NASA TECHNICAL MEMORANDUM

(48)

Report No. 53935

AN ANALYSIS OF CASSEGRAIN OPTICS

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September 16, 1969

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

MSFC - Form 3190 (September 1968)

IN-SSL-T-67-16 December 18, 1967
This Internal Note was changed to NASA TM X-53935 on Sept. 16, 1969

AN ANALYSIS OF CASSEGRAIN OPTICS

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I. INTRODUCTION

The equations presented in this note were derived by the author in an effort to clarify his understanding of the properties and behavior of a Cassegrain optical system, one of the more common systems used in astronomical telescopes. The final equations may be found in the literature, although infrequently.

It is hoped that this paper will be useful to the researcher using a Cassegrain system by helping him to better understand his instrument and by providing him with the tools to predict its behavior. For instance, Equation (6) is commonly used to describe the system. As is noted, one of the terms is a function of the other, and it would generally be necessary to solve the equation by measuring the dimensions during the experiment. Therefore, Equation (5) is offered as a more useful form. It can also be tabulated and graphed, along with some other functions, to provide a useful guide for telescope users.

II. THE EFFECTIVE FOCAL RATIO OF A CASSEGRAIN SYSTEM

Astronomical telescopes have traditionally been used for photographic work. The focal ratio, which is the focal length divided by the aperture, is not a matter of great concern in astrophotography since most objects of study are point sources and the speed of the system has no effect on the concentration of energy.

With the recent proliferation of radiometric techniques, the cone of light from a telescope is often passed through a series of apertures or stops which can easily eclipse the edges of the cone and affect the data. Instruments using radiometry are often matched to a specific cone angle. This cone angle is the measure of the focal ratio. Any calculation of effective focal ratio must be based upon the resultant cone angle of the focused light.

In using a Cassegrain system, an observer will generally attach his radiometer to the telescope and then focus by moving the secondary mirror. As the equations will show, this motion can have a drastic effect on the effective focal ratio.

An arbitrary sign convention has been adopted for mirror formulas. The following is taken from Jenkins and White*:

1. Distances measured from left to right are positive, while those measured from right to left are negative.

2. Incident rays travel from left to right and reflected rays from right to left.

3. The focal length is measured from the focal point to the vertex. This gives f a positive sign for concave mirrors and a negative sign for convex mirrors.

4. The radius is measured from the vertex to the center of curvature. This makes r negative for concave mirrors and positive for convex mirrors.

5. Object distances s and image distances s' are measured from the object and from the image, respectively, to the vertex. This makes both s and s' positive, and the object and image real when they lie to the left of the vertex, while they are negative and virtual when they lie to the right.

Obviously, rule 2 cannot be adhered to in a Cassegrain system, which has mirrors facing each other (Fig. 1). However, since most of the terms involve the secondary, this will face the left. The primary, therefore, will face the right and its focal length will be negative.

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*Jenkins, F. A. and White, H. E., <u>Fundamentals of Optics</u>, p. 86, McGraw Hill (1957).

The terms used in the following discussion of effective focal ratio are defined as follows:

F_{eff}	focal ratio of the system	
F_1	focal ratio of the primary	$(F_1 < 0)$
f_1	focal length of the primary	$(f_1 < 0)$
f	focal length of the secondary	$(f < 0)$
X_1	half aperture of the primary	
X_2	half cone size at the secondary	
α	half cone angle of the primary	
β	half cone angle of the system	
s	distance of secondary from prime focus	$(s < 0)$
s'	distance of secondary from Cassegrain focus	$(s' > 0)$

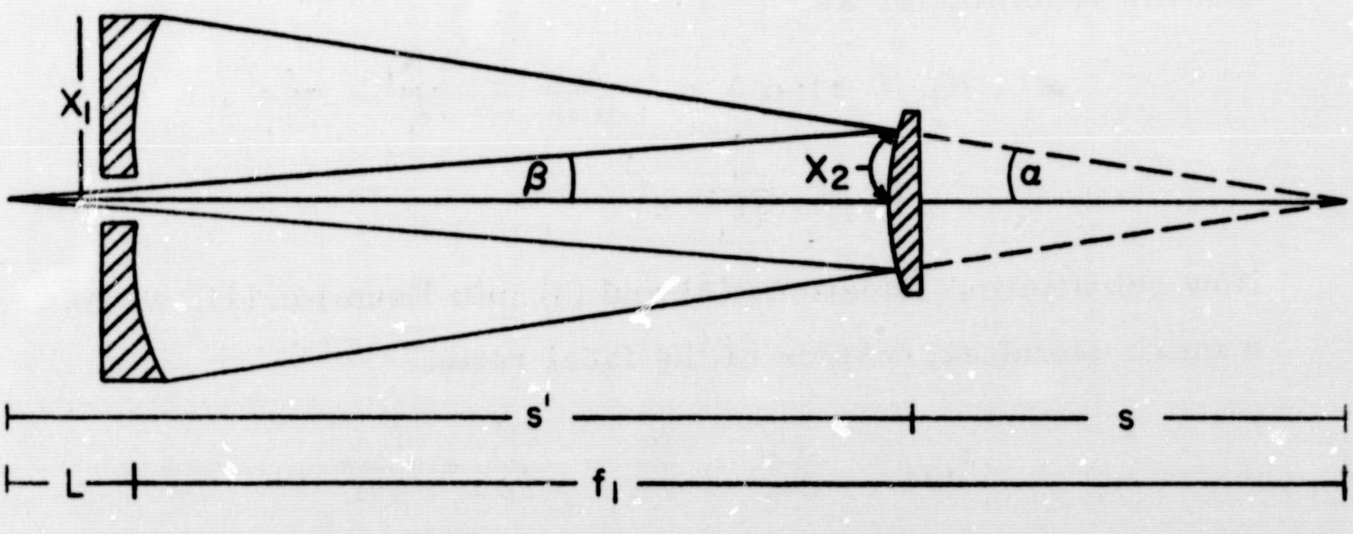

FIG. 1. Cassegrain Focusing System

The effective focal ratio of the system shown in Figure 1 is the cotangent of the cone angle. The focal ratio is expressed as

$$F_{eff} = \cot 2\beta = \frac{s'}{2X_2} .$$

$$\therefore F_{eff} = \frac{s'}{2X_2} . \tag{1}$$

According to the mirror formula*,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} . \tag{2}$$

The numerator in Equation (1) is obtained as follows:

$$s' = \frac{fs}{s-f} , \tag{3}$$

and the denominator as

$$X_2 = s \tan \alpha = \frac{s}{\cot \alpha} = \frac{s X_1}{f_1} = \frac{s}{2F_1} .$$

$$\therefore X_2 = \frac{s}{2F_1} . \tag{4}$$

Now substituting Equations (3) and (4) into Equation (1), we obtain a more useful expression of the focal ratio:

$$F_{eff} = \frac{s'}{2X_2} = \frac{fs}{s-f} \cdot \frac{2F_1}{2s} = \frac{F_1 f}{s-f} .$$

$$\therefore F_{eff} = \frac{F_1 f}{s-f} . \tag{5}$$

In focusing, one changes the value of s. Since s and f are nearly of the same value, the denominator of Equation (5) changes

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*Jenkins and White, p. 87

rapidly with small changes in s; F_{eff}, therefore, is extremely sensitive to changes in s (except in large systems). In one 12-inch F16 system, for example, F_{eff} changed to F14 when s changed by 0.6-inch.

Figure 2 shows an F16 and F14 beam focused upon a detector through a stop designed to admit an F16 beam. The terms

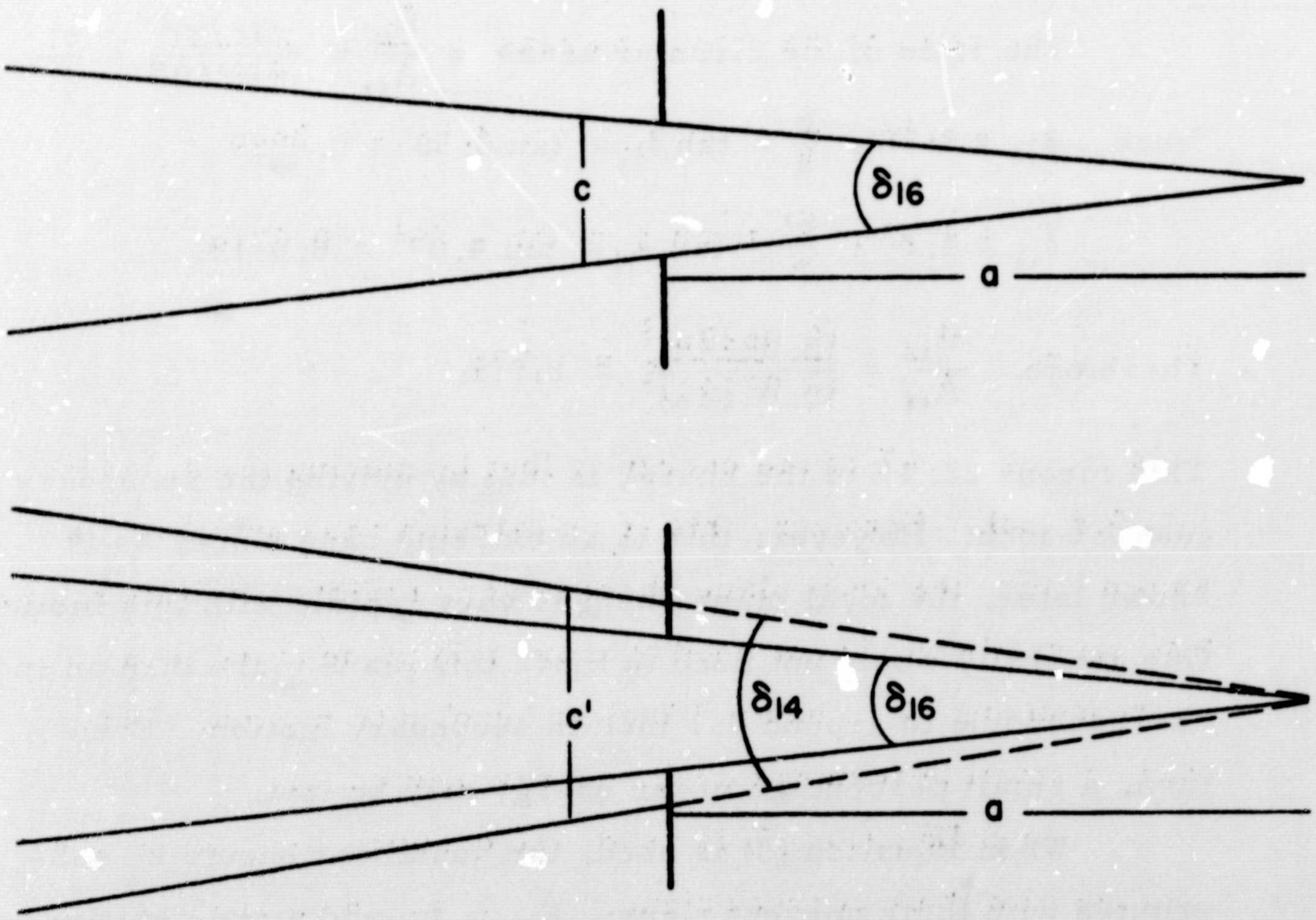

FIG. 2. Operation of Beam Stop

used are defined as follows:

a	distance from focal point to a stop
c	diameter of the stop
c'	diameter of the F14 beam at distance a
δ_{16}	cone angle of F16 beam
δ_{14}	cone angle of F14 beam
A_{16} and A_{14}	cross-section of the respective beam at the stop

The ratio of the circular areas $= \dfrac{A_{16}}{A_{14}} = \dfrac{\pi(c/2)^2}{\pi(c'/2)^2} = \dfrac{c^2}{c'^2}$.

Since $\delta_{16} = 3.58°$, $\dfrac{c}{a} = \tan\delta_{16} = \tan 3.58° = 0.0629$

$\delta_{14} = 4.09°$, $\dfrac{c'}{a} = \tan\delta_{14} = \tan 4.09° = 0.0714.$

Therefore, $\dfrac{A_{16}}{A_{14}} = \dfrac{(0.0629\,a)^2}{(0.0714\,a)^2} = 0.776.$

This means 22.4% of the energy is lost by moving the secondary just 0.6 inch. However, this is an extreme case since, as is shown later, the focal plane changes very rapidly with this motion. One generally would not need to move this plane more than an inch, corresponding to around 0.1 inch in secondary motion. Even then, a small percentage of the energy may be lost.

When Equation (5) is used, the quantities should be substituted with their correct signs. F_1, f, f_1, and s are negative, and s' is positive.

Since the following relationship may be obtained from Equation (3):

$$\frac{f}{s-f} = \frac{s'}{s},$$

Equation (5) may be written as

$$F_{eff} = F_1 \left(\frac{s'}{s}\right) . \tag{6}$$

The term s'/s is called the multiplying factor of the secondary, and Equation (6) is the form generally used in the literature. However, as explained earlier, s and s' are terms in the same function and in practice Equation (5) must be used.

III. MOVEMENT OF THE FOCAL PLANE

Important factors in using a Cassegrain system with a radiometer are the location of the focus and its rate of change as a function of s. The following relationships are obtained when L is the distance of the focus from the primary vertex.

$$L = s' - (f_1 - s) = |s'| - |f_1| + |s| ;$$

$$\therefore L = \left|\frac{sf}{s-f}\right| - |f_1| + |s| . \tag{7}$$

$$\frac{dL}{ds} = \left|\frac{(1)(fs) - f(s-f)}{(s-f)^2}\right| - |0| + |1|$$

$$= \left|\frac{fs - fs + f^2}{(s-f)^2}\right| + |1| .$$

Therefore,

$$\frac{dL}{ds} = 1 + \frac{f^2}{(s-f)^2} . \tag{8}$$

Although s' increases in a negative direction, according to rule 1 it is measured in a positive direction. Therefore, to prevent the negative value of s from being differentiated out and leaving a "-1" in Equation (8), absolute values were used.

The right hand term of Equation (8) is the derivative $\frac{ds'}{ds}$ and dL/ds may be expressed as

$$\frac{dL}{ds} = 1 + \frac{ds'}{ds} . \tag{9}$$

This form is a clearer expression of the motions involved, but should not be used in place of Equation (8).

In the 12-inch system mentioned before, dL/ds was equal to 13 under normal conditions. This increases rapidly, as shown in Table I (page 12).

IV. PLATE SCALE AND EFFECTIVE FOCAL LENGTH

As with the effective focal ratio, the effective focal length (f_{eff}) must be determined only by its effect on the final result or image. It connot be simply assumed that $f_{eff} = 2X_1 F_{eff}$ where $2X_1$ is the aperture of the primary. However, this equation is correct and is the final resultant of what the author believes to be a rigorous treatment of the problem.

The focal length of a system determines one property of the image produced, its scale. This is more properly termed plate scale, P. The dimensions of the plate scale are as follows:

$$\frac{\text{angle subtended by two points}}{\text{image size}} = \frac{\theta}{y} = P.$$

This definition is necessary in astronomy since all objects are at infinity and object distance cannot be used.

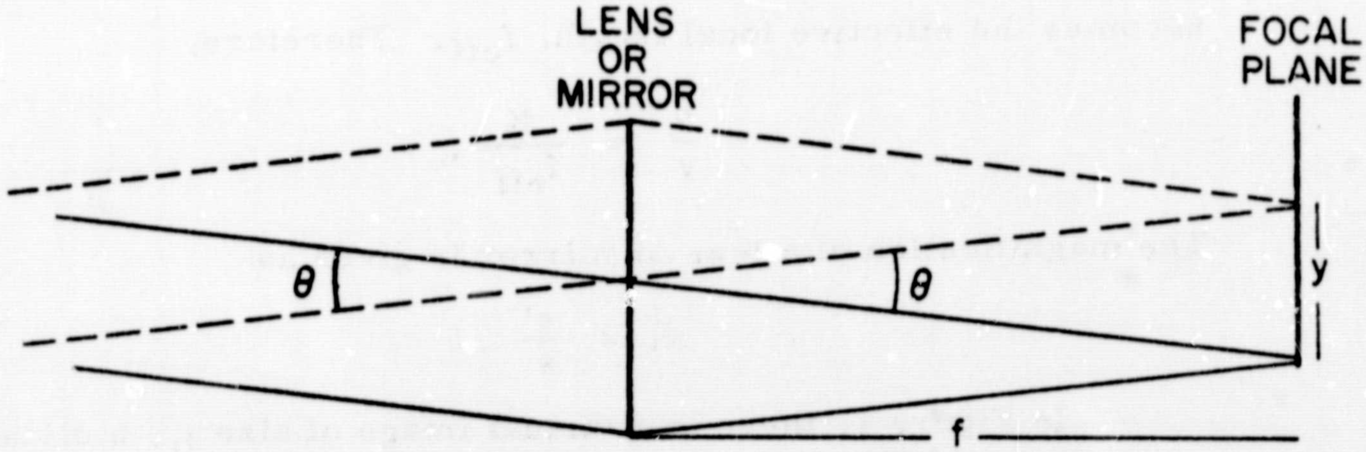

FIG. 3. Determination of Plate Scale

The two rays passing undeviated through the center of the system will be used, since the angle remains unchanged. In a normal Cassegrain, these rays do not exist because of the center hole and secondary obstruction, but this does not affect the theory. The angle θ in Figure 3 may be identified as follows:

$$\tan\theta = \frac{y}{f} = \theta \text{ radians for small angles;} \tag{10}$$

therefore, $y = \theta f$, or $\frac{\theta}{y} = \frac{1}{f}$.

To obtain θ in seconds of arc, convert radians to seconds:

$$1 \text{ radian} = 206,265 \text{ sec of arc;}$$

hence, $$P = \frac{\theta \text{sec}}{y} = \frac{206,265}{f} . \tag{11}$$

The values of y and f must be given in the same unit (mm, in., ft, etc), and the constant 206,265 is designated K.

Equation (11) defines f, and for complicated systems, f becomes the effective focal length, f_{eff}. Therefore,

$$\frac{\theta}{y} = \frac{K}{f_{eff}} .$$

The magnification of a lens or mirror is given as

$$m = \frac{s'}{s} .$$

In Figure 1, there is a virtual image of size y_1, a distance s behind the secondary. The secondary will produce a real image of size y where

$$y = y_1 \frac{s'}{s} .$$

From Equation (11),

$$f_1 = \frac{Ky_1}{\theta} \text{ , and}$$

$$f_{eff} = \frac{Ky}{\theta} = \frac{Ky_1}{\theta} \frac{s'}{s} ;$$

therefore, $f_{eff} = f_1 \left(\frac{s'}{s}\right)$, (12)

which is similar to Equation (6). As with Equation (6), the effective focal length can be put in the more usable form

$$f_{eff} = f_1 \left(\frac{f}{s-f}\right) . \tag{13}$$

It is important to note that θ remains unchanged, since it is the angle subtended by two points in the object plane.

An interesting relation can be obtained by multiplying Equation (13) by a form of unity:

$$f_{eff} = \frac{2X_1 f_1 f}{2X_1(s-f)} ,$$

where $2X_1$ is the aperture of the primary.

Since $\frac{f_1}{2X_1} = F_1$, we obtain $f_{eff} = \frac{2X_1 F_1 f}{s-f}$;

and since $\frac{F_1 f}{s-f} = F_{eff}$, we obtain

$$f_{eff} = 2X_1 F_{eff} . \tag{14}$$

Therefore, the assumption that the effective focal length is equal to the aperture times the effective focal ratio is verified.

V. CONCLUSION

In order to retain control over the parameters that may affect his data, the researcher using a Cassegrain telescope should find it useful to have a tabulation of these parameters. It would be a simple matter, with the use of a computer, to construct a table of the various functions of s in small increments. The table should include F_{eff}, $\tan^{-1} \frac{1}{F_{eff}}$ (cone angle), L, $\frac{dL}{ds}$, and P. It should also be useful to plot several or all of these functions on a graph. As an example: for a 12-inch F16 system (in which $2X_1 = 12$ in., $f = -17$ in., $F_1 = -4$, and $f_1 = -48$ in.), Table I values are computed and then plotted as in Figure 4.

Although the mathematical development of this paper deals with Cassegrain systems, with the proper sign changes it also is applicable to systems using a Barlow lens.

TABLE I

s	F_{eff}	$\tan^{-1}\frac{1}{F_{eff}}$ ($\theta°$)	L	$\frac{dL}{ds}$	P $\frac{sec}{mm}$
11.00	11.3	5.08	-5.8	9.00	60.0
11.25	11.8	4.85	-3.5	9.76	57.4
11.50	12.1	4.72	-0.9	10.48	54.7
11.75	12.9	4.45	+1.9	11.47	52.6
12.00	13.6	4.22	+4.8	12.57	49.8
12.25	14.3	4.02	+8.0	13.79	47.3
12.50	15.1	3.79	11.7	15.22	44.8
12.75	16.0	3.58	15.8	17.00	42.5
13.00	17.0	3.37	20.3	19.10	39.9
13.25	18.1	3.16	25.3	21.28	37.4
13.50	19.4	2.95	31.0	24.78	34.9
13.75	20.9	2.74	38.8	29.48	32.5
14.00	22.7	2.52	45.5	33.10	29.9

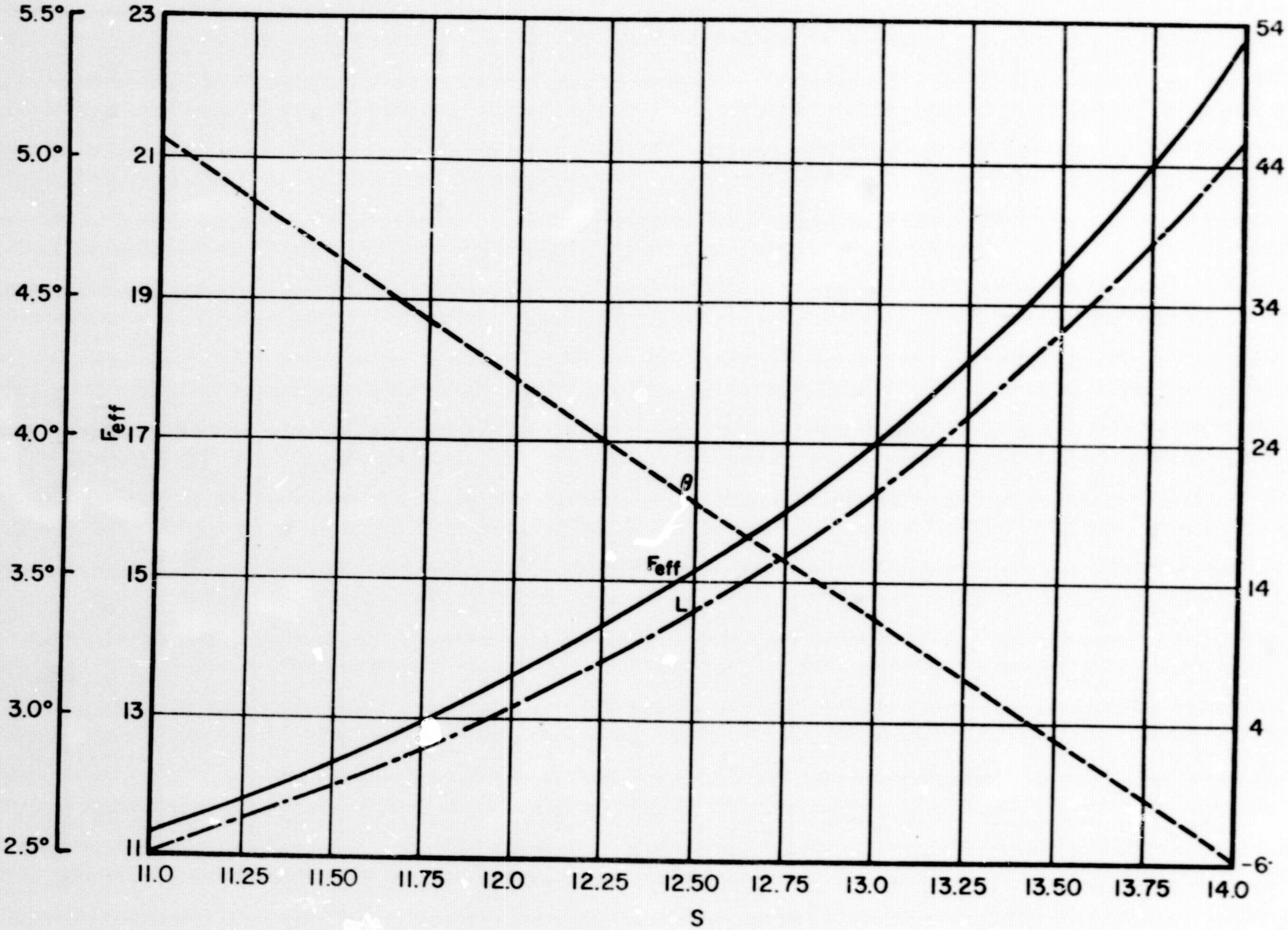

FIG. 4. Plot of Table I Functions of S