ALLMAT: A TSS/360 FORTRAN IV SUBROUTINE FOR EIGENVALUES AND EIGENVECTORS OF A GENERAL COMPLEX MATRIX

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Subroutine ALLMAT is described and listed. ALLMAT computes the eigenvalues and eigenvectors of a general (non-Hermitian) complex matrix. The program uses the complex QR algorithm to compute eigenvalues and inverse iteration to compute eigenvectors. The user has the option of computing only the eigenvalues, if desired. An entry point EVDATA is available to provide the user with timing and accuracy information, as well as the number of iterations necessary for each eigenvalue and eigenvector.
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SUMMARY

A subroutine is described and listed that computes the eigenvalues and eigenvectors of a general (non-Hermitian) complex matrix. The program, ALLMAT, uses the complex QR algorithm to compute eigenvalues and inverse iteration to compute eigenvectors. The user has the option of computing only the eigenvalues, if desired. An entry point EVDATA is available to provide the user with timing and accuracy information, as well as the number of iterations necessary for each eigenvalue and eigenvector.

INTRODUCTION

Many areas of physics, mathematics, statistics, and engineering require the eigenvalues and eigenvectors of square matrices. This area of numerical analysis, sometimes called the algebraic eigenvalue problem, holds a place that is as important as the more familiar areas such as numerical integration, curve fitting, and numerical integration of differential equations. A general library of subroutines for a computer installation is commonly limited to one such program, and quite often this one subroutine is of only limited applicability.

The ideal subroutine for the algebraic eigenvalue problem should have many features: it should be fast and accurate; it should give a matrix of eigenvectors that are linearly independent; it should be capable of computing only eigenvalues at a corresponding increase in speed; it should have minimal storage requirements; and it must be able to treat all matrices, real or complex, symmetric or nonsymmetric, regardless of the condition of the matrix. Unfortunately, probably no such procedure exists. The usefulness of any subroutine may be judged on the basis of how many of these criteria are fulfilled, as balanced against the needs of the individual user.
There are many different techniques for diagonalizing a square matrix. The treatise by Wilkinson (ref. 1) is evidence for this. Reference 1 describes the state-of-the-art for the algebraic eigenvalue problem as of 1965. Typically, one chooses a particular method because he believes that his matrix has some feature that requires special handling, because a subroutine is conveniently at hand, or because he knows only that one method. Two of the most commonly used procedures are the power method and the Jacobi transformation.

The power method is a special-purpose procedure that computes the largest eigenvalue of a matrix by the formation of a sequence of powers of the matrix acting upon an arbitrary vector. This procedure is useful for the computation of a few eigenvalues (the largest in magnitude) and their eigenvectors. The computation of a full set of eigenvalues and eigenvectors is both time consuming and inaccurate.

The Jacobi transformation, as it applies to a complex Hermitian matrix, consists of a sequence of unitary transformations that diagonalize $2 \times 2$ submatrices of the full matrix. This procedure generates the eigenvectors along with the eigenvalues and is particularly useful when the eigenvectors are required to be orthogonal to a high degree of accuracy. The limitations of the Jacobi transformation are that the accuracy of the eigenvectors is usually limited, and as yet no extension to nonsymmetric or non-Hermitian matrices has been made. The most common computer library subroutine for the algebraic eigenvalue problem is a real-symmetric version of the Jacobi method (ref. 2).

For a general (i.e., nonsymmetric or non-Hermitian) matrix, two procedures have been derived to compute the eigenvalues and eigenvectors, respectively. The input matrix is reduced to a Hessenberg form (ref. 1), and the QR transformation of Francis (refs. 3 and 4) is used to compute the eigenvalues. With a knowledge of the eigenvalues, the Wielandt inverse iteration method (ref. 1) generates the eigenvectors. The QR transformation and inverse iteration appear to be the best currently available for their respective tasks (ref. 1) in terms of accuracy and speed. This combined procedure has been coded at the Oak Ridge National Laboratory (ref. 5) for an IBM 360/50 using the H-level FORTRAN compiler and COMPLEX*16 arithmetic. This program was used as the basis for the subroutine to be described in this report.

In order to make the subroutine as general as possible, some modifications and additions were made to the ORNL program, as follows. The fact that for some matrices the Hessenberg form may be decomposed into disjoint submatrices is incorporated in both the QR transformation and the inverse iteration to reduce computational time. A perturbation method is used to obtain linearly independent eigenvectors when eigenvalues are either degenerate or very nearly the same value (meaning that the matrix itself may be ill-conditioned (ref. 1)). An auxiliary entry point is provided to give the user in-
formation about the number of iterations required, timing data (measured as central processor or CPU time elapsed in the computation), and error data for the resulting eigenvalues and eigenvectors. Finally, a flag has been provided to allow the user to compute only the eigenvalues, with the use of the QR transformation. The relative contribution made by the present work is seen from the observation that approximately 60 percent of the coding of the current form of the subroutine ALLMAT is the ORNL coding while the remaining 40 percent is new.

The end result of the work described here is a subroutine for the IBM/360 to compute the eigenvalues and eigenvectors of a square matrix. Certainly, this is not meant to be the final work in such procedures, the algebraic eigenvalue problem is an area of extensive research in numerical analysis. On the other hand, this subroutine does satisfy most of the criteria mentioned earlier for the ideal subroutine, at least to some degree. The criterion that is least satisfied is minimal storage. Because ALLMAT is written with COMPLEX*16 arithmetic and has some large scratch-pad arrays, the subroutine uses a large amount of storage. On a TSS/360 system this storage requirement is not a basic limitation on the subroutine, but it does imply that the CPU time is affected.

This brief mention of storage requirements is an opportunity to interpose a slight warning to the prospective user of ALLMAT. If only a small number of the (largest) eigenvalues of a matrix are desired, the power method is more efficient than ALLMAT. For a real, symmetric matrix, a problem that requires eigenvectors along with the eigenvalues would be better suited to a real Jacobi subroutine. On the other hand, for the computation of eigenvalues alone, or for the eigenvalues and eigenvectors of a real, nonsymmetric matrix or for a complex matrix, ALLMAT seems to be the best choice, at this time.

The next section of this report describes schematically the construction of the subroutine ALLMAT. This includes the information necessary for a programmer to use ALLMAT. Also included are brief descriptions of the mathematical procedures used in ALLMAT. The following section discusses the special features that have been incorporated in ALLMAT, including a description of the subsidiary ENTRY EVDATA that provides timing and accuracy information for the user. Finally, a number of test matrices are used as examples for ALLMAT. These examples give an indication of running times and accuracy obtainable with the program, even with some ill-conditioned input matrices. A FORTRAN listing of ALLMAT is given in the appendix.

This report is intended to be used as a user's manual for the subroutine ALLMAT, and as such it described the call vector for the subroutine and the rules for usage. In addition, enough information is provided the prospective user to allow an intelligent application of this program to his particular problem. The prospective user should not
apply this program to his problem without some understanding of the numerical methods involved and of the construction of the subroutine.

GENERAL CONSTRUCTION

Usage

The information to be discussed in this section is aimed at explaining the program as a FORTRAN subroutine, along with a description of the ENTRY EVDATA.

The user's access is through the statement (see the appendix for the complete FORTRAN listing of the subroutine):

```
CALL ALLMAT (AA, LAMBDA, M, MM, EVECT, NCAL)
```

where

- **AA**
  - input COMPLEX*16 matrix, of dimension $M \leq MM$. Upon return from ALLMAT, $i^{th}$ column of AA is $i^{th}$ eigenvector, corresponding to $i^{th}$ eigenvalue.

- **LAMBDA**
  - COMPLEX*16 vector of length $M$ that contains eigenvalues upon return from ALLMAT.

- **M**
  - actual dimension of input matrix AA.

- **MM**
  - dimension of AA as it appears in a dimension statement in the calling program. MM is the upper bound for the size of matrices used. As ALLMAT is currently written, MM must be no greater than 50.

- **EVECT**
  - a logical switch. If EVECT = .TRUE., the eigenvectors of AA are calculated, and returned in the matrix AA. If EVECT = .FALSE., no eigenvectors are calculated and AA contains no useful information upon return from ALLMAT.

- **NCAL**
  - number of eigenvalues successfully computed by ALLMAT. If NCAL < $M$ some attempts of the QR transformation did not converge within 10 iterations. The value of the element of LAMBDA that corresponds to this eigenvalue has been set to zero by ALLMAT.

In addition to the primary entry point, a secondary ENTRY EVDATA is available to give the user information on the CPU time taken for the eigenvalue and eigenvector procedures. Also available are the number of QR iterations required for each eigen-
value, the number of inverse iterations required for each eigenvector, and the Euclidean norms of the residual vectors. A more complete description of these quantities is given later. The usage for this optional entry point is

\begin{verbatim}
CALL EVDATA (ITS,KTS,NCO,MCO,RNORM)
\end{verbatim}

where

\begin{itemize}
  \item **ITS** elapsed time for QR transformation for eigenvalues, including time to reduce to upper Hessenberg form. ITS is an integer, in microminutes.
  \item **KTS** elapsed time for inverse iteration for eigenvectors. Does not include time represented by ITS. Also an integer in microminutes.
  \item **NCO** an integer vector of dimension MM that has as its \(i\)th element the number of QR iterations for the \(i\)th eigenvalue. NCO \((i) = 10\). If NCO \((i) = 0\), this eigenvalue was obtained along with another, no separate QR iteration was required. If NCO \((i) < 0\), no convergence was obtained for this eigenvalue within ten QR iterations.
  \item **MCO** integer vector of dimension MM that has as its \(i\)th element the number of inverse iterations necessary to obtain the \(i\)th eigenvector. MCO \((i) \leq 10\).
  \item **RNORM** REAL*8 vector of the norms of the residual vectors of \(AA\). See section SPECIAL FEATURES OF ALLMAT for a more complete description. RNORM also has a dimension MM.
\end{itemize}

As an example of the usage of ALLMAT, consider a 6x6 complex matrix \(AA\) that is to be diagonalized. Let us assume that the TYPE statement in the calling program that specifies the dimensions of \(AA\) and \(LAMBDA\) has the form

\begin{verbatim}
COMPLEX*16 AA (10,10), LAMBDA(10)
\end{verbatim}

The arrays have been overdimensioned for more generality. Let us further assume that eigenvectors are desired from ALLMAT, so that EVECT has been assigned a value .TRUE.. Then the call to ALLMAT is

\begin{verbatim}
CALL ALLMAT (AA,LAMBDA,6,10,EVECT,NCAL)
\end{verbatim}

Upon return from ALLMAT the integer variable NCAL contains the number of eigenvalues that have been successfully computed by ALLMAT. The \(i\)th column of AA (I.E.
AA (1,1) to AA (6,1)) contains the \( i^{th} \) eigenvector, corresponding to the eigenvalue \( \Lambda (1) \).

If the timing and error information provided by EVDATA are desired by the user, then the statement

\[
\text{CALL EVDATA (ITS, KTS, NCO, MCO, RNORM)}
\]

is used, where NCO, MCO, and RNORM have been dimensioned at least six in the calling program. The conversion from ITS or KTS (in microminutes) to milliseconds is obtained by multiplying either integer by 0.06 and assigning the result to a floating-point variable.

**QR TRANSFORMATION**

The basis of the QR transformation is a theorem by Francis that states any non-singular matrix \( A \) has a unique decomposition into the product of a unitary matrix \( Q \) and an upper triangular matrix \( R \) (ref. 3), or

\[
A = QR
\]

The QR algorithm consists of forming a sequence of matrices similar to \( A = A(1) \) such that

\[
A(K) = Q(K) R(K)
\]

and then

\[
A(K+1) = R(K) Q(K)
\]

where \( A(K) \) is the form of the matrix after the \( K^{th} \) decomposition. Francis (ref. 3) shows that this sequence of matrices has as its limit an upper triangular matrix, the diagonal elements of which are the eigenvalues of the original matrix \( A \). Furthermore, even if the original matrix is singular, the algorithm still gives convergence to a unique triangular matrix, even though some of the intermediate \( Q \) and \( R \) may not be unique.

A full description of the QR transformation is certainly not relevant to this report. A detailed discussion of the convergence properties and the error analysis of the QR algorithm is given in references 1, 3, and 4. It is sufficient to note for our purpose...
that the QR algorithm is an extremely stable, rapidly converging procedure to calculate the eigenvalues of a general matrix (ref. 1). The version of the QR transformation that is part of ALLMAT, one that includes origin shifts to accelerate convergence, is powerful enough to satisfy nearly all of the needs of the average user.

There is one unusual feature of the standard way in which the QR algorithm is employed that the prospective user should be aware of. A preliminary step in any implementation of the QR transformation is the reduction of the input matrix to Hessenberg form. An upper Hessenberg form (i.e., \( A_{ij} = 0 \) if \( i > j + 1 \)) is used in ALLMAT. The reduction is accomplished by a sequence of elementary transformations (ref. 1). The elements of these elementary transformations are stored in the unused portion of \( A \) (the lower subtriangle of \( A \)) and in the integer vector JNT. This information is used at the end of the inverse iteration to recover the eigenvectors of the original matrix from the eigenvectors of the Hessenberg matrix. The point of caution for the user is that the working matrix for the subroutine is the Hessenberg form, which in general bears no simple relation to the input matrix. Thus, if the user attempts to debug this subroutine at an intermediate stage, the relation between the Hessenberg form and the original form must be kept in mind.

The advantage of using the Hessenberg form is apparent in the time needed to complete the computation of the eigenvalues. Most methods that operate on the entire input matrix, such as the Jacobi method, require a number of operations that is approximately \( 30N^3 \) (ref. 2), where \( N \) is the order of the matrix. The reduction to Hessenberg form is a one-pass operation and requires \( \alpha N^3 \) operations, where \( \alpha \) is of order unity. The QR algorithm applied to the Hessenberg form only requires something of the order of \( N^2 \) operations. One interesting result of this is the observation (ref. 5) that under many conditions the QR transform produces eigenvalues in less time than the Jacobi transformation.

**Inverse Iteration**

The basis of the inverse iteration procedure is the observation that, if \( \lambda \) is an eigenvalue of the matrix \( A \), the quantity \( (A - \lambda I) \), where \( I \) is the unit matrix, will be singular. Thus, if \( \lambda \) is a good approximation to an eigenvalue of \( A \), the matrix \( (A - \lambda I)^{-1} \) may be iterated to obtain an approximation \( Y \) to the eigenvector \( X \). The iteration process is carried out until after the \( K^{th} \) iteration the norm of the iterated vector, \( (A - \lambda I)^{-1} Y_K \), is greater than some preselected value (see the appendix). This procedure is equivalent to the power method, but in inverse powers of the matrix \( (A - \lambda I) \). The speed with which this iteration produces an eigenvector depends on the accuracy of the estimate for the eigenvalue, but rarely does this procedure, combined
with the QR algorithm, require more than 2 iterations to produce eigenvectors to at least six or seven place accuracy. Again, the interested user is referred to Wilkinson (ref. 1) for a complete description of the method and the error analysis.

SPECIAL FEATURES OF ALLMAT

As mentioned in the introduction, the basic elements of ALLMAT, the reduction to Hessenberg form, the QR transformation, and the inverse iteration, are taken from an ORNL subroutine (ref. 5). There are several features that have been added to this basic program to either add effectiveness to the program or provide timing and accuracy information to the user. These special features will be discussed in this section, more or less in the order that they appear in the program.

Decomposed Hessenberg Form

The reduction of the original matrix to Hessenberg form is a procedure that decreases the number of operations necessary for the QR algorithm. In a large number of cases the nature of the Hessenberg form allows further simplifications. To illustrate this, sketch (a) shows an upper Hessenberg matrix, of order N. The X's in the sketch

\[
\begin{pmatrix}
\times & \times & \times & \times & \cdots & \cdots \\
\times & \times & \times & \times & \cdots & \cdots \\
0 & \times & \times & \times & \times & \cdots \\
0 & 0 & \times & \times & \times & \times \\
\cdots \\
0
\end{pmatrix}
\]

(a)
indicate matrix elements, generally nonzero, whose values are unimportant. Now let one of the subdiagonal elements vanish, for example $A(R, R-1) = 0$. Then the Hessenberg matrix may be decomposed into four submatrices as shown in sketch (b).

![Sketch of Hessenberg matrix decomposition](image)

The submatrices $B$ and $D$ are upper Hessenberg matrices of order $R-1$ and $N-R+1$, respectively. Submatrix $C$ is a nonzero matrix with $N-R+1$ columns and $R-1$ rows. The remaining submatrix of this partition of $A$ is entirely filled with zeroes.

The result of this decomposition is that the problem of finding the eigenvalues of $B$ and $D$ becomes entirely disjoint; that is, the eigenvalues of $B$ and $D$, collectively, are the eigenvalues of $A$. The submatrix $C$ plays no part in the eigenvalue problem. Thus, instead of the solution of a single matrix of order $N$, the problem has been reduced to the solution of two matrices, of order $N-R+1$ and $R-1$. Since $N^2 > (N-R+1)^2 + (R-1)^2$ for $N \geq 3$ and $R$ that is not trivial, this decomposition implies a significant reduction in the total number of operations in the QR transforma-
tion. For many input matrices, particularly those matrices that are sparse, a number of such decompositions may be performed and the gain in machine time is important.

This decomposition is not as important for the calculation of the eigenvectors, although some improvement is made. The eigenvectors corresponding to eigenvalues of D (see sketch (b)) depend upon the submatrices B and C, so that the entire matrix must be used in the inverse iteration procedure. The eigenvectors corresponding to the eigenvalues of B, on the other hand, do not require matrices C or D, so that only B is used in the inverse iteration. Thus, some advantage is gained from the decomposition for the calculation of the eigenvectors. On the whole, though, the main advantage of the decomposition enters in the QR transformation.

Perturbation of Close Eigenvalues

One difficulty with the inverse iteration method arises when two or more eigenvalues are very nearly the same. Since every calculated eigenvalue differs from the "true" eigenvalue by an amount that depends on many factors, these eigenvalues may not produce linearly independent eigenvectors. The way chosen to resolve this accidental degeneracy was to perturb each successive close eigenvalue by an amount small enough to not disturb the convergence of the iterative procedure, but large enough to resolve the eigenvectors into linearly independent vectors (ref. 1). The choice of the perturbation, EPSIL, is arbitrary and a better choice could be made for particular types of matrices.

Since the existence of close but distinct eigenvalues implies that the matrix may be ill-conditioned (ref. 1), the accuracy of the calculated eigenvectors will be in doubt. In this sense, the use of a perturbation to separate the eigenvalues is an attempt to recover some useful information from a badly posed problem. Thus, for most matrices encountered, the existence of close but distinct eigenvalues should be rare. The occurrence of multiple eigenvalues is more common.

Multiple Eigenvalues

The existence of a set of multiple eigenvalue is a not uncommon occurrence in physical problems. The existence of such a set implies that there is a subspace of eigenvectors that one desires the basis vectors of. In this situation the perturbation is of some help. If, by the process of perturbing the degenerate eigenvalues within the inverse iteration process, one can obtain a set of distinct eigenvectors, even if they are not linearly independent, then there is a standard solution to the problem of determining
the basis vectors. For this purpose ALLMAT takes the set of distinct eigenvectors produced by the perturbation technique just discussed and uses a Gram-Schmidt (ref. 1) orthogonalization procedure to give a set of linearly independent eigenvectors. Since the Gram-Schmidt process involves taking the differences of nearly equal numbers in many cases, the accuracy of such a procedure is less than the accuracy of an inverse iteration vector for a distinct eigenvalue. Again, however, this represents an attempt to salvage as much information as one can from an undesirable situation. In practice, as shall be seen in the section TESTS, the results of this perturbation and orthogonalization procedure are good.

**ENTRY EVDATA**

The remaining special feature of ALLMAT is represented by the secondary entry point, EVDATA, as discussed in general construction. A typical user of an installation-supplied mathematical subroutine is usually blissfully unaware of any error considerations for his problem. Since the accuracy of any matrix eigenvalue evaluation strongly depends upon the properties of the input matrix, ignoring error information is equivalent to shutting one's eyes to avoid an oncoming truck. Additionally, since some eigenvalues and eigenvectors may in fact be absent due to nonconvergence either in QR or inverse iteration, the information provided by EVDATA is important to a user. The use of the TSS/FORTRAN multiple-data set capability means that this information is readily available to the user, without so much as the disturbance of an artistic output format.

The information available in EVDATA includes the number of iterations, the CPU time elapsed for the eigenvalue and the eigenvector computations, and an error estimate for each eigenvalue-eigenvector pair. The timing and counting variable provided in EVDATA were discussed sufficiently under usage, but the error information requires some further comment.

If \( \lambda \) and \( X \) are an exact eigenvalue and an exact eigenvector of the matrix \( A \), then the vector \( AX - \lambda X \) will be identically zero. Since neither \( \lambda \) nor \( X \) can ever be computed exactly, this vector \( (AX - \lambda X) \), called the residual vector, will be nonzero. The magnitude of this vector is then a measure of the error in \( \lambda \) and \( X \). The length of a vector, as used in ALLMAT, is the Euclidean norm, \( ||X|| = (\sum |x(i)|^2)^{1/2} \). The vector RNORM of EVDATA contains the norm of the residual vector for each eigenvalue-eigenvector pair, scaled to the Euclidean norm of the input matrix.

The data entry point EVDATA may be used even if no eigenvectors are computed (i.e., if EVECT = .FALSE.) In this case only ITS and NCO contain meaningful values.
TESTS

Seven matrices were chosen as examples for ALLMAT. The dimensions of these matrices vary from four to 19. All but two matrices are real but not symmetric, one of the remaining matrices is Hermitian, and the final example matrix is complex, but not Hermitian. Some of these matrices were chosen to illustrate ill-conditioning of one type or another. Since the numerical values of the eigenvectors are not of general use, they are not displayed.

Matrix 1

\[
A_1 = \begin{pmatrix}
0.1 & -0.7 & -0.4 & -0.5 \\
-0.5 & 0.2 & -0.1 & -0.2 \\
0.4 & 0.5 & 0.5 & 0.7 \\
0.1 & 0.2 & 0.5 & 0.4 \\
\end{pmatrix}
\]

This real, but unsymmetric, matrix of order four has the exact eigenvalues 0.9, 0.6, -0.3, and 0. In addition, the computed eigenvalue corresponding to 0. is 0.14E-16.

The information available from EVDATA on this test includes:

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Number of QR iterations</th>
<th>Number of inverse iterations</th>
<th>RNORM</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.3</td>
<td>8</td>
<td>3</td>
<td>0.65E-16</td>
</tr>
<tr>
<td>.14E-16</td>
<td>1</td>
<td>3</td>
<td>.43E-16</td>
</tr>
<tr>
<td>.6</td>
<td>1</td>
<td>3</td>
<td>.16E-16</td>
</tr>
<tr>
<td>.9</td>
<td>1</td>
<td>3</td>
<td>.91E-16</td>
</tr>
</tbody>
</table>

The total time for the QR transformation, including the initial reduction to Hessenberg form was 0.053 second, and the time for the inverse iteration was 0.046 second.
This matrix has eigenvectors identical to those of matrix 1, but has a different set of eigenvalues. The exact eigenvalues of $A_2$ are given in the following table:

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Number of QR iterations</th>
<th>Number of inverse iterations</th>
<th>RNORM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>2</td>
<td>3</td>
<td>0.68E-16</td>
</tr>
<tr>
<td>.75000075</td>
<td>4</td>
<td>3</td>
<td>.10E-15</td>
</tr>
<tr>
<td>.75</td>
<td>0</td>
<td>3</td>
<td>.46E-14</td>
</tr>
<tr>
<td>-.75</td>
<td>1</td>
<td>3</td>
<td>.64E-15</td>
</tr>
</tbody>
</table>

The closeness of the second and third eigenvalues hint at some error problems with the eigenvectors. The time for QR transformation was 0.020 second, and the time for the inverse iteration was 0.090 second. The difficulties anticipated from the closeness of the eigenvalues are evidenced in the degradation of the third value of RNORM in the table.
This matrix $A_3$ is an example of an ill conditioned matrix, in contrast with the previous example. Here, $A_2$ had two nearly alike eigenvalues even though the matrix is not mathematically ill conditioned (ref. 1).

\[
\begin{array}{cccc}
\text{Eigenvalue} & \text{Number of QR iterations} & \text{Number of inverse iterations} & \text{RNORM} \\
0.01 & 3 & 3 & 0.25E-16 \\
.01001 & 3 & 3 & .14E-16 \\
.1 & 3 & 3 & .18E-16 \\
.05 & 1 & 3 & .23E-16 \\
\end{array}
\]

The time for QR transformation was 0.038 second; that for inverse iteration was 0.025 second. Apparently, the ill conditioning did not effect the inverse iterations, as all values of RNORM are satisfactory.

Matrix 4

\[
A_4 = \begin{pmatrix}
6. & 5. & 4. & 3. & 2. & 1. \\
2. & 3. & 2. & -2. & 4. & 3. \\
3. & 1. & -3. & -1. & 5. & 5. \\
4. & -4. & 2. & 0. & 1. & 4. \\
\end{pmatrix}
\]

Unlike the first three test matrices, $A_4$ has a pair of complex eigenvalues.

\[
\begin{array}{cccc}
\text{Eigenvalue} & \text{Number of QR iterations} & \text{Number of inverse iterations} & \text{RNORM} \\
3.0929 & 6 & 3 & 0.78E-16 \\
.1772+.95E-16i & 6 & 3 & .24E-15 \\
.42295+4.3954i & 5 & 3 & .11E-15 \\
.42295-4.3954i & 4 & 3 & .17E-15 \\
15.247+.11E-14i & 1 & 3 & .56E-15 \\
-7.3630 & 1 & 3 & .47E-15 \\
\end{array}
\]
The time for QR transformation was 0.142 second; that for inverse iteration was 0.314 second. The imaginary part of the sum of the eigenvalues (which should be 0.) is 0.355E-14.

Matrix 5

This test matrix is a 19 by 19 real, unsymmetric matrix given by Francis (ref. 4) to demonstrate the QR transformation. The matrix is too complicated to list here, but the error information is informative. The time to produce the eigenvalues was 2 seconds, and the time to calculate the 19 eigenvectors was 22 seconds. Although this time is large when compared with the previous examples, it is quite reasonable when compared with other methods (ref. 5). Even with a matrix of this order, the residual vectors all had norms less than 1.E-16.

Matrix 6

This matrix has several features that make it useful as an example. Each nonzero element of A6 is a purely imaginary number and, in addition, A6 is Hermitian. Thus, the eigenvalues of A6 are real and, since the trace of A6 vanishes, the eigenvalues occur in positive-negative pairs. There is a pair of degenerate eigenvalues with the value 0, so that the orthogonalization procedure must be used to obtain the eigenvectors. Finally, A6 is sufficiently sparse that the decomposition of the Hessenberg form is effective in reducing the time required for the computations.
<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Number of QR iterations</th>
<th>Number of inverse iterations</th>
<th>RNORM</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.91376103</td>
<td>3</td>
<td>3</td>
<td>0.26E-16</td>
</tr>
<tr>
<td>.91376103</td>
<td>3</td>
<td>3</td>
<td>.16E-16</td>
</tr>
<tr>
<td>-1.03872417</td>
<td>7</td>
<td>3</td>
<td>.31E-15</td>
</tr>
<tr>
<td>-.38452612</td>
<td>6</td>
<td>3</td>
<td>.96E-16</td>
</tr>
<tr>
<td>.38452612</td>
<td>5</td>
<td>3</td>
<td>.55E-15</td>
</tr>
<tr>
<td>1.03872417</td>
<td>4</td>
<td>3</td>
<td>.74E-15</td>
</tr>
<tr>
<td>-1.5371192</td>
<td>1</td>
<td>3</td>
<td>.15E-14</td>
</tr>
<tr>
<td>1.5371192</td>
<td>1</td>
<td>3</td>
<td>.21E-14</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

The time for QR transformation was 1.07 second; that for inverse iteration was 1.50 second. The eigenvalues appear in this table in the order in which they are calculated by the QR algorithm. Since the reduction to Hessenberg form and the use of the decomposed Hessenberg form rearrange the matrix, the eigenvalues are not computed in pairs, necessarily. This same effect caused the QR routine to take three iterations to compute a zero eigenvalue. The degenerate eigenvalues caused no loss of accuracy in the computation of the eigenvectors. Furthermore, the sum of the eigenvalues is purely imaginary, and has the magnitude 0.4E-14, reflecting the zero trace of \( A_6 \).

Matrix 7

The final example matrix was generated from matrix \( A_4 \) by taking each element of this 6 by 6 real, nonsymmetric matrix and multiplying by the imaginary unit \( i \). The result, \( A_7 \), is a complex non-Hermitian matrix whose eigenvalues are the eigenvalues

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Number of QR iterations</th>
<th>Number of inverse iterations</th>
<th>RNORM</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0929i</td>
<td>6</td>
<td>3</td>
<td>0.77E-16</td>
</tr>
<tr>
<td>.95E-16+.1772i</td>
<td>6</td>
<td>3</td>
<td>.28E-15</td>
</tr>
<tr>
<td>4.3954+.42295i</td>
<td>5</td>
<td>3</td>
<td>.10E-15</td>
</tr>
<tr>
<td>-4.3954+.42295i</td>
<td>4</td>
<td>3</td>
<td>.16E-15</td>
</tr>
<tr>
<td>.44E-15-7.3630i</td>
<td>1</td>
<td>3</td>
<td>.48E-15</td>
</tr>
<tr>
<td>15.247i</td>
<td>1</td>
<td>3</td>
<td>.52E-15</td>
</tr>
</tbody>
</table>
of $A_4$ multiplied by $i$. The QR transformation time was 0.140 second, and the inverse iteration time was 0.319 second.

A comparison of the results indicated in the preceding table with the results for example $A_4$ shows that ALLMAT handles the non-Hermitian form with comparable speed, at no loss of accuracy in the eigenvalues and eigenvectors. The sum of the eigenvalues is $0.31E-14+12.i$.

These seven examples were chosen to be representative of the application of ALLMAT. Some matrices ($A_1$, $A_4$, $A_5$, and $A_7$) pose no particular problems, while the remaining ($A_2$, $A_3$, and $A_6$) were included to demonstrate one of more special characteristics of the program. It is seen from the results given above that ALLMAT had no difficulty with any of these test matrices. The norm of the residual vectors is typically less than $1.E-15$, and all computed eigenvalues that were also known exactly were in agreement to at least 14 places. At no point did either the QR algorithm or the inverse iteration fail to give convergence within the allotted limit of 10 iterations. In fact, only once did the inverse iteration procedure require more than three iterations to satisfy the convergence criterion.

**CONCLUDING REMARKS**

This report is intended to be a user's guide for the prospective user of ALLMAT. The information presented here about the construction of ALLMAT should be considered a minimum for the use of this matrix eigenvector program. No program of the complexity of ALLMAT should be used without some understanding of the basic algorithms involved. Certainly, though, most users will apply ALLMAT without consideration of even the simplified discussion presented here. For these users the entry point EVDATA should be required usage as an indicator when ALLMAT does fail on a matrix.

Experience has shown that two inverse iterations are usually enough to give an eigenvector correct to sufficient accuracy. The current version of ALLMAT, however, iterates until the norm of the iterated vector, $(A - \lambda I)^{-1}X$, is greater than $1.E40$. If computing time is at a premium, this criterion can be easily changed to a test on the number of iterations. The current limitation on ALLMAT is to matrices of dimension no larger than 50. This restriction may also be changed easily.

ALLMAT was designed to be a general purpose matrix eigenvalue and eigenvector subroutine. Almost any matrix, including the most general case of a complex, non-Hermitian matrix, is amenable to diagonalization by ALLMAT. Furthermore, timing
test (ref. 5) indicate that the QR transform may be preferred to the Jacobi method for the eigenvalues of real and symmetric matrices.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 16, 1970,
129-02.
APPENDIX - FORTRAN LISTING

SUBROUTINE ALLMAT (AA, LAMBDA, M, MM, EVECT, NCAL)
  IMPLICIT REAL*8 (A-H, O-Z)
  COMPLEX*16 AA(MM, MM)

  IF THE USER REQUIRES DIMENSION LARGER THAN 50, THE
  DIMENSIONS IN THE 2 LINES FOLLOWING THIS COMMENT MUST
  BE CHANGED FROM 50 TO A SIZE TO SUIT THE USER.

  COMPLEX*16 A(50, 50), H(50, 50), HL(50, 50), LAMBDA(MM)
  COMPLEX*16 VECT(50), MULT(50), SHIFT(3), TEMP, SINT, COST, TEMP1, TEMP2
  COMPLEX*16 EIG, CONJ1
  LOGICAL EVECT, INTH(50)
  DIMENSION NCOUNT(50), MCOUNT(50)
  INTEGER JNT(50), R, RP1, RP2
  DO 1000 J = 1, M
  DO 1000 I = 1, M

  A(I, J) = AA(I, J)
  CALL CPU_TIM(ITIM)
  ITSUM = 0
  KTSUM = 0
  CONJ1 = (0., -1.)

  THE CONSTANT EPSIL DETERMINES THE CONVERGENCE OF THE
  QR ALGORITHM, AND ALSO IS THE PERTURBATION PARAMETER
  FOR THE INVERSE ITERATION.
  EPSIL = 1.0D-12

  THE CONSTANT EPSMAX DETERMINES THE CONVERGENCE OF THE
  INVERSE ITERATION. THIS NUMBER IS THE LOGARITHM OF
  NORM OF THE ITERATED EIGENVECTOR THAT IS SUFFICIENT
  FOR CONVERGENCE.
  EPSMAX = 40.
  NSTOP = M
  N = NSTOP
  NSTART = 1
  MN1 = 1
  NCAL = 0
  IF (N .NE. 1) GO TO 1
  LAMBDA(1) = A(1, 1)
  A(1, 1) = 1.0
  GO TO 92
1 ICOUNT = 1
SHIFT(1) = 0.
1 IF (N.NE.2) GOTO 4
2 NSPI = NSTART + 1
3 TEMP = (A(NSTART,NSTART)+A(NSPI,NSPI)+CDSQRT = 
4 1((A(NSTART,NSTART)+A(NSPI,NSPI))**2-4.*A(NSPI, - 
5 A(NSPI,A(NSTART,NSTART)-A(NSPI,NSTART))*A(NSTART, - 
6 A(NSPI)))**2).
0005200 RELTEM = TEMP
0005300 AMGTEM = CONJ*TEMP
0005400 IF (RELTEM.NE.0..OR.AMGTEM.NE.0.) GOTO 3
0005500 LAMBDA(MN1) = A(NSTART,NSTART)+A(NSPI,NSPI)+SHIFT(1)
0005600 LAMC(NSTOP) = ICOUNT
0005700 NCOUNT(NSTOP) = ICOUNT
0005800 NCOUNT(MN1) = ICOUNT
0005900 GO TO 37
0006000 3 LAMBDA(NSTOP) = TEMP + SHIFT(1)
0006100 LAMBDA(MN1)=(A(NSTART,NSTART)+A(NSPI,NSPI) - - 
0006200 1A(NSPI,NSTART)*A(NSTART,NSPI))/(LAMBDA(NSTOP) - 
0006300 2-SHIFT(1))+SHIFT(1)
0006400 NCOUNT(NSTOP) = ICOUNT
0006500 NCOUNT(MN1) = ICOUNT
0006600 ICOUNT = 1
0006700 GO TO 37
0006800C REDUCE MATRIX A TO HESSENBERG FORM.
0006900C
0007100 4 NM2 = N-2
0007200 DO 15 R=1,NM2
0007300 RP1 = R+1
0007400 RP2 = R+2
0007500 ABIG = 0.
0007600 JNT(R) = RP1
0007700 DO 5 I=RP1,N
0007800 RELAIR = A(I,R)
0007900 AMGAIR = CONJ*A(I,R)
0008000 ABSSQ = RELAIR**2 + AMGAIR**2
0008100 IF (ABSSQ.LE.ABIC) GOTO 5
0008200 JNT(R) = 1
0008300 ABIG = ABSSQ
0008400 5 CONTINUE
0008500 INTER = JNT(R)
0008600 IF (ABIG.EQ.0.) GOTO 15
0008700 IF (INTER.EQ.RP1) GOTO 8
0008800 DO 6 I=RP1,N
0008900 TEMP = A(RP1,I)
0009000 A(RP1,I) = A(INTER,I)
0009100 6 A(INTER,I) = TEMP
0009200 DO 7 I=1,N
0009300 TEMP = A(I,RP1)
0009400 A(I,RP1) = A(I,INTER)
0009500 7 A(I,INTER) = TEMP
0009600 DO 8 I=RP2,N
0009700 MULT(I) = A(I,R)/A(RP1,R)
0009800 8 A(I,R) = MULT(I)
0009900 DO 11 I=1,RP1

20
TEMP = 0.
DO 10 J=RP2,N
10 TEMP = TEMP + A(I,J)*MULT(J)
A(I,RP1) = A(I,RP1) + TEMP
DO 13 I=RP2,N
13 TEMP = 0.
DO 12 J=RP2,N
12 TEMP = TEMP + A(I,J)*MULT(J)
A(I,RP1) = A(I,RP1) + TEMP*MULT(I)*A(RP1,RP1)
DO 14 I=RP2,N
14 TEMP = 0.
DO 12 J=RP2,N
12 TEMP = TEMP + A(I,J)*MULT(J)
A(I,RP1) = A(I,RP1) + TEMP - PI*U*T(I)*A(RP1,RP1)
DO 14 I=RP2,N
14 A(I,J) = A(I,J) - MULT(1)*A(RP1,J)
CONTINUE
CALCULATE EPSILON.
EPS = 0.
DO 16 I=1,N
16 EPS = EPS + CDABS(A(I,I))
DO 18 I=2,N
18 SUM = 0.
IM1 = I - 1
DO 17 J=IM1,N
17 EPS = EPS + CDABS(A(I,J))
18 SUM = SUM + CDABS(A(I,J))
IF(SUM.GT.EPS) EPS=SUM
EPS = SQRT(FLOAT(N))*EPS/D20
IF (EPS. EQ. 0.) EPS=l.D-20
EPS
SAVE THE HESSENBERG FORM IN THE ARRAY H.
20 DO 19 I=1,N
19 R(I,J) = A(I,J)
NSM1 = NSTOP - 1
IF (NSM1.NE.0) GOTO 100
R = 1
100 CONTINUE
R = 1
102 NSTART = R
NSTART AND NSTOP ARE THE INDICES OF THE BEGINNING AND END OF A DECOMPOSED HESSENBERG BLOCK.
NS = NSTOP - NSTART + 1
NC = NS
MNI = NSTOP + NSTART - N
103 IF (NS.NE.1) GOTO 21
LAMBDA(MNI) = A(NSTART,NSTART) + SHIFT(1)
NCOUNT(MNI) = 1
GO TO 37
21 IF (NS.EQ.2) GOTO 2
22 RELANN = A(N,N)
AMGANN = CONJ*I*A(N,N)
RLNM1 = A(N,N-1)
AMNNM1 = CONJ*I*A(N,N-1)
RLDNI1 = A(N,N-1)/A(N,N)
AMDNI1 = CONJ*I*(A(N,N-1)/A(N,N))
IF (RELANN.NE.0. .OR. AMGANN.NE.0.) -
111 IF (NABS(RLNDNI1) + NABS(AMDDNI1) - 1,0 - 18) 24,24,23
23 IF (NABS(RLNNM1) + NABS(AMNNM1).GE.EPS) GOTO 25
24 LAMBDA(MNI) = A(N,N) + SHIFT(1)
NCOUNT(MNI) = 1
ICOUNT = 1
N = N - 1
NS = NS - 1,
MN1 = MN1 + 1
GO TO 21
DETERM = DETERM
SHIFT(2) = 2*(A(N-1,N-1)+A(N,N)+CDSQRT((A(N-1, -
1N-1)+A(N,N)*SQR(A(N,N)*A(N-1,N-1)-A(N,N-1) -
2*A(N-1,N))/2).
RLSHE = SHIFT(2)
AMGSHF = CONJ*SHIFT(2)
IF (RLSHE.NE.0. .OR. AMGSHF.NE.0.) GOTO 26
SHIFT(3) = A(N-1,N-1)+A(N,N)
GO TO 27
26 SHIFT(3) = (A(N,N)*A(N-1,N-1)-A(N,N-1)*A(N,N))/SHIFT(2)
27 IF (ABS1FT(2)-SHIFT(2)-A(N,N)).LT.0ABS(SHIFT(3) -
1-A(N,N)) GO TO 28
INDEX = 3
GO TO 29
INDEX = 2
IF (ABS1FT(2)-A(N,N)).GE.EPS) GOTO 30
LAMBDA(MNI) = SHIFT(2) + SHIFT(1)
LAMBDA(MNI+1) = SHIFT(3) + SHIFT(1)
NCOUNT(MNI) = 1
NCOUNT(MNI+1) = 0
ICOUNT = 1
N = N - 2
NS = NS - 2
MN1 = MN1 + 2
GO TO 103
SHIFT(1) = SHIFT(1) + SHIFT(INDEX)
DO 31 I=NSTART,N
31 A(I,I) = A(I,I) - SHIFT(INDEX)
PERFORM GIVENS ROTATIONS, OR ITERATES.
IF (ICOUNT .LE. 10) GOTO 32
NCOUNT(MNP) = -ICOUNT
NC = NC - NS
GO TO 37

32 NM1 = N - 1
TEMP1 = A(NSTART,NSTART)
TEMP2 = A(NSTART+1,NSTART)
DO 36 R=NSTART,NM1
NN = R
RP1 = R + 1
RELM1 = TEMP1
AMGTM1 = CONJG*TEMP1
RELM2 = TEMP2
AMGTM2 = CONJG*TEMP2
RHO = DSQRT(RELM1**2+AMGTM1**2+RELM2**2+AMGTM2**2)
IF (RHO.EQ.0.) GOTO 36
COST = TEMP1/RHO
SINT = TEMP2/RHO
DO 33 I=INDEX,N
TEMP = CONJG(COST)*A(NN,I)+CONJG(SINT)*A(RP1,I)
A(RP1,I) = -SINT*A(NN,I)+COST*A(RP1,I)
A(NN,I) = TEMP
33 A(NN,I) = TEMP
36 CONTINUE
ICOUNT = ICOUNT + 1
GO TO 22

CALCULATE VECTORS.

37 IF (.NOT.EVECT) GOTO 64
CALL CPUTIM(JTLM)
LTSUM = LTSUM + (JTIF4 - ITIM)
IF (NC.EQ.0) GOTO 64
NPNCAL = NSTART + NC - 1
N = NSTOP
NS = NSTOP - NSTART + 1
N = NSTOP
NS = NSTOP - NSTART + 1
NM1 = N - 1
IF (N .NE. 2) GOTO 38
EPS = QMAX1(CBABS(LAMBDA(1)),CBABS(LAMBDA(2)))*1.D-16
IF (EPS.EQ.0.) EPS=EPSIL
H(1,1) = A(1,1)
H(2,1) = A(2,1)
H(1,2) = A(1,2)
H(2,2) = A(2,2)
L=NSTART,NPNCAL
ABIG = 0.
EIG = LAMBDA(L)
IF (L.EQ.NSTART) GOTO 40
LM1 = L - 1
RELEIG = EIG
AMGEIG = CONJ*EIG
DO 39 I=NSTART,LM1
RELAM1 = LAMBDA(I)
AMGAM1 = CONJ*LAMBDA(I)
IF (DABS(RELEIG-RELAM1).GT.EPSIL) GOTO 39
IF (DABS(AMGEIG-AMGAM1).GT.EPSIL) GOTO 39
EIG = EIG + CONJ*EPSIL
39 CONTINUE
DO 42 I=1,N
DO 41 J=1,N
HL(J,I) = H(J,I)
HL(I,I) = HL(I,I) - EIG
DO 46 I=1,NM1
MULT(I) = 0.
INTH(I) = .FALSE.
IP1 = I + 1
IF (CDABS(HL(I+1,I)).LE.CDABS(HL(I,I))) GO TO 44
INTH(I) = .TRUE.
DO 43 J=I,N
TEMP = HL(I+1,J)
HL(I+1,J) = HL(I,J)
HL(I,J) = TEMP
43
HL(I,I) = TEMP
44 RELH!! = HL(I,I)
AMGHII = CONJ*HL(I,I)
IF (RELHII.EQ.0..AND.AMGHII.EQ.0.) GOTO 46
MULT(I) = -HL(I+1,I)/HL(I,I)
DO 45 J=IP1,N
MULT(I) = -HL(I+1,I)/HL(I,I) + MULT(I)*HL(I,J)
45
CONTINUE
DO 48 I=1,N
VECT(I) = 1.
48
DO 51 I=1,NM1
K = N-I
IF (RELHNN.EQ.0..AND.AMGHNN.EQ.0.) HL(N,N)=EPS
VECT(N) = VECT(N)/HL(N,N)
DO 50 I=1,NM1
K = N-I
DO 52 J=K,NM1
50 VECT(K) = VECT(K) - HL(K,J+1)*VECT(J+1)
51 VECT(K) = VECT(K)/HL(K,K)
BIG = 0.
DO 52 I=1,N
RELVEC = VECT(I)
AMGVEC = CONJ*VECT(I)
110 ICOUNT = 1
DO 49 I=1,N
49 RELHNN = HL(N,N)
AMGHNN = CONJ*HL(N,N)
IF (RELHNN.EQ.0..AND.AMGHNN.EQ.0.) HL(N,N)=EPS
VECT(N) = VECT(N)/HL(N,N)
K = N-I
DO 50 J=K,NM1
50 VECT(K) = VECT(K) - HL(K,J+1)*VECT(J+1)
51 VECT(K) = VECT(K)/HL(K,K)
AMGHKK = CONJ*HL(K,K)
IF (RELHKK.EQ.0..AND.AMGHKK.EQ.0.) HL(K,K)=EPS
51
0032400  SUM = DABS(RELVEC)+DABS(AMGVEC)
0032500  IF (SUM.LE.BIG) GOTO 52
0032600  BIG = SUM
0032700  II = 1
0032800  RELV = RELVEC
0032900  AMGV = AMGVEC
0033000  52 CONTINUE
0033100  IF (BIG.EQ.0.) GOTO 155
0033200  IF (AMGV.EQ.0.) GOTO 135
0033300  IF (DABS(AMGV).GT.DABS(RELV)) GOTO 125
0033400  RAT = AMGV/RELV
0033500  DEN = RELV + RAT*AMGV
0033600  DO 120 I=1,N
0033700  IF (I.EQ.11) GOTO 120
0033800  RELVEC = VECT(I)
0033900  AMGVC = CONJ*VECT(I)
0034000  RELVC = (RELVEC + RAT*AMGVEC)/DEN
0034100  AMGVC = (AMGVEC - RAT*RELVEC)/DEN
0034200  VECT(I) = DCMPLX(RELVC,AMGVC)
0034300  120 CONTINUE
0034400  VECT(11) = 1.
0034500  GO TO 150
0034600  125 RAT = RELV/AMGV
0034700  DEN = AMGV + RAT*RELV
0034800  DO 130 I=1,N
0034900  IF (I.EQ.11) GOTO 130
0035000  RELVEC = VECT(I)
0035100  AMGV = CONJ*VECT(I)
0035200  RELVC = (RELVEC + RAT*AMGVEC)/DEN
0035300  AMGVC = (AMGVEC - RAT*RELVEC)/DEN
0035400  VECT(I) = DCMPLX(RELVC,AMGVC)
0035500  130 CONTINUE
0035600  VECT(11) = 1.
0035700  GO TO 150
0035800  135 DO 53 I=1,N
0035900  53 VECT(I) = VECT(I)/BIG
0036000  150 ABIG = ABIG + DLOG10(BIG)
0036100  IF (ABIG.GT.EPSMAX1) GOTO 55
0036200  155 IF (ICOUNT.GE.10) GOTO 55
0036300  DO 54 I=1,M
0036400  IF (.NOT.INTH(I)) GOTO 54
0036500  TEMP = VECT(I)
0036600  VECT(I) = VECT(I+1)
0036700  VECT(I+1) = TEMP
0036800  54 VECT(I+1) = VECT(I+1)+MULT(I)*VECT(I)
0036900  ICOUNT = ICOUNT + 1
0037000  GO TO 48
0037100  55 IF (M.LE.2) GOTO 69
0037200  MCOUNT(L) = ICOUNT
0037300  MM2 = M-2
0037400  DO 57 I=1,MM2
0037500  M11 = M-1-I
0037600  M11 = M-I+1
0037700  DO 56 J=M11,M
0037800  56 VECT(J)=H(J,M11)*VECT(M11+1)+VECT(J)
0037900  INDEX = JNT(M11)
TEMP = VECT(M11+1)

VECT(M11+1) = VECT(INDEX)

57 VECT(INDEX) = TEMP

NORMALIZE EIGENVECTOR.

69 SUM = 0.

DO 58 I=1,M

58 RELVEC = VECT(I)

AMGVEC = CONJ*VECT(I)

58 SUM = SUM + RELVEC*RELVEC + AMGVEC*AMGVEC

58 SUM = DSQRT(SUM)

IF (SUM.EQ.0.) GO TO 60

DO 59 I=1,M

59 SUM = SUM + RELVEC*RELVEC + AMGVEC*AMGVEC

59 VECT(I) = VECT(I)/SUM

CONTINUE

60 A(I,L) = VECT(I)

DO 61 I=1,M

CALL CPUTIM(KTIM)

CONTINUE

63 NCAL = NCAL + NC

IF(NSTART.EQ.1) GOTO 70

SHIFT(1) = 0.

NSTOP = NSTART - 1

GO TO 20

70 DO 80 L=2,M

DO 79 I=1,M

79 JNT(I) = 0

RELAML = LAMBDA(L)

AMGAML = CONJ*LAMBDA(L)

LM1 = L - 1

R = 0

DO 71 I=1,LM1

RELAMI = LAMBDA(I)

AMGAMI = CONJ*LAMBDA(I)

IF (DABS(RELAML-RELAMI).GT.EPS) GOTO 71

IF (DABS(AMGAML-AMGAMI).GT.EPS) GOTO 71

JNT(I) = L

R = R + 1

CONTINUE

GOTO 80

R = R - 1

CONTINUE

GOTO 76

R = R - 1

CONTINUE

GOTO 76

SUM = 0.
DO 77 I=1,M
A(I,L) = A(I,L) - VECT(I)
77 SUM = SUM + A(I,L)*DCONJG(A(I,L))
IF (SUM.EQ.0.) GOTO 80
SUM = DSQRT(SUM)
DO 78 I=1,M
78 A(I,L) = A(I,L)/SUM
DO 92 J=1,M
IF (.NOT.EVECT) RETURN
DO 95 I=1,M
TEMP = A(I,J)
AA(I,J) = TEMP
RETURN
ENTRY EVDATA (ITS,KTS,NCO,MCO,RNORM)
DIMENSION MCO(I), NCO(I), RNORM(I)
DO 83 I=1,M
ITS = ITSUM
IF (.NOT.EVECT) RETURN
DO 84 I=1,M
ANORM = 0.
DO 85 I=1,M
DO 85 J=1,M
85 ANORM = ANORM + A(J,I)*DCONJG(A(J,I))
ANORM = DSQRT(ANORM)
IF (ANORM.EQ.0.) ANORM=1.
KTS = KTSUM
DO 90 L=1,M
90 RNORM(L) = DSQRT(VNORM)/ANORM
RETURN
END
REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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