UNIVERSITY OF SOUTHERN CALIFORNIA

RESEARCH ON NEW TECHNIQUES FOR THE
ANALYSIS OF MANUAL CONTROL SYSTEMS

PROGRESS REPORT NO. 9

George A. Bekey
Anil V. Phatak

June 15, 1969 - June 15, 1970

Prepared for the National Aeronautics and Space Administration under Grant Number NGL 05-018-022

ELECTRONIC SCIENCES LABORATORY
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I. INTRODUCTION

This progress report covers the past one year period and combines the two semi-annual reports for December 1969 and for June 1970. During this time major efforts were directed towards completing past research projects and to initiating work on projects proposed in the last progress report. Specifically, the work by Merritt on a geometric approach to the identification of human controller decision algorithms has been temporarily put aside. Research on synthetic electromyograms has been concluded and a paper on the topic was presented at the Sixth NASA-University Conference on Manual Control held at Dayton, Ohio in April 1970. An abstract of this paper is included in this report.

The primary topic of research reported herein deals with the application of statistical decision theory to the subject of manual adaptive control. The results presented are preliminary and of a theoretical nature. We are in the process of applying these results via computer simulations to certain well studied manual adaptive control situations. Results will be given in the next progress report.

Work on the modeling of neuromuscular systems is being continued by Les Ostroy under the direction of Dr. G.P. Moore. An outline of his proposed efforts is included.

In addition, there is a new project under way in collaboration with Dr. J. D. Smith of the Biomedical Engineering Program, dealing with the
study of human eye movements during fixation on a steady target. A statement of the problem along with a program of proposed experiments and analyses is presented. We hope to model quantitatively the information processing aspects of the visual feedback apparatus, and to gain physiological background for mathematical concepts such as observation or measurement noise. The latter concepts as we know are invariably involved in the formulation of mathematical models of human controllers.

II. APPLICATIONS OF DECISION THEORY TO MANUAL ADAPTIVE CONTROL

A. V. Phatak and W. H. Lai

A. Background

The subject of manual adaptive control has been a focus of considerable attention and interest since the pioneering work by Sheridan [1] in 1960. As a result, large numbers of publications are available dealing with the various aspects of human controller adaptive behavior. Consequently, no attempt is made here to review the entire literature or to list all the pertinent references. However, a recent survey article by Young [2] provides a comprehensive review of the work done in the field over the last decade.

One of the most practically significant areas studied in the past has been the adaptive behavior of human controllers in tracking tasks in response to a sudden change in the controlled plant dynamics. The basic structure of all the existing mathematical models of human controller adaptation
to step changes in plant dynamics is very similar and involves the partitioning of the adaptive process into the following four phases [3, 4]:

i) optimal control of pre-failure plant dynamics

ii) detection of a change (or failure) in plant dynamics

iii) identification of the post-failure plant dynamics

iv) modification of optimal control appropriate to post-failure plant dynamics.

An error in identifying the post-failure dynamics may be corrected subsequently by allowing for a looping of the sequence (iii)-(iv), in time [5]. It must be emphasized that the hypothesized model structure implicitly assumes the separation of identification and control as comprising the optimal overall solution (separation hypothesis). There exist counter examples that indicate the invalidity of this assumption, in general [6].

The combination of appropriate models for the four phases of adaptation, therefore, constitutes the desired adaptive human controller model. Models for phases (i) and (iv) are available in the form of describing functions appropriate for pre- and post-failure plant dynamics, respectively. Hence, the contribution to adaptive modeling lies in the mathematical description of human decision processes represented by phases (ii) and (iii). Unfortunately, the direct modeling approach based on processing experimental data has severe limitations because of the lack of identification techniques for rapidly varying nonstationary systems. As a result, the development of models for human adaptive control has remained inductive in approach and limited to general
schema. It is only recently [3, 4] that the formalism of statistical decision theory and some pattern recognition concepts were applied to describe human decision making in specific laboratory simulated control situations. The suggestions remain ad hoc and need testing on more generalized control tasks. Young [2] in concluding his survey paper summarizes the state of the art and possible future directions for research with the statement: "only by pressing the development of a theory of manual adaptive control for the unlikely and unexpected failure will we keep the theory of manual control relevant to the needs of the times." In keeping with this spirit, the subject of this section is a systematic comparative study of the various techniques of decision theory as they apply to understanding and modeling phase (ii) of the human adaptive strategy—specifically, the detection of the random occurrence of a change in plant dynamics from one known configuration to another. Extensions to phase (iii) will be carried out at a later date.

B. **Formulation of the Problem**

The human operator is involved in controlling a plant which may be described (after some manipulation) by the linear difference equation

\[ x(k+1) = A(k) x(k) + B(k) u(k) + w(k) \]  

where \( x(k) \) is the \( n \) vector of states at time \((k+1)T\), \( T \) is the sampling interval, \( u(k) \) is the \( r \) vector of deterministic control inputs and \( w(k) \) is an \( n \) vector of state noise.
The plant dynamics \([A(k), B(k)]\) may suddenly change from a known configuration \([A_0, B_0]\) to another fixed form \([A_1, B_1]\) at some unknown instant at time \(k = 6\). For a process of finite duration, \(0 \leq k \leq N\), \(\delta\) may have a value greater than \(N\), thus allowing for the case of no change in plant dynamics. In other words, let

\[
A(k) = \left\{ A_0 (1-\gamma_\delta(k)) + A_1 \gamma_\delta(k) \right\} \quad \text{and} \quad B(k) = \left\{ B_0 (1-\gamma_\delta(k)) + B_1 \gamma_\delta(k) \right\}
\]

where: \(\gamma_\delta(k) \triangleq u(k-\delta) = \begin{cases} 0 : k < \delta \\ 1 : k \geq \delta \end{cases}\)

\(A_0\) and \(A_1\) are \((n \times n)\) matrices, and \(B_0\) and \(B_1\) are \((n \times r)\) matrices, \(w(k)\) is a gaussian white noise sequence with known statistics given by

\[
E\ w(k) = 0
\]

\[
E\ w(k_1) w'(k_2) = K \Delta (k_1 - k_2)
\]

Also,

\[
E\ x(0) = 0
\]

\[
E\ x(0) x'(0) = P(0)
\]

Matrix \(K\) is the covariance of \(w\) and \(\Delta\) is the Kronecker delta function.

The human controller is displayed the \(s\) vector (\(s\) components)

\[
y(k) = C x(k) + v(k)
\]
where $C$ is an $(s \times n)$ matrix and $v(k)$ is a zero mean gaussian white observation noise sequence with covariance $L$. Thus,

$$E v(k) = 0$$

$$E v(k_1) v'(k_2) = L \delta(k_1 - k_2)$$  \hspace{1cm} (6)

Also, $w(k)$ and $v(k)$ are independent. That is,

$$E w(i) v'(j) = 0 \quad \text{for all } i, j$$  \hspace{1cm} (7)

All matrices $A_0, A_1, B_0, B_1, C, K$ and $L$ are, in general, functions of $k$; the argument being dropped here for simplicity of notation. Note that the human controller time delay is not included in the visual processing described by Eq. 5. However, it can be easily included without changing the structure of Eq. 1 and 5 by redefining the time scale $k' = k - kT$, where $kT$ is the time delay $\tau$ and $k$ is an integer.

The aim of this section is to discuss methods for detection of a step change in the plant dynamics given the observation vector $Y(k) = [y(0), y(1), \cdots, y(k)]'$ at any instant of time $k$. The problem is cast in the basic framework of statistical decision theory and the principal ideas applied are those of fixed length and sequential hypothesis testing.
C. Exact Solution to the Problem

The problem of detection of a change in plant dynamics may be solved by either of the following two ways depending on the context of the physical situation. They are

i) Off-line methods: Here, the observed data for the complete duration of the process are available before a decision is made. This situation may arise when the changes involved are not catastrophic and where the time delay between the actual occurrence of a change and detection of this change is not penalized. The concepts of fixed length hypothesis testing are readily applicable to this case.

ii) On-line methods: In these procedures the decision about the dynamical state of the system must be made following the acquisition of each new observation sample \( y(k) \) in time. It is important in such cases to detect the occurrence of a change as early as possible following failure with certain bounds on the various probabilities of misrecognition. The logical scheme for this purpose is a sequential hypothesis testing procedure.

Both methods discussed above require a computation of likelihood functions for the observation vector \( Y(k) \) conditioned on some appropriate hypothesis. In addition, the sequential procedures may be implemented on-line without large memory requirements only when the likelihood functions can be computed recursively. Unfortunately, the sequence of observed samples \( y(k) \) given by Eq. 5 is correlated thereby rendering the calculation of
the likelihood function in terms of $Y(k)$ difficult and its sequential implementation impossible. This difficulty, however, is superficial and can be easily circumvented by choosing a transformation operation on the observed sequence $Y(k)$ such that the corresponding mapped sequence is uncorrelated. Indeed, such a transformed sequence is available as shown by Schnepp [7] from the operations involved in the theory of Kalman filtering. The Kalman filter gives an optimal estimate of $x(k)$, denoted by $\hat{x}_\delta(k|k)$, in Eq. 1 for a fixed $\delta$ given the observation vector $Y(k) = [y(0), y(1), \cdots, y(k)]'$. During this filtering operation the sequence of observation errors defined by

$$e_\delta(k) = y(k) - \hat{y}_\delta(k|k-1)$$

where $\hat{y}_\delta(k|k-1) \triangleq$ optimal estimate of $y(k)$ given the observation vector $Y(k-1) = C(k) A(k) \hat{x}_\delta(k-1|k-1)$

has the property of being uncorrelated. It is the transformed observation error vector $\epsilon_\delta(k) \triangleq [e_\delta(0), \cdots, e_\delta(k)]'$ that will be used, henceforth, in the computation of likelihood functions. A discussion of the various off-line and on-line detection schemes follows next.

D. **Fixed-length Test Procedures**

Consider a sequence of observations $Y(n) = [y(0), y(1), \cdots, y(n)]'$ as defined by Eq. 5. A change in the plant dynamics may or may not have occurred during the finite interval zero to $n$. In terms of the model of Eq. 1,
there are (n+2) mutually exclusive and exhaustive hypotheses on the exact occurrence of change, namely:

i) Hypothesis that step change in plant dynamics occurred at \( t = i \);
\( i = 0, 1, \ldots, n \), denoted by \( H_i \) and

ii) Hypothesis that no step change in plant dynamics occurred during the process, that is, \( t > n \) (say \( n+1 \)) denoted by \( H_{n+1} \).

Note that \( t = 0 \) corresponds to the hypothesis that failure occurred before the start of the test. Let the a priori probabilities of these (n+2) hypotheses be denoted by \( P(H_i) \), \( i = 0, 1, \ldots, n+1 \) where \( \sum_{i=0}^{n+1} P(H_i) = 1 \).

Then if one is interested only in the detection of failure and not the failure instant then the following composite hypothesis test is applicable.

**Composite Binary Hypothesis Test.** [8]

Test the null hypothesis \( H_{n+1} \) versus the composite hypothesis \( H_f \) that failure occurred sometime during the process, that is, \( t = i \), \( i \in I_n \) where \( I_n = \{0, 1, \ldots, n\} \). Then, the likelihood ratio test is

\[
\Lambda(\epsilon(n)) = \frac{\sum_{i=0}^{n} P(H_i) p(\epsilon(n)|H_i)}{p(\epsilon(n)|H_{n+1})} 
\]

\[
\eta = \frac{P(H_{n+1})}{(1-P(H_{n+1}))} \left( \frac{C_{10} - C_{00}}{C_{01} - C_{11}} \right) 
\]

(10)
where

\[\begin{align*}
C_{00} &= \text{cost of accepting } H_{n+1} \text{ when } H_{n+1} \text{ is true} \\
C_{11} &= H_f \quad H_f \\
C_{01} &= H_{n+1} \quad H_f \\
C_{10} &= H_f \quad H_{n+1}
\end{align*}\] (11)

Usually \( C_{00} = C_{11} = 0 \) but \( C_{01} \) and \( C_{10} \) are set by the designer. Note that the probability densities in (10) may be written in terms of the known gaussian density of each component \( e(i) \), \( i = 0, 1, \ldots, n \) [7] of the error sequence \( \xi(n) \), given \( H_i \).

If the aim is to find the exact occurrence of failure \( \delta = i, i \in I_n \), then the following test is appropriate.

**Multi-Hypothesis Test.** [8]

The test involves the following decision rule: Accept hypothesis \( H_i \) corresponding to the index \( i \) for which the inequality

\[ A_i(\xi(n)) = \sum_{j=0}^{n+1} C_{ij} p(\xi(n)|H_j) P(H_j) < \sum_{j=0}^{n+1} C_{kj} p(\xi(n)|H_k) P(H_k) \equiv A_k(\xi(n)) \] (12)

is preserved for all \( k \neq i, i, k \in I_{n+1} \).

\( C_{ij} \) is the cost of accepting \( H_i \) when \( H_j \) is true.

Let the elements of the cost matrix be defined as follows:
\[ C_{i,n+1} \triangleq \text{cost of false alarm} = C_f, \quad \forall i \in I_n \]
\[ C_{n+1,i,j} \triangleq \text{"miss" cost} = C_m, \quad \forall j \in I_n \]
\[ C_{i,j} \triangleq \text{cost of misrecognition} = \begin{cases} L, & \text{if } i \neq j, \quad \forall i,j \in I_n \\ 0, & \text{if } i=j, \quad \forall i,j \in I_{n+1} \end{cases} \]

(13)

Thus, the test can be reduced to the following:

\[
\begin{align*}
\min_{j \in I_n} A_{j, n+1} &= A_{i, n} \quad A_{n+1, n} \quad \frac{H_{i, n+1}}{H_i} \quad A_{i, n} \quad A_{n+1, n} \\
\text{This implies that if}
\end{align*}
\]

\[
\begin{align*}
L \sum_{j=0}^{n} P(H_j) \left[ p(\xi(n)|H_j) + C_f \frac{P(H_{n+1})}{P(H_{n+1})} p(\xi(n)|H_{n+1}) \right] \\
\text{if further we assume that } C_m = L \text{ and } C_f \triangleq C, \text{ then Eq. 15 simplifies to}
\end{align*}
\]

\[
\frac{P(H_{i,n} \mid \xi(n))}{P(H_{n+1} \mid \xi(n))} \quad H_i \quad H_{n+1} \quad \frac{C}{L} \quad (16)
\]

Note that \( H_i \) satisfies the condition

\[
P(H_{i} \mid \xi(n)) = \max_{j \in I_n} P(H_j \mid \xi(n)) \quad (17)
\]

and that Eq. 16 involves the ratio of conditional a posteriori probabilities of \( H_i \) to \( H_{n+1} \). A similar approach has been proposed in a recent paper for
the IFAC Symposium 1970 [9] and was brought to this authors attention after the above work was done.

E. Sequential Test Procedures

Sequential procedures are essential when one desires an on-line monitoring of the dynamical state of the system. The specific tests suggested here are primarily extensions of Wald’s Sequential probability ratio test (SPRT) to composite and multihypotheses testing [8, 10].

At any instant of time $k$, the observation vector is $Y(k) = [y(0), y(1), \ldots, y(k)]'$ and there are three decision alternatives available; namely:

i) A failure has occurred at some $\delta \in I_k$,

ii) No failure "" in the past or $\delta \notin I_k$,

iii) Take another observation sample $y(k+1)$ and repeat decision steps (i)-(iii).

If the detection of the location of $\delta$ is not required then the appropriate detection procedure is the sequential composite binary hypothesis test. For the case where the location of $\delta$ is important, it is necessary to apply some form of a sequential multi-decision procedure. The generalized sequential probability ratio test (GSPRT) proposed by Reed [11] seems most reasonable from both the theoretical and computational points of view and is presented later in this section.
Sequential Composite Binary Hypothesis Test

Assume that $P(H_i)$ for all $i \in I_{N+1}$ are equal, where $H_i$ is the hypothesis that the change in plant dynamics occurs at $\delta = i$ and $N$ is the finite duration of the process. Then the sequential procedure to test the alternate hypothesis given by (i) versus the null hypothesis of (ii) is the probability ratio test given below.

Form the likelihood ratio after the $k^{th}$ sample:

$$\Lambda(\epsilon(k)) = \frac{p(\epsilon(k)|H_i, i \in I_k)}{p(\epsilon(k)|H_i, i \notin I_k)}$$

and compare it to the two thresholds $A$ and $B$.

If $\Lambda(\epsilon(k)) \geq A$: accept $H_i, i \in I_k$

$\Lambda(\epsilon(k)) \leq B$: accept $H_i, i \notin I_k$

$B < \Lambda(\epsilon(k)) < A$: take another sample

where $A = \frac{1-\beta}{\alpha}$, $B = \frac{\beta}{1-\alpha}$

$\alpha$ and $\beta$ being some specified false alarm and miss probabilities (probabilities of misrecognition). Note that

$$p(\epsilon(k)|H_i, i \in I_k) = \sum_{i=0}^{k} p(\epsilon(k)|H_i)$$

and
The independent error sequence vectors $\{\xi(k)\mid H_i\}$ are obtained by solving the Kalman filtering equations for each hypothesis $H_i$. Since $\delta = i \in I_{k+1}$, this implies a linear growth in the number of Kalman filters required; very simply, the number of Kalman filters needed at instant $k$ is $k+2$. Clearly this is an impractical solution and hence demonstrates the need for suboptimal detection schemes. Newbold and Ho [12] discuss this scheme in their paper and recognizing its lack of utility propose some ad hoc suboptimal detection procedures. These are presented later in this section for review purposes.

**Sequential Multihypothesis Test Procedures**

The only rigorous sequential procedure for multihypothesis testing known to this author is the generalized sequential probability ratio test [GSPRT] proposed by Reed [11] in 1960. The principal idea used is that of sequential rejection of hypotheses according to a prescribed decision rule until finally only one hypothesis remains and this is the one accepted by the test.

Any physical process may be assumed to have a finite duration, say $N$. Hence, at the start of the process there are $N+2$ hypotheses $H_i$, $i = 0, 1, \ldots, N+1$, to consider where $H_i$ is the hypothesis that failure occurred at $\delta = i$. $H_{N+1}$ is the hypothesis that no failure occurs during the entire length of the process. The GSPRT for detection of a step change in plant dynamics may be formulated as follows:

$$p(\xi(k)\mid H_i, i \notin I_k) = [1-(k+1) P(H_i)] p(\xi(k)\mid H_{k+1})$$ (21)
At instant $k$, there are $M_k$ hypotheses $H_i$ under consideration
($M_{k+1} \leq M_k \leq \cdots \leq M_1 \leq M_0 = N+2$). Let $\bigtriangleup_k = \text{set of indices } i \text{ corresponding to the hypotheses } H_i \text{ included in } M_k$, so that $\bigtriangleup_{k+1} \subset \bigtriangleup_k \subset \bigtriangleup_0 = I_{N+1}$.

Define a generalized likelihood ratio at the $k^{th}$ stage to be a vector with components

$$\Lambda_i(k) = \frac{\frac{p(\xi(k) \mid H_i)}{\prod_{j \in \bigtriangleup_k} p(\xi(k) \mid H_j)}}{M_k}, \quad i \in \bigtriangleup_k$$

For each hypothesis $H_i$ there is a corresponding stopping boundary given by

$$A(H_i) = \frac{1-p_{ii}}{\prod_{q \in \bigtriangleup_k} (1-p_{iq})}, \quad i \in \bigtriangleup_k$$

where $p_{ij}$ is the probability of accepting $H_i$ when indeed $H_j$ is true.

The decision rule at the $k^{th}$ stage is to compare each $\Lambda_i$, $i \in \bigtriangleup_k$, with its corresponding stopping boundary $A(H_i)$ and to eliminate the hypothesis $H_i$ from consideration if

$$\Lambda_i(\xi(k)) < A(H_i), \quad i \in \bigtriangleup_k$$

The hypotheses rejected after the $k^{th}$ sample are eliminated from the set $\bigtriangleup_k$ to give a new set $\bigtriangleup_{k+1}$ with $M_{k+1}$ elements to be tested at the
(k+1)\textsuperscript{st} stage. Note that even though the total number of alternative hypotheses at the \(k\textsuperscript{th}\) stage is \(M_k\), the number of Kalman filters required to generate observation error vectors \(\xi(k)\) given \(H_i, i \in \Omega_k\) equals the number of indices \(0 \leq i \leq k\) that belong to \(\Omega_k\) (at most \(k+1\)) plus one.

The GSPRT is intuitively very appealing and a detailed analysis of the properties of this test as applied to the problem at hand needs to be done. This is planned for the next reporting period. Initial efforts indicate that the test could be extremely significant in giving rigorous justification of some existing ad hoc schemes. This topic is discussed in greater detail next.

F. Ad Hoc Sequential Tests

The exact sequential detection schemes described in section E above are difficult to implement in practice because of their excessive computational and memory requirements. As a result suboptimal methods are warranted. A few ad hoc procedures [9, 12] have been suggested in the literature. These are interpreted below in light of the general theory discussed earlier. In addition, adaptive sequential decision procedures are put forth wherein the stopping boundaries are made functions of the a posteriori probabilities of the failure instant, that is \(A_i(H_i) = f(P(H_i) | \xi(k)), i \in I_{N+1}\). Details are given next.
1) **Occasional or Continuous Application of the Sequential Binary Hypothesis Test.**

These two tests were proposed by Newbold and Ho [12] as simplifications to the sequential composite binary hypothesis test given by Eq. 18 and 19. The basic idea is to allow only two hypotheses at the start of the test, namely:

i) No failure occurs throughout the test and

ii) Failure occurred before the start of the test

and apply the classical SPRT until a decision is reached. If the decision is that (i) is true then the test may be repeated again after a while or immediately; the former is called "occasional testing" and the latter "continuous testing" by Newbold and Ho [12]. Modifications in the two thresholds A and B are made before the start of each test to account for information gained from the earlier tests on the a priori probabilities of the two hypotheses. However, these tests have one major drawback. They assume that a failure occurs between tests and not during a test. If, indeed, a failure occurs during a test then there is not much that can be said about the response of these tests or their overall performance. Obviously if one waits long enough after the change then the number of samples from the failure hypothesis (ii) will be much larger than the alternative no failure hypothesis (i) and the test would eventually terminate with the decision that (ii) is true. Again, this long delay in detection is undesirable. The continuous test was compared [12] to the exact composite test via Monte Carlo methods for a problem of detecting
increases in the drift rate in a gyro system caused by incipient gyro failure. Results show that degradation of the ad hoc method from the exact scheme as measured by the average sample number is not great. An unexpected result is that the exact composite test takes an unusually long time to reach a decision that no failure has occurred. More analytical and computational work on these two tests as they apply to the problem discussed here is needed.

2) Sequential Application of Fixed-length Multi-hypotheses Testing Via a Sliding Observation Window.

This procedure involves recursive application of the fixed-length multihypothesis test to data samples from a sliding interval consisting of the observation vector \([y(k-\ell), y(k-\ell+1), \ldots, y(k-1), y(k)]\)' where \(k\) is the present instant of time. Failure could have occurred anywhere in this interval. The test proceeds as follows:

Hypothesis \(H_i\) that failure has occurred at \(i, i = k-\ell+1, \ldots, k-1\) is accepted if the "greater than" (>) inequality holds in the following two equations:

\[
\frac{P(H_i | \epsilon_T(k))}{P(H_k | \epsilon_T(k))} \sim \frac{\eta \equiv \frac{C}{L}}{H_k} ; \quad i = k-\ell+1, \ldots, k-1
\]  
\(25\)

\[
\frac{P(H_i | \epsilon_T(k))}{P(H_{k-\ell} | \epsilon_T(k))} \sim \frac{\eta \equiv \frac{C}{L}}{H_{k-\ell}} ; \quad i = k-\ell+1, \ldots, k-1
\]  
\(26\)
where \( \xi_T(k) = [e(k-\ell), \ldots, e(k)]' \)
as defined earlier

\[
C^\Delta = \begin{cases} 
  C_{i, k}, & i = k-\ell, k-\ell+1, \ldots, k-1 \\
  C_{i, k-\ell}, & i = k-\ell+1, \ldots, k.
\end{cases}
\tag{27}
\]

and

\[
C^\Delta_{ij} = \begin{cases} 
  L; & i = (k-\ell, \ldots, k), j = (k-\ell +1, \ldots, k-1), i \neq j \\
  0; & i = j = k-\ell, \ldots, k.
\end{cases}
\]

[Note that \( C^\Delta_{ij} \) = cost of accepting \( H_i \) when \( H_j \) is true.]

If either (or both) Eq. 25 or 26 does not hold with the ">" sign then one more observation error sample \( e(k+1) \) is taken and the two equations tested again for the observation vector \( \xi_T(k+1) \). This recursive test is continued until a particular hypothesis \( H_i \) is accepted. Most processes are of a finite duration and if by the end of the process a failure is not detected then the conclusion is that no failure occurred. A procedure similar to this has been suggested by Telksnys and Černiauskas in a recent paper [9] but without any justification of the kind given here. It can easily be shown that the sliding window technique discussed above may be obtained as a simplified case of the sequential multi-hypotheses decision procedure (GSPRT) given earlier. The window size can also be made adaptive by setting the boundaries of the test \( A(H_i) \) to be functions of the corresponding a posteriori probabilities \( p(H_i|\xi(k)) \). These suggestions need a thorough analysis to check on their practical feasibility.
G. Conclusions

In summary, various methods based on statistical decision theory have been suggested as being applicable to the problem of failure detection. The next step must be a computer simulation study of these proposed methods to check on their practical feasibility. More analytical work is also warranted wherever possible to obtain some theoretical estimates on the relative performances of each of the methods. This is planned for the next reporting period.

Note that the methods discussed above all require an explicit mathematical formulation of the plant model. In many realistic situations this may not be available. In such instances non-parametric detection schemes may be useful. There exists some literature in the field [13] that seems relevant to our problem. This approach will be explored in the future.
III. NEUROMUSCULAR SYSTEM

L. Ostroy, G.P. Moore

Several mathematical models of the human neuromuscular system have been proposed in the literature [14-18]. Some include afferent pathways from the peripheral sense organs, most importantly the muscle spindles followed in importance by the Golgi tendon organs and joint proprioceptive organs. Renshaw cells have also been included in tentative hypotheses. It has been impossible to get quantitative experimental data from such afferents or from gamma efferents to the spindles during voluntary movement since they are only accessible through surgery. What is known about these nerves and their physiological functioning has been obtained from experimental animals under anesthesia. A model which includes the afferents is therefore quite speculative, yet no definitive model can be developed which excludes them.

In the face of this very difficult problem, it is nevertheless desirable to evaluate the various models and even perhaps to include quantitative parameters into the sensory pathways of the models. One way to do this is to use an entire system hypothesis and then to compare its performance by computer simulation with that of the neuromuscular system it represents. This is the direction of the current research effort and its ultimate aim is the improved understanding of voluntary function of the neuromuscular system. More immediately, this method of research should provide additional insights
into what are currently controversial issues such as the physiological functioning of the spindel receptors and the servo-loop hypothesis of that function. What is planned for future work, therefore, is to use computer simulations of the many models, McRuer, Robinson, Houk, etc., in connection with experimental data from experiments on physiological tremor. This will enable sorting out of untenable features and ideas from the mathematical models.

Physiological tremor is a neuromuscular phenomenon which has been studied extensively in healthy subjects and in many different muscles and it promises to provide a great many clues into the working of the neuromuscular system [18,19]. A mathematical model of the neuromuscular system must be able to account for tremor. In turn, the parameters of the proposed system could perhaps be obtained by experimental tremor measurements made on human subjects. Measurements of tremor are not especially difficult. Computer facilities and software are now available to our group for the reduction of tremor data via autocorrelation, cross correlation and automatic spectral analysis. Tremor can arise from a large number of sources and it is by no means proven that the peripheral neuromuscular system is its source. There is, however, considerable circumstantial evidence that this is true. If tremor is to be used to understand the peripheral neuromuscular system then its characteristics must be dependent on the system, otherwise studying tremor would be irrelevant. It has been shown by Marsden, et al. [19], that the finger tremor of the two hands are
uncorrelated. This is very strong evidence for ruling out certain sources of tremor outside the neuromuscular system. It is strong evidence as well for ruling out sources within the neuromuscular system higher than spinal cord level, for if tremor were the result of outside sources or sources within the brain above the spinal cord, these sources would affect both hands and the physiological tremor would have to be correlated. There is further evidence that tremor is dependent on the topology of the peripheral neuromuscular system and independent of higher brain function, since i) tremor disappears with section of the afferents at their entrance to the spinal cord at the dorsal roots, and ii) finger tremor can be affected significantly by cooling of the hand and arm [16].

IV. PUBLICATIONS

A paper on synthetic electromyograms was presented at the Sixth Annual NASA-University Conference on Manual Control. An abstract of the paper is given below.
ON THE GRADING OF TENSION DURING VOLUNTARY CONTRACTION OF SKELETAL MUSCLE

L. Ostroy, A. V. Phatak, and G. A. Bekey

Abstract

The gradation of tension during voluntary contraction of skeletal muscle is due to two factors: (a) the firing statistics of individual motor units and (b) the temporal and spatial order in which motor units are activated. It is well known from experimental data that the standard deviation $\sigma$ of the surface EMG potential and the average tension $T$ of the muscle are linearly related, i.e., $\sigma_{\text{EMG}} = kT$. The purpose of this study is to investigate the inverse problem of determining possible firing statistics and recruitment order of single motor units which yield the above EMG-tension relationship.

It is sometimes assumed that the level of tension in skeletal muscle is set by neuron firing rate and random recruitment of motor units. This hypothesis was simulated on a digital computer and a synthetic EMG was produced. It was found that the tension was proportional to the variance of the EMG signal, rather than its standard deviation (as shown experimentally).

Recent physiological data suggest the following alternative hypotheses:

(1) Recruitment of motor units occurs at the center of the muscle at low tension levels and proceeds towards the surface as tension level increases.

(2) Recruitment of motor units is in order of size.

Preliminary analysis of these hypotheses indicates that they give results consistent with experimental data.

A reprint of a paper titled "Decision Processes in the Adaptive Behavior of Human Controllers" by A. V. Phatak and G. A. Bekey is included at the end of this report.
V. STUDY OF EYE MOVEMENTS DURING FIXATION ON A STEADY TARGET

J.D. Smith, A.V. Phatak

A. Background

Human eye movements during fixation on a steady target have been classified into three distinct types or categories. These are:

i) microsaccades—described as rapid eye movements of short duration (up to $15^\circ$ per second)

ii) drift or slower eye movements ($\approx 4'$ per second)

and

iii) tremor which describes random small amplitude eye movements of high frequency ($\approx 8$-10 Hz) which are also known to be common to other skeletal muscle processes in the body [14].

The saccades have been described as two types, recentering and short period fixation. The recentering saccades can be thought of as coupled to drifts [20]; a corrective recentering saccade often follows a drift and moves about the same distance as the drift, but in a direction $180^\circ$ opposite to it. In the past, emphasis has been on eye movement data analysis with no attempts at relating the origin of the three types of movement to candidate neurophysiological sites or system configurations. This project is undertaken to answer specific questions about the origin of these different types of movement. Experiments are proposed to investigate eye movements under different test conditions so as to test the validity of plausible hypotheses on the origin of these movements. The results of these experiments will be used to construct a quantitative model of the system that is anatomically isomorphic.
B. Eye Movement System

For the purpose of this study the eye movement fixation system is decomposed into the following three components: retina, higher centers and muscle dynamics. More elaborate eye movement control systems have been generated both in the direction of systems control [21] and in the direction of neuroanatomical detail [22]. During fixation on a target that is externally aided by the experimenter [23] to achieve an extremely small amount of error--termed image stabilization--the subject witnesses a disappearance of the target. This disappearance is proposed to be due to attenuation of the gain of the retinal elements (cones) sensing the target image. Immediately a rationale for constant small eye movements during fixation can be seen. Two possible sites of origin could inject some of the small eye movements into the system: i) the retinal network might initiate an error signal proportional to this gain attenuation or ii) the disappearance of the image may be avoided by eye movements intrinsic to the extraocular muscles or by noisy signals arising from the eye movement controller itself (the oculomotor nuclei). These and other hypotheses will be discussed in the next section.

C. Objectives

The eye movement fixation system is conceptually divided under two separate categories: types of eye movements observed and the neurophysiological sites within the system. The movements are microsaccades, drift and tremor and the sites considered here are the retina, central neural structures and the extraocular muscles. Hypothesis concerning the relation of the two categories are listed below.
a. Hypotheses

Retina
1. The retina originates small eye movements via a signal proportional to attenuation of retinal gain such as that occurring during image stabilization. (predicted type of movement involved: microsaccade)
2. The specialized retinal network in the fovea dictates to a great extent the amplitude and range of eye movements during fixation. (predicted type of movements: microsaccade and drift)

Central Neural Structures
1. A constant amount of noise in the excitation of extraocular muscles from the oculomotor nuclei is responsible for tremor.

Muscle
1. Imbalances in tension between opposing extraocular muscles beyond the control of oculomotor nuclei generate eye movements. (predicted type of eye movement involved: primarily tremor)

b. Experiments

Retina
1. If the retina initiates small eye movements proportional to attenuation of retinal gain, a target of a flashing light should obviate the necessity of re-excitation of the cones by movement since the target will 'move', through time. Differences in the power spectra of the three types of eye movements (see Section D) during flashing and steady targets may be related to retinal processes.
2. To further investigate the role of the retina in initiating eye movements the distribution of the three types of movement will be studied during fixation utilizing foveal and extrafoveal retina during fixation with flashing and steady targets.
Central Neural Structures

1. If the amount of noise in the oculomotor command signal is not constant but varies with tension and position in the muscles then comparisons of the power spectra of the eye movements during fixation at targets in different parts of the visual field may show differences related to the different tension settings in the extraocular muscles. (This experiment also applies in the investigation of hypothesis #1 under Muscle.)

Muscle

1. In order to eliminate the possible effects of intrinsic muscle sources of eye movement on the retinal-target image error the extraocular muscles of the input eye (target perceiving eye) will be paralyzed. The other eye, blocked from perceiving the target, will move via commands through a system lacking the injection of intrinsic muscle movements into the retinal-target image.

D. Analysis of Data

A program utilizing fast fourier analysis will be used to generate power spectra of eye movements in both the horizontal and vertical axes of the visual field. Amplitude, direction and velocity histograms will be plotted for movements in the two axes. Editing techniques will be programmed as necessary to remove eye blinks and occasional saccadic eye movements that may interfere with the measurements.
REFERENCES


Decision Processes in the Adaptive Behavior of Human Controllers

ANIL V. PHATAK, MEMBER, IEEE, AND GEORGE A. BEKEY, MEMBER, IEEE

ABSTRACT—A decision algorithm which simulates the rapid adaptive behavior of human controllers following sudden changes in plant dynamics is developed. The control of a VTOL aircraft in hover following failure of the stability augmentation system is used as a specific example. The decision algorithm is based on the assumption that the human controller recognizes certain pattern features in the error/error-rate phase plane. Experimental data, obtained from pilots facing four possible alternatives following the time of failure, are presented. The proposed decision algorithm is developed, and digital simulation results are discussed. A theoretical justification for the algorithm, based on statistical decision theory, is presented in the Appendix.

I. INTRODUCTION AND BACKGROUND

MATHEMATICAL models of the steady-state tracking behavior of human operators in manual control systems have existed for a number of years. However, only recently attempts have been made to obtain quantitative data on the tracking and decision-making behavior of human controllers in certain nonstationary situations. This paper deals with the mathematical modeling of the decision processes involved in the behavior of trained human controllers in response to sudden changes in the controlled element dynamics.

Specifically, the time-varying control task examined in this paper is that of a VTOL aircraft in hover whose stability augmentation may suddenly fail. These failures are so chosen as to result in overall closed-loop system instability unless the human controller rapidly modifies his control strategy to that appropriate for the new dynamics. This example provides a specific realistic control situation to study human controller adaptation.

A number of investigators in the past [3]-[13] have studied human controller adaptive behavior following a change in the controlled element dynamics. Early work performed by Sheridan [3], [4] was concerned with human controller adaptation to smooth changes in the plant parameters and did not focus on the decision processes in the controller.

Experiments by Young et al. [6] and Elkind et al. [7] were among the first to deal with adaptation to sudden changes in the order and gain of the controlled element dynamics. A schematic model incorporating various phases of human controller response and decision making was proposed in [7]. The partitioned phases of this model are

1) steady-state tracking of prefailure plant dynamics
2) detection of a change in plant dynamics
3) identification of the change
4) stabilization of overall closed-loop system
5) reduction of accumulated error
6) steady-state tracking of postfailure plant.

Elkind and Miller [10]-[12] went further and suggested a model for detection and identification of a change in plant dynamics based on statistical decision theory. The identification phase was considered as a multihypothesis detection problem whose outcome was the estimated state of the new plant dynamics. No feedback procedure was given to correct for probable errors in plant estimation, and neither was the proposed model simulated to show its overall feasibility.

Models based on the partitioning of the controller's adaptive response into a finite number of distinct phases are not complete unless the logic involved in switching between the models for different phases is identified. This study presents a complete on-line algorithm which models the decision control logic in human controller adaptation to a specific but realistic time-varying control task.

II. MATHEMATICAL FORMULATION OF THE ADAPTIVE CONTROL PROBLEM

The human controller's task is illustrated in Fig. 1. This type of manual control task, where the operator's visual input consists only of the difference between input and system response, is known as "compensatory tracking."

Assume that the plant in Fig. 1 may be represented by a linear vector differential equation of the form

\[ \dot{m} = F(t)m(t) + G(t)c(t) + H(t)r(t), \]

\[ 0 \leq t \leq T_s, m(0) = m_0 \]  

(1)

where \( m = [m_1, m_2, \ldots, m_n]^T \) is an \( (n \times 1) \) state vector, \( c \) is a scalar control variable, \( r = [r_1, r_2, \ldots, r_n]^T \) is a \( (q \times 1) \) transient plant input vector; \( F(t), G(t), \) and \( H(t) \) are \( (n \times n), (n \times 1), \) and \( (n \times q) \) piecewise constant matrices, respectively. \( T_s \) is defined to be the stopping time, that is, the time at which the adaptive process terminates.

Define \( t_f \) as time of occurrence of failure; then let

\[ F(t), G(t)\big|_{t < t_f} = F_0, G_0 \]  

(2)

and

\[ H(t)\big|_{t < t_f} = H_0 = [0] \]  

(3)

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where $F_0$, $G_0$, and $H_0$ are constant matrices and correspond to the prefailure plant configuration. Following failure, the plant dynamics become one of $k$ possible alternatives,

$$
[F(t),G(t)]_{i=1}^{k} = [F_i,G_i], \quad i = 1,2,\ldots,k
$$

and

$$
H(t)_{j=1}^{q} = H_j, \quad j = 0,1,2,\ldots,q
$$

where $q$ is the number of alternative transient plant input disturbances. Thus there are, in theory, $k \times (q + 1)$ number of failure control situations that a human controller could encounter and over which he must maintain control of the plant.

There are two different types of information available to the human controller on which he can base his control strategy. One class is the explicitly displayed information as in the displayed error state vector $e_d(t)$. It is assumed that error and error-rate are the only directly perceived variables for the human controller; hence

$$
e_d(t) = [e(t), \dot{e}(t)]^T.
$$

In general, the displayed error will differ from the observed error due to physiological limitations and observation noise. However, these effects will be ignored in the following discussion and the displayed and observed errors will be tacitly assumed equal to each other.

The second kind of information accessible to the controller is implicit in nature and might include such elements as a description of the observed error state patterns (via a pattern transformation operator acting on the observed error state), and some form of knowledge about his internal compensation or equalization strategy. Thus let

$$
y(t) = T_v(e_d(t)), \quad y(0) = y_0
$$

represent the mapping of the displayed state vector $e_d(t)$ into an implicit information pattern vector $y(t)$ of, say, dimension $p$. Similarly, define $z(t)$ as the vector denoting the controller’s estimate of his internal equalization structure. The dimension of $z(t)$ equals the number of discernible alternative failure control situations. The operator thus has a multivariable feedback signal

$$
z(t) = [e_d(t), y(t), z(t)]^T
$$

available to him for decision making and control. Note that the vector $e_d(t)$ is continuous in time while the vectors $y(t)$ and $z(t)$ are most likely to be discrete or piecewise constant. Also, $z(t)$ is a vector random process.

The aim in solving the adaptive control problem is to find a control policy in the form

$$
c(t) = v(x(t)), \quad v \in V
$$

where $v$ represents an acceptable adaptive controller structure from the set of admissible human controller adaptive structures $V$. Notice that even though the controller $v$ is assumed to be deterministic, $z(t)$ and hence $c(t)$ are random processes.

In principle, the determination of an optimum controller structure for the given task could be done analytically, provided that a criterion of performance is specified. However, the criterion used by a human controller in adaptive situations is unknown. Furthermore, even if a cost or criterion functional could be specified, solution of the problem may require a prior knowledge of the statistics of the time varying plant dynamics and results are difficult to obtain, except for low order systems without state variable constraints [1], [2]. In order to avoid the difficulties associated with a purely analytical approach, this study uses a heuristic interpretation of known empirical data from past research on manual control to hypothesize a human controller decision and adaptation strategy. A mathematical plausibility argument for the resulting algorithm is given in the Appendix.

### III. Experiments

The specific control task studied here is shown in Fig. 2. At failure, the outputs of the rate and/or attitude sensors (either a) go to zero suddenly, b) have steps, or c) ramp transients to a nonzero constant magnitude bias (bias = ±1.5 inches). The above three kinds of failure are referred to as soft, hard, and ramp types of failures, respectively. Since there are three possible plant changes and three likely feedback transients following failure, there are nine different failure situations that can be studied. However, only six of these situations were studied herein. Table I shows the types of changes in effective dynamics and failure transients encountered by the human controller in this simulation.

The failures listed in Table I can be analyzed as transitions among the corresponding prefailure and postfailure dynamics shown in Table II. Hereafter, the four different types of stability augmentation and the resulting plant configurations are referred to by letters $A$, $B$, $C$, and $D$, respectively, as indicated in Table II.

A typical experimental run lasted three to four minutes with the failure occurring at a random time, one to three minutes from the start. The subject was a well-trained pilot who was given ten hours of training (180 transitions) in controlling the plant for various system failures prior to actual experimental runs. He was also trained in controlling the various plant configurations in steady-state stationary tracking.

The instructions to the controller, an airline pilot, were as follows: “This is a single degree of freedom roll tracking task in a hovering VTOL. The flight control system will be failed in various ways with the failures occurring at
A bank angle of 50 degrees or more will imply that the vehicle has crashed and hence the run will be terminated."

The pilot was not given any details about the system dynamics; failure rate or any other information which would tend to bias his control strategy. Training was the only source of learning available to the controller.

IV. FORMULATION OF THE MODEL STRUCTURE

The human controller in response to a change in the plant dynamics must modify his prefailure control strategy to that appropriate to the new postfailure plant for successful adaptation. If one assumes that steady state is attained both before and after failure, then the controller's tracking behavior under both conditions can be characterized by a describing function model [14].

For the particular control tasks and input signals, it can be shown [14] that the human controller can be represented by a model of the form,

\[ Y_p(j\omega) = \frac{C(j\omega)}{E(j\omega)} = K_p \frac{(j\omega + Z_p)}{(j\omega + P_p)} e^{-j\omega \tau_p} \]  

(10)

where \( K_p \) is the controller's high-frequency gain, \( \tau_p \) is the controller's effective reaction time delay, and \( (Z_p)^{-1} \) and \( (P_p)^{-1} \) are the controller's lead and lag equalization time constants. The value of \( \tau_p \) is assumed to be constant for a given plant.

Estimated values of the four parameters in (10) were obtained from existing operator describing function data for plant dynamics similar to those in Table II [14]. These values are given in Table III.
Any adaptive model of the human controller must satisfy the boundary constraints of prefailure and postfailure steady-state operator models. This implies that, following failure, the human controller model must detect the occurrence of a failure and proceed with the modification required from its prefailure structure to postfailure steady-state requirements. Obviously the modification must be based on some kind of identification of postfailure plant dynamics, either explicitly or implicitly. Based on this inductive reasoning, the adaptive model must include the following phases and characteristics:

1) the prefailure steady-state control strategy
2) detection of failure in plant dynamics
3) identification of postfailure plant dynamics and appropriate modification of operator’s strategy
4) the postfailure steady-state control strategy.

The phase following failure and prior to failure detection has been termed the “retention phase” in [13].

The four partitioned phases of the adaptive process listed above are illustrated in Fig. 3, which is a typical example of the transition data as studied in this paper. It shows the human controller’s adaptive response to a failure of both the rate and attitude feedback loops which corresponds, over the frequency range of interest, to a change in plant dynamics from a simple gain to a double integrator (types A to C in Table II). The time at which the stability augmentation failed is indicated by T.F.

During the retention phase the overall controller-plant closed-loop system is unstable, resulting in the rapid divergence of the error-rate and hence the error in the second trace of Fig. 3. Hence the modification of the controller structure following detection of failure must be sufficiently fast compared to the dominant time constant of the system to avoid loss of control.

A hypothetical model structure incorporating these requirements is shown in Fig. 4. The model shown includes a higher level controller, called a supervisor, capable of decision logic and able to reorganize the steady-state controller structure. The supervisor inputs are the displayed error state vector \( e_d(t) \) and the stick movement \( c(t) \). It is postulated that the supervisory structure operating on \( e_d(t) \) can be subdivided into a sequence of four operations as follows.

1) Pattern transformation: an operator \( T_y \) that maps the displayed error state vector \( e_d(t) \) into an element \( y(t) \) of a space of information pattern vectors \( Y \). Thus \( y(t) = T_y(e_d(t)) \in Y \).

2) Pattern classification: the task of categorizing the incoming pattern vector \( y(t) \) into one of a finite number of pattern classes \( Y_i \). The number of pattern classes equals the number of discernible transition response characteristics.

3) Plant estimation: the estimator acts continuously on the incoming decisions of the pattern classifier to give a continuous estimate of the current discrete state \( g_i \) of the plant configuration and transient disturbances, thus \( g_i = F(y(t) \in Y_i) \).

4) Controller modification: a change in the control strategy that involves modifying the controller parameter vector \( p(t) = [K_p, Z_p, P_p, \tau_p] \) (corresponding to the four parameters of the human operator describing function of (10)) to that appropriate for the estimated state of the plant. Thus \( p(t) = p(t)|_{g_i} \).

The above sequence of four operations is shown diagrammatically in Fig. 5. The derivation of appropriate models for the four sequential operations is discussed in the following section.

V. IDENTIFICATION AND DISCOVERY OF PATTERN FEATURES

In order to identify the patterns generated by the pattern transformation operator, large amounts of actual human controller tracking data were analyzed. More than a hundred tracking records (supplied by Systems Technology, Inc.) were studied in detail in order to identify
patterns of system error, error-rate, and stick movement which followed the onset of the plant transition. This inductive approach proves to be necessary because of the lack of knowledge regarding the basis on which the human controller selects patterns [15], [16] among his observed variables. However, a theoretical (a posteriori) explanation can be given for the human controller’s choice of pattern classification, plant estimation, and controller modification procedures (see the Appendix).

Study of the error/error-rate phase plane data for various transitions in the plant configuration, reveals the following three pattern features. Together they constitute a pattern vector. They are identified as the following:

1) pattern feature PF-1: the differential change in the error phase trajectory from prefailure to postfailure conditions;
2) pattern feature PF-2: the path of the postfailure error phase trajectory after the occurrence of the first error peak;
3) pattern feature PF-3: the path of the postfailure error phase trajectory following the occurrence of the second error peak.

Note that occurrence of the peaks is expected since the overall system following failure is unstable in an oscillatory mode. Also PF-1, 2, and 3 follow chronologically, in that order.

VI. CLASSIFICATION OF PATTERN FEATURES

A sequence of pattern features PF-1, 2, and 3 constitutes a pattern vector. The task of the pattern classifier is to categorize the three features sequentially into separate regions that relate to discernible plant transitions. Decision regions DR-1, 2, and 3 shown in Fig. 6 have been identified by analyzing actual transition data and serve to categorize the error phase trajectory pattern in different classifications.

The classification of the pattern features PF-1, 2, and 3 in terms of the decision regions DR-1, 2, and 3 is identified and proceeds as follows.

Classification of Pattern Feature PF-1

On examining transition response data it was evident that there is a distinct change in the system response following the occurrence of failure. Before failure, the observed variables follow a trajectory, with an average value of zero, corresponding to the operator control strategy suitable for the fixed prefailure plant configuration. Following failure and prior to detection of failure the overall controller plant closed-loop system is unstable for all types of failures, and this results in large magnitudes of error-rate and error.

The stick response after failure shows the expected retention phase, the end of which is marked by a sudden movement of the stick. The error-rate and/or error at the end of retention are large. Detection of failure by the human controller seems to be based on the large differential change occurring in the course of the error/error-rate phase trajectory from prefailure to postfailure conditions. It was noticed that in all types of failures the error phase trajectory following failure emerged out of region DR-1. Thus the failure detection criterion is as follows. If

\[
d_{e}(t) = [e(t), e'(t)]' \in
\]

DR-1 decide no failure has occurred; plant configuration is of type \( A \)

\[
d_{e}(t) \not\in
\]

DR-1 decide failure has occurred; plant configuration is of type \( \bar{A} \)

(11)

where a bar over the expression signifies “not in.”
Let \( t = T1 \) be the first time instant at which 
\[
e_d(t) \in \text{DR-1}.
\]
Then
\[
\text{sgn}(\dot{e}(T1)) = \begin{cases} +1 & \text{if } \dot{e}(T1) > 0 \\ -1 & \text{if } \dot{e}(T1) < 0 \end{cases} (12)
\]
is a property of pattern feature PF-1 that is useful in the classification of the remaining pattern features. Thus (11) and (12) constitute the decision outcomes in response to pattern feature PF-1.

**Classification of Pattern Feature PF-2**

Pattern feature PF-2 is useful in recognizing if the post-failure plant configuration is \( B \), with or without failure transients, or \( \bar{B} \).

Thus given \( t > T1 \) and that \( \text{sgn}(\dot{e}(t)) = -\text{sgn}(\dot{e}(T1)) \), the following feature discrimination rule is examined.

**Classification of Pattern Feature PF-3**

Feature PF-3 allows one to discriminate between the postfailure dynamics of types \( C \) and \( \bar{C} \). Thus given \( t \geq T1 \), and that \( \text{sgn}(\dot{e}(t)) = \text{sgn}(\dot{e}(T1)) \), then the following rule is examined.

It is hypothesized that the pattern feature classification rules (11)–(14), discussed above, are used by the human controller in estimating the postfailure plant and as a result modifying his control strategy. The controller’s plant estimation procedures and his modification strategy are discovered on examining transition data and are given next.

**VII. Plant Estimation and Modification of Controller Strategy**

The human controller in the transition tasks studied herein must make decisions regarding the state of the plant as the response unfolds in time and does not have the luxury of continuing to make measurements of the response indefinitely. The decisions, therefore, must be made on partial or almost no information about the state of the plant configuration. The operator is thus assumed to choose a conservative strategy for plant estimation and controller modification which allows him the option of correcting perhaps incorrect decisions sequentially in time. Evidence from human operator data indicates the choice of the following schemes for plant estimation and controller modification.

Monitor pattern feature PF-1. If 
\[
e_d(t) \in \text{DR-1} \text{ decide failure has occurred and that the postfailure plant configuration is of type } B \text{ (15)}
\]
then change controller parameter vector from
\[
p_A(t) = [K_p, Z_p, P_p, \tau_p]_A^{T} \to p_B(t) = [K_p, Z_p, P_p, \tau_p]_B^{T}. (16)
\]

Continue thereafter to monitor pattern feature PF-2. If the plant is not of type \( B \),
\[
e_d(t) \in \text{DR-2} \text{ decide that the previous plant estimate was erroneous and that the new estimate of the plant configuration is of type } C \text{ (17)}
\]
Consequently modify controller parameter vector from
\[
p_B(t) = [K_p, Z_p, P_p, \tau_p]_B^{T} \to p_C(t) = [K_p, Z_p, P_p, \tau_p]_C^{T}. (18)
\]

Observe PF-3. If 
\[
e_d(t) \in \text{DR-3} \text{ decide that the previous plant estimate was wrong; new estimate of the plant configuration is of type } D \text{ (19)}
\]
Change controller parameter vector from
\[
p_C(t) = [K_p, Z_p, P_p, \tau_p]_C^{T} \to p_D(t) = [K_p, Z_p, P_p, \tau_p]_D^{T}. (20)
\]
Thus (15)–(20) constitute the complete estimation and modification rules that model human controller behavior for the given experimental situation.

**VIII. Proposed Sequential Decision Algorithm**

On the basis of the pattern classification, plant estimation and plant modification rules discussed above, a sequential decision algorithm shown in Fig. 7 is proposed for supervisory control.

The algorithm includes decision and modification elements in a sequential order. Note that the modifications in the operator model lead in discrete steps to the new structure. The model parameters on switching are allowed to vary from one trial to the next according to some suitable statistical distribution. This flexibility can provide the adaptive model with the ability to display run-to-run variability in response and hence make the algorithm inherently stochastic. The range of parameter variations, however, must fall within the stability boundaries.

The detection and identification scheme proposed is a sequential decision making process that yields binary yes/no type decisions as the error/error-rate trajectory...
unfolds in time. The specific criteria used are based on the categorization of evolving error trajectory pattern features according to the postfailure augmentation. The modification strategy is directly related to the sequential plant estimation policy and a change of the plant estimate is followed by a modification of operator controller strategy as required.

The complete algorithm for human controller adaptation shown in Fig. 7 was simulated on a digital computer. Results for two sample cases, namely (1) a soft failure from augmentation type $A$ to type $C$ and (2) a hard failure from augmentation type $A$ to type $B$, are presented in Fig. 8 and 9, respectively. The corresponding phase plane plots are given in Fig. 10 and 11, respectively. The trajectories in these figures demonstrate the typical characteristics of pattern features $PF-1$, $PF-2$, and $PF-3$ appropriate to the postfailure plant configurations. Also notice that the stick response in Fig. 9 shows the steady-state bias of 1.5 inches required to counteract the hard failure transient of 1.5 inches in the attitude feedback. For a more detailed comparison of model response to human controller response under similar conditions, see [17].

IX. Conclusions

A mathematical model of the human controller's adaptive behavior in response to sudden changes in plant dynamics has been synthesized. The proposed model consists of a hierarchic arrangement of two parallel controller structures. The principal controller represents the steady-state tracking behavior of the operator; the higher level controller, the supervisor, incorporates the decision processes necessary to modify the steady-state controller in response to the estimated state of the plant dynamics. A theoretical framework based on statistical decision theory (presented in the Appendix) provides a rationale for the proposed schemes for detection of failure, estimation of postfailure dynamics, and ensuing modification of the steady-state controller parameter vector. The question of choosing the best pattern features in the error phase trajectory, however, was not treated analytically in this study and the problem is open to investigation.

Finally, the complete adaptive model of the human controller was verified by comparison of simulated model responses to actual human controller data for similar conditions.
Fig. 8. Model response, transition A–C: soft.

Fig. 9. Model response, transition A–B: hard.
Fig. 10. Error-rate/error phase plane model response (corresponds to Fig. 8), transition A-C: soft.
Fig. 11. Error-rate/error phase plane model response (corresponds to Fig. 9), transition A-B: hard.

\[ Y_{H-0}(j\omega) |_{T<T1} = \frac{8(j\omega + 3)}{(j\omega + .05)} e^{-4j\omega} \]
\[ Y_{H-0}(j\omega) |_{T>T1} = 17.2 e^{-24j\omega} \]

NOTE: TRAJECTORY NEVER ENTERS DR-2 AFTER ERROR-RATE REVERSAL

Fig. 12. Functional form of the decision theory models.
Appendix

Theoretical Basis for the Proposed Algorithm

The algorithm for supervisory control developed in this paper incorporates the four sequential operations of 1) extraction of error-phase trajectory patterns, 2) classification of the chosen patterns, 3) estimation of the state of the plant, and 4) modification of the controller strategy to match the estimated plant configuration.

As was pointed out in the text, a mathematical theory for the optimum selection of the error, error-rate patterns is beyond the scope of this paper. It is also assumed here that the human operator has stored in memory a file of controller structures suitable to the finite number of allowable plant configurations, any one of which he can retrieve instantaneously (time of retrieval is assumed to be much smaller than the dominant system time constant), if required. Hence the operation of modification of controller strategy becomes merely one of retrieving a model structure suitable to the estimated state of the plant.

Pattern classification and plant estimation operations were identified and modeled after a careful study of over one hundred transition response data for the human operator. The objective of this Appendix is to present a mathematical basis for the proposed schemes for operator pattern classification and plant estimation within the framework of statistical decision theory.

The tasks of classifying the pattern features PF-1, 2, and 3, are considered as problems in sequential binary hypothesis testing. For example, classifying PF-1 involves making the decision whether the corresponding plant configuration is A or \( \bar{A} \); similarly classifying PF-2 and PF-3 requires choosing between plant configurations B or \( \bar{B} \) and C or \( \bar{C} \), respectively. Plant estimation follows classification of each of the three pattern features and is defined as a case of fixed length multihypothesis testing. Thus following the classification that the plant configuration is \( \bar{A} \), that is, failure has occurred, the estimation problem is to decide whether the plant configuration is B, C, or D. The number of alternative plant configurations for estimation reduces from three to two after classification of PF-2 and to one following classification of PF-3. Thus a successive elimination procedure is used to identify the post-failure plant dynamics.

Before proceeding with the formulation of the decision theory model, it is necessary to make the following assumptions.

1) System error \( e(t) \) and error rate \( \dot{e}(t) \) are the only sources of information available to the decision-maker of the supervisor, and both are assumed to be Gaussian.

2) The supervisor samples the error and error-rate periodically every \( AT \) seconds and uses samples of \( \Delta e \), the error increment, and \( \Delta e \), the error-rate increment, over each sampling interval, as independent inputs to the decision-maker.

3) The supervisor has available the separate probability densities for \( \Delta e \) and \( \Delta e' \), conditioned to the hypotheses that \( A \) or \( \bar{A} \) is true, \( B \) or \( \bar{B} \) is true and \( C \) or \( \bar{C} \) is true. Thus for \( \Delta e \), the supervisor knows the probability densities:

\[ l_i(\Delta e|A), \quad l_i(\Delta e|\bar{A}), \quad l_i(\Delta e|B), \quad l_i(\Delta e|\bar{B}), \quad l_i(\Delta e|C), \quad l_i(\Delta e|\bar{C}). \]

Similar expressions are assumed to be available for \( \Delta e' \).

4) The supervisor cannot distinguish between the densities for the individual hypotheses in \( A \) and \( B \). Hence

\[ l_i(\Delta e|A) = l_i(\Delta e|B) = l_i(\Delta e|C) = l_i(\Delta e|D) \]

and

\[ l_i(\Delta e|\bar{A}) = l_i(\Delta e|\bar{B}) = l_i(\Delta e|\bar{C}) = l_i(\Delta e|\bar{D}). \]

Similar assumptions hold for variable \( \Delta e' \).

5) Samples of \( \Delta e \) are statistically independent. The same is true for \( \Delta e' \).

6) The supervisor has knowledge of all prior probabilities of the various alternate hypotheses for the classification and estimation problems. For example,

\[ P(A) = \pi = 1, \quad P(\bar{A}) = 1 - \pi \]

for the failure detection case.

The background of hypothesis testing is given next.

Formalism of Statistical Decision Theory

The subject of hypothesis testing is best described by reference to Fig. 12, which shows the functional form of the decision process used by the supervisor. The various elements in the decision channel of Fig. 12 are described as follows:

- \( S \) the set of possible events of hypotheses \( s_i \), namely the hypothesis that the plant configuration is \( A, B, C, \) or \( D \)

- \( \sigma(s) \) a priori probability density on \( S \); \( \int_S \sigma(s) \, ds = 1 \)

- \( V \) the set of observables \( v = \Delta e' \) (or \( \Delta e \)); one decision channel operates on samples of \( \Delta e \), and another parallel channel on samples of \( \Delta e' \)

- \( l(v|s_i) \) probability density on \( V \), given \( s_i \) \( \in S; \int_V l(v|s_i) \, dv = 1 \)

- \( \Gamma \) the space of concluding hypotheses \( \gamma_i \); here the concluding hypothesis is that the plant configuration is \( A, B, C, \) or \( D \) (or any composite set such as \( \bar{A}, \bar{B} \))

- \( d(\gamma_i|v) \) the decision rule, that is, the criterion for selecting some \( \gamma_i \in \Gamma \) upon observing \( v \in V \).

In this section the criterion chosen is the Bayes rule.

Sequential Binary Hypothesis Testing

Suppose that

\[ S = \Omega_0 \cup \Omega_1, (\Omega_0 \cap \Omega_1 = 0) \]  \hspace{2cm} (21)

and let

\[ H_0 \] the null hypothesis that \( s \in \Omega_0 \)

\[ H_1 \] the alternate hypothesis that \( s \in \Omega_1 \). \hspace{2cm} (22)
Then the Bayes rule for sequentially testing the composite hypothesis $H_1$ versus $H_0$ is a sequential likelihood ratio test as follows.

At the $m$th stage, compute the generalized likelihood ratio

$$
\Lambda_m(v) = \frac{\int_{\Omega_1} l(v_1,v_2,\ldots,v_m|g \in \Omega_1) \sigma_1(s) \, ds}{\int_{\Omega_0} l(v_1,v_2,\ldots,v_m|g \in \Omega_0) \sigma_0(s) \, ds}
$$

(23)

where

$\sigma_0(s) =$ a priori probability density on $\Omega_0$;

$$
\int_{\Omega_0} \sigma_0(s) \, ds = 1
$$

$\sigma_1(s) =$ a priori probability density on $\Omega_1$;

$$
\int_{\Omega_1} \sigma_1(s) \, ds = 1
$$

and compare it to two thresholds $L_0$ and $L_1$. If

$$
\Lambda_m(v) \geq L_1: \text{accept } H_1
$$

$$
\Lambda_m(v) \leq L_0: \text{accept } H_0
$$

(24)

$L_0 < \Lambda_m(v) < L_1$: take another sample

and

$$
L_1 = \frac{1 - \beta}{\alpha}, \quad L_0 = \frac{\beta}{1 - \alpha}
$$

(25)

where

$\alpha =$ probability of rejecting $H_0$ when it is true; that is, probability of false alarm

$\beta =$ probability of accepting $H_0$ when it is false; that is, probability of miss.

The sequential likelihood ratio test (24) may be used for pattern classification of features $PF-1$, $2$, and $3$, respectively. Application to $PF-1$ provides the Bayes solution to the failure detection problem and is discussed next.

**The Process of Failure Detection**

It is assumed that there are two parallel decision processes operating, one on $\Delta e$ and the other on $\Delta \hat{e}$. Let

- $H_0 =$ the null hypothesis that the plant is in configuration $A$
- $H_1 =$ the alternate hypothesis that the plant is in configuration $\bar{A}$.

The sequential likelihood ratio test for $\Delta e$ as is follows: At the $m$th sample stage, the likelihood ratio (23) reduces to (assuming $B$, $C$, and $D$ are equally likely)

$$
\Lambda_m(\Delta e) = \frac{l(\Delta e_1,\Delta e_2,\ldots,\Delta e_m|A)}{l(\Delta e_1,\Delta e_2,\ldots,\Delta e_m|\bar{A})},
$$

(26)

Since $\Delta e_j$ are assumed to be independent of each other

$$
\Lambda_m(\Delta e) = \prod_{j=1}^{m} \frac{l(\Delta e_j|A)}{l(\Delta e_j|\bar{A})},
$$

(27)

Since $e$ is assumed Gaussian, let

$$
l(\Delta e|A) = \frac{1}{\sqrt{2\pi \delta_A^2}} \exp \left( - \frac{(\Delta e - \mu)^2}{2\delta_A^2} \right)
$$

(28)

$$
l(\Delta e|\bar{A}) = \frac{1}{\sqrt{2\pi \delta_{\bar{A}}^2}} \exp \left( - \frac{(\Delta e - \mu)^2}{2\delta_{\bar{A}}^2} \right)
$$

(29)

where $\mu =$ mean divergence rate following failure.

Assume $\delta_A = \delta_{\bar{A}} \equiv \delta$ as the standard deviation of the observed error-rate increment for both configurations $A$ and $\bar{A}$. Then,

$$
\Lambda_m(\Delta e) = \exp \left\{ \frac{\mu}{\delta^2} \sum_{j=1}^{m} \left( \Delta e_j - \frac{\mu}{2} \right) \right\}.
$$

(30)

From (24), if

$$
\epsilon(mT) \geq \sum_{j=1}^{m} \Delta e_j \geq \left( \frac{\delta^2}{\mu} \log L_1 + \frac{m}{2} \mu \right): \text{accept } H_1; \text{ that is, failure has occurred}
$$

$$
\epsilon(mT) \leq \sum_{j=1}^{m} \Delta e_j \leq \left( \frac{\delta^2}{\mu} \log L_0 + \frac{m}{2} \mu \right): \text{accept } H_0; \text{ that is, failure has not occurred}
$$

$$
\left( \frac{\delta^2}{\mu} \log L_0 + \frac{m}{2} \mu \right) < \epsilon(mT) < \left( \frac{\delta^2}{\mu} \log L_1 + \frac{m}{2} \mu \right): \text{take another sample}
$$

(31)

For the detection of failures, one always wants to continue testing until it is decided that failure has occurred. This can be guaranteed by making condition 2 in (31) very unlikely. For this to be true, choose $L_0$ to be very small; that is, make $\beta$, the probability of choosing $H_0$ when it is false, negligible and near zero.

A sequential likelihood ratio test for the second parallel channel operating on error may be similarly determined. A failure is said to have occurred when either of the tests [such as (31)] on error-rate or error terminates. The failure detection criteria of the form (31) for error and error rate give rise to a rectangular decision boundary such as DR-1 in Fig. 6.

Thus it has been shown that a theoretical basis exists for picking regions such as DR-1 for failure detection.

**Estimation of Postfailure Plant Dynamics**

Following the detection of failure one must estimate whether the plant configuration corresponds to $B$, $C$, or $D$. It is postulated that the estimation process is a fixed length multihypothesis testing problem, and that this length is one sampling interval. This assumption is necessary because the overall system is still unstable fol-
lowing detection and hence the identification must be rapid enough to avoid loss of control.

The Bayes solution requires the knowledge of the following conditional costs and a priori probabilities:

\[ C_{ij} = \text{cost to the operator if he decides hypothesis } H_i \text{ is true when hypothesis } H_j \text{ is true} \]

\[ q_i = P(H_i), \text{ a priori probability that } H_i \text{ is true.} \]

(32)

For the problem at hand,

\[ H_1 = \text{hypothesis that postfailure plant configuration is } B, \]

\[ H_2 = \text{hypothesis that postfailure plant configuration is } C, \]

\[ H_3 = \text{hypothesis that postfailure plant configuration is } D. \]

\[ q_i = P(H_i) = 1/3. \]

\[ B, C, \text{ and } D \text{ are equally likely.} \]

Then the Bayes decision rule is as follows: select hypothesis \( H_i \) for which

\[ A_i = \sum_{j=1}^{3} l(v|H_j)C_{ij}q_j \]

is minimum. Earlier it was assumed that

\[ l(v|H_j) = l(v|H_j) = l(v|H_j) = l(v|\bar{A}). \]

Thus choose \( H_i \) for which

\[ A_i = \frac{1}{3} l(v|\bar{A}) \sum_{j=1}^{3} C_{ij} \]

(34)

is minimum.

Decision rule (34) suggests the selection of \( H_i \) for which the total cost of false alarm is the smallest. Actual transition data indicate that the human operator chooses \( B \) as the estimate of the postfailure plant dynamics following detection of failure. This may be explained by saying that the cost to the operator for choosing \( B \) when it is indeed \( \bar{B} \) must be smaller than the cost of false alarms for choosing \( C \) when it is \( \bar{C} \) or \( D \) when it is \( \bar{D} \).

Thus an application of statistical decision theory has provided a justification of the proposed pattern classification rule for feature PF-1 and the proposed plant estimation strategy after detection. This approach can similarly explain the remainder of the proposed supervisory control algorithm.

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