AREAS OF CONTACT AND PRESSURE DISTRIBUTION
IN BOLTED JOINTS

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ABSTRACT

When two plates are bolted (or riveted) together these will be in contact in the immediate vicinity of the bolt heads and separated beyond it. The pressure distribution and size of the contact zone is of considerable interest in the study of heat transfer across bolted joints.

The pressure distributions in the contact zones and the radii at which flat and smooth axisymmetric, linear elastic plates will separate were computed for several thicknesses as a function of the configuration of the bolt load by the finite element method. The radii of separation were also measured by two experimental methods. One method employed autoradiographic techniques. The other method measured the polished area around the bolt hole of the plates caused by sliding under load in the contact zone. The sliding was produced by rotating one plate of a mated pair relative to the other plate with the bolt force acting.

The computational and experimental results are in agreement and these yield smaller zones of contact than indicated by the literature. It is shown that the discrepancy is due to an assumption made in the previous analyses.

In addition to the above results this report contains the finite element and heat transfer computer programs used in this study. Instructions for the use of these programs are also included.
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NOMENCLATURE

A, B, C  radii
D  thickness
E  modulus of elasticity
G  shear modulus
\( h_c, h_f \)  heat transfer coefficients
H  hardness
\( k, k_1, k_2 \)  thermal conductivities
P, p  pressure
r  coordinate
\( R_o \)  radius of separation
\( u, w \)  displacement in \( r \) and \( z \) directions
x  coordinate
\( X_c \)  length of contact
y  coordinate
y'  slope
z  coordinate

\( \dot{\varphi} \)  deflection
\( \varepsilon \)  dilation
\( \varepsilon_r, \varepsilon_t, \varepsilon_{rz} \)  strains
\( \sigma, \sigma_1, \sigma_2 \)  standard deviations
\( \sigma_r, \sigma_t, \sigma_z \)  stresses
\[ \lambda, \mu \] Lame's constants
\[ \nu \] Poisson's ratio
\[ \tau \] shear stress
\[ \theta \] angle

Subscripts
\[ r \] radial direction
\[ t \] tangential direction
\[ z \] z-direction
Chapter I
INTRODUCTION

When two plates are bolted (or riveted) together, these will be in contact in the immediate vicinity of the bolt heads and separated beyond it. The pressure distribution in the contact area and the separation of the plates is of considerable interest in the study of heat transfer across joints. Cooper, Mikic and Yovanovich [1] show that with assumed Gaussian distribution of surface heights, the microscopic contact conductance is related to the interface pressure, surface characteristics and the hardness of the softer material in

\[ h_c = 1.45 \frac{\tan \theta}{\sigma} k(\frac{P}{H})^{0.985} \]  
(1.1)

where

\[ k = \frac{2k_1 k_2}{k_1^2 + k_2^2} \]  
(1.2)

and \( k_1 \) and \( k_2 \) represent the thermal conductivities of two bodies in contact; \( \sigma \) is the combined standard deviation for the two surfaces which can be expressed as

\[ \sigma = (\sigma_1^2 + \sigma_2^2)^{1/2} \]  
(1.3)
where $\sigma_1$ and $\sigma_2$ are the individual standard deviation of height for the respective surfaces; $\tan \theta$ is the mean of the absolute value of slope for the combined profile and it is related, for normal distribution of slope, to the individual mean of absolute values of slopes as

$$\tan \theta = (\tan \theta_1^2 + \tan \theta_2^2)^{1/2}$$

(1.4)

where

$$\tan \theta_i = \lim_{L \to \infty} \frac{1}{L} \int_0^L |y_i'| \, dx; \quad i = 1, 2$$

(1.5)

and $y'$ is the slope of the respective surface profiles; $P$ represents the local interface pressure; and $H$ is the hardness of the softer material.

Relation (1.1), as written above, is applicable for contact in a vacuum. One can modify the expression by simply adding to it

$$h_f = \frac{\text{conductivity of interstitial fluid}}{\text{average distance between the surfaces}}$$

(1.6)

in order to account approximately for the presence of the interstitial fluid.

All parameters in relation (1.1), except for the pressure, are functions of the material and geometry and can be easily obtained. The determination of the pressure distribution and the extent of the contact area between two plates present both mathematical and experimental
difficulties. From the mathematical point of view, the difficulty stems from the fact that the theory of elasticity will yield a three dimensional (axisymmetric) problem with mixed boundary conditions. Experimentally, the discrimination between contact and gaps of the order of millionths of an inch is required.

Roetscher [2] proposed in 1927, a rule of thumb that the pressure distribution of two bolted plates, Fig. 1, is limited to the two frustums of the cones with a half cone angle of 45 degrees as shown in Fig. 2 and that at any level within the cone the pressure is constant. Also, for symmetric plates, according to Roetscher, separation will occur at the circle which is defined by the contact plane and the 45 degree truncated cone emanating from the outer radius of the bolt head.

Since 1961 Fernlund [3], Greenwood [4] and Lardner [5] among others reported solutions based upon the theory of elasticity. Although their solutions also yield separation radii at approximately 45 degrees as in Roetscher's rule, their solutions yield a much more reasonable pressure distribution as compared to Roetscher's constant pressure at each level of the frustrum. These investigators have made use of the Hankel transform method demonstrated by Sneddon [6] in his solution for the elastic stresses produced in a thick plate of infinite radius by the application of pressure to its free surfaces. The basic assumption in their approach is that two bolted plates can be represented by a single plate of the same thickness as the combined thickness of the two plates under the same external loading. It then follows that the $z$-stress distribution at the parting plane can be approximated by the $z$-stress distribution in the same plane of the single plate. It also follows that separation will occur at the smallest radius in that plane for which
the $z$-stress is tensile. In the case of two plates of equal thickness
the $\sigma_z$ stress at the midplane of the equivalent single plate is the
stress of interest.

Fernlund [3], for example, used the method of superposition in the
sequence shown in Figs. 3(a) to 3(c) to obtain annular loading. Then
by superposition of shear and radial stresses at radius $A$, Figs. 3(d)
and 3(e), opposite in sign of those due to the annular loading at the
free surfaces, Fernlund obtained the solution for a single plate with a
hole under annular loading (Fig. 3(f)).

Experimental work in this area included Bradley's [7] measurements
of the stress field by three dimensional photoelasticity techniques, and
the use of introducing pressurized oil at various radii in the contact
zone and measuring the pressure at which oil leaks out from the joint
[3,8]. Both of these experimental methods have uncertainties as indi-
cated by the authors.

Because of the cumbersomeness of the Hankel transform solution and
experimental difficulties, the body of work in this area has been very
limited and definite verification of analytical results by experiment is
not cited in the literature.

The research described in the succeeding chapters was undertaken
with the following primary objectives:

a) To provide a method of solution for the case of two bolted
plates without the simplifying assumption of the single
plate substitution.

b) To devise a test to validate the two plate analysis.

c) To test the validity of the single plate substitution.
A finite element computer program has been assembled for the analytical solution of two-plate problems. Experiments have been performed to verify the analytical results. Since in heat transfer calculations the extent of the radius of contact is of primary importance, and since by restricting the experimental effort to the verification of only this parameter, (rather than the verification of the entire pressure distribution,) many experimental uncertainties should be eliminated, the experiments were designed only for the determination of the contact area.

Agreement between analysis and experiment was obtained and the results show that the single plate substitution is not justified and the 45 degree rule is not valid for the flat and smooth surfaces studied.
Chapter II

ANALYSIS

A. Problem Statement

The objective of the analysis was to solve the linear elasticity problem of two plates in contact defined mathematically by the following equations for each plate:

The equations of equilibrium

\[
\frac{\partial}{\partial r} (r \sigma_r) - \sigma_t + r \frac{\partial \tau}{\partial z} = 0
\]

(2.1)

\[
\frac{\partial}{\partial r} (r \tau) + \frac{\partial}{\partial z} (r \sigma_z) = 0
\]

where \( \tau_{rz} = \tau_{zr} = \tau \) and \( \tau_{rt} = \tau_{tr} = \tau_{zt} = \tau_{tz} = 0 \).

The stress - strain relations, using standard notation for stress and strain,

\[
\sigma_r = \lambda \varepsilon_r + 2 \mu \varepsilon_r
\]

(2.2)

\[
\sigma_t = \lambda \varepsilon_t + 2 \mu \varepsilon_t
\]

\[
\sigma_z = \lambda \varepsilon_z + 2 \mu \varepsilon_z
\]

\[
\tau = 2 \mu \varepsilon_{rz}
\]
where $\lambda$ and $\mu$ are Lame's constants and

$$\lambda = \frac{2G\nu}{1-2\nu}$$  \hspace{1cm} (2.3)$$

$$\mu = G$$

if $G$ is the modulus of elasticity in shear and $\nu$ is Poisson's ratio; and $\varepsilon$ the volume expansion is defined by

$$\varepsilon = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}$$  \hspace{1cm} (2.4)$$

where $u$ is the displacement in the radial direction and $w$ is the displacement in the axial direction.

The strain - displacement relations

$$\varepsilon_r = \frac{\partial u}{\partial r}$$

$$\varepsilon_t = \frac{u}{r}$$  \hspace{1cm} (2.5)$$

$$\varepsilon_z = \frac{\partial w}{\partial z}$$

$$\varepsilon_{rz} = \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)$$

The above equations can be combined to yield the equilibrium equations in terms of displacements

$$\nabla^2 u - \frac{u}{r^2} + \frac{1}{1-2\nu} \frac{\partial \varepsilon}{\partial r} = 0$$  \hspace{1cm} (2.6)$$

$$\nabla^2 w + \frac{1}{1-2\nu} \frac{\partial \varepsilon}{\partial z} = 0$$
The applicable boundary conditions are (see Fig. 11)

\[ \sigma^{(1)}(r, z) = \sigma^{(2)}(r, z) = 0 \]
\[ \tau^{(1)}(r, z) = \tau^{(2)}(r, z) = 0 \]
\[ \sigma^{(1)}(r, z) = \sigma^{(2)}(r, z) = 0 \]
\[ \tau^{(1)}(r, z) = \tau^{(2)}(r, z) = 0 \]
\[ \tau^{(1)}(r, D_1) = \tau^{(2)}(r, -D_2) = 0, \]
\[ \tau^{(1)}(r, 0) = \tau^{(2)}(r, 0), \quad A \leq r \leq R \]
\[ \sigma^{(1)}(r, D_1) = \sigma^{(2)}(r, -D_2) = 0, \quad B \leq r \leq C \]
\[ \tau^{(1)}(r, 0) = \tau^{(2)}(r, 0) = 0, \quad R \leq r \leq C \]
\[ \sigma^{(1)}(r, 0) = \sigma^{(2)}(r, 0), \quad A \leq r \leq R \]
\[ \sigma^{(1)}(r, 0) = \sigma^{(2)}(r, 0) = 0, \quad R \leq r \leq R \]
\[ \sigma^{(1)}(r, D_1) = \sigma^{(2)}(r, -D_2) = P(r), A \leq r \leq B \]
\[ w^{(1)}(r, 0) = w^{(2)}(r, 0), \quad A \leq r \leq R \]

\[ 2\pi \int_A^B \Pr \, dr = 2\pi \int_A^{R_o} \Pr \, dr \]
Inspection of the above equations shows that the above constitutes a mixed boundary value problem and the most appropriate technique for solution is the finite element method.

B. Method of Analysis

A finite element computer program was assembled for the analytical solution of bolted plates. Descriptions of the finite element method are given in references [9,10], but for completeness, an outline of the mathematical formulation for this case is presented in Appendix A. A listing of the computer program and instructions for its use may be found in Appendix B. Appendix C contains user’s instructions and a listing of the finite element program modified to include thermal strains.

As in the previous work axial symmetry and isotropic linear elastic material behavior were assumed. However, the computer programs accommodate plates with different material properties in a bolted pair.

The basic concept of the finite element method is that a body may be considered to be an assemblage of individual elements. The body then consists of a finite number of such elements interconnected at a finite number of nodal points or nodal circles. The finite character of the structural connectivity makes it possible to obtain a solution by means of simultaneous algebraic equations. When the problem, as is the case here, is expressed in a cylindrical coordinate system and in the presence of axial symmetry in geometry and load, tangential displacements do not exist, and the three-dimensional annular ring finite element is then reduced to the characteristics of a two-dimensional finite element.
The analysis consists of (a) structural idealization, (b) evaluation of the element properties, and (c) structural analysis of the assembly of the elements. Items (b) and (c) are covered in the appendices and in the references quoted. The structural idealization and the criteria for acceptable solutions will be described in this chapter.

Fig. 4(a) shows two circular plates in contact under arbitrary axisymmetric loading. The plates are subdivided into a number of annular ring elements which are defined by the corner nodal circles (or node points when represented in a plane) as shown in Figs. 4(b) and 4(c). Unlike the cases described in Chapter I, which have been solved by the Hankel transform method, all plates solved by the finite element method have finite radii. The cross sections of each annular ring element is either a general quadrilateral or triangle. To improve accuracy smaller elements are used in zones where rapid variations in stress are anticipated than in zones of constant stress; thus the different size elements shown in Fig. 4(b). (However, the total number of elements allowable are subject to computer capacity.)

Figure 4(b) shows the two plates in contact for the radial distance $X_c$ and separated beyond it. It is to be noted that the nodal points on the parting line and within the length of contact $X_c$ are common to elements in both plates. The other elements adjacent to the parting line on each plate are separated from their corresponding elements in the mating plate and these elements have no common nodal points. Physically, it is equivalent to the welding together of the two plates in the contact zone. Mathematically, we are imposing the condition that
in the contact zone the displacements in the z and r directions be identical for both plates. In the case of bolted plates of equal thickness, i.e. in the presence of symmetry about the parting plane, these conditions apply exactly. Furthermore, because of this symmetry, one needs to analyze only one plate, as shown in Fig. 5(b), with the imposed boundary conditions on the contact zone of zero displacement in the z-direction and freedom to displace in the r-direction. It can also be observed that the solution of two plates with symmetry about the parting plane is equivalent to the solution of one of these plates under the same loading conditions, but resting on a frictionless infinitely rigid plane. Also, under the above conditions the shear stress in the contact zone is identically zero.

In the case of bolted plates of unequal thickness the model includes both plates as shown in Fig. 5(c). This model is an approximation because, in general, two plates of unequal thickness do not have the same displacement in the r-direction on the contact surface. The solution yields, therefore, a shearing stress distribution in the contact zone. The solution, however, should be exactly compatible with the physical model if the frictional forces in the joint prevent sliding.

The critical aspect of the approach used herein is the determination of the largest nodal circle on the parting plane which is common to an element on each plate. This nodal circle defines the contact zone and the radius, \( R_0 \), at which separation occurs.

The output of the finite element computer program includes the displacement of each node in the r and z directions and the average
\( \sigma_z, \sigma_r, \sigma_t \) and \( \tau_{rz} \) stresses for each element.

The computation is iterative and the objective is to achieve the lowest possible compressive \( \sigma_z \) stress in the outermost elements bordering the contact zone. Unacceptable solutions are shown in Fig. 6(a) and 6(b). If \( R_o \) for a given external load distribution is too small, then the solution will show that the two plates intersect (Fig. 6(a)). On the other hand, if \( R_o \) is assumed too large, the solution will show that the outer portion of the contact zone sustains a tensile \( \sigma_z \) stress (Fig. 6(b)). Neither of these two situations is physically feasible.

In general, the procedure employed was to commence the iterations with a value for \( R_o \) which would yield a tensile \( \sigma_z \) stress in the outer elements adjacent to the contact zone and then move \( R_o \) inward. The iteration ended as soon as no tensile \( \sigma_z \) stress was present at the contact zone. For example, for the case shown in Fig. 5(b), if the \( \sigma_z \) stress for the element in the last row and to the left of the last roller is tensile, then the following iteration will proceed without the last roller. Thus, the resolution is one nodal interval. Finer resolution can be obtained by reducing the interval between nodal circles by introducing more elements or shifting the grid locally. The same criteria apply to the model shown in Fig. 5(c).

In the finite element analysis of the Fernlund (3) model, i.e. single plate with external loads at the faces \( z = \pm D \) no iteration is required and the rollers shown in Fig. 5(c) would extend to the outer radius of the plate. (Although Fernlund's computations are based on infinite plates, computations show that there is no distinction between infinite plates and plates of radius greater than five times of the outer
radius, $B$, of the load. See Fig. 5(a).

Convergence was tested by subdividing elements further, with nodal points in the coarser grid remaining nodal points in the finer grid. Changing the mesh from 180 elements to 360 elements have shown no improvement in accuracy. Meshes from 180 to 300 elements were used in this analysis. Typical spacings between nodal points were 0.015 inch radially and 0.03 inch in the $z$-direction.
Chapter III

EXPERIMENTAL METHOD

The objective of the experiment was to determine the extent of contact between two plates when bolted together. Sixteen type 304 stainless steel plates, 4 inches in diameter, were machined to nominal thicknesses of 1/16, 1/8, 3/16 and 1/4 inch, 4 plates for each thickness. After rough machining these plates were stress relieved at 1875°F and ground flat to 0.0002 inch. One side of each plate was then lapped flat to better than one fringe of sodium light (11 micro-inches) in the case of the 1/8, 3/16 and 1/4 inch plates, and to better than two fringes in the case of the 1/16 inch plates. Disregarding scratches, the finish of the lapped surfaces was 5 micro-inches rms. Each plate had a central hole, 0.257 inch in diameter, for a 1/4 – 20 bolt, and two notches and two holes on the periphery (see Fig. 7). Two techniques were employed in determining the area of contact when two of these plates were bolted together. The first technique entailed the following procedure (see Fig. 7):

(a) The plates were cleaned with alcohol and lens tissue.

(b) One plate was placed on the base of the fixture shown in Fig. 7, lapped surface up and the two holes on the periphery of the plates engaged with two pins on the fixture. Spacers between the fixture base and plate prevented the pins from extending beyond the top surface of the plate.
(c) A second plate was placed on top of the first plate, lapped surfaces mating. The notches on the two plates were lined up with each other and with notches in the base of the fixture. Thus, rotation of the plates was prevented.

(d) A standard 1/4 - 20 hex-nut with its annular bearing surface (0.42 inch O.D.) lapped flat was engaged on a high strength 1/4 - 20 bolt. The nut was located about two threads away from the head of the bolt and served in lieu of the bolt head. The lapped surface of the nut faced away from the bolt head and since the nut was not sent home against the bolt head, the looseness of fit between nut and bolt offered a degree of self alignment.

(e) The bolt and nut assembly described in (d) above was then inserted through the 1/4 inch central holes of the two plates and a second 1/4 - 20 lapped nut was engaged on the bolt. Thus the two plates were captured by the two 1/4 -20 nuts with the lapped surfaces of the nuts bearing against the plates.

(f) With the torque wrench shown on the right in Fig. 7, the nuts were torqued down to 70 pound-inches of torque to yield a 1100 pound force in the bolt [11].

(g) The position of the keys was changed to engage with only the lower plate and the fixture and a special spanner wrench, as shown in Fig. 7, was engaged with the top plate. The spanner wrench was restrained to move in the horizontal plane and it was set into motion by the screw pressing against the wrench handle.
(h) With the aid of the spanner wrench the upper plate was rotated relative to the lower plate several times approximately + 5 degrees.

Thus, the above procedure allowed for the rubbing of one plate relative to its mate while under a bolt force of approximately 1100 lbs. The remaining steps were the disassembly and the measurement of the extent of the contact zone which was defined by the shine due to the rubbing in the contact zone. It is to be noted that the boundaries of the contact zone as measured by the naked eye and by searching for marks of "polished" or "damaged" surface under a 10.5 power magnification are essentially the same.

The above test was performed on 5 pairs of specimen. These were
1. One 0.07 in. plate mated to a 0.65 in. plate
2. One 0.126 in. plate mated to a 0.126 in. plate
3. One 0.191 in. plate mated to a 0.192 in. plate
4. One 0.253 in. plate mated to a 0.256 in. plate
5. One 0.124 in. plate mated to a 0.257 in. plate

The identical tests were repeated for
1. One 0.124 in. plate mated to a 0.126 in. plate; and
2. One 0.191 in. plate mated to a 0.192 in. plate,
but in lieu of the 1/4 - 20 nuts in direct contact with the plates special washers, 1.000 in. O.D., 0.257 in. I.D. and 0.620 in. high, were interposed between the bolt head and nut.

The diameters of the contact zones were measured with a machinist ruler with 100 divisions to the inch and with a Jones and Lamston Vertac 14 Optical Comparator.
The second technique used the same parts and fixture, but it involved autoradiographic measurements.

Four plates, 1/4, 3/16, 1/8 and 1/16 inch thick were sent to Tracerlab, Inc., Waltham, Mass., for electrolytic plating with radioactive silver Ag \(^{110}\) (half life of 8 months). Each plate was masked except for an area on the lapped face one inch in radius. The plates then received a plating of copper about 5 microinches thick and then approximately a 5 microinch plating of silver containing the radioactive isotope. The resultant activity on each plate was about 2 millicuries.

These plates were then mated to plates of equal thickness (not plated) and assembled in a shielded hood as indicated in steps (a) to (h) above except that in the case of the pair of 1/4 inch plates care was taken not to rotate the plates during and after assembly and in the remaining cases the rotation specified in step (h) was done only once in one direction.

The plates were then disassembled and the radioactive contamination on the plates which were in contact with the radioactive plates measured. The transferred activity was:

- 1/4 in. plate approximately 0.05 microcuries
- 3/16 in. plate approximately 3. microcuries
- 1/8 in. plate approximately 0.1 microcuries
- 1/16 in. plate approximately 0.4 microcuries

It was also observed in handling that the adhesion of the silver on the 3/16 in. plate was poor.
Kodak type R single coated industrial x-ray film was then placed on the contaminated plates under darkroom conditions. The sensitive side of the film was pressed against the radioactive sides of the plates with a uniform load of about five pounds and left for exposure for three days. After three days, the film was removed and developed. The results are shown in Fig. 10.
Chapter IV
RESULTS

A. Pressure Distribution and Radii of Separation from Single Plate and Two Plate Finite Element Models.

Using the finite element procedure described in Chapter II, the midplane stress distribution of single circular plates of thickness 2D, outer radii of 1.54 in., inner radii of 0.1 in., Poisson ratio of 0.3, and loaded by a constant pressure between radii A and B, Fig. 3(f), was computed. Computations were performed for D values of 0.1, 0.1333 and 0.2 in. For each value of D the radius B, which defines the region of the symmetric external load, assumed the values of 0.31, 0.22, 0.16 and 0.13 in. The \( \sigma_z \) stress distribution at the midplane, from the inner radius to the radius at which the above stress is no longer compressive, is shown in Figs. 12, 13 and 14 as a function of radius.

The identical cases were then recomputed, using again the finite element method, in accordance with the two plate model shown in Figs. 4(b) and 5(b). These results are given in Figs. 15, 16 and 17.

Inspection of the above figures show that the two plate model yields a somewhat different stress distribution in the contact zone than the stress distribution approximated from the single plate model, and more significantly, from the heat transfer point of view, the two plate model yields a lower value for the radius of separation, \( R_0 \), which
results in a reduction in area for heat transfer. Table 1 gives a comparison of the values for $R_o$ obtained from the two models.

It may be observed that the single plate result of Fernlund (Ref. 3, pp. 56, 124) is in fair agreement with the finite element results obtained for the single plate model.

B. Radii of Separation from Experiment and Their Predicted Values from the Two Plate Finite Element Computation.

As described in Chapter III, stainless steel circular plate specimen (Fig. 7) were bolted together, rotated relative to each other with the bolt force acting, and after disassembly the contact area of the joint was determined by measuring the footprints (the shiny, polished areas) on each plate due to the plates rubbing against each other. Photographs of these footprints are shown in Fig. 8. Fig. 9 also shows a typical footprint of the annular bearing surface of the 1/4 - 20 nut against a plate. All plates tested were of 304 stainless steel, 4 inch O.D., .257 I.D., and the nominal thicknesses of the plates were 1/16, 1/8, 3/16 and 1/4 inch. In addition to the plates fastened with standard nuts which gave a loading circle of radius $B$ (Fig. 5) of 0.211 inch, plates fastened by the special nuts described in Chapter III for which $B$ was 0.5 inch were also tested.

Figure 10 shows the results of the autoradiographic tests described in Chapter III. For all plate pairs tested, i.e. 1/16, 1/8, 3/16 and 1/4 inch nominal, the value of $B$ was 0.211 inch.

The pressure distributions and radii of separation for all the
above test cases were computed independently by the two plate model finite element analysis. Table 2 gives the test and analytical results for all test cases. The test results are an average of all measurements (minimum of six readings). A description of the analyses follows.

Figure 18 shows the results of a two plate and a single plate model analysis for the 0.253 inch bolted test specimen. For Figure 19 the external pressure distribution between radii A and B is triangular. (The total force, however, is equal to the force exerted in the case of uniform pressure.) In one case, the peak external pressure is at A, Fig. 20(a), and in the other case at B, Fig. 20(b). Results of another computation which assumed a uniform displacement of 50 microinches under each nut is shown in Fig. 21. It is interesting to note that the point of separation obtained by using the two plate model for all variations of loading given above occurs in the range of \( r/A \) values of 2.73 to 2.93 while the two plate model yields separation at a value for \( r/A \) of 3.5. The computed deflections under the nuts are given in Fig. 22.

The finite element analysis results for the 0.191 in. plate pair specimen are given in Fig. 23. Figures 24 and 25 show the computed pressure distribution and deflection patterns in the joint, respectively, for the 1/8 in. plate pair. In order to investigate the possible influence misalignments of the spanner wrench, i.e. vertical forces or restraints exerted at edge of plate, may have on the results of the experiment, the extreme case of fixing the outer edges of the plate as shown in Fig. 20(c) was considered. As Fig. 24 shows, within the
resolution of the finite element grid size, the effect is negligible. This model, Fig. 20(c), and result also indicate that the influence of additional fasteners 2 inches away would not have an influence on the contact zone for the geometry considered. (However, if the distance between bolts is considerably reduced, then the contact area should increase.) The computed results for the 1/16 inch plate pair is given in Fig. 26.

Figure 27 gives the finite element analysis results for the asymmetric case of a 1/8 in. plate bolted to a 1/4 in. plate. The model shown in Fig. 5(c) was used and as discussed in Chapter II, this model is strictly valid only if the friction in the joint prevents sliding between the plates. Nevertheless, the percent discrepancy between the computed value and tested value (see Table 2) falls within the range of the symmetric cases analyzed and tested.

In summary, the results obtained from the two plate finite element model and from experiment are in good agreement (Fig. 28).
Chapter V

APPLICATION

An application of the above results for the evaluation of the thermal contact conductance, $h_c$, and the determination of the heat transferred in a specific, but typical, lap joint section is illustrated in this chapter.

An aluminum lap joint in a vacuum environment, the relevant section and boundary conditions as shown in Fig. 29, was analyzed by means of a nodal analysis. The plate thickness was 0.1 in. and the hole diameter, $2A$, was 0.2 in. The bearing surface of the bolt, $2B$, was 0.26 in. in diameter. Because of the high conductivity and small thickness of the plates, no $z$ dependence (see Fig. 29) was assumed for the temperature in the main body of the plate. However, heat flow in the $z$-direction in the nodes above and below the contact zone is considered. Qualitatively, the heat flow in the joint proceeds in the $x$-$y$ plane from the left end (Fig. 29) toward the 0.2 in. diameter hole. In the vicinity of the hole, a macroscopic constriction for heat flow is encountered because the flow is being channeled toward the small contact zone. The flow of heat then encounters the microscopic constrictions at the contacting asperities (which determine $h_c$) in the contact zone; spreads out in the $x$-$y$ directions in the second plate; and continues to the right edge of the lap joint.
The material properties assumed were (refer to equation 1.1):

\[
\begin{align*}
H &= 150,000 \text{ psi} \\
k &= 100 \text{ Btu/hr-°F-ft} \quad (k_1 = k_2 = 100) \\
\sigma &= 5.9 \times 10^{-6} \text{ ft.} \quad (\sigma_1 = \sigma_2 = 50 \times 10^{-6} \text{ in.}) \\
\tan \theta &= 0.1
\end{align*}
\]

Assuming further, a uniform load of 46,500 psi on the loading surface (#10 screw; 1000 lb. bolt force) and referring to Fig. 15, curve \( \frac{R}{A} = 1.3 \), the following interface stresses, \( \sigma_z \), contact heat transfer coefficient, \( h_c \), and conductance, \( (\text{area})(h_c) \), were obtained as a function of inner and outer radii. (These radii define increments of area, the sum of which define one quarter of the contact zone.):

<table>
<thead>
<tr>
<th>( r_{\text{outer}} ) inch</th>
<th>( r_{\text{inner}} ) inch</th>
<th>( \sigma_z ) psi</th>
<th>( \frac{h_c}{\text{Btu/hr-°F-ft}^2} )</th>
<th>( \text{Area} \times h_c ) Btu/hr-°F-ft(^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.13</td>
<td>.1</td>
<td>27,900</td>
<td>446,000</td>
<td>16.6</td>
</tr>
<tr>
<td>.16</td>
<td>.13</td>
<td>14,000</td>
<td>223,000</td>
<td>10.6</td>
</tr>
<tr>
<td>.175</td>
<td>.16</td>
<td>3,950</td>
<td>63,100</td>
<td>1.7</td>
</tr>
</tbody>
</table>

The conductance between nodal points were then computed and with the aid of the steady state heat transfer program listed in Appendix D, the nodal temperatures for the conditions given in Fig. 29 were computed. The heat transferred from the edge maintained at 20°F to the edge at 0°F (Fig. 29) for this case was 2.88 Btu/hour. The same computation was repeated for the case of a bearing surface between the plate
and the bolt (2B) of 0.44 in. in diameter, but the bolt force was left unchanged. The heat transferred from the 20°F edge to the 0°F edge in this case was 3.15 Btu/hour. In the absence of the joint the heat transfer along an equivalent 7 inch length of solid aluminum would have been 3.58 Btu/hour. This data shows that the thermal resistance of the contact zone (not entire 7 inch lap joint) was decreased from 1.52 to 0.92 °F·hr/Btu by the increase of the effective bolt head diameter from .26 to .44 in. It should be observed that the change in thermal resistance of the joint is primarily due to the increase in contact area and the resulting decrease in macroscopic constriction resistance at the hole. Also, the heat flux in this example is mainly controlled by the 7 inch length and 0.1 inch thickness rather than the joint resistance. This emphasizes the importance of a balanced thermal design.

For large heat fluxes where thermal strains may have an influence on the radii of separation, the finite element program given in Appendix C may be used. Also, in a non-vacuum environment the effect of the interstitial fluid is added in two ways. Firstly, equation (1.6) is applied to account for the presence of interstitial fluid in the contact zone, and secondly, conduction across the gaps between the plates and convection from the plates is considered. (Radiation heat transfer, if applicable, should also be included.)
Chapter VI

CONCLUSIONS

The finite element technique used in this work for the analysis of the pressure distribution and deformation of smooth and flat bolted plates under conditions of axial symmetry predicts contact areas in joints considerably lower than reported previously in the literature. These results were verified experimentally. The discrepancy between the previously reported results and the results reported here is due to the simplifying assumption made by earlier researchers that a joint can be modeled as a single plate.

The computer programs listed in the appendices will also accommodate joints made up of plates of dissimilar materials and the presence of thermal gradients.

Of the eleven tests performed, only one (case 3, autoradiographic) yielded inconsistent results. (This data point could probably be ignored because of the poor adhesion of the plating material which manifested itself by the high radioactive contamination count during test.)

The finite element analysis performed for the test specimen show that the gap between the 1/4 inch bolted steel specimen is 98.6 microinches at the outer radius of the plate of 2 inches, and 1/32 of an inch away from the radius of separation (0.35 in.), the gap is
only 3 microinches for the test load. This data indicates the difficulties previous workers have encountered in their experiments. (This also explains the oval shape of several of the footprints.) Furthermore, this data shows that the effects of surface roughness and the lack of flatness could have a significant effect on the size of contour area.

An application of the above work to a heat transfer problem is illustrated in Chapter V.
REFERENCES


APPENDIX A

FINITE ELEMENT ANALYSIS OF AXISYMMETRIC SOLIDS

The finite element method and the equations which govern the stresses and displacements in axisymmetric solids is given in the literature [9,10,12,13,15] and the procedure will be briefly summarized in this appendix.

The procedure for the standard stiffness analysis method is as follows [15]:

(a) The internal displacements, \( \mathbf{v} \), are expressed as

\[
\{v(r,z)\} = \{M(r,z)\} \{\alpha\} \tag{A.1}
\]

where \( M \) is a displacement function and \( \alpha \) are the generalized coordinates representing the amplitudes of the displacement functions.

(b) The nodal displacements \( v_i \) are expressed in terms of the generalized coordinates

\[
\{v_i\} = [A] \{\alpha\} \tag{A.2}
\]

where \( A \) is obtained by substituting the coordinates of the nodal points into \( M \).

(c) The generalized coordinates are expressed in terms of the nodal displacements

\[
\{\alpha\} = [A]^{-1} \{v_i\} \tag{A.3}
\]
(d) The element strains, $\varepsilon$, are evaluated

$$\{\varepsilon\} = [B(r,z)] \{\alpha\} \quad (A.4)$$

where $B$ is obtained from the appropriate differentiation of $M$.

(e) The element stresses are expressed in terms of the stress-strain relation $D$

$$\{\sigma(r,z)\} = [D] \{\varepsilon\} = [D] [B] \{\alpha\} \quad (A.5)$$

(f) Assuming a virtual strain $\bar{\varepsilon}$ and a generalized virtual coordinate displacement $\bar{\alpha}$ the internal virtual work, $W_i$, in the differential volume, $dV$, is given by

$$dW_i = \{\varepsilon\}^T \{\sigma\} dV = \{\alpha\}^T [B]^T [D] [B] \{\alpha\} dV \quad (A.6)$$

and the total internal virtual work is

$$W_i = \{\bar{\alpha}\}^T \left[ \int_{Vol} [B]^T [D] [B] dV \right] \bar{\alpha} \quad (A.7)$$

(g) The external work, $W_e$, associated with the generalized displacement $\bar{\alpha}$ is

$$W_e = \{\alpha\}^T \{\beta\} \quad (A.8)$$

where $\beta$ are generalized forces corresponding with the displacements $\alpha$.  

(h) After equating $W_i$ and $W_e$ and setting the $\overline{\alpha}$ displacement to unity

$$\{ \varepsilon \} = \int_{\text{Vol}}^{} [B]^T [D] [B] \, \alpha = [\overline{k}] \{ \alpha \} \quad (A.9)$$

where $$[\overline{k}] = \int_{\text{Vol}}^{} [B]^T [D] [B] \, dV \quad (A.10)$$

and which transforms to the nodal point surfaces

$$k = [A^{-1}]^T [\overline{k}] [A^{-1}] \quad (A.11)$$

(i) The stiffness matrix for the complete system is then

$$[K] = \sum_{m=1}^{n} [k]_m \quad (A.12)$$

where $n$ equals the number of elements and the equilibrium relationship becomes

$$\{Q\} = [K] \{v_1\} \quad (A.13)$$

where

$$\{Q\} = \sum_{m=1}^{n} \{R\}_m \quad (A.14)$$

$$\{R\} = \int_{\text{Area}}^{\ } [A^{-1}]^T [M]^T \{P\}_m \, dA \quad (A.15)$$

and $P$ are the surface forces.

The above procedure applies with minor modification to problems with thermal and body force loading.
The expression
\[
\{Q\} = [K] \{v_i\}
\]  
(A.16)
represents the relationship between all nodal point forces and all nodal point displacements. Mixed boundary conditions are considered by rewriting this equation in the partitioned form
\[
\begin{pmatrix}
Q_a \\
Q_b
\end{pmatrix} =
\begin{bmatrix}
K_{aa} & K_{ab} \\
K_{ba} & K_{bb}
\end{bmatrix}
\begin{pmatrix}
u_a \\
u_b
\end{pmatrix}
\]
(A.17)
where \( v_i = u \).

The first part of the partitioned equation can be written as
\[
\{Q_a\} = [K_{aa}] \{u_a\} + [K_{ab}] \{u_b\}
\]  
(A.18)
and then expressed in the reduced form
\[
\{Q^*\} = [K_{aa}] \{u_a\}
\]  
(A.19)
where
\[
\{Q^*\} = \{Q_a\} - [K_{ab}] \{u_b\}
\]  
(A.20)

The matrix equation (A.19) is solved for the nodal point displacements by standard techniques. Once the displacements are known the strains are evaluated from the strain displacement relationship and the stresses in turn are evaluated from the stress strain relations.

Both triangular and quadrilateral elements are used. The displacements in the \( r-z \) plane in the element are assumed to be of the form
\[ v_r = \alpha_1 + \alpha_2 r + \alpha_3 z \]  
\[ v_z = \alpha_4 + \alpha_5 r + \alpha_6 z \] (A.21)

This linear displacement field assures continuity between elements since lines which are initially straight remain straight in their displaced position. Six equilibrium equations are developed for each triangular element.

A quadrilateral element is composed of four triangular elements and ten equilibrium equations correspond to each element.
# APPENDIX B

## FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC AXISYMMETRIC PLATES (ref. 13,14)

**Input Instructions:**

<table>
<thead>
<tr>
<th>Card Sequence</th>
<th>Item</th>
<th>Format</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Title</td>
<td>18A4</td>
<td>1-72</td>
</tr>
<tr>
<td>2</td>
<td>Total number of nodal points</td>
<td>I5</td>
<td>1-5</td>
</tr>
<tr>
<td></td>
<td>Total number of elements</td>
<td>I5</td>
<td>6-10</td>
</tr>
<tr>
<td></td>
<td>Total number of materials</td>
<td>I5</td>
<td>11-15</td>
</tr>
<tr>
<td></td>
<td>Normalizing stress (NORM)</td>
<td>I5</td>
<td>16-20</td>
</tr>
<tr>
<td></td>
<td>Number of pressure cards</td>
<td>I5</td>
<td>21-25</td>
</tr>
</tbody>
</table>

(If NORM = 0, put in value of \( E \) in material card;
if NORM = 1, put in value \( E/\sigma_{\text{vertical}} \);
if NORM = -1, put in value \( E/\sigma_{\text{octahedral}} \);

**NOTE:** Use NORM = 0 for this application.)

<table>
<thead>
<tr>
<th>Card Sequence</th>
<th>Item</th>
<th>Format</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(Material property cards - one set of (a) and (b) for each material)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a) 1st card</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Material No.</td>
<td>I5</td>
<td>1-5</td>
</tr>
<tr>
<td></td>
<td>Initial ( \sigma_z ) stress</td>
<td>F10.0</td>
<td>6-15</td>
</tr>
<tr>
<td></td>
<td>Initial ( \sigma_r ) stress</td>
<td>F10.0</td>
<td>16-25</td>
</tr>
<tr>
<td></td>
<td>(b) Second Card</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( E )</td>
<td>F10.0</td>
<td>1-10</td>
</tr>
<tr>
<td></td>
<td>( \nu )</td>
<td>F10.0</td>
<td>11-20</td>
</tr>
</tbody>
</table>
Card Sequence | Item | Format | Column
--- | --- | --- | ---
4 | Nodal point information (One for each node) | 215,4F10.0 | 
| | Node number | 1-5 | 
| | CODE | 6-10 | 
| | r-coordinate | 11-20 | 
| | z-coordinate | 21-30 | 
| | XR | 31-40 | 
| | XZ | 41-50 | 

If the number in column 10 is

<table>
<thead>
<tr>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Remarks

The following restrictions are placed on the size of problems which can be handled by the program.

<table>
<thead>
<tr>
<th>Item</th>
<th>Maximum Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodal Points</td>
<td>450</td>
</tr>
<tr>
<td>Elements</td>
<td>450</td>
</tr>
<tr>
<td>Materials</td>
<td>25</td>
</tr>
<tr>
<td>Boundary Pressure Cards</td>
<td>200</td>
</tr>
</tbody>
</table>
All loads are considered to be total forces acting on a one radian segment. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. The boundary code (column 10), XR and XZ are set equal to zero.

If the number in columns 6-10 of the nodal point cards is other than 0, 1, 2 or 3, it is interpreted as the magnitude of an angle in degrees. The terms in columns 31-50 of the nodal point card are then interpreted as follows:

- XR is the specified load in the s-direction
- XZ is the specified displacement in the n-direction

The angle must always be input as a negative angle and may range from -.001 to -180 degrees. Hence, +1.0 degree is the same as -179.0 degrees. The displacements of these nodal points which are printed by the program are

\[
\begin{align*}
\mathbf{u}_r &= \text{the displacement in the s-direction} \\
\mathbf{u}_z &= \text{the displacement in the n-direction}
\end{align*}
\]

Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information by incrementing by one the preceding I, J, K and L. The material identification code for the generated cards is set equal to the value given on the last card. The last element card must always be supplied.

Triangular elements are also permissible; they are identified by repeating the last nodal point number (i.e. I, J, K, K).

One card for each boundary element which is subjected to a normal pressure is required. The boundary element must be on the left as one
progresses from $I$ to $J$. Surface tensile force is input as a negative pressure.

Printed output includes:

1. Reprint of input data.
2. Nodal point displacement
3. Stresses at the center of each element.

Nodal point numbers must be entered counterclockwise around the element when coding element data.

The maximum difference between the nodal point numbers on an element must be less than 25. However, on a nodal diagram elements and nodes need not be numbered sequentially.
Listing:

```
C  *****************************************************************************************
C  FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC
C  AXYSYMMETRIC PLATES REF. FEAST 1,3 SAAS 2
C
C  IMPLICIT REAL*8 (A-H,O-Z)
C  IMPLICIT INTEGER*2(I-N)
C
C  COMMON /STOP,HED(18),SIGIR(25),SIGI7(25),GAMMA(25),ZKNOT(25),
C        1 DEPTH(25),E(1,25),SIG(7),R(450),Z(450),UR(450),
C        2 UZ(450),STOTAL(450,4),KSW
C  COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPH,GNRM,MTYPE,ICGCE(450)
C  COMMON /ARG/ RRR(5),P(10),LM(4),DD(3,3),
C        1 HH(6,10),HR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),Xi(6),
C        2 E(10),IX(450,5)
C
C  COMMON /BANARG/ R(900),A(900,54),MBAND
C  COMMON/PRESS/ IBC(200),JBC(200),PR(200),NUMPC
C  DATA STRS /='***/
C
C  READ AND PRINT CONTROL INFORMATION
C  *****************************************************************************************
C
C  50 READ (5,1000,END=950) HED
C    WRITE (6,2000) HED
C
C    READ(5,1001) NUMNP,NUMEL,NUMMAT,GNRM,NUMPC
C    WRITE (6,2006) NUMNP,NUMEL
C    IF (GNRM) 65,65,66
C
C    WRITE (6,2041)
C
C  READ AND PRINT MATERIAL PROPERTIES
C  *****************************************************************************************
C
C  65 CONTINUE
C
C    DO 80 M=1,NUMMAT
C    READ (5,10C2) MTYPE,
C    SIGIZ(MTYPE),SIGIR(MTYPE)
C    WRITE (6,2007) MTYPE,SIGIZ(MTYPE),SIGIR(MTYPE)
C    READ (5,10C3) (E(J,MTYPE),J=1,2)
```

FENT0001  FENT0002  FENT0003  FENT0004  FENT0005  FENT0006  FENT0007  FENT0008  FENT0009  FENT0010  FENT0011  FENT0012  FENT0013  FENT0014  FENT0015  FENT0016  FENT0017  FENT0018  FENT0019  FENT0020  FENT0021  FENT0022  FENT0023  FENT0024  FENT0025  FENT0026  FENT0027  FENT0028  FENT0029  FENT0030  FENT0031  FENT0032  FENT0033  FENT0034  FENT0035  FENT0036
WRITE (6, 2051) (E(J, MTYPE), J=1, 2)
80 CONTINUE
C----------------------------------------------------------------------
C READ AND PRINT NODAL POINT DATA
C----------------------------------------------------------------------
100 WRITE (6, 2013)
   (L = 0
105 READ (5, 1006) N, ICODE(N), R(N), Z(N), UR(N), UZ(N)
106   NL=L+1
       IF (L.EQ.0) GO TO 110
       ZX=N-L
       DR=(R(N)-R(L))/ZX
       DZ=(Z(N)-Z(L))/ZX
110   L=L+1
       IF (N-L) 113, 112, 111
111   ICODE(L)=0
       R(L)=R(L-1)+DR
       Z(L)=Z(L-1)+DZ
       UR(L)=0.0
       UZ(L)=0.0
       GO TO 110
112 WRITE (6, 2014) (K, ICODE(K), R(K), Z(K), UR(K), UZ(K), K=NL, N)
113 WRITE (6, 2015) N
   IF (NUMP-N) 113, 120, 105
   GO TO 900
C----------------------------------------------------------------------
C RFAC AND PRINT ELEMENT PROPERTIES
C----------------------------------------------------------------------
120 WRITE (6, 2016)
   N=0
130 READ (5, 1007) M, (IX(M, I), I=1, 5)
140 N=N+1
       IF (M-N) 170, 170, 150
150   TX(N, 1)=IX(N-1, 1)+1
       IX(N, 1)=IX(N-1, 1)+1
       IX(N, 2)=IX(N-1, 2)+1
       IX(N, 3)=IX(N-1, 3)+1
IX(N,4)=IX(N-1,4)+1
IX(N,5)=IX(N-1,5)
170 WRITE (6,2017) N,(IX(N,I),I=1,5)
180 IF (M-N) 180,180,140
180 IF (NUMEL-N) 300,300,130
C
C READ AND PRINT THE PRESSURE CARDS
C
C
300 IF(NUMPC) 290,210,290
290 WRITE(6,9000)
DO 200 L=1,NUMPC
READ(5,9001) IRC(L),JAC(L),PR(L)
200 WRITE(6,9002) IRC(L),JAC(L),PR(L)
210 CONTINUE
C
C DETERMINE PAND WIDTH
C
C
J=0
DO 340 N=1,NUMEL
DO 340 I=1,4
DO 325 L=1,4
KK=IX(N,I)-IX(N,L)
IF (KK.LT.0) KK=-KK
IF (KK.GT.J) J=KK
325 CONTINUE
340 CONTINUE
MRAIND=2*J+2
C
C SOLVE FOR DISPLACEMENTS AND STRESSES
C
C
KSW=0
CALL STIFF
IF (KSW.NE.0) GO TO 900
C
CALL RANSCL
WRITE(6,2052)
WRITE (6,2025) (N,B (2*N-1),B (2*N),N=1,NUMNP)

CALL STRESS(SPLIT)

PROCESS ALL DFCKS EVEN IF ERROR

GO TO 910

WRITE (6,4000)

WRITE (6,4001) NED

READ (5,1000) CHK

IF (CHK.NE.STRS) GO TO 920

GO TO 50

CONTINUE

WRITE (6,4002)

EXIT

FORMAT (18A4)

FORMAT (1215)

FORMAT (15,2F10.0)

FORMAT (2F10.0)

FORMAT (3F10.0)

FORMAT (215,4F10.0)

FORMAT (6I5)

FORMAT (1H1,2OA4)

FORMAT (28HM0NUMBER OF NODAL POINTS---- 13/ 1 2AH NUMBER OF ELEMENTS-------- 13]

FORMAT (2OHM0ATERIAL NUMBER---- 13/ 1 25H INITIAL VERTICAL STRESS= FIO.3, 5X, 2 26H INITIAL HORIZONTAL STRESS= FIO.3)

FORMAT (12H0NAL POINT ,4X, 4HTYPE ,4X, 10HR-ORDINATE ,4X, 1 10HZ-ORDINATE ,10X, 6HR-LOAD ,10X, 6HZ-LOAD )

FORMAT (112,18,2F14.3,2E16.5)
2015 FORMAT (26HOMOLOGICAL POINT CARD ERROR N = I5)
2016 FORMAT (49HIELEMENT NO., I J K L MATERIAL )
2017 FORMAT (1113,416,1112)
2025 FORMAT (12HOMOLOGICAL POINT ,6X, 14HR-DISPLACEMENT ,6X, 14HZ-DISPLACEMENT)
2041 FORMAT (76HOMOLOGUS AND YIELD STRESS NORMALIZED WITH RESPECT TO INITIAL VERTICAL STRESS )
2051 FORMAT(140,10X,'E',8X,'NU',/3X,F11.1,F10.4/)
2052 FORMAT(1H1)
C ******************************************************************************
3003 FORMAT (1615)
C ******************************************************************************
4000 FORMAT (1111 ' ABNORMAL TERMINATION')
4001 FORMAT (1111 ' END OF PROBLEM ' 20A4)
4002 FORMAT (1111 ' END OF JOB')
C ******************************************************************************
9000 FORMAT(29HOPRESSURE BOUNDARY CONDITIONS/ 24H I J PRESSURE)
9001 FORMAT(215,F10.0)
9002 FORMAT(216,F12.3)
END
SURROUNDD STIFF
C
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON STTCP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
DEPHT(25),F(10,25),SIG(7),R(450),Z(450),UR(450),
2 UZ(450),STOTAL(450,4),KSH
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPHT,NORM,MTYPE,ICODE(450)
COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),R(10),LM(4),DD(3,3),
1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,5),F(6,10),TP(6),XI(6),
2 EE(10,10),IX(450,5)
COMMON /BANARG/ B(900),A(900,54),MBAND
COMMON/PRFSS/ IRC(200),JBC(200),PR(200),NUPRC
DIMENSION CODE(450)
C ******************************************************************************
C INITIALIZATION
C

NB = 27
ND = 2 * NB
ND2 = 2 * NUMNP
DO 50 N = 1, ND2
B(N) = 0.0
DO 50 M = 1, ND
50 A(N, M) = 0.0
C FORM STIFFNESS MATRIX
C

DO 210 N = 1, NUMEL
C
C 90 CALL QUAD(N, VOL)
   IF (VOL) 142, 142, 144
142 WRITE (6, 2003) N
   KSW = 1
   GO TO 210
C
144 IF (IX(N, 3) - IX(N, 4)) 145, 165, 145
145 DO 150 II = 1, 9
   CC = S(II, 10) / S(10, 10)
   DO 150 JJ = 1, 9
150 S(II, JJ) = S(II, JJ) - CC * S(10, JJ)
C
   DO 160 II = 1, 8
   CC = S(II, 9) / S(9, 9)
   DO 160 JJ = 1, 8
160 S(II, JJ) = S(II, JJ) - CC * S(9, JJ)
C
ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS
C
165 DO 166 I = 1, 4
166 LM(I) = 2 * IX(N, I) - 2
C
DO 200 I=1,4
DO 200 K=1,2
II=LM(I)+K
KK=2*II-2+K
DO 200 J=1,4
DO 200 L=1,2
JJ=LM(J)+L-II+1
LL=2*J-2+L
IF (JJ) 700,200,175
175 IF (NO-JJ) 180,195,195
180 WRITE (6,2004) N
KSW=1
GO TO 210
195 A(II,JJ)=A(II,JJ)+S(KK,LL)
200 CONTINUE
210 CONTINUE
IF(KSW.EQ.1) GO TO 500
C
ADD CONCENTRATED FORCES
C
DO 250 N=1,NUMNP
K=2*N
B(K)=B(K)+U7(N)
B(K-1)=B(K-1)+UR(N)
250 CONTINUE
C
PRESSURE BOUNDARY CONDITIONS
C
IF(NUMPC) 260,310,260
260 DO 300 L=1,NUMPC
I=IR(L)
J=JR(L)
CODE(I)=ICODE(I)
CODE(J)=ICODE(J)
300 CONTINUE
PP=PR(I)/6.
DZ=(Z(I)-Z(J))*PP
DR=(R(J)-R(I))*PP
RX=2.*R(I)+R(J)
ZX=R(I)+2.*C*R(J)
264 II=2*I
JJ=2*J
270 SINA=C.*0
COSA=1.*0
IF(CODE(I)) 271,272,272
271 SINA=DSIN(CODE(I))
COSA=DCOS(CODE(I))
272 B(I-I-1)=B(I-I-1)+RX*(COSA*DZ+SINA*DR)
B(I)=B(I)-RX*(SINA*DZ-COSA*DR)
280 SINA=C.*0
COSA=1.*0
IF(CODE(J)) 291,292,292
291 SINA=DSIN(CODE(I))
COSA=DCOS(CODE(I))
292 B(JJ-1)=B(JJ-1)+ZX*(COSA*DZ+SINA*DR)
B(JJ)=B(JJ)-ZX*(SINA*DZ-COSA*DR)
300 CONTINUE
310 CONTINUE
C DISPLACEMENT B.C.
C
DO 400 M=1,NUMNP
U=UR(M)
N=2*M-1
KX=ICODE(M)+1
GO TO (400,370,390,380),KX
370 CALL MODIFY(N,U,ND2)
GO TO 400
380 CALL MODIFY(N,U,ND2)
390 U=UZ(M)
N=N+1
CALL MODIFY(N,U,ND2)
CONTINUE
C
RETURN
C
STOP
C
END
C
SUBROUTINE QUAD(N, VOL)
C
IMPLICIT REAL*8 (A-H, O-Z)
IMPLICIT INTEGER*2 (I-N)
COMMON STOP, HD(18), SIGR(25), SIGZ(25), GAMMA(25), ZKNOT(25),
1 DEPTH(25), E(1C, 25), SIG(7), R(450), Z(450), UR(450),
2 U2(450), ZTOTAL(450, 4), KSW
COMMON /INTEGR/ NUMNP, NUMEL, NUMMAT, NDEPTH, NORM, MTYPE, ICODE(450)
COMMON /ARG/ KR(5), ZZ(5), S(10, 10), P(10), LM(4), DD(3, 3),
1 HH(6, 10), RR(4), ZZ(4), C(4, 4), H(6, 10), C(6, 6), F(6, 10), TP(6), XI(6),
2 FE(10), IX(450, 5)
COMMON /BANARG/ B(900), A(900, 54), MBAND
C
I=IX(N, 1)
J=IX(N, 2)
K=IX(N, 3)
L=IX(N, 4)
C
I1=1
I2=2
I3=3
I4=4
I5=5
C
DETERMINE ELASTIC CONSTANTS AND STRESS-STRAIN RELATIONSHIP
C
CALL MPROP(N)
FORM QUADRILATERAL STIFFNESS MATRIX

210 RRR(5) = (R(I) + R(J) + R(K) + R(L))/4.0
Z77(5) = (Z(I) + Z(J) + Z(K) + Z(L))/4.0
DO 94 M = 1, N
MM = IX(N, M)
IF(R(MM) .EQ. 0.0 .AND. ICODE(MM) .EQ. 0) ICODE(MM) = 1
93 RRR(M) = R(MM)
94 Z77(M) = Z(MM)

DO 100 II = 1, 10
DO 95 JJ = 1, 6
95 HH(JJ, II) = 0.0
DO 100 JJ = 1, 10
560 S(II, JJ) = 0.0
100 IF (K - L) .LT. 125, 120, 125
120 CALL TRISTF(I1, I2, I3)
RRR(5) = (RRR(I) + RRR(J) + RRR(K) + RRR(L))/3.0
Z77(5) = (Z77(I) + Z77(J) + Z77(K) + Z77(L))/3.0
VOL = XI(I)
GO TO 160
125 VOL = 0.0
CALL TRISTF(I4, I1, I5)
IF(XI(I) .EQ. 0.0) WRITE(6, 2000) N
VOL = VOL + XI(I)
CALL TRISTF(I1, I2, I5)
IF(XI(I) .EQ. 0.0) WRITE(6, 2000) N
VOL = VOL + XI(I)
CALL TRISTF(I3, I4, I5)
IF(XI(I) .EQ. 0.0) WRITE(6, 2000) N
VOL = VOL + XI(I)
CALL TRISTF(I2, I3, I5)
IF(XI(I) .EQ. 0.0) WRITE(6, 2000) N
VOL = VOL + XI(I)
C C C
140 \text{RETURN FORMAT(15*ZEROA,ELEMENT(15))}
200 \text{RETURN FORMAT(15* ELEMENT(15))}

**INITIALIZATION**

\text{C C C}

\begin{align*}
\text{LM(1)} &= I1 \\
\text{LM(2)} &= JJ \\
\text{LM(3)} &= KK
\end{align*}

\begin{align*}
\text{RR(1)} &= \text{RR(I1)} \\
\text{RR(1)} &= \text{RR(JJ)} \\
\text{RR(1)} &= \text{RR(KK)}
\end{align*}
C

95 DO 100 I=1,6
   DO 90 J=1,10
      F(I,J)=0.0
90   H(I,J)=0.0
   DO 100 J=1,6
100  D(I,J)=0.0

C
FORM INTEGRAL (g)T*(c)*(g)
CALL INTR(XI,RR,ZZ)

C
D(2,6)=XI(1)*(C(1,2)+C(2,3))
D(3,5)=XI(1)*C(4,4)
D(5,5)=D(3,5)
D(6,6)=XI(1)*C(2,2)
D(1,1)=XI(3)*C(3,3)
D(1,2)=XI(2)*(C(1,3)+C(3,3))
D(1,3)=XI(5)*C(3,3)
D(1,6)=XI(2)*C(2,3)
D(2,7)=XI(1)*(C(1,1)+2.0*C(1,3)+C(3,3))
D(2,3)=XI(4)*C(1,3)+C(3,3))
D(3,3)=XI(6)*C(3,3)+XI(1)*C(4,4)
D(3,6)=XI(4)*C(2,3)
   DO 110 I=1,6
   DO 110 J=1,6
110  D(I,J)=D(I,J)

C
FORM COEFFICIENT-DISPLACEMENT MATRIX

C
CCM=RR(2)*(ZZ(3)-ZZ(1))+RR(1)*(ZZ(2)-ZZ(3))+RR(3)*(ZZ(1)-ZZ(2))
DD(1,1)=(RR(2)*ZZ(3)-RR(3)*ZZ(2))/CCM
DD(1,2)=(RR(3)*ZZ(1)-RR(1)*ZZ(3))/CCM
DD(1,3)=(RR(1)*ZZ(2)-RR(2)*ZZ(1))/CCM
DD(2,1)=(ZZ(2)-ZZ(3))/CCM
DD(2,2)=(ZZ(3)-ZZ(1))/CCM
DO 120 I=1,3
    J=2*LM(I)-1
    H(I,J)=DD(I,1)
    H(2,J)=DD(2,1)
    H(3,J)=DD(3,1)
    H(4,J+1)=DD(1,1)
    H(5,J+1)=DD(2,1)
    120 H(6,J+1)=DD(3,1)

C FORM STIFFNESS MATRIX \( H^T D H \)

DO 130 J=1,10
    IF (H(K,J)) 128,130,128
128 DO 129 I=1,6
    129 F(I,J)=F(I,J)+D(I,K)*H(K,J)
    130 CONTINUE

DO 140 I=1,10
    IF (H(K,I)) 138,140,138
138 DO 139 J=1,10
    139 S(I,J)=S(I,J)+H(K,I)*F(K,J)
    140 CONTINUE

C FORM STRAIN TRANSFORMATION MATRIX

DO 410 I=1,6
    DO 410 J=1,10
        HH(I,J)=HH(I,J)+H(I,J)
    410 CONTINUE
500 RETURN
FNC

SUBROUTINE MPRCP(N)
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
1 DEPTH(25),E(1C,25),SIG(7),R(450),Z(450),UR(450),
2 UZ(450),STOTAL(450,4),KSW
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NCEPTh,NORM,MTYPE,ICODE(450)
COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),C(6,6),F(6,10),TP(6),XI(6),
2 EE(10),IX(450,5)
COMMON /BANARG/ B(900),A(900,54),MBAND

I=IX(N,1)
J=IX(N,2)
K=IX(N,3)
L=IX(N,4)
MTYPE=IX(N,5)

DO 5 II=1,4
DO 5 JJ=1,4
5 C(II, JJ)=0.0

C DETERMINE ELASTIC CONSTANTS

C

40 DO 55 KK=1,2
55 EF(KK)=E(KK,MTYPE)
60 TF(NORM) 65,75,65
65 FF(1)=EE(1)*SIGIZ(MTYPE)

C FORM STRESS STRAIN RELATIONSHIP

C

75 COFF=EF(1)/(1.-EF(2)-2.*EE(2)*EF(2))
COMMON
SITC0, HEDO (1), SIGIR (25), SIG12 (25), GANMPA (25), ZKNCGL (25),

IMPLICIT INTEGER (1-N)
IMPLICIT REAL*8 (A-H-Z)

SUBROUTINE BANSL
END
RETURN
B(N) = 1.0
Q(N,N) = 0.0
Continue
250
IF (N-2) .LT. 0.0
220
PRINT 1, (K(N), N=1, N-2)
220
F(N-1) = F(N)
220
K = N-1

COMMON /BANAG5/ R(900), A(900, 54), W(MBAND)
IMPLICIT INTEGER(1-N)
IMPLICIT REAL*8 (A-H-Z)

SUBROUTINE MODIFY (N, U, ND2)
END
RETURN
C(4:4) = CDFE(4.5-EE(2))
C(3:3) = CDFE(3.1-EE(1))
C(2:2) = CDFE(2.1-EE(1))
C(1:1) = CDFE(1.1-EE(1))
C(0:0) = CDFE(0.1-EE(1))
C(1:1) = CDFE(1.0-EE(2))
C(2:2) = CDFE(2.0-EE(2))
C(3:3) = CDFE(3.0-EE(2))
C(4:4) = CDFE(4.0-EE(2))
1 DEPTH(25),E(1C,25),SIG(7),R(450),7(450),UR(450),
2 U7(450),STOTAL(450,4),KSW
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICCDE(450)
COMMON /BANARG/ R(500),A(900,54),MBAND
ND2=2*NUMNP

C
DO 280 N=1,ND2
DO 260 L=2,MBAND
C=A(N,L)/A(N,1)
I=N+L-1
C
IF (ND2.LT.1) GC TC 260
C
J=O
DO 250 K=L,MBAND
J=J+1
250 A(I,J)=A(I,J)-C*A(N,K)
B(I)=B(I)-C*B(N)
260 A(N,L)=C
280 B(N)=B(N)/A(N,1)
C
BACKSUBSTITUTION
C
N=ND2
300 N=N-1
C
IF (N.LE.0) GO TO 500
DO 400 K=2,MBAND
L=N+K-1
IF (ND2.LT.L) GO TO 400
B(N)=B(N)-A(N,K)*B(L)
400 CONTINUE
C
GO TO 300
C
500 RETURN
SUBROUTINE STRESS(SPLCT)
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON STTOP,HD0(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
1 DEPTH(25),E(1C,25),SIG(7),R(450),Z(450),UR(450),
2 UZ(450),STOTAL(450,4),KSW
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),E(6,6),F(6,10),TP(6),XI(6),
2 EE(10),IX(450,5)
COMMON /RANARG/ B(900),A(900,54),MRAND
C
C ************
C COMPUTE ELEMENT STRESSES AND STRAINS
C
DO 300 N=1,NUMEL
CALL QUAD(N,VOL)
C
C FIND ELEMENT COORDINATES
C
II=IX(N,1)
J1=IX(N,2)
K1=IX(N,3)
L1=IX(N,4)
C
IF (K1-L1,EQ,0) GO TO 50
RRR(5)=(R(II)+R(J1)+R(K1)+R(L1))/4.0
ZZZ(5)=(Z(II)+Z(J1)+Z(K1)+Z(L1))/4.0
GO TO 100
C
50 RRR(5)=(R(II)+R(J1)+R(K1))/3.0
ZZZ(5)=(Z(II)+Z(J1)+Z(K1))/3.0
C
C COMPUTE STRAINS
C
100 DO 120 I=1,4
II=2*I
FENT0577
FENT0578
FENT0579
FENT0580
FENT0581
FENT0582
FENT0583
FENT0584
FENT0585
FENT0586
FENT0587
FENT0588
FENT0589
FENT0590
FENT0591
FENT0592
FENT0593
FENT0594
FENT0595
FENT0596
FENT0597
FENT0598
FENT0599
FENT0600
FENT0601
FENT0602
FENT0603
FENT0604
FENT0605
FENT0606
FENT0607
FENT0608
FENT0609
FENT0610
FENT0611
FENT0612
C*******CALCULATE ROTATION OF PRINCIPAL PLANES*******

C****IF(DABS(SIG(4)) .LT. 1.0E-09) SIG(4)=0.0
C****IF(DABS(BB) .GT. 1.0E-09) GO TO 510
C****BB=0.0
C****IF (((SIG(4) .NE. 0.0) .OR. (BB .NE. 0.0)) GC TO 520
C****ANG=0.0
C****GO TO 530
C****ANG=DATAN2(SIG(4),BB)/2.0
C****SIG(8)=57.396*ANG
C****SIG(7)=((SIG(5)-SIG(6))/2.0
C**********OUTPUT STRESSES**********
C****IF(N .NE. 1) GO TO 615
C****WRITE (6,2000)
C****WRITE (6,2001) N,RRR(5),ZZZ(5),(SIG(I),I=1,4)
C****CONTINUE
C****FORMAT (8H1,ELEMENT,8X,'R',8X,'Z',6X,'SIG(R)',6X,'SIG(Z)',5X,'SIG(T')
C****RETURN
C****END
C****SUBROUTINE INTER(XI,RR,ZZ)
C****IMPLICIT REAL=A-H,C-Z
C****IMPLICIT INTEGER*2(I-J)
C****DIMENSION RR(1),ZZ(1),XI(1)
C****DIMENSION XX(7),RR(7),ZZ(7),XX(9)
C****XX(1)=.1259391805448
C****XX(2)=XX(1)
XX(3)=XX(1)
XX(4)=1323941527884
XX(5)=XX(4)
XX(6)=XX(4)
XX(7)=225
XX(8)=696140478028
XX(9)=410426152314
R(7)=(RR(1)+RR(2)+RR(3))/3.
Z(7)=(ZZ(1)+ZZ(2)+ZZ(3))/3.

C
50 100 I=1,3
J=I+3
R(I)=XX(8)*RR(I)+(1.-XX(8))*R(7)
R(J)=XX(9)*RR(I)+(1.-XX(9))*R(7)
Z(I)=XX(8)*ZZ(I)+(1.-XX(8))*Z(7)
100 Z(J)=XX(9)*ZZ(I)+(1.-XX(9))*Z(7)

C
50 200 I=1,7
200 XM(I)=XX(I)*R(I)

C
50 300 I=1,6
300 XI(I)=0.

C
AREA=5*(RR(1)*(ZZ(2)-ZZ(3))+RR(2)*(ZZ(3)-ZZ(1))+RR(3)*(ZZ(1)-ZZ(2))

C
50 400 I=1,7
XI(1)=XI(1)+XM(I)
XI(2)=XI(2)+XM(I)/R(I)
XI(3)=XI(3)+XM(I)/(R(I)**2)
XI(4)=XI(4)+XM(I)*Z(I)/R(I)
XI(5)=XI(5)+XM(I)*Z(I)/(R(I)**2)
400 XI(6)=XI(6)+XM(I)*(Z(I)**2)/(R(I)**2)

C
50 500 I=1,6
500 XI(I)=XI(I)*AREA
APPENDIX C

FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC
ELASTIC AXISYMMETRIC PLATES - THERMAL STRAINS INCLUDED (Ref. 13, 14)

Program Capabilities:

The following restrictions are placed on the size of problems which can be handled by the program.

<table>
<thead>
<tr>
<th>Item</th>
<th>Maximum Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodal Points</td>
<td>450</td>
</tr>
<tr>
<td>Elements</td>
<td>450</td>
</tr>
<tr>
<td>Materials</td>
<td>25</td>
</tr>
<tr>
<td>Boundary Pressure Cards</td>
<td>200</td>
</tr>
</tbody>
</table>

Printed output includes:

1. Reprint of Input Data
2. Nodal Point Displacements
3. Stresses at the center of each element.

Input Data Format:

A. Identification card - (18A4)
   Columns 1 to 72 of this card contain information to be printed with results.

B. Control card - (515,F10.0)
   Columns 1 - 5  Number of nodal points
   6 - 10  Number of elements
   11 - 15  Number of different materials
Normalizing stress (see NORM, Appendix B)
Number of boundary pressure cards
Reference temperature (stress free temperature)

C. Material Property information

The following group of cards must be supplied for each different material:

First Card - (215, 2F10.0)
Columns 1 - 5 Materials identification - any number from 1 to 12.
6 - 10 Number of different temperatures for which properties are given = 8 maximum.
11 - 20 Initial Z stress.
21 - 30 Initial R stress.

Following Cards - (4F10.0) One card for each temperature
Columns 1 - 10 Temperature
11 - 20 Modulus of elasticity - E
21 - 30 Poisson's ratio - v
31 - 40 Coefficient of thermal expansion

D. Nodal Point Cards - (215, 5F10.0)
One card for each nodal point with the following information:
Columns 1 - 5 Nodal point number
10 Number which indicates if displacements or forces are to be specified.
11 - 20 R - ordinate
21 - 30 Z - ordinate
31 - 40 XR
41 - 50 XZ
51 - 60 Temperature
If the number in column 10 is

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>free</td>
<td>XR is the specified R-load and XZ is the specified Z-load.</td>
</tr>
<tr>
<td></td>
<td>XR is the specified R-displacement and XZ is the specified Z-load.</td>
</tr>
<tr>
<td></td>
<td>XR is the specified R-load and XZ is the specified Z-displacement.</td>
</tr>
<tr>
<td>fixed</td>
<td>XR is the specified R-displacement and XZ is the specified Z-displacement.</td>
</tr>
</tbody>
</table>

All loads are considered to be total forces acting on a one radian segment. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. The necessary temperatures are determined by linear interpolation. The boundary code (column 10), XR and XZ are set equal to zero.

Skew Boundaries:

If the number in columns 5-10 of the nodal point cards is other than 0, 1, 2 or 3, it is interpreted as the magnitude of an angle in degrees. The terms in columns 31-50 of the nodal point card are then interpreted as follows:

- XR is the specified load in the s-direction
- XZ is the specified displacement in the n-direction

The angle must always be input as a negative angle and may range from -0.001 to -180 degrees. Hence, +1.0 degree is the same as -179.0 degrees. The displacements of these nodal points which are printed by the program are
\( u_r \) = the displacement in the s-direction
\( u_z \) = the displacement in the n-direction

E. Element Cards - (615)

One card for each element

<table>
<thead>
<tr>
<th>Columns</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 5</td>
<td>Element number</td>
</tr>
<tr>
<td>6 - 10</td>
<td>Nodal Point I</td>
</tr>
<tr>
<td>11 - 15</td>
<td>Nodal Point J</td>
</tr>
<tr>
<td>16 - 20</td>
<td>Nodal Point K</td>
</tr>
<tr>
<td>21 - 25</td>
<td>Nodal Point L</td>
</tr>
<tr>
<td>26 - 30</td>
<td>Material Identification</td>
</tr>
</tbody>
</table>

1. Order nodal points counter-clockwise around element.
2. Maximum difference between nodal point I.D. must be less than 25.

Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information by incrementing by one the preceding I, J, K and L. The material identification code for the generated cards is set equal to the value given on the last card. The last element card must always be supplied.

Triangular elements are also permissible; they are identified by repeating the last nodal point number (i.e., I, J, K, K).

F. Pressure Cards - (215, 1F10.0)

One card for each boundary element which is subjected to a normal pressure.

<table>
<thead>
<tr>
<th>Columns</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 5</td>
<td>Nodal Point I</td>
</tr>
<tr>
<td>6 - 10</td>
<td>Nodal Point J</td>
</tr>
<tr>
<td>11 - 20</td>
<td>Normal Pressure</td>
</tr>
</tbody>
</table>

The boundary element must be on the left as one progresses from I to J. Surface tensile force is input as a negative pressure.
Listing:

**FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC AXYSYMMETRIC PLATES REF FEAST 1,3 SAAS 2**

**IMPLICIT REAL*8 (A-H,O-Z)**
**IMPLICIT INTEGER*2(I-N)**
**COMMON STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNCT(25),**
**1 DEPTH(25),F(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),**
**2 UZ(450),STOTAL(450,4),**
**3 T(450),TFMP,Q,KSW**
**COMMON /INTEGER/ NUMNP,NUMEL,NUMMAT,NDPETH,NORM,MTYPE,ICODE(450)**
**COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),**
**1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),**
**2 EE(10),IX(450,5)**
**COMMON /BANARG/ R(900),A(900,54),MBAND**
**COMMON/PRESS/ JBC(200),JBC(200),PR(200),NUMPC**
**DATA STRS /*****/

READ (5,1000,END=950) HED
WRITE (6,2000) HED

READ(5,1001) NUMNP,NUMEL,NUMMAT,NORM,NUMPC,Q
WRITE(6,2006) NUMNP,NUMEL,NUMMAT,NUMPC,Q
IF (NORM) 65,65,66
WRITE (6,2041)

WRITE (6,2041)

READ(5,1012) MTYPE,NUMTC,SIGIZ(MTYPE),SIGIR(MTYPE)
WRITE (6,2111) MTYPE,NUMTC,SIGIZ(MTYPE),SIGIR(MTYPE)
N=0

120 WRITE (6, *016)

* READ AND PRINT ELEMENT POINT CRITERES *

* READ AND PRINT NODAL POINT DATA *

80 CONTINUE

81 IF (J+TYPE=ENUMTC. J+TYPE)

DO 81 J=1, I+NUMTC

WRITE (6, *2010) (EJ+J+TYPE(P), J=I, 4), I=1, I+NUMTC

READ (5, 0111) (EJ+J+TYPE(P), J=I, 4), I=1, I+NUMTC
130 READ (5,1CC7) M,(IX(M,I),I=1,5)
140 N=N+1
      IF (M-N) 170,170,150
150 IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
170 WRITE (6,?07) N,(IX(N,I),I=1,5)
      IF (M-N) 180,180,140
180 IF (NUMEL-N) 300,300,130
      READ AND PRINT THE PRESSURE CARDS
      ************************************************************
      300 IF (NUMP'C) 290,210,290
      290 WRITE (6,9000)
            DO 200 L=1,NUMPC
      READ(5,9001) IBC(L),JBC(L),PR(L)
      200 WRITE (6,9002) IBC(L),JBC(L),PR(L)
      CONTINUE
      ************************************************************
      DETERMINE BAND WIDTH
      ************************************************************
      J=0
      DO 340 N=1,NUMEL
            DO 325 I=1,4
            KK=IX(N,I)-IX(N,L)
            IF (KK.LT.0) KK=-KK
            IF (KK.GT.J) J=KK
      325 CONTINUE
      CONTINUE
      340 CONTINUE
      MBAND=2*J+2
      SOLVE FOR DISPLACEMENTS AND STRESSES
      ************************************************************
KSW=0
CALL STIFF
IF (KSW.NE.0) GO TO 900

CALL BANSDOL
WRITE (6,2052)
WRITE (6,2025) (N,B) (2*N-1),B (2*N),N=1,NUMNP)

450 CALL STRESS(SPLOT)

**

PROCSS ALL DECKS EVEN IF ERROR

**

GO TO 910

900 WRITE (6,4000)
910 WRITE (6,4001) HED

920 READ (5,1000) CHK
IF (CHK.NE,STRS) GO TO 920
GO TO 50

950 CONTINUE
WRITE (6,4002)
CALL EXIT

**

1000 FORMAT (18A4)
1001 FORMAT (5I5,F10.0)
1002 FORMAT (15,2F10.0)
1003 FORMAT (2F10.0)
1004 FORMAT (2F10.0)
1005 FORMAT (3F10.0)
1006 FORMAT (2I5,5F1C.0)
1007 FORMAT (6I5)
1011 FORMAT (4F10.0)
1012 FORMAT (2I5,2F1C.0)

**

2000 FORMAT (1H1,20A4)
2006 FORMAT (//,
1 30HO NUMBER OF NODAL POINTS------ I3 /
2 30HO NUMBER OF ELEMENTS------- I3 /
3 30HO NUMBER OF DIFF. MATERIALS--- I3 /
4 30HO NUMBER OF PRESSURE CARDS---- I3 /
5 30HO REFERENCE TEMPERATURE------ E12.4)
2010 FORMAT (15HO TEMPERATURE 15X 5HE 15X 6HNU 15X 6HALPHA 9X 1/4F20.8)
2011 FORMAT (17HOMATERIAL NUMBER= I3, 30H, NUMBER OF TEMPERATURE CARDS= I3, 25H INITIAL VERTICAL STRESS= F10.3, 5X,
2 27H INITIAL HORIZONTAL STRESS= F10.3)
2013 FORMAT (12HINODAL POINT ,4X, 4HTYPE ,4X, 10HR-ORDINATE ,4X,
1 10H7-ORDINATE ,10X, 6HR-LOAD ,10X, 6HZ-LOAD,10X, 4HTEMP )
2014 FORMAT (I12,18,2F14.3,2F16.5,F14.3)
2015 FORMAT (26HONOAL POINT CARD ERROR N= I5)
2016 FORMAT (49H1ELEMENT NO. I J K L MATERIAL )
2017 FORMAT (I113,4I6,1112)
2025 FORMAT (12HINODAL POINT ,6X, 14HR-DISPLACEMENT ,6X, 14HZ-DISPLACEMENT / (112,1P2D20.7))
2041 FORMAT (76HOMOCULUS AND YIELD STRESS NORMALIZED WITH RESPECT TO INITIAL VERTICAL STRESS )
2051 FORMAT (1HO,10X,'E',8X,'NU',/ ,3X,F11.1,F10.4/)
2052 FORMAT (1H1)
C  ******************************************************
3003 FORMAT (16I5)
C  ******************************************************
4000 FORMAT (/// ' ABNORMAL TERMINATION')
4001 FORMAT (/// ' END OF PROBLEM ' 20A4)
4002 FORMAT (/// ' END OF JCB')
C  ******************************************************
5000 FORMAT(29HOPRESSURE BOUNDARY CONDITIONS/ 24H I J PRESSU
1RE )
9001 FORMAT(2I5,F10.0)
9002 FORMAT(2I6,F12.3)
FND
SUBROUTINE STIFF
C
IMPLICIT REAL*8 (A-H,0-C-Z)
IMPLICIT INTEGER*2(I-N)
COMMON STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNT(25),
1 DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
2 UZ(450),STOTAL(450,4),
3 T(450),TEMP,Q,KSW
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPT,H,NORM,MTYPE,ICODE(450)
COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
1 HH(6,10),RR(4),ZZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
2 EE(10),IX(450,5)
COMMON /BARG/ B(900),A(900,54),MBAND
COMMON/PRESS/ IBC(200),JBC(200),PR(200),NUMPC
DIMENSION CODE(450)
C
INITIALIZATION
C
************************************************************************************
C
NB=27
ND=2*NB
ND2=2*NUMNP
DO 50 N=1,ND2
B(N)=0.0
DO 50 M=1,ND
50 A(M,N)=0.0
C
*************************************************************************************
C
FORM STIFFNESS MATRIX
C
*************************************************************************************
C
DO 210 N=1,NUMEL
C
90 CALL QUAD(N,VOL)
IF (VOL) 142,142,144
142 WRITE (6,2003) N
KSW=1
GO TO 210
C
144 IF (IX(N,3)-IX(N,4)) 145,165,145
145 DO 150 II=1,9
    CC=S(II,10)/S(10,10)
P(II)=P(II)-CC*P(10)
    DO 150 JJ=1,9
150 S(II,JJ)=S(II,JJ)-CC*S(10,10)
C
   DO 160 II=1,8
    CC=S(II,9)/S(9,9)
P(II)=P(II)-CC*P(9)
   DO 160 JJ=1,8
160 S(II,JJ)=S(II,JJ)-CC*S(9,9)
C
C   ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS
C
165 DO 166 I=1,4
166 LM(I)=2*IX(N,I)-2
C
   DO 200 I=1,4
   DO 200 K=1,2
   II=LM(I)+K
   KK=2*I-2+K
   P(II)=R(II)+P(KK)
   DO 200 J=1,4
   DO 200 L=1,2
   JJ=LM(J)+L-II+1
   LL=2*J-2+L
   IF (JJ) 200,200,175
175 IF (ND-JJ) 180,195,195
180 WRITE (6,2004) N
   KSW=1
   GO TO 210
195 A(II,JJ)=A(II,JJ)+S(KK,LL)
200 CONTINUE
210 CONTINUE
   IF(KSW.EQ.1) GO TO 500
29) SIN = DSIN(CODEL(J))  29) 291.292.293.
   IF(CODEL(J) 291.292.293.
   COFA=I+0
   290) SIN=COFA
   272) R(I,J)=RI(I,J)+R*(1+1)
   271) SIN=DSIN(CODEL(J))
   IF(CODEL(J) 271.)272.)273.
   COFA=1+0
   270) SIN=0
   J=J+1
   LI=L+1
   264) R*(0)/R*(1)
   Z=Z+1
   260) DO 300 L=I,NUMPC
   IF(NUANC=260.310.260
   PRESSURE BOUNDARY CONDITIONS
   C
   CONTINUE
   BK(K)=BK(K)+L*U(R(N))
   AK(K)=AK(K)+L*Z(U(N))
   N=N+1
   DO 250 N=1,NUMPD
   C
   ADD CONCENTRATED FORCES
   C
COSA=DCOS(CODE(I))
B(JJ-1)=B(JJ-1)+Z*X*(COSA*DZ+SINA*DR)
B(JJ)=B(JJ)-Z*X*(SINA*DZ-COSA*DR)
300 CONTINUE
310 CONTINUE
C DISPLACEMENT B*C.
C
DO 400 M=1,NUMNP
U=UR(M)
N=2*M-1
KX=ICODE(M)+1
GO TO (400,370,390,380),KX
370 CALL MODIFY(N,U,ND2)
GO TO 400
380 CALL MODIFY(N,U,ND2)
390 U=UZ(M)
N=N+1
CALL MODIFY(N,U,ND2)
400 CONTINUE
C
500 RETURN
C
*************************************************************************
2003 FORMAT (26HNEGATIVE AREA ELEMENT NO. I4)
2004 FORMAT (29HORANO WIDTH EXCEEDS ALLOWABLE I4)
C
*************************************************************************
END
SUBROUTINE QUAC(N,VOL)
C
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
1 DFPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
2 UZ(450),STOTAL(450,4),
3 T(450),TEM,Q,KS
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NCEPHT,NORM,MTYPE,ICODE(450)
COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DO(3,3),
1 HH(6,10), RR(4), Z(4), C(4,4), H(6,10), D(6,6), F(6,10), TP(6), XI(6),
2 EE(10), IX(450,5)

COMMON /BANARG/ R(900), A(900,54), MRAND

C *********************************************************
I=IX(N,1)
J=IX(N,2)
K=IX(N,3)
L=IX(N,4)
C
I1=1
I2=2
I3=3
I4=4
I5=5
C THERMAL STRESSES
TEM=((T(I)+T(J)+T(K)+T(L))/4.0
DO 103 M=7,8
IF(E(M,1,MTYPE)-TEMP) 103,104,104
103 CONTINUE
104 RATIO=0.0
DEN=E(M,1,MTYPE)-E(M-1,1,MTYPE)
IF(DEN) 70,71,70
70 RATIO=(TEMP-E(M-1,1,MTYPE))/DEN
71 DO 105 KK=1,3
105 FE(KK)=E(M-1,KK+1,MTYPE)+RATIO*(E(M,KK+1,MTYPE)-E(M-1,KK+1,MTYPE))
TEMP=TEMP-Q
C *********************************************************
C DETERMINE ELASTIC CONSTANTS AND STRESS-STRAIN RELATIONSHIP
C*********************************************************
C
CALL MPROP(N)
C
98 DO 110 M=1,3
110 TT(M)=(C(M,1)+C(M,2)+C(M,3))*EE(3)*TEMP
C *********************************************************
C FCRM QUADRILATERAL STIFFNESS MATRIX
```plaintext
C

210 RRR(5) = (R(1) + R(J) + R(K) + R(L)) / 4.0
ZZZ(5) = (Z(1) + Z(J) + Z(K) + Z(L)) / 4.0
DO 94 MM = 1, N
   MM = IX(N, MM)
   IF (R(MM) .EQ. 0.0 .AND. ICODE(MM) .EQ. 0) ICODE(MM) = 1
94  RRR(MM) = R(MM)
   ZZZ(MM) = Z(MM)
C
   DO 100 II = 1, 10
      P(II) = 0.0
   DO 95 JJ = 1, 6
      HH(JJ, II) = 0.0
   DO 100 JJ = 1, 10
100   S(II, JJ) = 0.0
   IF (K-L) 125, 120, 125
120   CALL TRISTF(I1, I2, I3)
      RRR(5) = (RRR(1) + RRR(2) + RRR(3)) / 3.0
      ZZZ(5) = (ZZZ(1) + ZZZ(2) + ZZZ(3)) / 3.0
      VOL = XI(1)
      GO TO 160
125   VOL = 0.0
      CALL TRISTF(I4, I1, I5)
      IF (XI(1) .EQ. 0.0) WRITE(6, 2000) N
      VOL = VOL + XI(1)
      CALL TRISTF(I1, I2, I5)
      IF (XI(1) .EQ. 0.0) WRITE(6, 2000) N
      VOL = VOL + XI(1)
      CALL TRISTF(I3, I4, I5)
      IF (XI(1) .EQ. 0.0) WRITE(6, 2000) N
      VOL = VOL + XI(1)
      CALL TRISTF(I2, I3, I5)
      IF (XI(1) .EQ. 0.0) WRITE(6, 2000) N
      VOL = VOL + XI(1)
C
   DO 140 II = 1, 6
```

DO 140 JJ=1,10
140 HH(I,JJ)=HH(I,JJ)/4.0
C
C
160 RETURN
C
****
2000 FORMAT ( ' ZERO AREA ELEMENT',I5)
END
SUBROUTINE TRISTF(IJ,JJ,KK)
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2 (I-N)
COMMON * STOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNCT(25),
1 DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
2 UZ(450),STOTAL(45C,4),
3 T(450),TEMP,Q,KSW
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NCEPHT,NORM,MTYPE,ICODE(450)
COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
2 EE(10),IX(450,5)
COMMON /BANARG/ R(900),A(900,54),MBAND
C
*--------------------------------------------------------*
C
INITIALIZATION
C
LM(1)=II
LM(2)=JJ
LM(3)=KK
C
RR(1)=RRR(IJ)
RR(2)=RRR(JJ)
RR(3)=RRR(KK)
RR(4)=RRR(IJ)
ZZ(1)=ZZZ(IJ)
ZZ(2)=ZZZ(JJ)
ZZ(3)=ZZZ(KK)
ZZ(4)=ZZZ(IJ)
C
85 DO 100 I=1,6
86 DO 90 J=1,10
87 F(I,J)=0.0
88 90 H(I,J)=0.0
89 100 D(I,J)=0.0
C
C FORM INTEGRAL G*T*(C)*(G)
C CALL INTER(XI,RR,ZZ)
C
C D(2,6)=XI(1)*(C(1,2)+C(2,3))
C D(3,5)=XI(1)*C(4,4)
C D(5,5)=D(3,5)
C D(6,6)=XI(1)*C(2,2)
C D(1,1)=XI(3)*C(3,3)
C D(1,2)=XI(2)*(C(1,3)+C(3,3))
C D(1,3)=XI(5)*C(3,3)
C D(1,6)=XI(2)*C(2,3)
C D(2,2)=XI(1)*(C(1,1)+2.0*C(1,3)+C(3,3))
C D(2,3)=XI(4)*C(3,3)
C D(2,6)=XI(4)*C(4,4)
C D(3,6)=XI(4)*C(2,3)
C DD 110 I=1,6
C DD 110 J=1,6
110 D(J,I)=D(I,J)
C
C FORM COEFFICIENT-DISPLACEMENT MATRIX
C
C COMM=RR(2)*(ZZ(3)-ZZ(1))+RR(1)*(ZZ(2)-ZZ(3))+RR(3)*(ZZ(1)-ZZ(2))
C DD(1,1)=(RR(2)*ZZ(3)-RR(3)*ZZ(2))/COMM
C DD(1,2)=(RR(3)*ZZ(1)-RR(1)*ZZ(3))/COMM
C DD(1,3)=(RR(1)*ZZ(2)-RR(2)*ZZ(1))/COMM
C DD(2,1)=(ZZ(2)-ZZ(3))/COMM
C DD(2,2)=(ZZ(3)-ZZ(1))/COMM
C DD(2,3)=(ZZ(1)-ZZ(2))/COMM
C
C FEWT0433
C FEWT0434
C FEWT0435
C FEWT0436
C FEWT0437
C FEWT0438
C FEWT0439
C FEWT0440
C FEWT0441
C FEWT0442
C FEWT0443
C FEWT0444
C FEWT0445
C FEWT0446
C FEWT0447
C FEWT0448
C FEWT0449
C FEWT0450
C FEWT0451
C FEWT0452
C FEWT0453
C FEWT0454
C FEWT0455
C FEWT0456
C FEWT0457
C FEWT0458
C FEWT0459
C FEWT0460
C FEWT0461
C FEWT0462
C FEWT0463
C FEWT0464
C FEWT0465
C FEWT0466
C FEWT0467
C FEWT0468
DD(3,1)=(RR(3)-RR(2))/COMM
DD(3,2)=(RR(1)-RR(3))/COMM
DD(3,3)=(RR(2)-RR(1))/COMM

C
DO 120 I=1,3
  J=2*LM(I)-1
  H(1,J)=DD(1,I)
  H(2,J)=DD(2,I)
  H(3,J)=DD(3,I)
  H(4,J+1)=DD(1,I)
  H(5,J+1)=DD(2,I)
120 H(6,J+1)=DD(3,I)
C
FORM STIFFNESS MATRIX (H)^T*(D)*(H)
C
DO 130 J=1,10
  DO 130 K=1,6
    TF(H(K,J))=128,130,128
128 DO 129 I=1,6
129 F(I,J)=F(I,J)+D(I,K)*H(K,J)
130 CONTINUE
C
DO 140 I=1,10
  DO 140 K=1,6
    TF(H(K,I))=138,140,138
138 DO 139 J=1,10
139 S(I,J)=S(I,J)+K(K,I)*F(K,J)
140 CONTINUE
  TP(1)=X(2)*T(3)
  TP(2)=X(1)*(T(1)+T(3))
  TP(3)=X(4)*T(3)
  TP(4)=0.0
  TP(5)=0.0
  TP(6)=X(1)*T(2)
DO 160 I=1,10
  DO 160 K=1,6
160 P(I) = P(I) + H(K, I) * TP(K)
C
C FORM STRAIN TRANSFORMATION MATRIX
C
DO 410 I = 1, 6
DO 410 J = 1, 10
410 HH(I, J) = HH(I, J) + H(I, J)
C
C 500 RETURN
END

SUBROUTINE MPROP(N)
IMPLICIT REAL*8 (A-H, C-Z)
IMPLICIT INTEGER*2 (I-N)
COMMON STTOP, HD(18), SIGIR(25), SIGIZ(25), GAMMA(25), ZKNOT(25),
DFPHT(25), E(8, 4, 25), SIG(7), R(450), Z(450), UR(450), TT3(3),
2 U(450), STD(450, 4),
3 T(450), TEMP, C, KSW
COMMON /INTEGR/, NUMLP, NUMML, NUMMAT, NDEPTH, NORM, MTYPE, ICODE(450)
COMMON /ARG/, RRR(5), ZZZ(5), S(10, 10), P(10), LMM(4), DD(3, 3),
1 HH(6, 10), RR(4), ZZ(4), C(4, 4), H(6, 10), D(6, 6), F(6, 10), TP(6), XT(6),
2 EE(10), IX(450, 5)
COMM I/BNARG/ B(900), A(900, 54), MBAND
C
I = IX(N, 1)
J = IX(N, 2)
K = IX(N, 3)
L = IX(N, 4)
MTYPE = IX(N, 5)
C
DO 5 II = 1, 4
DO 5 JJ = 1, 4
5 CII, JJ = 0, 0
C
C DETERMINE ELASTIC CONSTANTS
C
C
FEWT0505
FEWT0506
FEWT0507
FEWT0508
FEWT0509
FEWT0510
FEWT0511
FEWT0512
FEWT0513
FEWT0514
FEWT0515
FEWT0516
FEWT0517
FEWT0518
FEWT0519
FEWT0520
FEWT0521
FEWT0522
FEWT0523
FEWT0524
FEWT0525
FEWT0526
FEWT0527
FEWT0528
FEWT0529
FEWT0530
FEWT0531
FEWT0532
FEWT0533
FEWT0534
FEWT0535
FEWT0536
FEWT0537
FEWT0538
FEWT0539
FEWT0540
C
60 IF (NORM) 65,75,65
65 FE(1)=EE(1)*SIGIZ(MTYPE)

C
FORM STRESS STRAIN RELATIONSHIP

C
75 COFF=EE(1)/(1.-EE(2)-2.*EE(2)*EE(2))
   C(1,1)=COEF*(1.-EE(2))
   C(1,2)=COEF*EE(2)
   C(1,3)=EE(2)*COEF
   C(2,1)=C(1,2)
   C(2,2)=C(1,1)
   C(2,3)=C(1,2)
   C(3,1)=C(1,3)
   C(3,2)=C(1,2)
   C(3,3)=C(1,1)
   C(4,4)=COEF*(0.5-EE(2))
RETURN
END
SUBROUTINE MODIFY(N,U,ND2)

C
IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON /RANARG/ R(900),A(900,54),MBAND
DO 250 M=2,MBAND
   K=N-M+1
   IF (K) 235,235,230
   230 R(K)=B(K)-A(K,M)*U
   A(K,M)=0.0
235 K=N+M-1
   IF (NC2-K) 250,240,240
   240 B(K)=R(K)-A(N,M)*U
   A(N,M)=0.0
   CONTINUE
A(N,1)=1.0
A(N)=U
RETURN
END
SUBROUTINE BANSOL

IMPLICIT REAL*8 (A-H,O-Z)
IMPLICIT INTEGER*2 (I-N)
COMMON STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNCT(25),
1 DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
2 U7(450),USTOTAL(450,4),
3 T(450),TEMP,Q,KS
COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDDEPTH,NORM,MTYPE,ICODE(450)
COMMON /RANARG/ B(900),A(900,54),MBAND
ND2=2*NUMNP

DO 280 N=1,ND2
DO 260 L=2,MBAND
C=A(N,L)/A(N,1)
I=N+L-1

IF (ND2.LT.1) GO TO 260

J=0
DO 250 K=L,MBAND
J=J+1
250 A(I,J)=A(I,J)-C*A(N,K)
R(I)=R(I)-C*B(N)
260 A(N,L)=C
280 R(N)=R(N)/A(N,1)

C
BACKSUBSTITUTION

C
N=ND2
300 N=N-1

IF (N.LE.0) GO TO 500
DO 400 K=2,MBAND

FEWT0577
FEWT0578
FEWT0579
FEWT0580
FEWT0581
FEWT0582
FEWT0583
FEWT0584
FEWT0585
FEWT0586
FEWT0587
FEWT0588
FEWT0589
FEWT0590
FEWT0591
FEWT0592
FEWT0593
FEWT0594
FEWT0595
FEWT0596
FEWT0597
FEWT0598
FEWT0599
FEWT0600
FEWT0601
FEWT0602
FEWT0603
FEWT0604
FEWT0605
FEWT0606
FEWT0607
FEWT0608
FEWT0609
FEWT0610
FEWT0611
FEWT0612
L=N+K-1
IF (ND2.LT.L) GO TO 400
B(N)=B(N)-A(N,K)*R(L)
400 CONTINUE
C
GO TO 300
C
500 RETURN
END
SUBROUTINE STRESS(SPLT)
IMPLICIT REAL*8 (A-H, O-Z)
IMPLICIT INTEGER*2 (I-N)
COMMON STTOP, HED(18), SIGIR(25), SIGIZ(25), GAMMA(25), ZKNOT(25),
1 DEPTH(25), E(8425), SIG(7), R(450), Z(450), UR(450), TT(3),
2 UZ(450), STOTAL(450,4),
3 T(450), TEMP, Q, KSW
COMMON /INTEGR/ NUMNP, NUMEL, NUMMAT, NCEPHT, NORM, MTYPE, ICODE(450)
COMMON /ARG/ RRR(5), ZZ(5), S(10,10), P(10), LM(4), DD(3,3),
1 HH(6,10), RR(4), ZZ(4), C(4,4), H(6,10), D(6,6), F(6,10), TP(6), XI(6),
2 EE(10), IX(450,5)
COMMON /BANG/ R(900), A(900,54), MBAND
C
**************************************************************************
C COMPUTE ELEMENT STRESSES AND STRAINS
C
**************************************************************************
DO 300 N=1,NUMEL
CALL QUAD(N, VOL)
C
C FIND ELEMENT COORDINATES
C
I1=IX(N,1)
J1=IX(N,2)
K1=IX(N,3)
L1=IX(N,4)
C
IF (K1-L1.EQ.0) GO TO 50
RRR(5)=(R(I1)+R(J1)+R(K1)+R(L1))/4.
C
FEWT0613
FEWT0614
FEWT0615
FEWT0616
FEWT0617
FEWT0618
FEWT0619
FEWT0620
FEWT0621
FEWT0622
FEWT0623
FEWT0624
FEWT0625
FEWT0626
FEWT0627
FEWT0628
FEWT0629
FEWT0630
FEWT0631
FEWT0632
FEWT0633
FEWT0634
FEWT0635
FEWT0636
FEWT0637
FEWT0638
FEWT0639
FEWT0640
FEWT0641
FEWT0642
FEWT0643
FEWT0644
FEWT0645
FEWT0646
FEWT0647
FEWT0648
$ZZ(5) = (Z(I1) + Z(J1) + Z(K1) + Z(L1))/4.0$

GO TO 100

50 $RRR(5) = (R(I1) + R(J1) + R(K1))/3.0$

$7ZZ(5) = (Z(I1) + Z(J1) + Z(K1))/3.0$

C

COMPUTE STRAINS

C

100 DO 120 I=1,4

II=2*I

JJ=2*IX(N,I)

P(I1-I)=B(JJ-I)

120 P(I1)=B(JJ)

C

P(9)=0.0

P(10)=0.0

130 DO 150 I=1,2

RR(I)=P(I+8)

DO 150 K=1,8

150 RR(I)=RR(I) - S(I+8,K)*P(K)

C

COMM=S(9,9)*S(10,10) - S(9,10)*S(10,9)

IF (COMM) 155,160,155

155 P(9)=S(10,10)*RR(1) - S(9,10)*RR(2))/COMM

P(10)=(-S(10,9)*RR(1) + S(9,9)*RR(2))/COMM

C

160 DO 170 I=1,6

TP(I)=0.0

DO 170 K=1,10

170 TP(I)=TP(I) + HH(I,K)*P(K)

C

RR(1)=TP(2)

RR(2)=TP(6)

RR(3)=(TP(1)+TP(2)*RRR(5)+TP(3)*ZZZ(5))/RRR(5)

RR(4)=TP(3)+TP(5)

C

COMPUTE STRESSES
SUBROUTINE INTER(XI, RR, ZZ)
IMPLICIT REAL*8 (A-H, I-Z)
IMPLICIT INTEGER*2(I-N)
DIMENSION RR(1), ZZ(1), XI(1)
DIMENSION XM(7), R(7), Z(7), XX(9)
C
XX(1) = 1.759391805448
XX(2) = XX(1)
XX(3) = XX(1)
XX(4) = 1.323941527884
XX(5) = XX(4)
XX(6) = XX(4)
XX(7) = .225
XX(8) = 69619478028
XX(9) = 410426192314
R(7) = (RR(1) + RR(2) + RR(3))/3.
Z(7) = (ZZ(1) + ZZ(2) + ZZ(3))/3.
C
DO 100 I = 1, 3
   J = I + 3
   R(I) = XX(8) * RR(I) + (1.0 - XX(8)) * R(J)
   R(J) = XX(9) * RR(I) + (1.0 - XX(9)) * R(J)
   Z(I) = XX(8) * ZZ(I) + (1.0 - XX(8)) * Z(J)
100   Z(J) = XX(9) * ZZ(I) + (1.0 - XX(9)) * Z(J)
C
DO 200 I = 1, 7
   XM(I) = XX(I) * R(I)
C
DO 300 I = 1, 6
300   XI(I) = 0.
C
AREA = .5 * (RR(1) * (ZZ(2) - ZZ(3)) + RR(2) * (ZZ(3) - ZZ(1)) + RR(3) * (ZZ(1) - ZZ(2))
C
DO 400 I = 1, 7
END
APPENDIX D

STEADY STATE HEAT TRANSFER PROGRAM FOR BOLTED JOINT

Program Capacity: 50 nodal points

Output Data:
(a) Input data
(b) Inverse of matrix
(c) Nodal temperature
(d) Given and calculated augmenting vector and residual error

Input Data Sequence:
A. Case identification (12A4) followed by two blank cards
B. Card (II) with a 1
C. Card (I7) with dimension of matrix
D. Card (II) with a 1
E. Cards (II, 3(I3, E15.8)) with node indices started in the first I3 field followed by conductance between these nodes. Only input from lower node number to higher node number required (since the conductance from node i to j equals the conductance from j to i.) Each card has three groups of 3 node numbers followed by a conductance value except the last card. Last card could have 1, 2 or 3 groups and has a 1 in column 1.
F. Cards (II, 3(I6, E15.8)) with number of node followed by conductance from the node to ground node which is at specified temperature. Each card has 3 groups of node number followed by conductance. The II field is skipped except for the last card for ground conductances which can have 1, 2 or 3 fields and the first column has a 1. A
node can be connected to only one ground node.

G. Same as F above, but code temperature specified for ground node instead of the conductance value.

H. Same as F above, but code internal power dissipation for the particular node instead of the conductance value.
Listing:

C STEADY STATE HEAT TRANSFER PROGRAM BOLTED JOINT
DIMENSION IDENT(12), AA(050,050), AA(050,050), B(50), BI(50),
IBC(50), RES(50), ACON(50), TACCN(50), Q(50)
101 WRITE(6,23)
41 READ(5,51) K, IDENT
51 FORMAT(11,12A4)
   WRITE(6,111) IDENT
111 FORMAT(12A6)
   IF(K .NE. 1) GO TO 41
   READ(5,55) N,K
55 FORMAT(17/I1)
   M = N+1
   DO 3 I = 1,N
3   DO 3 J = 1,N
   AA(I,J) = 0.0
   ACON(I) = 0.
   Q(I) = 0.
   TACCN(I) = 0.
   CONTINUE
C READ IN COEFF. MATRIX ELEMENTS
42 READ(5,52) K,(I,J,AA(I,J),JM=1,3)
52 FORMAT(11,3(2I3,E15.8))
   IF(K .NE. 1) GO TO 42
43 READ(5,53) K,(I,ACON(I),JM=1,3)
   IF(K .NE. 1) GO TO 43
44 READ(5,53) K,(I,TACCN(I),JM=1,3)
   IF(K .NE. 1) GO TO 44
45 READ(5,53) K,(I,Q(I),JM=1,3)
   IF(K .NE. 1) GO TO 45
53 FORMAT(11,3(16,E15.8))
   DO 500 I=1,N
500 B(I) = -(Q(I) + ACON(I) * TACCN(I))
   DC 1000 I=1,N
   DC 1000 J=I,N
1000 AA(J,I) = AA(I,J)
   DC 3000 I=1,N
IN=I
AA(I, I) = 0.
DC 2001 J=1,N
IJ=J
IF (IJ .NE. IN) GO TO 2001
2000 AA(I, I) = AA(I, I) + AA(I, J)
2001 CONTINUE
3000 AA(I, I) = (AA(I, I) + AC(D(I))) * (-1.)
WRITE(6, 26)
26 FORMAT(1H1, 27X, 1HI, 12X, 1HQ, 15X, 1CHGRD, CCEND., 1CX, 1CHGRD, TEMP. //)
WRITE(6, 25) (I, Q(I), ACON(I), TACON(I), I=1, N)
25 FORMAT(1H1, 26X, 1I3, 7X, F10.5, 10X, F1C.5, 10X, F10.5, 10X)
MM = 1
DO 4 I=1, N
DO 4 J = 1, N
A(I, J) = AA(I, J)
4 CONTINUE
WRITE(6, 5)
5 FORMAT(1H1, 39X17HA = COEFF., MATRIX //
1 40X21HB = AUGMENTING VECTOR //
2 40X19HT = SOLUTION VECTOK //
3 40X16HAI = INVERSE OF A //
4 40X33HBC = AUGMENTING VECTOR CALCULATED //
5 40X21H( A ) * ( T ) = ( R ) ///)
DO 7 I = 1, N
WRITE(6, 6) ( I, J, A(I, J), J = 1, N )
6 FORMAT(1H1 / (4: 5H A(I3, 1H, I3, 2H) = F10.5, 5X))
DO 9 I = 1, N
BI(I) = A(I)
9 CONTINUE
CALL MAT(N, M, A, B)
WRITE(6, 22)
22 FORMAT(1H1)
C WRITE INVERSE MATRIX
DO 9 I = 1, N
9 WRITE(6, 10) ( I, J, A(I, J), J = 1, N )
10 FORMAT(1H / (4, 5H AI(I3, 1H, I3, 2H)=E15.8))
WRITE(6,23)
23 FORMAT(1H1)
WRITE SOLUTION VECTOR
12 WRITE(6,11)(J, B(J), J=1,N)
11 FORMAT(1H / 4(5H T(I3, 2H)=F10.5,9X))
DO 13 I = 1,N
BC(I) = J*J
DO 13 J = 1,N
BC(I) = BC(I) + (AI(I,J) * B(J))
13 CONTINUE
DO 15 J = 1,N
RES(J) = ABS(BI(J)) - ABS(BC(J))
15 CONTINUE
WRITE(6,16)
16 FORMAT(1H1,30X76H AUGMENTING VECTOR CALCULATE AUGMENTING VECTOR
RESIDUAL ERROR //)
WRITE (6,18) (J, I, BI(J), J, I, BC(J), RES(J), J=1,N)
18 FORMAT(25X4H B(I3,1H, I3,2H)=E15.8,2X4H BC(I3,1H,I3,2H)=E15.8,6XE15.8 /)
GO TO 101
END
SUBROUTINE MAT (N,M,A,B)
M = N + 1
N = SIZE OF MATRIX TO BE INVERTED
TO SOLVE AX = B, WHERE INPUT A = A, INPUT B = B
OUTPUT B= X, OUTPUT A = A INVERSE

DIMENSION A(50,50), B(50)
N1 = N - 1
TEMP 15 = A(1,1)
A(M,N) = 1.C / TEMP 15
B(M) = A(1,2) / TEMP 15
DO 1 I=2,N1
A(M,I-1) = A(1,I+1) / TEMP 15
1 CONTINUE
A(M,N1) = B(1) / TEMP 15
DO 10 I=1,N1
TEMP 6 = A(I+1,1)
B(I) = A(I+1,2) - TEMP 6 * B(M)
DO 5 J=2,N1
A(I,J-1) = A(I+1,J+1) - TEMP 6 * A(M,J-1)
5 CONTINUE
A(I,N1) = B(I+1) - TEMP 6 * A(M,N1)
A(I,N) = -TEMP6 / TEMP 15
10 CONTINUE
B(N) = B(M)
DO 15 I=1,N
A(N,I) = A(M,I)
15 CONTINUE
C REPEATS N - 1 TIMES
DO 1CO K=1,N1
TEMP 15 = B(I)
A(M,N) = 1.0 / TEMP 15
B(M) = A(1,1) / TEMP 15
DO 51 I=2,N
A(M,I-1) = A(1,1) / TEMP 15
51 CONTINUE
DO 60 I =1,N1
TEMP 6 = B(I+1)
B(I) = A(I+1,1) - TEMP 6 * B(M)
DO 55 J=2,N
A(I,J-1) = A(I+1,J) - TEMP 6 * A(M,J-1)
55 CONTINUE
A(I,N) = -TEMP 6 / TEMP 15
60 CONTINUE
B(N) = B(M)
DO 65 I=1,N
A(N,I) = A(M,I)
65 CONTINUE
C CONTINUE
100 CONTINUE
RETURN
END
### Table 1

Separation Radius Comparison - Single and Two Plate Models

*(see Figs. 12 - 17)*

<table>
<thead>
<tr>
<th>A/B</th>
<th>B/A</th>
<th>( R_o/A ) Single Plate Model</th>
<th>( R_o/A ) Two Plate Model</th>
<th>Percent Discrepancy Between Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.1</td>
<td>4.2</td>
<td>3.7</td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>3.3</td>
<td>2.7</td>
<td>22.2</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>2.7</td>
<td>2.1</td>
<td>28.6</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>2.4</td>
<td>1.7</td>
<td>41.7</td>
</tr>
<tr>
<td>.75</td>
<td>3.1</td>
<td>4.5</td>
<td>3.8</td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>3.6</td>
<td>2.8</td>
<td>28.9</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>3.0</td>
<td>2.2</td>
<td>36.4</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>2.7</td>
<td>2.0</td>
<td>35.0</td>
</tr>
<tr>
<td>.5</td>
<td>3.1</td>
<td>5.1</td>
<td>4.1</td>
<td>24.4</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>4.2</td>
<td>3.2</td>
<td>31.3</td>
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<td>1.6</td>
<td>3.6</td>
<td>2.8</td>
<td>28.6</td>
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<tr>
<td></td>
<td>1.3</td>
<td>3.3</td>
<td>2.5</td>
<td>32.0</td>
</tr>
</tbody>
</table>
TABLE 2

Test and Analytical Results for Radii of Separation of Bolted Plates (see Fig. 5)

<table>
<thead>
<tr>
<th>Case</th>
<th>D in.</th>
<th>2B in.</th>
<th>Separation Diameters, (2R_o) - in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>&quot;Rubbing Test&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Range</td>
</tr>
<tr>
<td>1</td>
<td>.065</td>
<td>.422</td>
<td>.42-.48</td>
</tr>
<tr>
<td>2</td>
<td>.124</td>
<td>.422</td>
<td>.50-.53</td>
</tr>
<tr>
<td>3</td>
<td>.191</td>
<td>.422</td>
<td>.58-.64</td>
</tr>
<tr>
<td>4</td>
<td>.253</td>
<td>.422</td>
<td>.70-.76</td>
</tr>
<tr>
<td>5. Unmatched Pair</td>
<td>.124/ .257</td>
<td>.422</td>
<td>.54-.58</td>
</tr>
<tr>
<td>6.</td>
<td>.124</td>
<td>1.0</td>
<td>1.06-1.10</td>
</tr>
<tr>
<td>7.</td>
<td>.191</td>
<td>1.0</td>
<td>1.11-1.17</td>
</tr>
</tbody>
</table>

% Discrepancy Between Computed Values and Tested Values

Rub. Test | Autorad. Test

- Original x-ray film shows hole in plate and 0.6 inch diameter zone more distinctly than remainder of area sensitized by the radioactive contamination. Loose radiographic contamination observed during test.

** Assembled and disassembled radioactive and non-radioactive plates without rotating plates relative to each other.
FIG. 1. BOLTED JOINT

FIG. 2. ROETSCHER's RULE OF THUMB FOR PRESSURE DISTRIBUTION IN A BOLTED JOINT
FIG. 3(d)

FIG. 3(e)
FIG. 3(f)

FIG. 3. FERNLUND'S SEQUENCE OF SUPERPOSITION
FIG. 4. FINITE ELEMENT IDEALIZATION OF TWO PLATES IN CONTACT

(a) Actual Plates

(b) Finite Element Idealization

(c) Single Annular Ring Element

Nodal Circle

Element

$X_c$
FIG. 5. FINITE ELEMENT MODELS

(a) Plates of Equal Thickness Under Load

(b) Finite Element Model for Plates of Equal Thickness

(c) Finite Element Model for Plates of Unequal Thickness
FIG. 6. EXAMPLES OF UNACCEPTABLE SOLUTIONS

(a) Plates Intersect, \( R_0 \) too small

(b) Contact Zone Sustains Tension, \( R_0 \) too large
FIG. 7. PLATE SPECIMEN, BOLT AND NUTS, FIXTURE AND TOOLS.
FIG. 8(a). FOOTPRINTS ON MATED PAIR OF 1/16 INCH PLATES.
FIG. 8(b). FOOTPRINTS ON MATED PAIR OF 1/8 INCH PLATES.
FIG. 8(c). FOOTPRINTS ON MATED PAIR OF 3/16 INCH PLATES.
FIG. 8(d). FOOTPRINTS ON MATED PAIR OF 1/4 INCH PLATES.
FIG. 8(e). FOOTPRINTS ON MATED PAIR OF 1/8 AND 1/4 INCH PLATES.

FIG. 8. FOOTPRINTS ON THE MATING SURFACES OF 1/16 - 1/16, 1/8 - 1/8, 3/16 - 3/16, 1/4 - 1/4, and 1/8 - 1/4 PAIRS. (A = .128, B = .21)
FIG. 9. FOOTPRINT OF NUT ON PLATE.
FIG. 10 (a). 1/16 INCH PAIR

FIG. 10(b). 1/8 INCH PAIR
FIG. 10. X-RAY PHOTOGRAPHS OF CONTAMINATION TRANSFERRED FROM RADIOACTIVE PLATE TO MATED PLATE. 1/16, 1/4, 3/16, 1/4 INCH PAIRS. (A = .128 in., B = .21 in.)
FIG. 11. FREE BODY DIAGRAM FOR TWO PLATES IN CONTACT.
Fig. 12. Single plate analysis - midplane stress.
FIG. 13. SINGLE PLATE ANALYSIS-MIDPLANE $\sigma_z$ STRESS DISTRIBUTION (D = 0.133 in.)

- $\nu = 0.3$
- $A = 0.1$ in.
- $A/D = 0.75$
- $C = 1.54$ in.
FIG. 16. INTERFACE PRESSURE DISTRIBUTION IN A BOLTED JOINT

\( \frac{D}{D} \) = 0.3
\( A = 0.1 \text{ in.} \)
\( A/D = 0.75 \)
\( C = 1.54 \text{ in.} \)

\( r/A \) vs. \( \frac{d}{D} \)
FIG. 17. INTERFACE PRESSURE DISTRIBUTION IN A BOLTED JOINT

(D = 0.2 in.)
FIG. 18. FINITE ELEMENT ANALYSIS RESULTS FOR 1/4 INCH PLATE PAIR.
FIG. 19. PRESSURE IN JOINT, TRIANGULAR LOADING
FIG. 20. VARIATIONS OF LOADING AND BOUNDARY CONDITIONS.
FIG. 21. PRESSURE IN JOINT, UNIFORM DISPLACEMENT UNDER NUT.

\[ \nu = 0.305 \]
\[ D = 0.253 \text{ in.} \]
\[ A = 0.1285 \text{ in.} \]
\[ B = 0.211 \text{ in.} \]

50 MICROINCH DISPLACEMENT BETWEEN RADI Us A AND B.
Fig. 22. Deflection of plate under nut.

- Deflection under nut:
  \[ v = 0.305 \]
  \[ D = 0.253 \text{ in.} \]
  \[ A = 0.1285 \text{ in.} \]
  \[ B = 0.211 \text{ in.} \]

- Total load: 879\#.

- Deflection of free surface beyond nut, \( z = D, r > B \).

- Peak external pressure at \( A \), triangular distribution.

- Uniform pressure.

- Peak external pressure at \( B \).
FIG. 25. GAP DEFORMATION FOR FREE AND FIXED EDGES — FINITE ELEMENT ANALYSIS, 1/8 INCH PLATE PAIR.
FIG. 27. FINITE ELEMENT ANALYSIS RESULTS FOR 1/8 INCH PLATE MATED WITH 1/4 INCH PLATE.
FIG. 28. COMPARISON BETWEEN TESTED AND MEASURED SEPARATION RADIi.