by more than a few tenths of 1 percent. In most cases the major sources of inaccuracy in the determination of gravitational torque will result from uncertainties in the determination of the inertial dyadic and the exact orientation of the spacecraft relative to local vertical.

2.6 Inertial Dyadic

To calculate the torque on a satellite, using equation (2), a coordinate frame must be established and **I** and R must be expressed in this frame. Since **I** is constant when expressed in a body-fixed frame (for a rigid body with nonmovable appendages), this frame is most commonly used. If the body-fixed frame is defined as a right-handed, orthogonal coordinate frame (x, y, z) with origin at the mass center, the nine components of **I**, arranged in a matrix form, are:

$$\begin{bmatrix} I_{xx} = \int_m dm(y^2 + z^2) & I_{xy} = -\int_m dm\, xy & I_{xz} = -\int_m dm\, xz \\ I_{yx} = \int_m dm\, yx & I_{yy} = \int_m dm\, (z^2 + x^2) & I_{yz} = -\int_m dm\, yz \\ I_{zx} = -\int_m dm\, zx & I_{zy} = -\int_m dm\, zy & I_{zz} = \int_m dm\, (x^2 + y^2) \end{bmatrix} \tag{4}$$

Because the inertial dyadic is symmetric, i.e., $I_{xy} = I_{yx}$ and so forth, it is always possible to find a body-fixed frame in which only the diagonal components, I_{xx}, I_{yy}, and I_{zz}, have nonzero values. This particular frame is called the principal axes of the body and the three nonzero components of the inertial dyadic are called the principal moments of inertia.

For control system purposes it is often most convenient to select a body-fixed coordinate frame that coincides with the orientation of one or more of the hardware elements, e.g., sensor optical axis, torque axis, gyro input axis, etc.; the axes thus selected will not necessarily coincide with the body's principal axes. In this case the diagonal elements, I_{xx},

I_{yy}, and I_{zz}, are not the principal moments of inertia and the inadvertent use of this term may create confusion.

Misunderstanding or lack of communication between control system engineers and the group supplying data on the inertial properties of the spacecraft have led to incorrect determination of gravitational torques. The three major sources of difficulty have been (1) the use of left-handed coördinate frames in inertial determination, (2) the use of a station rather than a mass centered location of the origin in the inertial determination, and (3) confusion regarding the sign of the product of inertial term. I_{xy} is defined as $-\int_m dm\, xy$. Because the integral itself (without the minus sign prefixed) can have either positive or negative value for a particular configuration, it is often difficult to ascertain from a table listing the inertial properties of the spacecraft whether the prefixed minus sign was included in the original calculation.

2.7 Gyroscopic Terms

The inclusion of inertial or gyroscopic terms in the gravitational torque equations has caused confusion. In the development of the equations for the attitude motion of a rigid body subject to gravitational torque, application of Newton's second law yields:

$$\frac{d\mathbf{H}}{dt} = \mathbf{L}_g \tag{5}$$

where $\mathbf{H} = \mathbf{I} \cdot \boldsymbol{\omega}$ is the angular momentum of the satellite. Because $\mathbf{I}$ is expressed in a body-fixed frame, equation (5) becomes:

$$\frac{\delta \mathbf{H}}{\delta t} = -\boldsymbol{\omega} \times \mathbf{H} + \mathbf{L}_g = -\boldsymbol{\omega} \times \mathbf{I} \cdot \boldsymbol{\omega} + \mathbf{L}_g \tag{6}$$

where all vectors are expressed in a frame attached to the body and $\frac{\delta \mathbf{H}}{\delta t}$ indicates that the derivative is relative to the body frame. The similarity between the $\boldsymbol{\omega} \times \mathbf{I} \cdot \boldsymbol{\omega}$ term in equation (6) and the $R \times \mathbf{I} \cdot R$ term in equation (2) indicates that in the equations defining the components of torque, inertial and gravitational torques will have the same form and could, therefore, be combined. Combining these terms entails the risk of including the inertial torque twice when the dynamic equations are written in full, or, for an inertially stabilized spacecraft, including torques that do not exist. The separation of gravitational and gyroscopic torques is particularly important when consideration must be given to the momentum change produced by the torque as, for example, in determining impulse storage requirements for a momentum wheel-gas jet control system. In this case the integrated effect of the torque must be computed with respect to inertial space and the gyroscopic terms do not appear. Further discussion of torque impulse and the integrated effect of gravitational torque will be found in section 4.1.7 and appendix B of this document, and in reference 38.

2.8 Mass Distribution

2.8.1 Determination

Computer tabulation of mass properties is almost universal in the aerospace industry. The inputs to the computer programs are mass and location of small assemblies, and mass, inertias, and center-of-mass location for major structures. Some programs approximate distributed structure by numerous small parallelepipeds, cylinders, etc. Program outputs include spacecraft weight, coordinates of the mass center, axial and cross products of inertia relative to a set of normally body-fixed axes (plan axes) plus the principal axes, principal moments, and angles between the principal axes and the body-fixed set.

Programs for the evaluation of mass properties typically use coordinate axes whose origin and orientation are plan determined; these may not coincide with the origin and orientation that would normally be chosen for control system analysis. The inertia cross products are frequently tabulated without the minus sign that appears in the defining equations (see equation (4)). Before using a computed tabulation of spacecraft inertial properties for the determination of gravitational torques, the designer should ascertain that the coordinate axes, mass center, and sign conventions are properly chosen.

2.8.2 Verification

Measurement of spacecraft moments of inertia is generally confined to compact structures or the rigid portion of the spacecraft having extendable booms, antennas, solar arrays, etc. For spin-stabilized satellites and some dual-spin satellites where portions of the satellite spin, the alinement of the spin axis relative to one of the principal axes is important. Spin balancing machines have been designed to locate the principal axis of inertia. Several such machines, manufactured in this country and abroad, are now available. Spin balancing is also required when the satellite is attached to a spinning stage of the launch vehicle even though the satellite itself may be despun after injection into orbit. In addition to locating a principal axis, spin balancing equipment can be used to measure the inertia about that axis but this is not the most commonly used technique for inertial determination.

The method most widely used for measuring the moment of inertia about a spacecraft body axis employs a torsion pendulum. The body is suspended so that it is subjected to a restoring torque porportional to the angular deflection (for small angle deflections) and the period of oscillation T (in seconds), is measured. If the spring constant K is known, the inertia J about the symmetry axis is $J = KT^2/4\pi^2$. A common practice for a torsional pendulum with an unknown K is to calibrate the suspension using an object whose inertia is known or easily determined, e.g., a cylinder of uniform density. The spacecraft's inertia is then $J = J_r\,[(T/T_r)^2 - 1]$ where J_r and T_r refer to the reference object used. When bifilar and trifilar suspensions are used, precision is required to correctly locate the axis about which the measurement is made. More detailed descriptions of these test techniques are in reference 39.

2.9 Summary

The state of knowledge of gravitational disturbance torques is generally adequate for the formulation of suitable design criteria. Recent flight experience indicates that the effects of gravitational torque on spacecraft control systems can be accurately predicted in most instances. For a few configurations, e.g., extremely long flexible booms, the analytical techniques are still being refined. Otherwise the analytical methods and knowledge of the gravitational field are sufficient for spacecraft design purposes.

Accurate determination of the inertial dyadic of the spacecraft is the limiting factor in the accuracy of the gravitational torque determination. In most cases the inertial dyadic for the spacecraft in its orbital configuration cannot be measured and must be determined analytically. The designer should check on the origin and orientation of the coordinate frame used and ascertain that the products of inertia are properly defined.

The analysis described in this section is sufficient for most preliminary design applications and for detailed calculations on compact spacecraft. For cases where the equations must be developed with more generality, gravitational torque expressions that include multiple, hinge-connected bodies, damping terms and other external disturbances, and fields other than the central inverse square field, are given in references 17 and 40 through 44.

3. CRITERIA

Disturbance torque arising from the gravitational field acting on the distributed mass of the spacecraft shall be considered in the design of attitude control systems. It shall be demonstrated that gravitational torques acting in combination with all other disturbance torques do not degrade the performance of the attitude control system.

Where it has been determined that the gravitational disturbance torque is an important factor in the attitude control system design, procedures for the control of the spacecraft mass properties shall be initiated and followed during spacecraft design, development, fabrication, and test.

3.1 Gravitational Torque Analysis

Analytical studies should be conducted to determine the gravitational torque that will act on a spacecraft. The following data will be required; their accuracy should be consistent with the phase of the development program and the sensitivity of the attitude control system to gravitational disturbances.

3.1.1 Gravitational Field Model

The complexity of the field model should be consistent with the accuracy of the orbital and the spacecraft inertial data.

3.1.2 Radius Vector

The radius vector should be determined from the orbital parameters. Variations from nominal values should be considered.

3.1.3 Mass Distribution of the Spacecraft

For a single rigid body or for several rigid bodies whose positions relative to one another are essentially fixed, the mass distribution properties should be characterized by the inertial dyadic (that is, the three axial moments of inertia plus the three cross products of inertia) referred to fixed body axes with origin at the mass center. When the spacecraft consists of multiple bodies whose orientations and/or positions relative to one another are not fixed, the representation of mass distributions should account for this variation. Where applicable, in-flight variation of the spacecraft inertial properties due to equipment motion, deployment and jettison of equipment, propellant depletion, flexing of extended booms, etc., should be evaluated.

3.1.4 Orientation of the Spacecraft

The orientation of the spacecraft body axes relative to the radius vector (local vertical) should be determined for all predicted orientations of the spacecraft. Where the orientation cannot be accurately established, the orientation that produces the greatest demands on the control system should be used.

3.2 Evaluation of Disturbance Torque Effects

The evaluation of the effects of gravitational disturbance torques on the attitude control system for spinning and nonspinning spacecraft should include, where applicable, the following:

(1) Control system actuator requirements; viz. peak torque, momentum storage, momentum transfer

(2) Deformation of extended structures

(3) Dynamic interactions and resonances caused by torque variations on spacecraft and appendages

(4) Precession and nutation of the spin axis

(5) Perturbation of spin rate

3.3 Gravity Torque Control

Whenever the gravitational torques are found to be important or dominant compared to other disturbance torques, measures for reducing the gravitational torque through vehicle reorientation, orbital altitude increase, or mass redistribution should be investigated. If such measures are not practicable, the control system should be designed to accomodate these torques, and procedures should be instituted for the accurate assessment and control of spacecraft mass and inertial properties throughout the spacecraft design, development, fabrication, and checkout. Spacecraft moment of inertia determinations should be based on calculated or measured mass properties. For compact spacecraft, critical moments of inertia and, if applicable, alinement of geometrical and principal moment axes should be verified by measurement of the entire spacecraft assembly.

4. RECOMMENDED PRACTICE

Analysis of the gravitational disturbance torques should be accomplished in the early design phase of spacecraft development. This will require close coordination between the control system and structural design groups to evaluate the effects of changes in configuration and hardware. Estimation of the inertial dyadic at this stage will be based on the gross mass properties of the spacecraft.

When the analysis indicates gravitational disturbance torque to be a dominant or significantly contributing disturbance to the attitude stabilization of the spacecraft, greater accuracy in the estimation of the inertial dyadic will be necessary and a continuing program for the determination and, if necessary, control of spacecraft mass properties will be required.

Experience has shown that the inertial data generated by the group responsible for determining the mass properties of the spacecraft will not always be in a form that is useful to the control system engineer. Particular attention should, therefore, be given to obtaining the necessary data in the proper form each time the mass properties are updated.

4.1 Recommended Practice for Torque Analysis

4.1.1 General Procedure

Analytical studies of the disturbance torques that act on a proposed spacecraft configuration are necessary in the very preliminary phase of design, prior to the final selection of a configuration, to obtain (1) a reasonable approximation of the magnitude of the disturbance torques for a given configuration, (2) identification of the dominant torque (or torques), and (3) determination of the design constraints that may be required to control the magnitude of a specific disturbance torque.

Determination of the gravitational torque at this stage will most likely be based on the estimated values of the principal moments of inertia, using approximation techniques to ascertain the maximum torque and, if required, the maximum angular impulse added per orbit. When the analysis indicates that the gravitational torque is of consequence in the design, further analytical studies of a more detailed nature should be initiated. At this time, the pertinent parameters should be determined more accurately and the probable extent of variations from nominal values ascertained. Computer simulation using numerical techniques will be essential except where the configuration is compact and rigid and the attitude history of the spacecraft with respect to local vertical easily determined.

4.1.2 Gravitational Field Model

Because the Earth is not a homogeneous sphere, an accurate model of the gravitational field involves a spherical harmonic expansion and a large number of harmonic coefficients. Although the higher harmonics of the gravitational field can cause significant perturbation of the satellite's orbit, they have negligible effect on the magnitude and direction of the gravitational torque. Ignoring the higher order terms in the harmonic expansion of the Earth's gravitational potential introduces a maximum magnitude error in the gravitational torque of less than 0.5 percent and an angular error of less than 0.1°. In determining the gravitational torque on a spacecraft, adequate accuracy is obtained using the inverse square field as in equation (1).

Values of the gravitational constant and mass ratio for the Sun, the planets of the solar system, and the Earth's Moon are given in table I.

Table I.–Gravitational Constants and Mass Ratios of Planets, Sun, and Moon

Body	Mass Ratio, M_{Sun}/M_{planet}	Gravitational constant, m^3/sec^2
Sun	1	$1.327\,125\,0 \times 10^{20}$
Mercury	5 983 000 ± 25 000	$2.218\,159 \times 10^{13}$
Venus	408 522 ± 3	$3.248\,60 \times 10^{14}$
Earth and Moon	328 900.1 ± 0.3	$4.035\,040 \times 10^{14}$
Mars	3 098 700 ± 100	$4.282\,84 \times 10^{13}$
Jupiter	1047.3908 ± 0.0074	$1.267\,076 \times 10^{17}$
Saturn	3499.2 ± 0.4	$3.792\,651 \times 10^{16}$
Uranus	22 930 ± 6	$5.787\,722 \times 10^{15}$
Neptune	19 260 ± 100	$6.890\,574 \times 10^{15}$
Pluto	1 812 000 ± 40 000	$7.324\,088 \times 10^{14}$
Earth	332 945.6	$3.986\,012 \times 10^{14}$
Moon		$4.902\,78 \times 10^{12}$

4.1.3 Radius Vector

The radius vector (magnitude and direction of the line joining the mass center of the body that sets up the gravitational field and the mass center of the spacecraft) is determined from the orbital parameters. Any consistent set of parameters can be used. Orbit inclination, orbit eccentricity, and the radius vector at periapses are typical for elliptical orbits. For circular orbits the vector to the ascending node is commonly used as the principal direction.

The trajectory data will generally provide values that are more accurate than required for gravitational torque analysis, where directional accuracy of greater than 0.1° and a magnitude accuracy exceeding 0.1 percent are unwarranted (sec. 4.1.4). In the evaluation of the effects of variations in orbit parameters, deviations that lower the periapsis or increase eccentricity will tend to increase the gravitational torque or associated angular impulse.

4.1.4 Spacecraft Mass Properties

Determination of the spacecraft inertial dyadic to an accuracy of better than 1 percent will rarely be possible. The difficulties associated with the accurate determination of mass, location, and moments of inertia of individual assemblies and similar properties for the mechanical structure preclude greater accuracy for even compact spacecraft. When deployable structures and flexible appendages are present, accuracy in the determination of spacecraft inertial dyadic is dependent on the accuracy to which the deflected shape of the appendage can be determined. In this case the achievement of accuracies of better than 10 percent is seldom practical. Because the accuracy of the torque calculation will always be dominated by the limitations in the knowledge of the inertial dyadic, it is unnecessary to obtain highly accurate values for the radius vector, field model, etc.

When the spacecraft contains assemblies that are free to move relative to one another, e.g., a gimbaled antenna, or a deployable boom, one of two different approaches may be employed in the analysis: either (1) the components of the inertia dyadic for the composite vehicle are varied to account for the motion (refs. 45 and 46), or (2) the two or more bodies are considered independently and interactions between the bodies are accomodated using suitable constraints and external torques at the hinge (refs. 16, 17, and 47). Approach (1) is frequently used when the purpose of the analysis is the determination of upper limits on torque or accumulated angular impulse. This approach is especially suitable when it is possible to determine the particular orientation or orientations of movable bodies that maximize the inertial components that are of primary concern (see app. B). Approach (2) is usually preferred when the relative motion between the bodies is large. Either approach is suitable for machine computation.

4.1.5 Spacecraft Orientation

During preliminary design, gravitational torque can be computed assuming a nominal orientation of the spacecraft. The orientation selected will depend on the mission, the nature of the control system, accuracy of control, etc. The assumed orientation should be based on conditions that tend to maximize the torque but, at the same time, are realizable when the control system is functioning normally. On an Earth-pointing spacecraft, for example, the assumption that one of the principal axes is maintained coaxial with the radius vector is unreasonable because any condition that caused a small deviation from the assumed orientation would increase the torque. Thus variations arising from unavoidable errors in sensor mounting or alinement, shifts in the spacecraft mass center, or offset errors in the control system would not be properly accounted for. Conversely, when the spacecraft is stellar or solar oriented, the angles between principal axes and local vertical are large, and small angle variations introduced by the attitude control system will be of little consequence.

Determination of the spacecraft orientation is often dependent on the authority of the control system. If the disturbance torques are inconsequential compared to the actuator capability, the orientation will depend primarily on the characteristics of the control system, i.e., sensor errors, servo errors, drifts, etc., and the primary purpose of the disturbance torque analysis will be the evaluation of accumulated effects. When the control is passive or the torque capability of the system is of the same order of magnitude as the disturbances, it is necessary to choose an initial orientation and simulate the rotational motion to determine the attitude behavior. Facilities and programs capable of executing large scale simulations of this nature will invariably include the routines for the determination of gravitational torques.

4.1.6 Instantaneous Torque

When a central inverse square field is assumed, the gravitational torque expression is given by equation (2), repeated here for convenience

$$\mathbf{L}_g = \frac{3k}{\mathrm{R}^3}\left(R \times \mathbf{I} \cdot R\right) \tag{7}$$

To compute the components of torque, a coordinate system must be established and scalar equations derived. For composite spacecraft, where the time-varying components of the inertial dyadic are available, or for a compact spacecraft equation (7) readily yields the components of gravitational torque in body coordinates. If the coordinate frame (x, y, z) is right handed, orthogonal, and fixed in the spacecraft with origin at the mass center, the components of gravitational torque along the x, y, and z axes are

$$L_{gx} = \frac{3k}{R^3}\left[(I_{zz} - I_{yy})a_y a_z + I_{yz}(a_y^2 - a_z^2) + I_{xz}a_x a_y - I_{xy}a_x a_z\right] \quad (8a)$$

$$L_{gy} = \frac{3k}{R^3}\left[(I_{xx} - I_{zz})a_z a_x + I_{zx}(a_z^2 - a_x^2) + I_{yx}a_y a_z - I_{yz}a_y a_x\right] \quad (8b)$$

$$L_{gz} = \frac{3k}{R^3}\left[(I_{yy} - I_{xx})a_x a_y + I_{xy}(a_x^2 - a_y^2) + I_{zy}a_z a_x - I_{zx}a_z a_y\right] \quad (8c)$$

where

k is the gravitational constant
R is the distance from the planet mass center to the spacecraft mass center
I_{ij} with $i, j = x, y, z$ are the components of the inertial dyadic (sec. 2.4)

and

a_i with $i = x, y, z$ are the direction cosines of R with respect to the x, y, z coordinate frame (fig. 4).

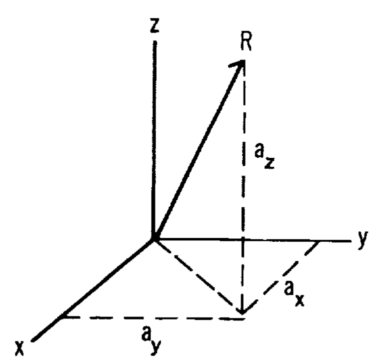


Figure 4.—Direction cosines.

When the attitude of the spacecraft relative to local vertical is not fixed, the direction cosines vary with time. In those cases where the control system maintains one of the body axes alined to the center of the Earth, say the x-axis, then $a_x = 1$ and $a_y = a_z = 0$. Equations (8a), (8b), and (8c) then reduce to

$$L_{gx} = 0 \tag{9a}$$

$$L_{gy} = \frac{3k}{R^3} (-I_{zx}) \tag{9b}$$

$$L_{gz} = \frac{3k}{R^3} (I_{xy}) \tag{9c}$$

When these equations are applicable, relative motions of various portions of the spacecraft can be examined to find those orientations that maximize the two inertial cross products. The restrictive assumption with respect to the spacecraft attitude, i.e., perfect alinement between the x-axis and local vertical, must be examined to insure that the torque components are not highly sensitive to small angular deviations away from this orientation.

When the x, y, and z body coordinates coincide with the principal axes, the products of the inertial terms are all zero and only the first terms on the right-hand side of equations (8a), (8b), and (8c) remain. In this form the expressions can be used to establish an upper bound on the gravitational torque. The torque is determined at perigee (minimum R) using the difference between the maximum and the minimum moment of inertia multiplied by 1/2 (the maximum value attainable by $a_x a_z$, etc.):

$$L_{g(max)} = \frac{3k}{2(R_{min})^3}\left(I_{max} - I_{min}\right) \tag{10}$$

For preliminary design, estimation of the gravitational torques can be obtained by considering an elementary satellite in a circular orbit. As illustrated in figure 5, the satellite is composed of six point masses separated by three massless rods of unequal length (ref. 48). If this satellite is displaced only in the orbit (pitch) plane, or only in the cross orbit (roll) plane, the gravitational torques are given by the pitch and roll expressions in the second column of table II. As indicated by the blank opposite yaw in the second column, no gravitational torque results from the displacement of the x-axis out of the orbit plane.

When the spacecraft is stabilized in space, the roll and pitch angles (and hence the gravitational torque) will vary as the spacecraft moves in its orbit. When the spacecraft is stabilized with respect to the orbital coordinates, the gravitational torque may appear constant in the body-fixed frame but inertial or gyroscopic torques arise because of rotation of the spacecraft and these are listed in the third column of table II. From table II it can be observed that the 4/2 term that occasionally appears in the literature as the coefficient for the roll torque is the result of combining the gravitational with the inertial torque.

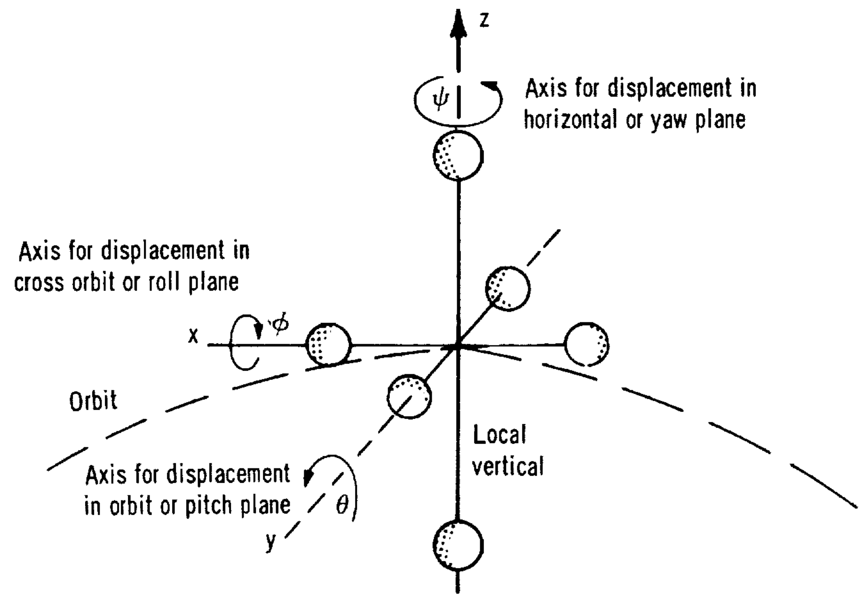


Figure 5.–Angular displacements for a three-axis gravitational-stabilized satellite.

Table II.–Gravitational and gyroscopic torque expressions

Displacement axis	Gravitational torque	Inertial or gyroscopic torque
Pitch (θ)	$-(3/2)\,\omega_0^2\,(I_R - I_Y)\sin 2\theta$	------------------
Roll (φ)	$-(3/2)\,\omega_0^2\,(I_p - I_Y)\sin 2\varphi$	$-(1/2)\,\omega_o{}^2\,(I_p - I_Y)\sin 2\varphi$
Yaw (ψ)	------------------	$-(1/2)\,\omega_o{}^2\,(I_p - I_R)\sin 2\psi$

I_R = moment of inertia about the satellite roll (x) axis, kg–m^2

I_p = moment of inertia about the satellite pitch (y) axis, kg–m^2

I_Y = moment of inertia about the satellite yaw (z) axis, kg–m^2

θ = angular displacement of the satellite z-axis off the local vertical direction in the orbit (pitch) plane, radians

φ = angular displacement of the satellite z-axis off the local vertical direction in the cross-orbit (roll) plane, radians

ψ = angular displacement of the satellite x-axis from the orbit plane (a yaw displacement), radians

ω_o = angular rate for a circular orbit, rad/sec

4.1.7 Torque Impulse

When determining the gas storage requirements for a spacecraft that uses gas jets to dump accumulated angular momentum, the average torque or torque-impulse over one orbit is of concern. The change in angular momentum must be computed using an inertially fixed coordinate system. For example, the set of equations (9) indicate that, for an Earth-oriented spacecraft, the gravitational torque components as seen in the body-fixed frame are constant for a circular orbit (R = constant). The angular momentum accumulated over an orbit is, however, not equal to the product of the constant torque components and the orbital period. When the torque components given by equations (9) are transformed into an inertial frame and integrated over an orbit, the result is (ref. 38)

	Elliptical orbit	*Circular orbit*
ΔH_x per orbit	0	0
ΔH_y per orbit	$-\frac{3k}{ph} e\pi I_{xz}$	0
ΔH_z per orbit	$\frac{3k}{ph} 2\pi I_{xy}$	$6\pi\omega_o I_{xy}$

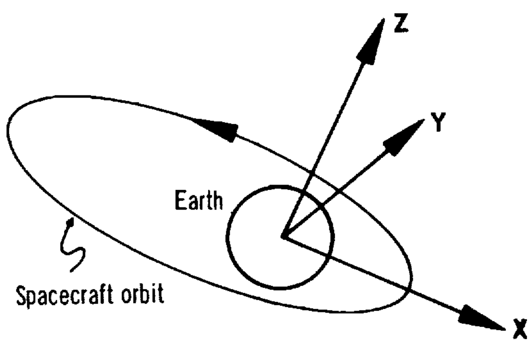
where

p = semilatus rectum of the orbit
h = magnitude of orbital angular momentum per unit spacecraft mass
e = eccentricity
ω_o = orbital angular velocity (circular orbit)

When the satellite is solar or stellar oriented, the body frame is an inertial frame and the torque equations can be integrated directly. Different formulations of the problem are found in references 10, 28, and 38. When the inertial coordinate system of figure 6 is used and orbital precession neglected, the angular moment accumulated over an orbit is

	Elliptical orbit	*Circular orbit*
ΔH_x per orbit	$\frac{3k}{ph}\pi I_{yz}$	$3\pi\omega_o I_{yz}$
ΔH_y per orbit	$-\frac{3k}{ph}\pi I_{xz}$	$-3\pi\omega_o I_{xz}$
ΔH_z per orbit	0	0

When the body axes do not coincide with the reference frame shown in figure 6, the components of the inertial dyadic in the reference frame can be determined by means of a similarity transformation (app. B). The cyclical terms of the angular momentum **H** average to zero over an orbit. For the cases of Earth and inertially oriented spacecraft these terms can be found in appendix B.



X = positive in the direction of perigee

Y = completes right-handed set

Z = normal to orbit plane positive in the direction of positive orbital angular momentum

Figure 6.—Inertial coordinates used to determine accumulated angular momentum.

4.2 Control of Gravitational Torques

Examination of the gravitational torque equation (2) or (8) indicates the three possible ways in which the gravitational disturbance torque can be reduced: (1) increasing the orbital altitude, (2) maintaining close alinement between local vertical and one of the principal axes of the vehicle, and (3) minimizing the difference between the moments of inertia along the principal axes.

Generally both the orbital altitude and the orientation of the spacecraft are determined by the mission requirements, and the only remaining option is to maintain moments of inertia by the principal axes as close to equal as possible within the configuration restraints. This is accomplished by careful arrangement of equipment and, to the extent permitted by weight restraints, the inclusion of adjustable balance masses.

4.2.1 Spin – Stabilized Satellites

The effect of gravitational torque on a spin-stabilized satellite is to produce a precession of the spin axis about the orbit normal (ref. 49). Where the spin axis is desired to be normal to the orbit plane, gravitational torque is beneficial and minimization is not indicated. In all other cases minimization of gravitational torque is required to keep the spin axis from precessing away from its desired orientation.

When the spacecraft consists of a single rigid body, symmetrical about the principal axes, stability considerations require that the spin axis be the axis of maximum moment of inertia, and minimization of gravitational torques imposes the constraint that the difference between the spin moment of inertia I_s and the transverse moment of inertia I_t be made as small as possible

Another factor that must be taken into consideration in specifying the ratio of spin axis to transverse axis moments of inertia is the wobble or torque free precession frequency of the spacecraft. The concern here is for the effectiveness of the wobble damper which must damp out the small residual motion of the spin axis after an operation of the attitude control system that changes the orientation of the spin axis, e.g., the firing of a jet. The effectiveness of the wobble damper in dissipating energy is highly dependent on the wobble frequency. As the inertial difference becomes small, the wobble frequency decreases and the damping time constant becomes inordinately long.

To reduce the disturbance effect of gravitational torques while retaining spin stability about the spacecraft's spin axis, the ratio of I_s to I_t should be as small as practical. A recommended range for this ratio is

$$1.03 \geqslant \frac{I_s}{I_t} \geqslant 1.07 \tag{11}$$

A ratio of at least 1.02 is required to obtain reasonable effectiveness for a wobble damper. Because of the difficulty of measuring moments of inertia with an accuracy of better than 1 percent, a lower limit of 1.03 is recommended. Unless the spin axis is normal to the orbit, the ratio should not exceed 1.07.

The spin axis precession caused by the gravitational torques will be inversely proportional to the spin angular momentum H. Thus, attitude disturbances are reduced by increasing the satellite's spin rate to the limit that can be tolerated by mission and other attitude control requirements. One method for increasing H without increasing the spin rate of the main spacecraft body is to spin only a portion of the body or install an angular momentum flywheel. With this arrangement, called a gyrostat, the spinning portion provides essentially all of the total angular momentum of the combined bodies.

In a gyrostat the gravitational torque can be completely eliminated while still retaining the desirable characteristics of spin stabilization. The spinning section of the satellite can be designed for a high I_s/I_t ratio such as 1.4 to 1.7 (the ratio for an infinitely thin disk is 2.0) while the I_s/I_t ratio for the entire satellite (main body plus spinning sections) can be made as close to 1.0 as practical. Damping of the wobble motion is best accomplished by mounting the wobble damper on the despun section. This is generally also required by stability considerations (ref. 50).

A gyrostat is recommended when the mission requires ultra-low spin rates for the main section of the satellite, as was the case for the Orbiting Solar Observatory satellites (ref. 51) and the Small Astronomy Satellites. The Small Astronomy Satellite (SAS-A) scheduled for launch in mid-1970 employs a small, high-speed wheel to provide the angular momentum necessary to spin stabilize the satellite and, at the same time, allow for an extremely low rotation rate of the outer body. In the event of failure of the high speed wheel, provision is made for maintenance of 0.25 rpm vehicle spin rate and removal of the angular momentum that would otherwise be transferred to the outer body. In this circumstance (i.e., the "fall-back" mode) gravitational torque becomes a significant factor in determining the attitude behavior of the satellite. Figure 7 shows the results of a digital simulation to determine the motion of the spin axis at the low spin rate where the gravitational torque causes a considerable precession of the spin axis.

As previously noted, the most favorable orientation of the spin axis from the standpoint of reducing the effects of gravitational torque is normal to the orbit plane. For missions requiring Earth observations, this orientation of the spin axis is favorable from a number of other considerations as well. The most recent TIROS and ESSA satellites and the Intelsat series are examples of the above.

Spinning spacecraft with unequal moments about the two transverse axes can exhibit irregularity in spin rate because of gravitational torque. This irregularity will generally be insignificant for spin rates that are greater than 10 times the orbital rate. Where small variations in spin rate cannot be tolerated it will be necessary to insure equality of the two transverse moments by proper arrangements of internal equipment or the addition of balance masses.

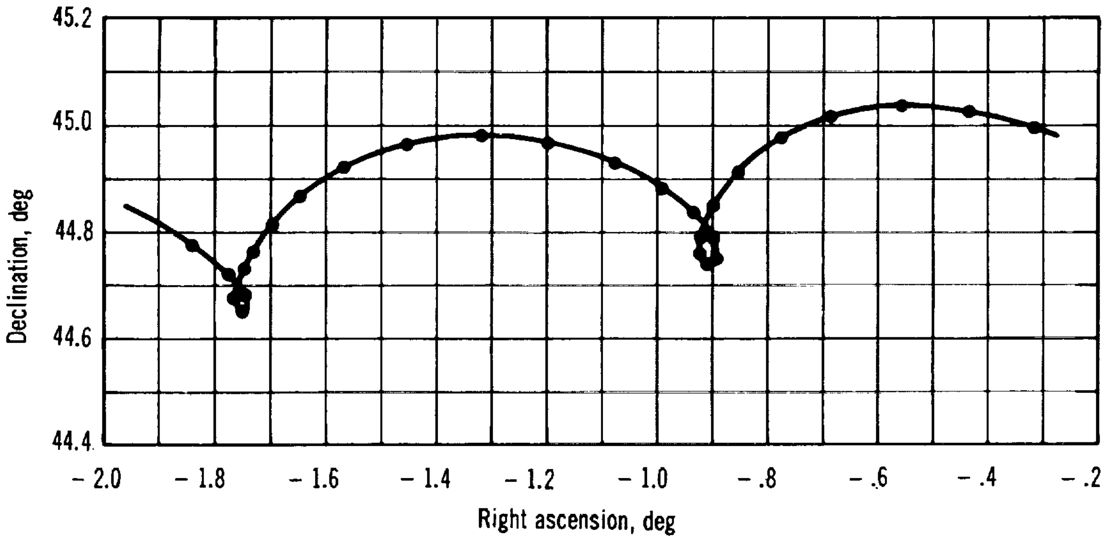


Figure 7.–Effects of gravity gradient torque on SAS-A spin axis attitude.

4.2.2 Three-Axis Actively Stabilized Spacecraft

Gravitational torques can be eliminated by (1) equalizing the three principal moments of inertia, (2) alining the symmetry axis of an axially symmetric spacecraft normal to the orbit plane, or (3) alining the principal axes of an asymmetric Earth-oriented spacecraft to the orbit axes. However, the designer does not normally have any of these prerogatives at his disposal and therefore must design the spacecraft control system to accommodate the gravitational torque.

For orientations that are relatively fixed with respect to an inertial frame, the gravitational torques will vary at twice orbital frequency and will usually have a nonzero average value over an orbit (app. B). The cyclical fluctuations will generally be controlled by some type of momentum storage device, e.g., reaction wheels or control moment gyros, that require only electric power for their operation. However, the average or secular torque will produce a continuously increasing component of angular momentum which would saturate any momentum storage device. Therefore, the control system must periodically react with the external environment to remove the excess angular momentum. Gas expulsion and magnetic torquing are commonly used.

When expendables are employed to counteract momentum accumulation. the average value of the gravitational torque should be carefully considered because it may be a major factor in determining the total stored impulse. The average torque depends on (1) orbit parameters, (2) spacecraft orientation, and (3) spacecraft inertia properties; the designer has generally no control over (1) and only limited control over (2) and (3).

It is often possible to reduce one component of the average torque to zero by making the spacecraft axially symmetrical. Reduction of the difference between the axial and transverse moments of inertia, however, is often limited by other considerations (viz, equipment length, shroud dimensions, etc). A feasible solution when deviations from the required orientation can be tolerated over certain portions of the orbit, for example, when the Sun is eclipsed by the Earth, is to reorient the spacecraft to a new position where the gravitational torque will tend to decrease the excess angular momentum. For further details see reference 52.

4.2.3 Spacecraft With Extended Structures

If the spacecraft has extended booms or has several sections connected together by long rods, gravitational torques can cause significant bending moments in these structures (refs. 26 and 53). If compatible with mission requirements, the longest axis of the spacecraft should be oriented as close to one of the orbital axes as practical. This orientation will minimize the gravitational torque on at least one axis and possibly on all three. Except for placing the spacecraft in a higher orbit, the only other alternative is to provide sufficient structural rigidity to resist the bending caused by the gravitational torques.

Appendix A

DERIVATION OF THE GRAVITATIONAL TORQUE EQUATION FOR A RIGID BODY

The vector torque equation is derived by determining the gravitational force **F** acting on each infinitesimal mass element dm; forming the vector cross product of this force with the vector to the body mass center; and integrating over all mass elements of the body. When a central inverse square field with gravitational constant k is assumed, the torque $\mathbf{L}_g$ is (ref. 9)

$$\mathbf{L}_g = \int_m \rho \times d\mathbf{F} = \int_m \rho \times \frac{-k\,dm}{|\mathbf{R}'|^3}\mathbf{R}' = \int_m \frac{-k}{|\mathbf{R}'|^3}\rho \times \mathbf{R}'dm \qquad \text{(A-1)}$$

where

ρ = the vector distance from the mass center of the spacecraft to the mass element dm

$\mathbf{R}'$ = the vector distance from the mass center of the planet to the mass element dm

and

$\int_m$ = indicates integration over all mass elements of the spacecraft

Let **R** be the radius vector from the mass center of the planet to the mass center of the spacecraft. Then

$$\mathbf{R}' = \mathbf{R} + \rho \qquad \text{(A-2)}$$

and

$$|\mathbf{R}'|^{-3} = (\mathbf{R}' \cdot \mathbf{R}')^{-3/2} = \mathbf{R}^{-3}\left[1 + 2\frac{\rho \cdot \mathbf{R}}{\mathbf{R}^2} + \left(\frac{\rho}{\mathbf{R}}\right)^2\right]^{-3/2} \qquad \text{(A-3)}$$

Neglecting terms of the order $(\rho/\mathbf{R})^2$ and higher, equation (A-3) becomes

$$|\mathbf{R}'|^{-3} \cong \mathbf{R}^{-3}\left(1 + \frac{2\rho \cdot \mathbf{R}}{\mathbf{R}^2}\right)^{-3/2} \cong \mathbf{R}^{-3}\left(1 - 3\frac{\rho \cdot \mathbf{R}}{\mathbf{R}^2}\right) \qquad \text{(A-4)}$$

Hence:

$$\mathbf{L}_g = -\frac{3k}{\mathbf{R}^3}\left(\frac{\mathbf{R}}{R} \times \int_m \rho\rho\,dm \cdot \frac{\mathbf{R}}{R}\right) \qquad \text{(A-5)}$$

Equation (A-5) is simplified by substituting the unit vector in the vertical direction R for $\mathbf{R}/R$, and observing that the inertial dyadic $\mathbf{I}$ can be used to replace the term $\int_m \rho\rho\, dm$. The inertial dyadic is defined as

$$\mathbf{I} = \int_m dm\, (\rho^2 \mathbf{E} - \rho\rho) \tag{A-6}$$

where $\mathbf{E}$ is the unit dyadic or idemfactor (ref. 54). Because $R \times \mathbf{E} \cdot R = R \times R = 0$, the inertial dyadic can be substituted for the integral term in equation (A-5) to obtain

$$\mathrm{L}_g = \frac{3k}{\mathrm{R}^3} (R \times \mathbf{I} \cdot R) \tag{A-7}$$

Appendix B

TORQUE AND ANGULAR IMPULSE DETERMINATION

Coordinate Systems

For the determination of gravitational torques, a number of coordinate systems are used. Each is a right-handed orthogonal frame related to the others by a rotation about one or more of the axes.

To simplify the notation the three base vectors of a coordinate frame will be identified by the subscripts 1, 2, and 3. The progression from one frame to the next is indicated by a specified rotation ${}^{i}A(\alpha)$, meaning a rotation about the ith axis through an angle α.

The coordinate frames are as follows:

A celestial inertial frame (c) (fig. B-1)

$(c) = c_1, c_2, c_3$
c_1 = first point of Aries
c_3 = north celestial pole
c_2 = completes a right-handed set ($= c_3 \times c_1$)

Orbital inertial frame (i) (fig. B-1)

$(i) = i_1, i_2, i_3$
$(i) = {}^{3}A(\omega)\, {}^{1}A(i)\, {}^{3}A(\Omega)\, (c)$
ω = argument of perigee
i = inclination (angle between orbit normal and north celestial pole)
Ω = angle between ascending node and c_1
i_1 = direction of perigee
i_3 = orbit pole, i.e., normal to the orbit in the direction of the orbital angular momentum
$i_2 = i_3 \times i_1$

Local orbital (l) (fig. B-1)

$l = l_1, l_2, l_3$
$= {}^{3}A(A)\, i$
A = true anomaly (angle from perigee to point in orbit)
l_1 = direction of local vertical

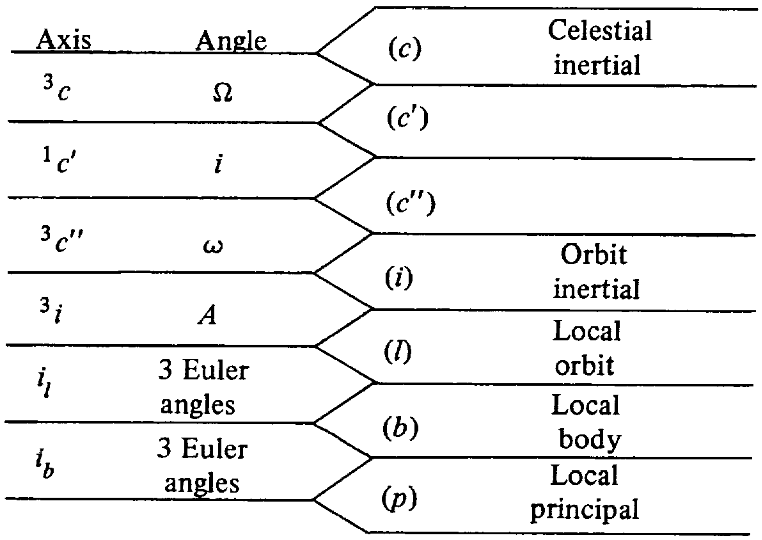
Local body axes (b) (fig. B-2)

$(\mathbf{b}) = \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$
$(\mathbf{b}) = {}^1A(\lambda)\,{}^2A(\varphi)\,{}^3A(\theta)(\mathbf{l})$
$(\mathbf{b}_1)$ = a control axis of the body (not necessarily a principal axis)

Principal body axis (p) (fig. B-2)

$(\mathbf{p}) = \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$
$(\mathbf{p}) = {}^1A(\xi)\,{}^2A(\chi)\,{}^3A(\psi)(\mathbf{b})$

Schematically, this can also be represented as follows:



The transformation of a vector from one coordinate frame to another is written schematically as:

$$\mathbf{V}_l = T_{l/i}\,\mathbf{V}_i$$

or in terms of the components of the vectors

$$\begin{bmatrix} V_{l1} \\ V_{l2} \\ V_{l3} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \begin{bmatrix} V_{i1} \\ V_{i2} \\ V_{i3} \end{bmatrix}$$

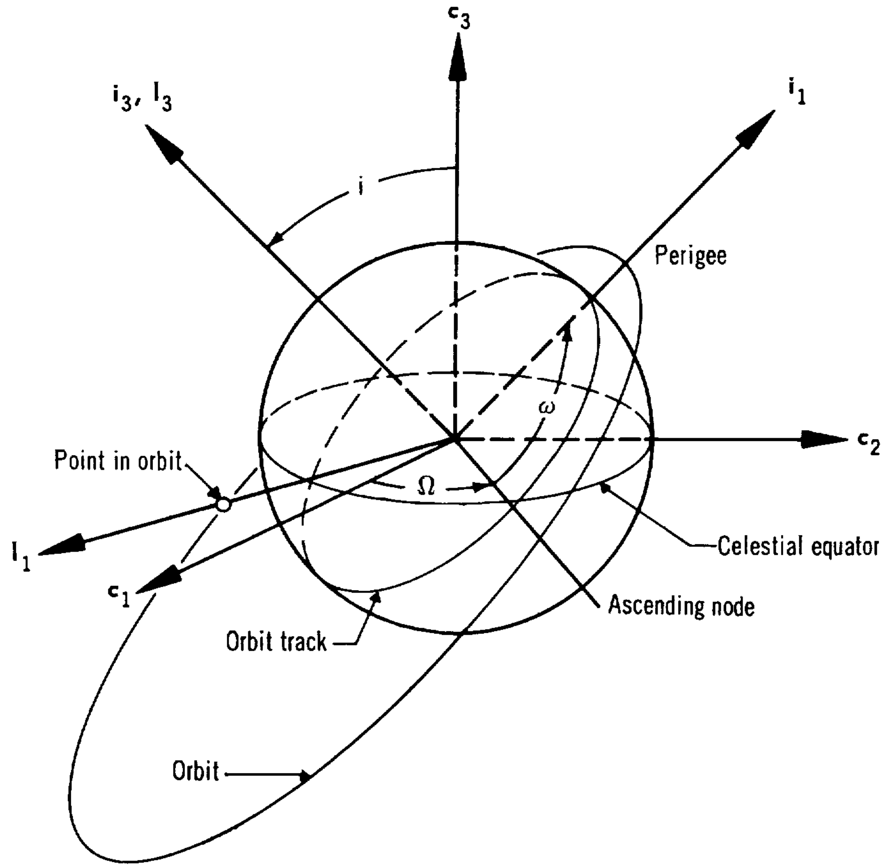


Figure B-1.–Coordinate geometry.

The inverse transformation is written

$$\mathbf{V}_i = T_{i/I}\,\mathbf{V}_I$$

where

$$T_{i/I} = (T_{I/i})^{-1}$$

When the transformation involves only rotations, i.e., $T_{j/k}$ is an orthonormal transformation, then

$$(T_{j/k})^{-1} = T_{k/j}$$

For example:

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

and

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

Transformation of the inertial dyadic from one coordinate frame to another is accomplished by means of a similarity transformation, i.e.,

$$\mathbf{I}_b = T_{b/p} \cdot \mathbf{I}_p \cdot (T_{b/p})^{-1} = T_{b/p}\, \mathbf{I}_p\, T_{p/b}$$

Gravitational Torque and Angular Impulse Per Orbit for Earth-Oriented Spacecraft

The inertial dyadic is written in local body coordinates. Because $R = \mathbf{l}_1$, the components of torque in body coordinates are

$$\mathbf{L}_b = \frac{3k}{\mathbf{R}^3} R \times \mathbf{I} \cdot R = \begin{bmatrix} L_{b1} \\ L_{b2} \\ L_{b3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} L_{b1} \\ L_{b2} \\ L_{b3} \end{bmatrix} = \frac{3k}{R^3} \begin{bmatrix} 0 \\ -I_{31} \\ I_{21} \end{bmatrix}$$

Transforming the torques into the orbital inertial frame:

$$\begin{bmatrix} L_{i1} \\ L_{i2} \\ L_{i3} \end{bmatrix} = \frac{3k}{R^3} \begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -I_{31} \\ I_{21} \end{bmatrix}$$

$$\begin{bmatrix} L_{i1} \\ L_{i2} \\ L_{i3} \end{bmatrix} = \frac{3k}{R^3} \begin{bmatrix} I_{31} \sin A \\ -I_{31} \cos A \\ I_{21} \end{bmatrix}$$

To compute the increment to angular momentum the integral of the torque is required:

$$\Delta \mathbf{H} = \int_0^t \mathbf{L}_i dt = \int_0^t \mathbf{L}_i \frac{dA}{\dot{A}} = \int_0^A \frac{3k}{R^3} \begin{bmatrix} (I_{31} \sin A)\, \mathbf{i}_1 \\ (-I_{31} \cos A)\, \mathbf{i}_2 \\ (I_{21})\, \mathbf{i}_3 \end{bmatrix} \frac{dA}{\dot{A}}$$

where

$$\dot{A} = \frac{dA}{dt}$$

The integration is performed using the following relation

$$\frac{1}{R^3 \dot{A}} = \left(\frac{1}{R^2 \dot{A}}\right) \cdot \left(\frac{1}{R}\right) = \frac{1}{h} \left(\frac{1 + e \cos A}{p}\right)$$

where

- h = magnitude of orbital angular momentum (constant) (the angular momentum of a unit mass traveling in the spacecraft orbit)
- p = the semilatus rectum = $a(1-e^2)$
- e = eccentricity
- A = true anomaly

Thus

$$\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \end{bmatrix} = \frac{3k}{ph}\left\{\begin{bmatrix} 0 \\ \dfrac{I_{31} eA}{2} \\ I_{21} A \end{bmatrix} + \begin{bmatrix} I_{31}(1-\cos A) + \dfrac{I_{31} e}{4}(1-\cos 2A) \\ -I_{31}\sin A + \dfrac{I_{31} e}{4}\sin 2A \\ I_{21} e \sin A \end{bmatrix}\right\}$$

The first term is the secular portion of the angular momentum (i.e., the part that does not have a zero average value over a complete orbit) and the second term is the cyclical component. The values given in section 4.1.6 are obtained using $A = 2\pi$; i.e., a complete orbit.

The above equations can be used to determine the angular impulse caused by gravitational torque that results when there is a misalinement between the control (or sensor) axis and the principal axis of the spacecraft. The situation is illustrated in figure B-2.

$$\mathbf{b} = {}^2A(\psi)\mathbf{p}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 0 & 1 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

Using the similarity transformation:

$$\mathbf{I}_b = T_{b/p}\, \mathbf{I}_p\, T_{p/b}$$

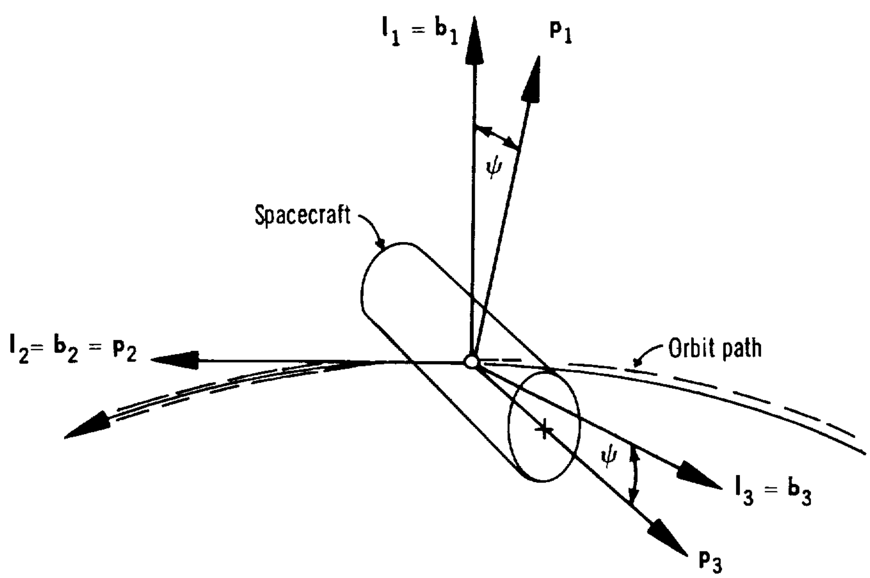


Figure B-2.–Earth-pointing geometry.

Or, in the matrix form:

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence:

$$I_{31} = 0$$

$$I_{21} = (I_2 - I_1)\cos\psi \sin\psi$$

when ψ is small, $I_{21} = (I_2 - I_1)\,\psi$ and the angular impulse per orbit due to the misalinement is

$$\Delta H_3 = \frac{6\pi}{ph}(I_2 - I_1)\,\psi$$

Gravitational Torque and Angular Impulse Per Orbit for Inertially Oriented Spacecraft

The inertial dyadic is written in the orbital inertial frame. The transformation from the local orbital frame to the orbital inertial frame is

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Therefore, since $\mathrm{R} = \bar{I}_1$

$$\begin{bmatrix} L_{i1} \\ L_{i2} \\ L_{i3} \end{bmatrix} = \frac{3k}{\mathrm{R}^3} \begin{bmatrix} 0 & 0 & \sin A \\ 0 & 0 & -\cos A \\ -\sin A & \cos A & 0 \end{bmatrix} \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \cos A \\ \sin A \\ 0 \end{bmatrix}$$

$$= \frac{3k}{2\mathrm{R}^3} \begin{bmatrix} I_{32} - I_{32} \cos 2A + I_{31} \sin 2A \\ -I_{31} - I_{31} \cos 2A - I_{32} \sin 2A \\ (I_{32} - I_{11}) \sin 2A + 2I_{12} \cos 2A \end{bmatrix}$$

The components of angular momentum are determined as in the previous example and, after separating secular and cyclical terms, the following is obtained:

$$\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \end{bmatrix} = \frac{3k}{2ph} \begin{bmatrix} I_{32} A \\ -I_{31} A \\ 0 \end{bmatrix} + \frac{3k}{4ph} \begin{bmatrix} I_{32}\left(-\sin 2A + e \sin A - \frac{e \sin 3A}{3}\right) - I_{31}\left[\left(\cos 2A - 1\right) + e\left(\cos A - 1\right) + \frac{e}{3}\left(\cos 3A - 1\right)\right] \\ I_{32}\left[\left(\cos 2A - 1\right) + e\left(\cos A - 1\right) + \frac{e}{3}\left(\cos 3A - 1\right)\right] - I_{31}\left(\sin 2A + 3e \sin A + \frac{e \sin 3A}{3}\right) \\ I_{12}\left(2 \sin 2A + 2e \sin A + \frac{2e \sin 3A}{3}\right) + \left[\left(\cos 2A - 1\right) + e\left(\cos A - 1\right) + \frac{e}{3}\left(\cos 3A - 1\right)\right] \end{bmatrix}$$

For a complete orbit the secular terms are

$$\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \end{bmatrix} = 3\pi\omega_0 \begin{bmatrix} I_{32} \\ -I_{31} \\ 0 \end{bmatrix}$$

For a solar-oriented spacecraft, the transformation from an inertial[1] to a local orbital frame is accomplished using five successive rotations (fig. B-3).

$$(\mathrm{I}) \doteq {}^3A(\omega)\,{}^1A(i)\,{}^3A(\lambda)\,{}^1A(\epsilon)\,{}^3A(\Omega)(\mathrm{s})$$

$$(\mathrm{I}) = T_{i/c}(\mathrm{s})$$

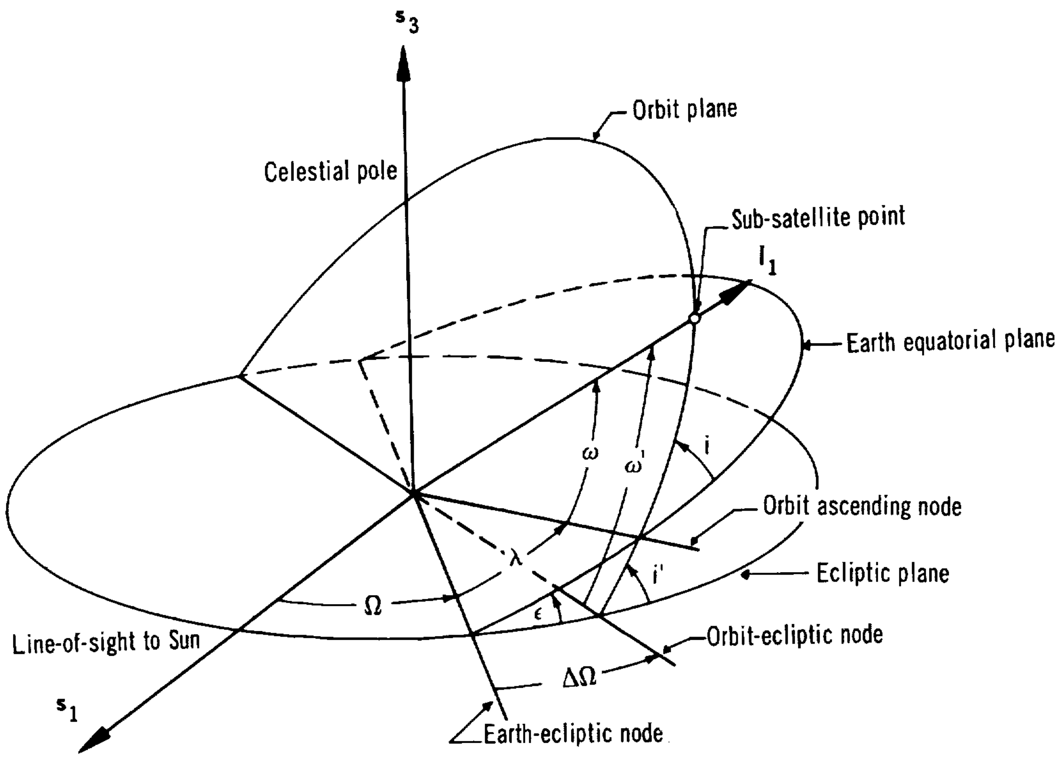


Figure B-3.—Coordinate geometry for solar orientations.

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[1]In this analysis a quasi-inertial system s is employed

s_1 = line of sight to the Sun

$s_3 = c_3$ = north celestial pole

$s_2 = c_2$ = completes a right-handed set

Note that an equivalent transformation can be accomplished using the angles Ω', i', and ω', that is

$$\mathbf{l} = {}^3A(\omega')\,{}^1A(i')\,{}^3A(\Omega')\,\mathbf{s} = T_{i/c}\,\mathbf{s}$$

where the relationship among the primed and unprimed variables is

$$\Omega' = \Omega + \Delta\Omega$$

$$\cos i' = \cos\epsilon \cos i - \cos\lambda \sin i \sin\epsilon$$

$$\sin \Omega = \frac{\sin\lambda \sin i}{\sin i'}$$

$$\sin \omega' \doteq \frac{\sin\lambda \sin\epsilon}{\sin i'}$$

If the control system maintains the alinement between one of the principal axes and the line of sight to the Sun and, further, if this axis is the symmetry axis, that is,

$$I_2 = I_3 \qquad \text{and} \qquad I_2 > I_1$$

then the angular impulse per orbit can be found using the similarity transformation to determine I_{32} and I_{31} in terms of I_1 and I_2. The transformation ${}^3A\,(\omega')$ is not required, hence:

$$\mathbf{I}_i = {}^1A(i')\,{}^3A(\Omega')\,\mathbf{I}_p\,[{}^3A(\Omega')]^T\,[{}^1A(i')]^T$$

and

$$\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \Delta H_3 \end{bmatrix} = \frac{3\pi\omega_o}{2} \begin{bmatrix} (I_2 - I_1)\sin 2\Omega' \sin 2i' \\ (I_2 - I_1)\sin 2\Omega' \sin i' \\ 0 \end{bmatrix}$$

The angle between the ecliptic and orbital planes i' varies because of orbital precession. The range of variation in i' depends on orbit inclination i decreasing with smaller values of i. For equatorial orbits, i' equals the angle between the ecliptic and equatorial planes ϵ, thus assuming the constant value of 23.45°. In the course of a year, Ω' will vary from 0 to 2π.

An alternate approach to this problem is given in reference 10 together with graphs showing the annual variation in ΔH_1 and ΔH_2.

Appendix C

SYMBOLS

A	true anomaly
$\dot{A}$	time derivative of A, (dA/dt)
a	semimajor axis of elliptical orbit
$\mathbf{b}$	Local body axes $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$
$\mathbf{c}$	celestial inertial frame $(\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3)$
e	orbit eccentricity
$\mathbf{F}$	vector force acting on a mass particle in a central inverse square field
$\mathbf{H}$	satellite's angular momentum
h	magnitude of orbital angular momentum
I_R	moment of inertia about the satellite roll axis
I_p	moment of inertia about the satellite pitch axis
I_s	spin moment of inertia
I_t	transverse moment of inertia
I_y	moment of inertia about the satellite yaw axis
$\mathbf{i}$	orbital inertial frame $(\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3)$
i	inclination angle from equatorial plane
i'	inclination angle from ecliptic plane
$\mathbf{I}$	inertial dyadic
J	spacecraft's inertia
J_r	inertia of reference object
k	gravitational constant of attracting body
K	spring constant
$\mathbf{l}$	local orbital frame $(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3)$
$\mathbf{L}_g$	gravitational torque
m	mass of particle
N-m	newton-meter
p	semilatus rectum
$\mathbf{p}$	principal body axes
R	unit vector in the direction of $\mathbf{R}$
$\mathbf{R}$	vector distance from planet's mass center to satellite's mass center
$\mathbf{R}'$	vector distance from planet's mass center to mass element dm $(\mathbf{R} + \rho)$
t	time measured from perigee
T	satellite's period of oscillation
T_r	period of oscillation of reference object
$\mathbf{V}$	vector quantity
ϵ	angle between the equatorial and the ecliptic plane
θ	angular displacement of the satellite yaw axis off the local vertical direction in the pitch plane

λ	angle between the intersection of the ecliptic and equatorial plane and the ascending node (measured on the equatorial plane)
$\boldsymbol{\rho}$	vector distance from spacecraft's mass center to mass element *dm*
φ	angular displacement of the satellite yaw axis off the local vertical direction in the roll plane
ψ	angular displacement of the satellite roll axis from the orbital plane (a yaw displacement
ω	angle between the ascending node and the subsatellite point (measured on the orbital plane)
ω_o	mean orbital angular velocity
$\boldsymbol{\omega}$	satellite's angular velocity about the spin axis
ω'	angle between the intersection of orbital and ecliptic plane and the subsatellite point (measured on the orbital plane)

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