NUMERICAL ANALYSIS OF FLOW AND PRESSURE FIELDS IN AN IDEALIZED SPIRAL-GROOVED PUMPING SEAL

by John Zuk and Harold E. Renkel

Lewis Research Center
Cleveland, Ohio 44135

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SUMMARY

A computer program is presented for finding the flow and pressure fields in a spiral-grooved pumping seal model for the limiting case of zero clearance. The governing nonlinear partial differential equations are solved numerically using the method of finite differences and over-relaxation. The program is written in FORTRAN IV and is completely described including the program listing, flow charts, and sample problem. The computer program calculates the net volume flow rate, velocity profiles, and pressure distributions for specific axial pressure gradients, Reynolds numbers, and aspect ratios. Other biharmonic problems may be solved using this program.

INTRODUCTION

In a companion paper (ref. 1) an analysis is given for the flow and pressure fields in a spiral-groove pumping seal model for the limiting case of zero clearance. A spiral-groove face seal (ref. 1) is a member of a general class of pressure generation devices that are characterized by two surfaces moving relative to each other with very small film thicknesses and with one or both surfaces grooved. Several geometric forms are found; for example, the cylindrical form (viscoseal), the herringbone groove bearing, and the conical and spherical bearing forms. Variations and combinations of these are also found. The numerical solutions of the exact governing equations using the numerical analysis and computer program presented herein are compared with classical models of the groove axial flow which neglect the coupling of the groove cross flow. The solutions presented in reference 1 included the following results: The cross flow shifts the pumping flow toward the land leading edge. Conditions under which the classical models give good approximations for the relation between axial pressure gradient and net volume flow are shown to depend on the Reynolds number and aspect ratio. The groove cross-
section static pressure is nearly constant except near the moving surface region. A low pressure region suggests the possibility of degassing and cavitation; a high-pressure region near the land leading edge results in a lift force acting on the moving surface.

The objective of this report is to present a method of solution and to present a computer program for numerical solutions for the flow and pressure fields in a spiral-grooved pumping seal model whose analysis is given in reference 1. Also, other physical problems that can be solved by the computer program are discussed. The computer program is written in FORTRAN IV for the Lewis Research Center IBM 7094II/7044 direct-couple system.

NUMERICAL ANALYSIS

Seal Model and Equations

As described in reference 1, the spiral-groove pumping seal model is a stationary rectangular cross-section groove with a wall (upper plate) moving at an oblique angle to the groove edges as illustrated in figure 1. In addition, a pressure gradient is imposed in the groove axial direction. A rectilinear Cartesian coordinate system is used. The flow is fully developed in the groove axial direction (z*-direction); that is, end effects are neglected in the z*-direction. The flow studied is for a homogeneous, incompressible Newtonian fluid under steady laminar flow conditions.

In reference 1 the flow field variables and resulting equations were nondimensionalized. (All symbols are defined in appendix A including dimensionless scaling values.)

In order to facilitate numerical analysis, the flow field equations across the groove (x*-y*-plane) are expressed in terms of a cross flow stream function \( \psi^* (x^*, y^*) \) and the vorticity \( \xi^*(x^*, y^*) \). The derivatives of the stream function are related to the groove cross flow velocity components \( u^* \) and \( v^* \) such that

\[
\begin{align*}
\mathbf{u}^* & = \frac{\partial \psi^*}{\partial y^*} \\
\mathbf{v}^* & = -\frac{\partial \psi^*}{\partial x^*}
\end{align*}
\]

In figure 2, the groove cross flow plane and the stream function direction are shown.

Since the flow is fully developed in the z*-direction (groove axial direction), \( \partial w^*/\partial z^* = 0 \); thus, the use of the stream function automatically satisfies the dimensionless incompressible continuity equation

\[\partial \psi^*/\partial t = 0\]
\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \cot^2 \alpha \frac{\partial w^*}{\partial z^*} = 0
\] (2)

The component of vorticity in the groove axial direction (z*-direction) is

\[\xi^* = \lambda \frac{\partial v^*}{\partial x^*} - \frac{1}{\lambda} \frac{\partial u^*}{\partial y^*} = -\left(\lambda \frac{\partial^2 \psi^*}{\partial x^* 2} + \frac{1}{\lambda} \frac{\partial^2 \psi^*}{\partial y^* 2}\right)\] (3)

Two important dimensionless parameters were found from nondimensionalizing the governing equations:

\[\text{Re} = \frac{bU \sin \alpha}{\nu}\]

and

\[\lambda = \frac{d}{b}\]

**Groove Cross Flow Plane (x*-y* plane)**

For the stated restrictions the appropriate flow field equation in the groove cross flow plane is the two-dimensional vorticity transport equation (see ref. 1) which reduces to

\[\frac{\partial^2 \xi^*}{\partial x^* 2} + \frac{1}{\lambda^2} \frac{\partial^2 \xi^*}{\partial y^* 2} = \text{Re} \left(\frac{\partial \psi^*}{\partial y^*} \frac{\partial \xi^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial \xi^*}{\partial y^*}\right)\] (4)

**Groove Axial Flow Direction (z*-direction)**

For fully developed flow in the z*-direction, the Navier-Stokes equation is

\[\frac{\partial \psi^*}{\partial y^*} \frac{\partial w^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*} \frac{\partial w^*}{\partial y^*} = -\frac{\partial P^*}{\partial z^*} + \frac{1}{\text{Re}} \left(\frac{\partial^2 w^*}{\partial x^* 2} + \frac{1}{\lambda^2} \frac{\partial^2 w^*}{\partial y^* 2}\right)\] (5)

where
\[ \frac{\partial P^*}{\partial z^*} = C_1 = \text{Constant} \]

Hence,

\[ P^* = C_1 z^* + C_2(x^*, y^*) \]  

The boundary conditions will now be stated: For convenience the stream function is chosen to be zero on the walls, hence,

\[ \psi^*(x^*, 0) = 0 \quad \psi^*(x^*, 1) = 0 \quad \psi^*(0, y^*) = 0 \quad \psi^*(1, y^*) = 0 \]  

The fluid velocity no-slip and impermeability condition on the walls expressed in stream function form is

\[ \frac{\partial \psi^*}{\partial y^*}(x^*, 0) = 0 \quad \frac{\partial \psi^*}{\partial y^*}(x^*, 1) = 1 \quad \frac{\partial \psi^*}{\partial x^*}(0, y^*) = 0 \quad \frac{\partial \psi^*}{\partial x^*}(1, y^*) = 0 \]  

And the no-slip condition for the groove axial direction velocity is

\[ w^*(x^*, 0) = 0 \quad w^*(x^*, 1) = -1 \quad w^*(0, y^*) = 0 \quad w^*(1, y^*) = 0 \]  

Once the flow field is found in the \( x^*-y^* \) plane, the static pressure field can be found from the dimensionless Navier Stokes equations modified in the following way. Using the dimensionless stream function and vorticity:

For \( \text{Re} > 1 \) (ref. 1):

\[ \frac{\partial P^*}{\partial x^*} = - \left( \frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial y^* \partial x^*} + \lambda \frac{2}{\text{Re}} \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} \right) - \frac{1}{\text{Re}} \frac{1}{\lambda} \frac{\partial \zeta^*}{\partial y^*} - \lambda \frac{\zeta^*}{\partial x^*} \]  

\[ \frac{\partial P^*}{\partial y^*} = - \left( \frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial y^* \partial x^*} + \lambda \frac{2}{\text{Re}} \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} \right) + \frac{1}{\text{Re}} \lambda \frac{\partial \zeta^*}{\partial x^*} - \lambda \frac{\zeta^*}{\partial y^*} \]  

For \( 0 < \text{Re} \leq 1 \) (as stated in ref. 2, the pressure has to be rescaled for small values of Reynolds number):

\[ \frac{1}{\text{Re}} \frac{\partial P^*}{\partial x^*} = - \left( \frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial y^* \partial x^*} + \lambda \frac{2}{\text{Re}} \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} \right) - \frac{1}{\text{Re}} \frac{1}{\lambda} \frac{\partial \zeta^*}{\partial y^*} - \lambda \frac{\zeta^*}{\partial x^*} \]
\[
\frac{1}{\text{Re}} \frac{\partial P^*}{\partial y^*} = - \left( \frac{\partial \psi^*}{\partial y^*} \frac{\partial^2 \psi^*}{\partial y^*^2} + \lambda^2 \frac{\partial \psi^*}{\partial x^*} \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} \right) + \frac{1}{\text{Re}} \lambda \frac{\partial \zeta^*}{\partial x^*} - \lambda \zeta^* \frac{\partial \psi^*}{\partial y^*}
\]

(13)

where \( P^* = \text{Re} P^* \).

In reference 2 results of the integration of the pressure distribution on the moving wall surface which yields a net lift force are shown. This net lift force per axial length is found from

\[
\frac{F^*}{\text{Axial length}} = \int_0^1 P^* \, dx^*
\]

(14)

In reference 2 it was further stated that if degassing occurred at the trailing edge (this region would be at ambient pressure) then a net lift force would occur only along the leading edge interval. The leading edge interval is defined as the range of \( x^* \) values from the point \( x^* \geq 0 \) (where \( P^* \) is always greater than 0) to the point \( x^* = 1 \).

\[
\frac{\text{Leading-edge force}}{\text{Axial length}} = \int_{x^* \geq 0}^1 P^* \, dx^*
\]

(15)

Outline of Solution

Due to the geometrical configuration and the nonlinearity of the flow field equation (4), analytical solutions are extremely difficult to obtain; however, equations (3), (4), (10), and (11) can be solved numerically.

The basic nondimensional flow field equations (3) and (4) are solved for the stream function \( \psi^* \) and vorticity \( \zeta^* \) distributions using finite difference techniques and successive overrelaxation, similar to that used by Lieberstein (ref. 3). Once \( \psi^* \) and \( \zeta^* \) are known, the normalized pressure field is calculated from equations (10) and (11), using a finite difference scheme suggested by Burggraf (ref. 4). From the results of equations (3) and (4) the dimensionless \( z^* \)-directional flow \( w^* \) and net volume flow \( Q^*_z \) can be calculated for specified values of the constant groove axial pressure gradient \( \partial P^*/\partial z^* \). Equation (5) is solved by the method of finite differences for the \( w^* \)-field and \( Q^*_z \) is calculated from

\[
Q^*_z = \int_0^1 \int_0^1 w^* \, dy^* \, dx^*
\]

(16)
Finite Difference Method

A grid of mesh points \((i, j)\) is constructed over the positive \(x^*-y^*\) plane (fig. 3) with \(i\) increasing for decreasing \(y^*\) values and \(j\) increasing for increasing \(x^*\) values. With this mesh, equations (3) to (5) can be developed into appropriate difference forms for solution on a digital computer. Central differencing techniques were used in the equations whenever possible; however, forward or backward differences were sometimes used especially at the walls. The FORTRAN IV computer programs are described in appendix C. The first program for the numerical solution of an idealized spiral-grooved pumping seal model calculates the vorticity and stream function, the \(u^*-\) and \(v^*-\) velocity profiles, the normalized pressure field, and the net lift forces. The second program calculates the \(w^*-\) velocity profile and dimensionless net volume flow rate along the groove axis.

The initial distribution for the stream function is assumed as a linear function with \(\psi^* = 0\) at the walls and \(\psi^* = -0.1\) at the center of the rectangular groove. For the case \(\lambda = 1\), square groove, the stream function contours are squares of constant value. Various other distributions including \(\psi^*\) equal to a constant for all interior points were examined but were found to be less efficient in that more iterations were required to obtain convergence. The vorticity initial distribution is calculated at all interior points using equation (3) which in difference form becomes

\[
\xi^*_{i,j} = -\frac{\lambda}{\Delta x^* 2} \left( \psi^*_{i,j+1} - 2\psi^*_{i,j} + \psi^*_{i,j-1} \right) - \frac{1}{\lambda \Delta y^* 2} \left( \psi^*_{i-1,j} - 2\psi^*_{i,j} + \psi^*_{i+1,j} \right)
\]

(17)

At the boundaries the initial vorticity values are calculated from equations (B3) and (B4). In finite difference notation these equations in dimensionless form, become

(1) Lower stationary wall

\[
\xi^*_{i, \text{max}, j} = \frac{2}{\lambda \Delta y^* 2} \left( \psi^*_{i, \text{max}, j} - \psi^*_{i, \text{max}-1, j} \right)
\]

(18)

(2) Left stationary wall

\[
\xi^*_{1, i} = \frac{2\lambda}{\Delta x^* 2} \left( \psi^*_{1, i} - \psi^*_{1, 2} \right)
\]

(19)
(3) Right stationary wall

\[ \xi_{i,j}^{*} = \frac{2\lambda}{\Delta x_*^2} \left( \psi_{1,j}^{*} - \psi_{1,j}^{*} \right) \]  

(4) Upper moving wall

\[ \xi_{1,j}^{*} = \frac{2}{\lambda \Delta y_*^2} \left( \psi_{1,j}^{*} - \psi_{2,j}^{*} - \Delta y_*^{*} \right) \]  

The solutions to equations (3) and (4) are calculated in an iterative routine in which the stream function or vorticity field is scanned once before entering the other field. In the literature, other authors (e.g., ref. 5), who have solved similar boundary value problems, have scanned each field a various number of times (from two to 50) before entering the other field. The present authors feel that this only overcorrects the values in that particular field, especially when a relaxation factor is used in the calculations. In the iterative process equation (4), which in difference form is

\[ \frac{1}{\Delta x_*^2} \left( \xi_{i,j}^{*} - 2\xi_{i,j}^{*} + \xi_{i,j-1}^{*} \right) + \frac{1}{\lambda \Delta y_*^2} \left( \xi_{i,j}^{*} - 2\xi_{i+1,j}^{*} + \xi_{i-1,j}^{*} \right) = \frac{\text{Re}}{4\Delta x_* \Delta y_*} \left[ \left( \psi_{i,j}^{*} - \psi_{i+1,j}^{*} \right) \left( \xi_{i,j}^{*} - \xi_{i-1,j}^{*} \right) - \left( \psi_{i,j}^{*} - \psi_{i,j}^{*} \right) \left( \xi_{i,j}^{*} - \xi_{i+1,j}^{*} \right) \right] \]  

serves as the basis for the vorticity calculations at the interior points; equations (18) to (21) are used for the vorticity boundary values. The values of the stream function at the interior mesh points in the iterative process are calculated from equation (17), while the boundary values are fixed at \( \psi^* = 0 \).

The successive overrelaxation technique used in this analysis is based on a paper by Lieberstein (ref. 3). The basic iterative relaxation equation is

\[ y_i^{n+1} = y_i^n - \omega \frac{f(y_1, y_2, \ldots, y_k)}{f'(y_1, y_2, \ldots, y_k)} \]  

where \( y_i^n \) and \( y_i^{n+1} \) are either \( \psi_{i,j}^* \) or \( \xi_{i,j}^* \) at the \( n \) or \( n+1 \) iteration; \( f(y_1, y_2, \ldots, y_k) \) is either equation (17) or (22); \( f'(y_1, y_2, \ldots, y_k) \) is the combined coefficients of \( \psi_{i,j}^* \) from equation (17) or the combined coefficients of \( \xi_{i,j}^* \) from equa-
tion (22); and \( \omega \) is the relaxation factor. In evaluating \( f(y_1, y_2, \ldots, y_k) \) and \( f'(y_1, y_2, \ldots, y_k) \) the most recent available values for the \( y \)'s are used. Substituting the appropriate terms into equation (23) yields the following equations for the vorticity and stream function as coded in the computer program:

\[
\xi_{i,j}^{n+1} = (1 - \omega)\xi_{i,j}^n - K_1 \left\{ \frac{\xi_{i,j}^n + \xi_{i+1,j}^n + \xi_{i,j-1}^n + \xi_{i,j-1}^n}{\Delta x^2} \right\} - \frac{\text{Re}}{4\Delta x^* \Delta y^*} \left[ \left( \psi_{i,j+1}^n - \psi_{i,j-1}^n \right) \left( \xi_{i-1,j}^n - \xi_{i+1,j}^n \right) - \left( \psi_{i-1,j}^n - \psi_{i+1,j}^n \right) \left( \xi_{i,j-1}^n - \xi_{i,j+1}^n \right) \right]
\]

\[
\psi_{i,j}^{n+1} = (1 - \omega)\psi_{i,j}^n + \frac{K_1}{\lambda} \left[ \frac{\lambda}{\Delta x^2} \left( \psi_{i,j+1}^n + \psi_{i,j-1}^n \right) \right] + \frac{1}{\lambda \Delta y^2} \left( \psi_{i-1,j}^{n+1} + \psi_{i+1,j}^{n+1} \right) + \xi_{i,j}^{n+1}
\]

where

\[
K_1 = \frac{\omega}{-2 \left( \frac{1}{\Delta x^2} + \frac{1}{\lambda^2 \Delta y^2} \right)}
\]

Values of \( \omega \) between 0 and 2 are used as relaxation factors, but the optimum value for most rapid convergence depends on the aspect ratio and mesh size. For \( \omega = 1 \), this iterative process reduces to the Liebmann iterated forms as used by Mills (ref. 5).

Once the stream function field has been found the \( u^* \) and \( v^* \) velocity components in the \( x^* \)- and \( y^* \)-directions, respectively, can be calculated from equation (1).
The stream function boundary conditions (eqs. (7) to (9)) specified in terms of velocity boundary conditions are

\[ u^*(0, y^*) = u^*(1, y^*) = u^*(x^*, 0) = 0, \quad u^*(x^*, 1) = 1 \]

\[ v^*(0, y^*) = v^*(1, y^*) = v^*(x^*, 0) = v^*(x^*, 1) = 0 \] (26)

In central difference notation these equations become

\[
\begin{align*}
\frac{\psi_i^*-\psi_{i+1,j}^*}{2 \Delta y^*} & \quad \text{and} \\
\frac{\psi_{i+1,j}^* - \psi_{i,j}^*}{2 \Delta x^*}
\end{align*}
\]

These equations are used in this form to calculate the velocities at all interior mesh points.

The static pressure field is obtained by a point-to-point integration of equations (10) and (11) over a single mesh width. At each mesh point the values obtained from the two equations are averaged to find one pressure for each point. Substituting equation (1) into equations (10) and (11) and rearranging terms yields the following forms from which the static pressure is calculated:

\[ \frac{\partial P^*}{\partial x^*} = -\frac{1}{\text{Re}} \frac{1}{\lambda} \frac{\partial \zeta^*}{\partial y^*} + \lambda \zeta^* v^* - u^* \frac{\partial^2 \psi^*}{\partial y^* \partial x^*} + \lambda^2 v^* \frac{\partial^2 \psi^*}{\partial x^*^2} \] (28)

\[ \frac{\partial P^*}{\partial y^*} = \frac{1}{\text{Re}} \frac{\lambda}{\partial x^*} - \lambda \zeta^* u^* - u^* \frac{\partial^2 \psi^*}{\partial y^*^2} + \lambda^2 v^* \frac{\partial^2 \psi^*}{\partial x^* \partial y^*} \] (29)

For the case \( 0 < \text{Re} \leq 1 \), the corresponding static pressure \((P^*)'\) equations are obtained by multiplying the right side of equations (28) and (29) by \text{Re}.

In the finite-difference representation of these equations the pressure partial derivatives are approximated using forward or backward differences and the values of the pressure at adjacent mesh points. Each of the terms on the right side is calculated at the same pair of adjacent points and then corresponding terms are averaged to give one approximation for each term. For example, in equation (28), the pressure term is approxi-
imated using points \((i, j)\) and \((i, j+1)\) on the grid. Each term on the right side is then evaluated at \((i, j)\) and at \((i, j+1)\); the value of a term from \((i, j)\) is averaged with the value from \((i, j+1)\) to give one approximation in the finite-difference representation.

To adjust for the discontinuity in the vorticity at the junctions of the moving wall with the stationary walls the following scheme was adopted for these two points:

1. If the term being defined is a partial derivative, the value of the vorticity is chosen in the same direction as the derivative; for example, the values for \(\partial \zeta^*/\partial x^*\) are calculated from equation (21), and the values for \(\partial \zeta^*/\partial y^*\) are calculated from equations (19) or (20).

2. If the term being defined is the vorticity itself, the value chosen is dependent on the direction of the pressure derivative term; for example, the value of the vorticity used in equation (28) is calculated from equation (21), and the value in equation (29) is calculated from either equation (19) or (20). The finite-difference equations representing equations (28) and (29) at the interior mesh-points, along the walls, and at the corners are thus given by

(1) Interior points

\[
\frac{P_{i,j}^* - P_{i,j-1}^*}{\Delta x^*} = \frac{1}{\text{Re} \lambda} \frac{1}{2} \left( \frac{\xi_{i-1,j}^* - \xi_{i+1,j}^*}{2 \Delta y^*} + \frac{\xi_{i-1,j-1}^* - \xi_{i+1,j-1}^*}{2 \Delta y^*} \right) + \frac{\lambda}{2} \left( \xi_{i,j}^*, \xi_{i,j+1}^* - \xi_{i,j-1}^* \right)
\]

\[
- \frac{1}{2} u_{i,j}^* \left( \frac{\psi_{i-1,j+1}^* - \psi_{i+1,j+1}^*}{4 \Delta x^* \Delta y^*} + \frac{\psi_{i+1,j-1}^* - \psi_{i+1,j}^*}{4 \Delta x^* \Delta y^*} \right)
\]

\[
+ \frac{\lambda}{2} v_{i,j}^* \left( \frac{\psi_{i,j+1}^* - 2 \psi_{i,j}^* + \psi_{i,j-1}^*}{\Delta x^*} \right)
\]

\[
+ \frac{\lambda}{2} v_{i,j-1}^* \left( \frac{\psi_{i,j+1}^* - 2 \psi_{i,j}^* + \psi_{i,j-1}^*}{\Delta x^*} \right)
\]

(30)
Boundary points

In equations (32) through (47) which follow, the zero-valued terms due to the boundary conditions of the spiral-grooved pumping seal have been omitted.

(a) Stationary wall \((x^* = 0, \ 0 < y^* < 1)\)

\[
\frac{P^*_{i, j} - P^*_{i+1, j}}{\Delta y^*} = \frac{\lambda}{\text{Re}} \frac{1}{2} \left( \frac{\zeta^*_{i, j+1} - \zeta^*_{i, j-1}}{2 \Delta x^*} + \frac{\zeta^*_{i+1, j+1} - \zeta^*_{i+1, j-1}}{2 \Delta x^*} \right) - \frac{\lambda}{2} \left( \frac{\zeta^*_{i, j} + \zeta^*_{i+1, j} + \zeta^*_{i+1, j+1} + \zeta^*_{i+2, j+1}}{4 \Delta x^* \Delta y^*} \right) \\
\frac{u^*_{i, j}}{\Delta y^*} - \frac{1}{2} \left( \frac{\psi^*_{i-1, j} - 2\psi^*_{i, j} + \psi^*_{i+1, j}}{\Delta y^*^2} \right)
\]

\[
\frac{u^*_{i+1, j}}{\Delta y^*} + \left( \frac{\psi^*_{i, j} - 2\psi^*_{i+1, j} + \psi^*_{i+2, j}}{\Delta y^*^2} \right)
\]

\[
+ \frac{\lambda^2}{2} \left( \frac{\psi^*_{i-1, j+1} - \psi^*_{i-1, j-1} + \psi^*_{i+1, j+1} - \psi^*_{i+1, j-1}}{4 \Delta x^* \Delta y^*} \right)
\]

\[
\frac{v^*_{i, j}}{\Delta x^*} + \left( \frac{\psi^*_{i+1, j+1} - \psi^*_{i+1, j-1} + \psi^*_{i+2, j+1} - \psi^*_{i+2, j-1}}{4 \Delta x^* \Delta y^*} \right)
\]
\[
\frac{P^*, 1 - P^*, 1, 1}{\Delta y^*} = \frac{1}{\text{Re} \lambda} 2 \left( \frac{\zeta^*, 2 - \zeta^*, 1}{\Delta x^*} + \frac{\zeta^*, 1, 2 - \zeta^*, 1, 1}{\Delta x^*} \right)
\] (33)

(b) Stationary wall \((x^* = 1, 0 < y^* < 1)\)

\[
\frac{P^*, j_{\text{max}} - P^*, j_{\text{max}} - 1}{\Delta x^*} = -\frac{1}{\text{Re} \lambda} 2 \left( \frac{\zeta^*, j_{\text{max}} - \zeta^*, j_{\text{max}} - 1}{\Delta y^*} + \frac{\zeta^*, j_{\text{max}} - 1, j_{\text{max}} - \zeta^*, j_{\text{max}} - 1}{\Delta y^*} \right) + \frac{\lambda}{2} \frac{\zeta^*, j_{\text{max}} - 1, j_{\text{max}} - 1}{2} \psi^*, j_{\text{max}} - 1, j_{\text{max}} - 1
\]

\[
	imes \left( \frac{\psi^*, j_{\text{max}} - \psi^*, j_{\text{max}} - 2 + \psi^*, j_{\text{max}} - 2 - \psi^*, j_{\text{max}} - 1}{2 \Delta x^* \Delta y^*} \right) + \frac{\lambda}{2} \frac{\psi^*, j_{\text{max}} - 1}{2} \left( \frac{\psi^*, j_{\text{max}} - 2 \psi^*, j_{\text{max}} - 2 + \psi^*, j_{\text{max}} - 3}{\Delta x^*} \right)
\]

\[
\] (34)

\[
\frac{P^*, j_{\text{max}} - P^*, j_{\text{max}}}{\Delta y^*} = \frac{1}{\text{Re} \lambda} 2 \left( \frac{\zeta^*, j_{\text{max}} - \zeta^*, j_{\text{max}} - 1}{\Delta x^*} + \frac{\zeta^*, j_{\text{max}} - 1, j_{\text{max}} - \zeta^*, j_{\text{max}} - 1}{\Delta x^*} \right)
\] (35)

(c) Stationary wall \((y^* = 0, 0 < x^* < 1)\)

\[
\frac{P^*, j_{\text{max}} - P^*, j_{\text{max}}}{\Delta x^*} = -\frac{1}{\text{Re} \lambda} 2 \left( \frac{\zeta^*, j_{\text{max}} - \zeta^*, j_{\text{max}}}{\Delta y^*} + \frac{\zeta^*, j_{\text{max}} - 1, j_{\text{max}} - 1}{\Delta y^*} \right)
\]

\[
\times \left( \frac{\zeta^*, j_{\text{max}} - 1, j_{\text{max}} - 1, j_{\text{max}} - 1}{\Delta y^*} \right)
\] (36)
\[
\frac{P_{i, \text{max}-1, j} - P_{i, \text{max}, j}}{\Delta y^*} = \lambda \frac{1}{\text{Re}} \frac{1}{2} \left( \frac{\zeta_{i, \text{max}, j+1} - \zeta_{i, \text{max}, j-1}}{2 \Delta x^*} + \frac{\zeta_{i, \text{max}-1, j+1} - \zeta_{i, \text{max}-1, j-1}}{2 \Delta x^*} \right)
\]

\[-\frac{\lambda}{2} \frac{u_{i, \text{max}-1, j}^*}{2} \]

\[
\times \left( \frac{\psi_{i, \text{max}-3, j}^* - 2\psi_{i, \text{max}-2, j}^* + \psi_{i, \text{max}-1, j}^*}{\Delta y^*^2} \right) + \lambda^2 \frac{v_{i, \text{max}-1, j}^*}{2}
\]

\[
\times \left( \frac{\psi_{i, \text{max}-2, j+1}^* - \psi_{i, \text{max}-2, j-1}^* + \psi_{i, \text{max}-1, j+1}^* - \psi_{i, \text{max}-1, j-1}^*}{2 \Delta x^* \Delta y^*} \right) \quad (37)
\]

(d) Moving wall (\(y^* = 1, \ 0 < x^* < 1\))

\[
\frac{P_{i, j} - P_{i, j-1}}{\Delta x^*} = \frac{1}{\text{Re}} \frac{1}{2} \left( \frac{\zeta_{i, j}^* - \zeta_{i, j+1}^*}{\Delta y^*} + \frac{\zeta_{i, j-1}^* - \zeta_{i, j}^*}{\Delta y^*} \right)
\]

\[-\frac{1}{2} \left( \frac{\psi_{i, j+1}^* - \psi_{i, j-1}^* + \psi_{i, j+1}^* - \psi_{i, j-1}^*}{2 \Delta x^* \Delta y^*} \right) \]

\[+ \frac{\psi_{i, j}^* - \psi_{i, j+2}^* + \psi_{i, j-2}^* - \psi_{i, j+2}^*}{2 \Delta x^* \Delta y^*} \quad (38)\]
\[
\frac{P_{1,j}^* - P_{2,j}^*}{\Delta y^*} = \frac{\lambda}{\text{Re}} \frac{1}{2} \left( \frac{\xi_{1,j+1}^* - \xi_{1,j-1}^*}{2 \Delta x^*} + \frac{\xi_{2,j+1}^* - \xi_{2,j-1}^*}{2 \Delta x^*} \right) - \frac{\lambda}{2} \left( \frac{\xi_{1,j}^* + \xi_{2,j}^* u_{2,j}^*}{2} \right) - \frac{1}{2} \left[ \frac{\psi_{1,j}^* - 2 \psi_{2,j}^* + \psi_{3,j}^*}{\Delta y^*} \right] + u_{2,j}^* \left( \frac{\psi_{3,j}^* - 2 \psi_{4,j}^*}{\Delta y^*} \right) + \lambda^2 \frac{v_{2,j}^*}{2} \left( \frac{\psi_{2,j+1}^* - \psi_{2,j-1}^* + \psi_{3,j-1}^* - \psi_{3,j+1}^*}{2 \Delta x^* \Delta y^*} \right)
\]

(39)

(e) Corner point \((x^* = 0, \ y^* = 1)\)

\[
\frac{P_{1,2}^* - P_{1,1}^*}{\Delta x^*} = -\frac{1}{\text{Re} \lambda} \frac{1}{2} \left( -\frac{\xi_{2,1}^*}{\Delta y^*} + \frac{\xi_{1,2}^* - \xi_{2,2}^*}{\Delta y^*} \right) - \frac{1}{2} \left( \frac{\psi_{1,2}^* - \psi_{1,1}^* + \psi_{2,1}^* - \psi_{2,2}^*}{\Delta x^* \Delta y^*} \right) + \lambda^2 \frac{v_{2,1}^*}{2} \left( \frac{\psi_{1,3}^* - \psi_{1,2}^* + \psi_{2,2}^* - \psi_{2,3}^*}{\Delta x^* \Delta y^*} \right)
\]

(40)

(f) Corner point \((x^* = 1, \ y^* = 1)\)

\[
\frac{P_{1,1}^* - P_{2,1}^*}{\Delta y^*} = \frac{\lambda}{\text{Re}} \frac{1}{2} \left( \frac{\xi_{1,2}^* - \xi_{1,1}^*}{\Delta x^*} + \frac{\xi_{2,2}^* - \xi_{2,1}^*}{\Delta x^*} \right)
\]

(41)
\[
\frac{P_{1, \text{jmax}} - P_{1, \text{jmax}-1}}{\Delta x^*} = -\frac{1}{\text{Re} \lambda} \frac{1}{2} \left( -\frac{\xi_{1, \text{jmax}}^* - \xi_{1, \text{jmax}-1}^*}{\Delta y^*} + \xi_{2, \text{jmax}}^* - \xi_{2, \text{jmax}-1}^* \right)
\]

\[-\frac{1}{2} \left( \frac{\psi_{1, \text{jmax}}^* - \psi_{1, \text{jmax}-1}^* + \psi_{2, \text{jmax}}^* - \psi_{2, \text{jmax}-1}^*}{\Delta x^* \Delta y^*} \right)
\]

\[
+ \frac{\psi_{1, \text{jmax}-1}^* - \psi_{1, \text{jmax}-2}^* + \psi_{2, \text{jmax}-2}^* - \psi_{2, \text{jmax}-1}^*}{\Delta x^* \Delta y^*}
\]

\[
\frac{P_{1, \text{jmax}} - P_{2, \text{jmax}}}{\Delta y^*} = \lambda \frac{1}{\text{Re} \lambda} \frac{1}{2} \left( \frac{\xi_{1, \text{jmax}}^* - \xi_{1, \text{jmax}-1}^*}{\Delta x^*} + \frac{\xi_{2, \text{jmax}}^* - \xi_{2, \text{jmax}-1}^*}{\Delta x^*} \right)
\]

\[
\text{(g) Corner point (} x^* = 0, \ y^* = 0)\]

\[
\frac{P_{\text{imax}, 2} - P_{\text{imax}, 1}}{\Delta x^*} = -\frac{1}{\text{Re} \lambda} \frac{1}{2} \left( \frac{\xi_{\text{imax}-1, 1}^* - \xi_{\text{imax}, 1}^*}{\Delta y^*} + \frac{\xi_{\text{imax}-1, 2}^* - \xi_{\text{imax}, 2}^*}{\Delta y^*} \right)
\]

\[
\frac{P_{\text{imax}, 1} - P_{\text{imax}, 1}}{\Delta y^*} = \lambda \frac{1}{\text{Re} \lambda} \frac{1}{2} \left( \frac{\xi_{\text{imax}, 2}^* - \xi_{\text{imax}, 1}^*}{\Delta x^*} + \frac{\xi_{\text{imax}, 1}^* - \xi_{\text{imax}-1, 1}^*}{\Delta x^*} \right)
\]

\[
\text{(h) Corner point (} x^* = 1, \ y^* = 0)\]

\[
\frac{P_{\text{imax}, 1} - P_{\text{imax}, \text{jmax}-1}}{\Delta x^*} = -\frac{1}{\text{Re} \lambda} \frac{1}{2} \left( \frac{\xi_{\text{imax}, \text{jmax}}^* - \xi_{\text{imax}, \text{jmax}-1}^*}{\Delta y^*} + \frac{\xi_{\text{imax}-1, \text{jmax}}^* - \xi_{\text{imax}-1, \text{jmax}-1}^*}{\Delta y^*} \right)
\]
\[
\frac{P_{i_{\text{max}-1},j_{\text{max}}} - P_{i_{\text{max}},j_{\text{max}}}}{\Delta y^*} = \frac{\lambda}{Re} \frac{1}{2} \left( \frac{\xi_{i_{\text{max}},j_{\text{max}}}}{\Delta x^*} - \frac{\xi_{i_{\text{max}-1},j_{\text{max}}}}{\Delta x^*} \right)
\]

The initial distribution across the entire static pressure field is constant at \( P^* = 1 \). The program then iterates on the pressure field from top-to-bottom and left-to-right until convergence is achieved. The normalized static pressure field is calculated by subtracting from all the pressures in the field the value at one reference point specified as a program input. It is felt that this normalization method presents more useful results rather than the pressure ratios presented in references 1 and 2.

Both integrations in equations (14) and (15) in finding the net lift force per axial length and leading edge force were performed numerically using Simpson's rule.

In program two, the \( w^* \)-velocity profile is calculated based on the stream function distribution calculated in program one. The stream function data are read in to program two, and the \( w^* \)-field is initially equal to one, except for the boundary conditions presented with equation (5). If for a certain set of parameters, \( Re \) and \( \lambda \), the \( w^* \)-field is to be calculated for more than one value of \( \partial P^*/\partial z^* \), the initial value of the \( w^* \)-field for the succeeding \( \partial P^*/\partial z^* \) value is set equal to the converged value from the preceding \( \partial P^*/\partial z^* \). Lieberstein's (ref. 3) successive overrelaxation technique is again used in the solution of equation (5) with the \( y_i \)'s of equation (23) being equal to the latest available values of \( w^*_{i,j} \). The \( f(y_1, y_2, \ldots, y_n) \) is evaluated from the following equation which is the finite difference form of equation (5):

\[
\frac{w_{i,j+1}^* - w_{i,j-1}^*}{2 \Delta x^*} - \frac{w_{i+1,j}^* - w_{i,j}^*}{2 \Delta y^*} = \frac{\partial P^*}{\partial z^*}
\]

\[
- \frac{1}{Re} \left( \frac{w_{i,j}^* - 2w_{i,j}^* + w_{i+1,j}^*}{\Delta x^*^2} \right) + \frac{1}{\lambda^2} \left( \frac{w_{i-1,j}^* - 2w_{i,j}^* + w_{i+1,j}^*}{\Delta y^*^2} \right) = 0
\]

And \( f'(y_1, y_2, \ldots, y_n) \) is the combined coefficients of \( w_{i,j}^* \) from the preceding equation.
Substituting the appropriate expressions for \( f \) and \( f' \) into equation (23) yields the \( z^* \)-directional flow equation as coded in program two:

\[
\begin{align*}
\psi_{i,j}^* - (1 - \omega)\psi_{i,j}^* + \text{Re } K_1 \left( \frac{\psi_{i-1,j}^* - \psi_{i+1,j}^*}{2 \Delta y^*} \right) \left( \frac{w_{i,j}^* - w_{i+1,j}^*}{2 \Delta x^*} \right) \\
- \left( \frac{\psi_{i,j}^* - \psi_{i-1,j}^*}{2 \Delta x^*} \right) \left( \frac{w_{i,j}^* + w_{i+1,j}^*}{2 \Delta y^*} \right) \\
+ \frac{\partial P^*}{\partial z^*} - \frac{1}{\text{Re}} \left( \frac{w_{i,j}^* + w_{i+1,j}^* + w_{i-1,j}^* + w_{i,j-1}^*}{\Delta x^* \Delta y^*} \right) + \frac{1}{\lambda^2} \left( \frac{w_{i,j}^* + w_{i+1,j}^* + w_{i,j-1}^* + w_{i,j+1}^*}{\Delta x^* \Delta y^*} \right)
\end{align*}
\]

where \( K_1 \) is defined in connection with (24) and (25). The range of the relaxation factor \( \omega \) is between 0 and 2.0 with an optimum value of approximately 1.3 for \( \lambda = 1 \) and 29 mesh points in the \( x^* \)- and \( y^* \)-directions. Other combinations of \( \lambda \) and grid size have different optimum relaxation factors.

After the \( w^* \)-field has iterated to convergence within the prescribed error condition, the net volume flow is calculated from equation (16) using the method of mechanical cubature. A requirement of this method, which is based on Simpson's Rule of integration for two dimensions, is that there be an odd number of mesh points in both the \( x^* \)- and \( y^* \)-directions so that the proper weight factors will be applied in the cubature scheme. The double integral in equation (16) is thus approximated by the double summation as given by

\[
Q^*_z = \frac{\Delta x^* \Delta y^*}{9} \sum_{i=2,4,6,\ldots}^{\text{imax}-1} \sum_{j=2,4,6,\ldots}^{\text{jmax}-1} 16w_{i,j}^* + 4 \left( w_{i-1,j}^* + w_{i,j+1}^* + w_{i-1,j+1}^* + w_{i,j-1}^* + w_{i+1,j}^* + w_{i,j-1}^* \right) \\
+ w_{i-1,j-1}^* + w_{i-1,j+1}^* + w_{i+1,j-1}^* + w_{i+1,j+1}^*
\]

(50)
Convergence Remarks

A solution was considered to have converged when the following criteria were met:

1. The relative change in the values of the stream function, pressure distribution, or axial velocity distribution between two successive iterations was less than a prescribed maximum for all mesh points; for example,

\[
\frac{f_{n+1} - f^n}{f_{n+1}} < \text{Prescribed maximum}
\]

2. The values of the stream function and vorticity at any mesh point changed less than 1 percent when the number of mesh points was doubled in either direction.

In addition to the convergence criteria, the effect of round-off error on the results of reference 1 was checked by doubling the precision of the computing machine calculations.

Computer Program Formulation

As mentioned in the previous section, the equations in finite difference form are solved on a high-speed digital computer. A complete description of the program is presented in appendix C. The program listing is given in appendix D. Figures 4 to 12 present the computer program flow charts. Flow charts of the main program, stream function and vorticity, u* and v*-velocities, pressure field, w*-velocities, and net volume flow Qz calculations are shown. A sample problem for the case where \( \lambda = 1 \), \( \text{Re} = 100 \), and \( \frac{\partial P*}{\partial z*} = -0.115 \) with its input and output is given in appendix E. (Plots of this case are in ref. 1.)

Program Use for Other Physical Problems

The computer program finds the stream function and vorticity in the groove cross-flow plane by solving equations (3) and (4), which are the following:

\[
\nabla^2 \psi* = -\zeta*
\]

\[
\frac{\partial^2 \zeta*}{\partial x^2} + \frac{1}{\lambda^2} \frac{\partial^2 \zeta*}{\partial y^2} = \text{Re} \left( \frac{\partial \psi*}{\partial y*} \frac{\partial \zeta*}{\partial x*} - \frac{\partial \psi*}{\partial x*} \frac{\partial \zeta*}{\partial y*} \right)
\]
These equations can be combined and placed in the following form (The aspect ratio \( \lambda \) can be eliminated by redefining the independent variables):

\[
\nabla^4 \psi^* = \text{Re} \frac{\partial (\psi^*, \nabla^2 \psi^*)}{\partial (x^*, y^*)}
\]

(51)

The computer program can be used to solve many problems in mathematical physics that appear in this form or are reducible to this form. For example, the biharmonic equation (This is the creeping flow case discussed in ref. 1) \( \nabla^4 \psi^* = 0 \)

Laplace's equation

\[
\nabla^2 \psi^* = 0
\]

Poisson's equation

\[
\nabla^2 \psi^* = -\zeta^*
\]

Of course, the proper transformation of the variables must be made to the form solved in the computer program. The proper boundary conditions must be specified.

The coding in the FORTRAN IV computer program contains all the boundary terms including the zero-valued terms omitted in equations (32) through (47) which were specifically written for the spiral-grooved pumping seal. The configuration must be rectangular.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 29, 1970,
126-15.
APPENDIX A

SYMBOLS

b  groove width
C  numerical coefficient
d  groove depth
F  net moving surface lift force
F* dimensionless lift force, (F/bρU^2 sin^2 α) × (total axial length)
K_1 numerical coefficient
n  iteration number
P  static pressure
P*' dimensionless pressure, Pb/μU sin α (for 0 < Re ≤ 1)
P* dimensionless pressure, P/ρU^2 sin^2 α (for Re > 1)
\frac{∂P*}{∂z*} dimensionless groove axial pressure gradient, constant
Q_z  net volume flow rate, groove axial direction
Q_z* dimensionless net volume flow rate, Q_z/(dbU cos α)
Re  Reynolds number, (bU sin α)/ν
U  moving wall velocity
u  velocity in x-direction
u* dimensionless velocity, u/(U sin α)
v  velocity in y-direction
v* dimensionless velocity, b/d\left[v/(U sin α)\right]
w  velocity in z-direction
w* dimensionless velocity, w/(U cos α)
x  cross groove coordinate
x* dimensionless coordinate = x/b
Δx* mesh width in x*-direction
y  groove depth direction coordinate
y* dimensionless coordinate = y/d
20
\( \Delta y^* \)  mesh width in \( y^*-\)direction
\( z \)  groove axial coordinate
\( z^* \)  dimensionless coordinate, \( z/(b \tan \alpha) \)
\( \alpha \)  angle between moving wall direction and groove axis
\( \lambda \)  aspect ratio, equal to groove depth to groove width, \( d/b \)
\( \mu \)  absolute viscosity of fluid
\( \nu \)  kinematic viscosity of fluid
\( \psi \)  stream function, defined in \( x-y \) plane
\( \psi^* \)  dimensionless stream function, \( \psi/(dU \sin \alpha) \)
\( \zeta \)  vorticity component in \( z \)-direction
\( \zeta^* \)  dimensionless vorticity, \( \zeta b/(U \sin \alpha) \)
\( \omega \)  relaxation factor

\( \nabla^2 \)  Laplacian operator,
\[
\left( \lambda \frac{\partial^2}{\partial x^*^2} + \frac{1}{\lambda} \frac{\partial^2}{\partial y^*^2} \right)
\]

\( \nabla^4 \)  biharmonic operator,
\[
\left( \lambda \frac{\partial^4}{\partial x^*^4} + \frac{2}{\lambda} \frac{\partial^4}{\partial x^*^2 \partial y^*^2} + \frac{\partial^4}{\partial y^*^4} \right)
\]

\( \frac{\partial (\psi^*, \nabla^2 \psi^*)}{\partial (x^*, y^*)} \)  Jacobian operator,
\[
\left( \frac{\partial \psi^*}{\partial x^*} \frac{\partial \nabla^2 \psi^*}{\partial y^*} - \frac{\partial \psi^*}{\partial y^*} \frac{\partial \nabla^2 \psi^*}{\partial x^*} \right)
\]

Subscripts:

\( i \)  mesh point in \( y^*-\)direction
\( \text{imax} \)  maximum value of mesh point in \( y^*-\)direction \( (y^* = 0) \)
\( j \)  mesh point in \( x^*-\)direction
\( j\text{max} \)  maximum value of mesh point in \( x^*-\)direction \( (x^* = 1) \)
\( 0 \)  boundary reference point
APPENDIX B

FIRST ORDER VORTICITY BOUNDARY VALUE APPROXIMATION

The boundary conditions for the stream function are known on the boundary because the normal and tangential velocities must satisfy the impermeability and no-slip conditions at the wall. The vorticity value on the boundary must be calculated and will vary from iteration to iteration until a specified convergence criterion is satisfied. The vorticity at the boundary can be found by the following first order approximation.

Consider the boundary along the x*-axis as shown in figure 13. The stream function expanded in a Taylor series about point 0 is

\[ \psi_1 = \psi_0^* + (\Delta y^*) \left( \frac{\partial \psi^*}{\partial y^*} \right)_0 + \frac{(\Delta y^*)^2}{2!} \left( \frac{\partial^2 \psi^*}{\partial y^*^2} \right)_0 + \mathcal{O}[(\Delta y^*)^3] \]  

(B1)

Using equation (3) which relates the stream and vorticity functions yields

\[ \lambda \left( \frac{\partial^2 \psi^*}{\partial x^*^2} \right)_0 + \frac{1}{\lambda} \left( \frac{\partial^2 \psi^*}{\partial y^*^2} \right)_0 = -\xi_0^* \]  

(B2)

Since \( \psi^* = \psi^*(x^*) \) on the boundary shown in figure 13,

\[ \left( \frac{\partial^2 \psi^*}{\partial x^*^2} \right) = 0 \]

Substituting equation (B2) into the truncated Taylor series expansion (B1) results in

\[ \xi_0^* = \frac{2}{\lambda(\Delta y^*)^2} \left[ \psi_0^* - \psi_1^* + (\Delta y^*) \left( \frac{\partial \psi^*}{\partial y^*} \right)_0 \right] \]  

(B3)

In like manner if the wall were along the y*-axis, the wall vorticity equation is

\[ \xi_0^* = \frac{2\lambda}{(\Delta x^*)^2} \left[ \psi_0^* - \psi_1^* + (\Delta x^*) \left( \frac{\partial \psi^*}{\partial x^*} \right)_0 \right] \]  

(B4)
The resulting vorticity boundary conditions are found by substituting the boundary conditions equations (7) and (8) into equations (B3) and (B4) and are for stationary walls:

\[
\xi_0^* = \frac{-2\psi_1^*}{\lambda(\Delta y^*)^2} \quad \text{(horizontal)}
\]

\[
\xi_0^* = \frac{-2\lambda \psi_1^*}{(\Delta x^*)^2} \quad \text{(vertical)}
\]

and for a moving wall

\[
\xi_0^* = \frac{2(-\psi_1^* + \Delta y^*)}{\lambda(\Delta y^*)^2}
\]

where \( \psi_1^* \) is the stream function value in the flow field which is \( \Delta x^* \) or \( \Delta y^* \) away from the boundary whose stream function has a value \( \psi_0^* \). (See fig. 13.)
APPENDIX C

DESCRIPTION OF COMPUTER PROGRAMS

Program ISGPSM

The computer program for the numerical solution of an idealized spiral grooved pumping seal model (ISGPSM) consists of the MAIN program and the following subroutines:

SVCalc enters the input data and calculates the stream function and vorticity distributions and many of the constant terms used throughout the program.

INDIST generates the stream function initial distribution.

ZETAF generates the vorticity initial distribution.

VORTWL calculates the values of the vorticity along the moving and stationary boundary walls.

UVFUNC computes the \( u^* \) and \( v^* \) velocity profiles in the \( x^* \) and \( y^* \) directions, respectively.

PRESS calculates the static pressure field based on the stream function and vorticity distributions and then the normalized pressure field with reference to a predetermined point.

BCDUMP punches data in sequentially numbered absolute binary cards with a maximum of 22 words per card.

Program ISGPSM is structured to make use of the overlay feature of the computing machine loading system (IBLDR) which saves in auxiliary storage those sections of the program currently not being executed. The use of this feature thus provides for a maximum of 65 mesh points in both the \( x^* \) and \( y^* \)-directions. Figure 14 is a chart of the overlay structure of the entire ISGPSM program.

The input to program ISGPSM is a comparatively few number of variables which are entered on one data card. The following is a list of the variables and the definition of each:

- LAMBDA aspect ratio
- RE Reynolds number
- OMEGA relaxation factor \([0, 2.0]\)
PCT  maximum allowable relative change between two successive iterations of the
stream function or static pressure field calculations at any mesh point

NOPTY number of mesh points in y*-direction (maximum number = 65)

NOPTX number of mesh points in x*-direction (maximum number = 65)

NOLINY number of lines of output data in y*-direction (maximum number = NOPTY)

NOLINX number of columns of output data in x*-direction (maximum number = 15)

NOLINY and NOLINX should be chosen such that (NOPTY - 1)/(NOLINY - 1) = K_y
and (NOPTX - 1)/(NOLINX - 1) = K_x where K_x and K_y are integers. The program will
then print lines 1, 1 + K_y, 1 + 2K_y, . . . , NOPTY and columns 1, 1 + K_x, 1 + 2K_x, . . . ,
NOPTX.

IREF mesh point number in the y*-direction for predetermined reference point in
the normalized pressure field calculations

JREF mesh point number in the x*-direction for the predetermined reference point
in the normalized pressure field calculations.

NOW a control word for the w*-velocity calculations; if NOW equals any nonzero
integer, the converged values of the stream function at all mesh points
will be punched into cards for use in w*-velocity profile program, but if
NOW equals zero, no card punching will occur.

NOPLOT a control word for the normalized static pressure field plots. If NOPLOT
equals any nonzero integer, the normalized static pressure field is
punched using program BCDUMP for use with Canright and Swigert (ref. 6)
three dimensional plotting program, but if NOPLOT equals zero, no card
punching will occur.

PROBNO a control word for the type of problem to be solved. If PROBNO ≠ 0, the
program will solve Laplace's equation (ξ* = 0) or Poisson's equation
(ξ* = f(x, y)). If PROBNO = 0 and Re = 0, a solution to the biharmonic
equation is obtained.

FLUFLG a control word for additional fluid flow calculations. If FLUFLG = 0, the
u* and v* velocity profiles and pressure fields will be calculated.

The format for this card is (4F8.0, 10I3).

<table>
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<th>32 35 38 41 44 47 50 53 56 59 62</th>
<th>72 73 80</th>
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<td>X, XXX</td>
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</table>
The printed output from the ISGPSM program includes the aspect ratio, Reynolds number, relaxation factor, maximum allowable relative change, number of iterations to convergence of the stream function, and the number of increments in the $x^*$ and $y^*$ directions. Also printed are paragraphs of data NOLINY lines long and NOLINX columns wide of the stream function and vorticity, the $u^*$ and $v^*$ profiles, and the static and normalized pressure fields with the number of iterations to convergence of the static pressure field. Following the normalized pressure field are the leading edge region and whole surface net forces.

Program WFIELD

Program WFIELD is a FORTRAN IV computer code for calculating the $z^*$-direction velocity profile $w^*$ and the dimensionless net volume flow rate $Q_{z}^*$ along the groove axis. LAMBDA, RE, OMEGA, PCT, NOPTY, NOPTX, NOLINY, and NOLINX are the first input variables to WFIELD and are entered on one data card. The definition of these variables is the same as for the first eight variables in the ISGPSM program and the format for this card is (4F6.0, 413)

|   | 6 | 7 | 12 | 13 | 18 | 19 | 24 | 27 | 30 | 33 | 36 | 37 | 72 | 73 | 80 |
|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|
| X | .XXX | XXX. | X | XX | .XXX | XX | XX | XX | XX | XX | XX | XX | X | XXX |

The second set of input cards is the values of the stream function (PSI) at all mesh points. These cards are punched in the format (9F8.2) by the ISGPSM program when the variable NOW is not equal to zero. The number of cards required is dependent on the grid size with a maximum of nine data words per card. The dimensionless groove axial pressure gradient $\partial P^*/\partial x^*$, which in the program is denoted by DPDZ, is the last input variable. For a specific geometry and stream function distribution, the $w^*$-velocity profile and net volume flow may be calculated for several $\partial P^*/\partial z^*$ values; however, each value must be entered on a separate data card in the format (F8.0).
The output from the WFIELD program is the aspect ratio, Reynolds number, relaxation factor, maximum relative change, number of iterations to convergence, number of increments in the \( x^* \) and \( y^* \) directions, and pressure gradient. Associated with a particular pressure gradient, the \( w^* \)-velocity at the specified mesh points and the net volume flow along the groove axis are also printed.

**Program GRAPH**

The three-dimensional plots of the normalized pressure field are generated using program GRAPH, subroutine PSURF, and the PLOT3D package of subroutines of Canright and Swigert (ref. 6). (It is assumed that the users of program GRAPH have the appropriate Calcomp plotter available although this is not necessary to use programs TSGPSM and WFIELD.) The data to be plotted is read by the computer via the FORTRAN IV program GRAPH. The PLOT3D subroutines then analyze the data to establish the array of coordinates of each point to be plotted, scale these values to the size of the plotting paper, and set up the figure axes. If the axes are to be rotated to present a nonstandard three dimensional projection, the data points and figure axes are transformed to the new coordinate system. The information on the data points, figure axes, and figure labels is then written on a magnetic tape for further processing by a California Computer Products (Calcomp) magnetic tape plotting system.

The input to program GRAPH is LAMBDA, NOPTY, NOPTX, and

- NOYPLS number of planes parallel to \( x^*-P^* \) plane or perpendicular to the \( y^*-axis \)
- NOXPLS number of planes parallel to \( y^*-P^* \) plane or perpendicular to the \( x^*-axis \)
- XSCALF \( x^*-\), \( y^*-\) and \( P^*-\)direction scale factors used to adjust the coordinates of each point to match the size of the plotting paper and present the plot in the proper perspective (These scale factors are dependent on the amount of hardware on the Calcomp plotter.)
- YSCALF
- PSCALF

These variables are entered on one data card in the format \((F6.0,4I3,3F6.0)\).

\begin{array}{cccccccccccc}
| x,xxx & xx & xx & xx | xx & xx | x.x | x.x | x.x & x.x |
\end{array}
In addition, the normalized pressure distribution at all mesh points is read from punched cards via the BCREAD input routine. These cards are generated in subroutine PRESS of the ISGPM and are punched by the BCDUMP routine in column binary format.
APPENDIX D

PROGRAM LISTING

$IBFTC MAIN LIST, DECK
C
COMMENT--NUMERICAL SOLUTIONS OF CONVECTIVE INERTIA EFFECTS FOR AN
C IDEALIZED, RECTANGULAR, PARALLEL GROOVED PUMP-SEAL MODEL.
C
COMMON PSI(65,65), ZETA(65,65), NOPTX, NOPTY, NPTXM1, NPTYM1, DX, DY,
1 TOODXS, TOODYS, LAMBDAX, TWODX, TWODY, RE, XINDX, YINDX, PCT, IREF, JREF,
2 U(65,65), V(65,65), NOW, NOPLOT, FLUFLG, PARTSI, LMODXS, LMDYSQ, PROBNO
INTEGER FLUFLG
1 CALL SVCALC
IF (FLUFLG .EQ. 0) CALL PRESS
GO TO 1
END

$IBFTC CALCSV LIST, DECK
SUBROUTINE SVCALC
C
COMMENT--STREAM AND VORTICITY DISTRIBUTIONS.
C
COMMON PSI(65,65), ZETA(65,65), NOPTX, NOPTY, NPTXM1, NPTYM1, DX, DY,
1 TOODXS, TOODYS, LAMBDAX, TWODX, TWODY, RE, XINDX, YINDX, PCT, IREF, JREF,
2 U(65,65), V(65,65), NOW, NOPLOT, FLUFLG, PARTSI, LMODXS, LMDYSQ, PROBNO
DIMENSION PSIDUT(65,65)
REAL LAMBDAX, LMODXS, LMDYSQ, LMSDYS
INTEGER XINDX, YINDX, PROBNO, FLUFLG
LOGICAL JAIL
WRITE (6,100)
100 FORMAT (1H18X, 38HIDEALIZED SPIRAL GROOVED PUMPING SEAL, / )
75 READ (5,3) LAMBDAX, RE, OMEGAX, PCT, NOPTX, NOPTY, NOLINX, NOLINX, IREF, JREF
A , NOW, NOPLOT, PROBNO, FLUFLG
3 FORMAT (4F8.0,10I3)
NPTXM1 = NOPTX-1
NPTYM1 = NOPTY-1
210 XINDX = NPTXM1/(NOLINX-1)
YINDX = NPTYM1/(NOLINX-1)
NPTXP1 = NOPTX+1
NPTYYP1 = NOPTY + 1
DX = .1/FLOAT(NPTXM1)
DY = .1/FLOAT(NPTYM1)
DXSQ = DX * DX
DYSQ = DY * DY
TWODY = DY + DY
TWODX = DX + DX
LMODXS = LAMBDAX/DXSQ
LMDYSQ = LAMBDAX*DYSQ
LMSDYS = LAMBDAX*LMDYSQ
RE4DXY = RE/4.*DX/DY
PSIMLT = LMDYSQ*DXSQ/2.*(LMSDYS+DXSQ)
ZETMLT = LAMBDAX*PSIMLT

29
TOODYS = 2./DYSQ
TOOXS = 2./DXSQ
PARTSI = -2.*((LAMBDA/DXSQ + 1./LAMBDA/DYSQ)
PARTZE = PARTSI/LAMBDA
OMOPSI = OMEGA/PARTSI
OMOPZE = OMEGA/PARTZE
TERM = OMOPZE *RE4DXY
ONEMOM= 1.-OMEGA

C COMMENT--STREAM FUNCTION INITIAL DISTRIBUTION.
C CALL INDIST

C COMMENT--VORTECITY INITIAL DISTRIBUTION.
C CALL ZETAF

15 ITKONT = 0

C COMMENT--ITERATIVE SOLUTIONS.
C
17 ITKONT= ITKONT+1
JAIL= .FALSE.
IF (PRORNO .NE. 0) GO TO 21
CALL VORTWL
DO 20 J=2,NPTXM1
DO 20 I=2,NPTYM1
ZETA(I,J) = ZETA(I,J)*ONEMOM - OMOPZE*((ZETA(I,J+1)+ZETA(I,J-1))
1 DXSQ + (ZETA(I+1,J)+ZETA(I-1,J))/LMDYSQ)
20 IF (R.E. .GT. 0.)ZETA(I,J) = ZETA(I,J)-TERM*(((PSI(I,J+1)-PSI(I,J-1))
1 *(ZETA(I-1,J)-ZETA(I+1,J)) - (PSI(I-1,J)-PSI(I+1,J))*(ZETA(I,J+1)
2 -ZETA(I,J-1)))
21 DO 22 J=2,NPTXM1
DO 22 I=2,NPTYM1
PSIF= PSI(I,J)*ONEMOM - OMOPSI*((LMODXS*(PSI(I,J+1)+PSI(I,J-1)))+
1 (PSI(I+1,J)+PSI(I-1,J))/LMDYSQ + ZETA(I,J))
IF (JAIL) GO TO 22
23 IF (ABS((PSIF-PSI(I,J))/PSIF) .GT. PCT) JAIL = .TRUE.
22 PSI(I,J) = PSIF
IF (JAIL) GO TO 17
45 WRITE (6,46) LAMBDA,RE,OMEGA,PCT,ITKONT,NPTXM1,NPTYM1
46 FORMAT (9X,BHLAMBDAC =F6.3,5X,5HRE =E12.4,5X, 19HRELAXATION FACTOR
A=F5.2/9X,24HMAXIMUM RELATIVE ERROR =F9.6,5X,22HNUMBER OF ITERATION
BS =15/9X,37HNUMBER OF INCREMENTS IN X-DIRECTION =14,5X,16HNUMBER Y-CIR
CECTION =14/1HO8X,24HSTREAM FUNCTION X 10E+4./IX)

C COMMENT--STREAM FUNCTION AND VORTECITY OUTPUT.
C
DO 50 J=1,NOPTX
DO 50 I=1,NOPTY
50 PSIOUT(I,J)=PSI(I,J)*1.E+4
IF (NOW .NE. 0) PUNCH 49,((PSIOUT(I,J),I=1,NOPTY),J=1,NOPTX)
48 FORMAT(9F8.2)
DO 51 I=1,NOPTY,YINDX
51 WRITE (6,52) (PSIOUT(I,J),J=1,NOPTX,YINDX)
52 FORMAT(9X,15F8.2)
WRITE (6,64)
64 FORMAT(1H18X,10HVORTICITY,/IX)
DO 67 I=1,NOPTY,YINDX
67 WRITE (6,68) (ZETA(I,J),J=1,NOPTX,YINDX)
68 FORMAT(9X,15F8.3)
   IF (FLUFLG .EQ. 0) CALL UVFUNC
   RETURN
   END

$IBFTC DISTI LIST,DECK
SUBROUTINE DISTI
   COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
       1 TOODXS,TOODYS,LAMBDA,TOODX,TOODY,RE,XINDX,YINDX,PCT,IREF,JREF,
       2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBN0
   NPTXP1 = NOPTX+1
   NPTYP1 = NOPTY+1
   MIDPTX = NPTXP1/2
   DO 8 J = 1,MIDPTX
   PSITRM = -.2*FLOAT(J-1)*DX
   MAX = NPTXP1 - J
   DO 8 I = 1,NOPTY
   PSI(I,J) = PSITRM
   8 PSI(I,MAX) = PSITRM
   MIDPTY = NPTYP1/2
   JMIN = 0
   JMAX = NPTYP1
   DO 10 I = 1,MIDPTY
   PSITRM = -.2*FLOAT(I-1)*DY
   MAX = NPTYP1 - I
   JMIN = JMIN+1
   JMAX = JMAX - 1
   IF (JMIN .LE. JMAX) GO TO 9
   JMIN = NPTXP1/2
   JMAX = (NPTXP1 + 1)/2
   9 DO 10 J = JMIN,JMAX
   PSI(I,J) = PSITRM
   10 PSI(MAX,J) = PSITRM
   RETURN
   END

$IBFTC ZETAXY LIST,DECK
SUBROUTINE ZETAXY
   COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
       1 TOODXS,TOODYS,LAMBDA,TOODX,TOODY,RE,XINDX,YINDX,PCT,IREF,JREF,
       2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARTSI,LMODXS,LMDYSQ,PROBN0
   REAL LMODXS,LMDYSQ
   14 DO 16 J = 2,NPTXM1
   DO 16 I = 2,NPTYM1
   ZETA(I,J) = PARTSI*PSI(I,J) - (LMODXS*(PSI(I,J+1)+PSI(I,J-1)) +
       1(PSI(I+1,J)+PSI(I-1,J))/LMDYSQ)
   DO 12 J = 1,NOPTX,NPTXM1
   DO 12 I = 1,NOPTY,NPTYM1
   12 ZETA(I,J) = 0.0
   RETURN
   END

31
$IBFTC VRTWAL LIST,DECK
SUBROUTINE VORTWL
C
COMMENT--VORTICITY BOUNDARY VALUES.
C
COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARSI,LMODXS,LMDYSQ,PROBNC
REAL LAMBDA

DO 10 J=ZrNPTXM1
C
COPMENT--MOVING WALL.
C
ZETA1,J) = (PSI(I,J)-PSI(2,J)-DY)*TOODYS/LAMBDA
C
COMMENT--STATIONARY WALLS.
C
10 ZETA(NOPTY,J) = (PSI(NOPTY,J)-PSI(NPTYM1,J))*TOODYS/LAMBDA
DO 20 I=2,NPTYM1
ZETA(I,1) = (PSI(I,1)-PSI(I,2))*TOODXS*LAMBDA
20 ZETA(I,NOPTX) = (PSI(I,NOPTX)-PSI(I,NPTXM1))*TOODXS*LAMBDA
RETURN
END

$IBFTC FUNC UV LIST,DECK
SUBROUTINE UVFUNC
C
COMMENT -- U* AND V* VELOCITY DISTRIBUTIONS.
C
COMMON PSI(65,65),ZETA(65,65),NOPTX,NOPTY,NPTXM1,NPTYM1,DX,DY,
1 TOODXS,TOODYS,LAMBDA,TWODX,TWODY,RE,XINDX,YINDX,PCT,IREF,JREF,
2 U(65,65),V(65,65),NOW,NOPLOT,FLUFLG,PARSI,LMODXS,LMDYSQ,PROBNC
INTEGER XINDX,YINDX

101 DO 102 I=1,NOPTY
U(I,1)=0.0
V(I,1)=0.0
U(I,NOPTX)=0.0
102 V(I,NOPTX)=0.0
DO 104 J=1,NOPTX
U(I,J) = 1.0
V(I,J) = 0.0
U(NOPTY,J) = 0.0
104 V(NOPTY,J) = 0.0
DO 106 J=2,NPTXM1
DO 106 I=2,NPTYM1
U(I,J) = (PSI(I-1,J) - PSI(I+1,J))/TWODY
106 V(I,J) = -(PSI(I,J+1)-PSI(I,J-1))/TWODX
WRITE (6,1000)
1000 FORMAT(1H18X,20HU* VELOCITY PROFILE,/1X)
DO 1002 I=1,NOPTY,YINDX
1002 WRITE (6,1004) (U(I,J),J=1,NOPTX,YINDX)
1004 FORMAT(9X,15F8.4)
WRITE (6,1007)
1007 FORMAT(1H18X,20HV* VELOCITY PROFILE,/1X)
DO 1010 I=1,NOPTY,YINDX
1010 WRITE (6,1004) (V(I,J),J=1,NOPTX,YINDX)
RETURN
END

32
SUBROUTINE PRESS

COMMON PSI(65,65), ZETA(65,65), NOPTX, NOPTY, NPTXM1, NPTYM1, DX, DY,
1 TDODX, TDODY, LAMBDAX, TWODX, TWODY, RE, XINDEX, YINDEX, PCT, IREF, JREF,
2 U(65,65), V(65,65), NOW, NOPLOT, FLUFLG, PARTSI, LMODX, LMODY, PNBNO
DIMENSION XTERM(65,65), YTERM(65,65), P(65,65), PNBNO(65,65)
EQUIVALENCE (P, PSI), (PNORM, ZETA)
INTEGER XINDEX, YINDEX
INTEGER PNBNO
LOGICAL JAIL
REAL LAMBDAX
REAL RE

FOURDY = TWODY + TWODY
FOURDX = TWODX + TWODX
NPTXM2 = NPTXM1 - 1
NPTYM2 = NPTYM1 - 1
NPTXM3 = NPTXM2 - 1
NPTYM3 = NPTYM2 - 1

COMMENT -- INTERIOR POINTS.

CON2 = LAMBDAX/2.
CON3 = TDODX*FOURDY
CON4 = (LAMBDAX/DX)**2/2.
CON6 = TDODY*DY
CON7 = LAMBDAX**2/CON3

IF (RE * GT. 1.) GO TO 7
CON2 = CON2*RE
CON3 = CON3*RE
CON4 = CON4*RE
CON6 = CON6*RE
CON7 = CON7*RE
CON1 = -1.*LAMBDAX/FOURDY
CON5 = LAMBDAX/FOURDX

GO TO 8

7 CON1 = -1./RE/LAMBDAX/FOURDY
CON5 = LAMBDAX/RE/FOURDX

8 DO 10 I=2, NPTXM1
   DO 9 J=3, NPTXM1
      DIF1 = PSI(I-1, J+1) - PSI(I-1, J-1) + PSI(I+1, J-1) - PSI(I+1, J+1)
      9 XTERM(I, J) = DX*(CON1*(ZETA(I-1, J) - ZETA(I-1, J-1) + ZETA(I-1, J-1) - ZETA
                  A(I+1, J-1))*CON2*(V(I, J)*ZETA(I, J)+V(I, J-1)*ZETA(I, J-1)) - (U(I, J)*
                  B*DI1+U(I, J-1)*PSI(I-1, J)-PSI(I-1, J-2)+PSI(I+1, J-2)-PSI(I+1, J))/
                  CCONS3 + CON4*(V(I, J)*PSI(I, J+1)-2.*PSI(I, J)+PSI(I, J-1)+V(I, J-1)*
                  D*PSI(I, J-2)*PSI(I, J-1)+PSI(I, J-2))
   10 XTERM(I, 2) = XTERM(I, 3)
   DO 20 J=2, NPTXM1
   DO 19 I=2, NPTYM2
      19 YTERM(I, J) = DY*(CON5*(ZETA(I, J+1) - ZETA(I, J-1) + ZETA(I+1, J-1) - ZETA
               A(I+1, J-1))*CON2*(U(I, J)*ZETA(I, J)+U(I+1, J)*ZETA(I+1, J)) - (U(I, J)*
               B*DI1+U(I, J-1)*PSI(I-1, J)+PSI(I-1, J-2)+PSI(I+1, J+2)-PSI(I+1, J+1)+
               CJ+PSI(I+2, J))*CON6 + CON7*(V(I, J)*DIF1+V(I+1, J)*PSI(I, J+1)-PSI
               D(I, J-1)+PSI(I+2, J-1)-PSI(I+2, J+1))
   20 YTERM(NPTXM1, J) = YTERM(NPTYM2, J)

C
COMMENT -- VERTICAL BOUNDARY WALL POINTS.

C

CON3 = CON3/2.
CON5 = CON5/2.
CON7 = 2.*CON7
DO 30 I=2,NPTYM1
XTERM(I,1) = DX*(CON1*(ZETA(I-1,1)-ZETA(I+1,1)+ZETA(I-1,2)-ZETA(I+1,2)) + CON2*(ZETA(I,1)+V(I,1)+ZETA(I,2)+V(I,2)) - U(I,1)+B * (PSI(I-1,2)-PSI(I-1,1)-PSI(I+1,2)+PSI(I+1,1))+U(I,2)*(PSI(I-1,1,3)
C -2.*PSI(I-1,2)+PSI(I+1,2)-PSI(I+1,3))/CON3 + CON4*(V(I,1)+PSI(I,1,3))
D -2.*PSI(I,2)+PSI(I,1,1)+V(I,2)*(PSI(I,4)-2.*PSI(I,3)+PSI(I,2,1)))
30 XTERM(I,NOPTX) = DX*(CON1*(ZETA(I-1,NOPTX)-ZETA(I+1,NOPTX)+ZETA(I,NPTXM1)-(I+1,NOPTX)+ZETA(I,NPTXM1)-ZETA(I+1,NOPTX)) + CON2*(ZETA(I,NOPTX)+V(I,NOPTX)+B ZETA(I,NPTXM1)-V(I,NPTXM1)) -U(I,NOPTX)*(PSI(I-1,NOPTX)-PSI(I+1,1),NOPTX)+C NPTXM1)+PSI(I+1,1,NOPTX)-PSI(I+1,NOPTX)+U(I,NOPTX)*((ZETA(I,1,1)-NOPTX)) /CON3 + E + CON4*(V(I,NOPTX)+PSI(I,NOPTX)-2.*PSI(I,NPTXM1)+PSI(I,NPTXM2))
F + V(I,NPTXM1)*(PSI(I,NPTXM1)-2.*PSI(I,NPTXM2)+PSI(I,NPTXM3)))
DO 40 I=2,NPTYM2
YTERM(I,1) = DY*(CON5*(ZETA(I,2)-ZETA(I,1))+ZETA(I+2,1)-ZETA(I,1))
A - CON2*(ZETA(I,1)+U(I,1)+ZETA(I+1,1)+U(I+1,1)) - (U(I,1)+B *(I-1,1)-2.*PSI(I,1)+PSI(I,1,1)+U(I,1)+C PSI(I+1,1))/(CON6 + CON7*(V(I,1)+PSI(I-1,2)-PSI(I,1,1)+PSI(I+1,1)
D - PSI(I+1,2)+V(I,1,1))+(PSI(I,2)-PSI(I,1)+PSI(I+2,1)-PSI(I+2,2))
E )
40 YTERM(I,NOPTX) = DY*(CON5*(ZETA(I,1,NOPTX)-ZETA(I,NOPTX)+ZETA(I,1,1),NOPTX)) - CON2*(ZETA(I,NOPTX)+U(I,NOPTX)+ZETA(I+1
B ,NOPTX)+U(I+1,NOPTX)) - (U(I,NOPTX)+PSI(I-1,NOPTX)-2.*PSI(I,1)
C NOPTX)+PSI(I+1,NOPTX)+U(I+1,NOPTX)+PSI(I,NOPTX)-2.*PSI(I,1)
D NOPTX)+PSI(I+2,NOPTX)) /CON6 + CON7*(V(I,NOPTX)+PSI(I-1,NOPTX)+E PSI(I-1,NOPTX)+PSI(I+1,NPTXM1)-PSI(I+1,NOPTX)+V(I+1,NOPTX)*PSI
F (I,NOPTX)=PSI(I,NPTXM1)+PSI(I+1,NPTXM1)-PSI(I+2,NPTXM1)) -PSI(I+2,NOPTX))

C

YTERM(NPTYM1,1) = YTERM(NPTYM2,1)
YTERM(NPTYM1,NOPTX) = YTERM(NPTYM2,NOPTX)

C

COMMENT -- HORIZONTAL BOUNDARY WALL POINTS.

C

IF(PROBNO .NE. 0) GO TO 49
ZETA(1,1) = -2./DY/LAMBDAL
ZETA(1,NOPTX) = ZETA(1,1)
49 CON1 = 2.*CON1
CON5 = CON5/2.
DO 50 J=3,NPTYMX1
XTERM(1,J) = DX*(CON1*(ZETA(1,J)-ZETA(2,J)+ZETA(1,J-1)-ZETA(2,J-1)) + CON2*(ZETA(1,1)+V(1,1)+ZETA(1,1)+V(1,1-1)) + (U(1,1)+B *(1,J)-2.*PSI(1,J)+PSI(1,J-1)-PSI(2,J-1)+V(1,J-1)) + (U(1,1)+C PSI(1,J)-PSI(1,J-1)+V(1,J-1)) + (PSI(1,J-1)-2.*PSI(1,J)+PSI(1,J-2))
D + PSI(1,J-1)+V(1,J-1)+PSI(1,J-1)+V(1,J-2)) /CON3 + CON4*(V(1,1)+PSI(1,J-1)-2.*PSI(1,J-2)+PSI(1,J-3))
50 XTERM(1,NOPTY) = DX*(CON1*(ZETA(NPTYM1,1,J)-ZETA(NOPTY,J)+ZETA(A
B NPTYM1,J-1)-ZETA(NOPTY,J-1)+CON2*(ZETA(NOPTY,J)+V(NOPTY,J)+ZETA
B (NOPTY,J-1)+V(NOPTY,J-1)) - U(NOPTY,J)*PSI(NPTYM1,J+1)-PSI(C
NPTYM1,J-1)+PSI(NOPTY,J-1)-PSI(NOPTY,J+1)+U(NOPTY,J)*PSI(D
NPTYM1,J)-PSI(NPTYM1,J-2)+PSI(NOPTY,J-2)-PSI(NOPTY,J))/CON3+CCN4
E + V(NOPTY,J)+PSI(NOPTY,J)+V(NOPTY,J)-2.*PSI(NOPTY,J)+PSI(NOPTY,J-1)+V(F
OPTY,J-1)+PSI(NOPTY,J-1)-2.*PSI(NOPTY,J-2)+PSI(NOPTY,J-3))
XTERM(1,2) = XTERM(1,3)
XTERM(1,2) = XTERM(1,3)
DO 60 J=2,NPTYMX1
YTERM(1,J) = DY*(CON5*(ZETA(1,J)+ZETA(1,J-1)-ZETA(2,J+1)-ZETA(2
A J-1))-CON2*
I
E NPTYM2,1)+PSI(NPTYM1,1)-PSI(NPTYM1,2))
DIF1= PSI(NPTYM1,NOPTX)-PSI(NPTYM1,NPTXM1)+PSI(NOPTY,NPTYM1)-PSI
A (NOPTY,NOPTX)
XTERM(NOPTY,NOPTX)= DX*(CON1*(ZETA(NPTYM1,NOPTX)-ZETA(NOPTY,NOPTX)
A +ZETA(NPTYM1,NPTXM1)-ZETA(NOPTY,NPTX))+CON2*(ZETA(NOPTY,NOPTX)
B )-V(NOPTY,NOPTX)+ZETA(NOPTY,NPTXM1)*V(NOPTY,NPTXM1)-(U(NOPTY,NOPTX
C )-DIF1+U(NOPTY,NPTXM1)*(PSI(NPTYM1,NPTXM1)-PSI(NPTYM1,NPTXM2)+PSI
D (NOPTY,NPTXM2)-PSI(NOPTY,NPTYM1))/CON3*CON4*(V(NOPTY,NOPTX)*(PSI
E (NOPTY,NOPTX)-2.*PSI(NOPTY,NPTXM1)+PSI(NOPTY,NPTXM2))+V(NOPTY,
F NPTXM1)*(PSI(NOPTY,NPTXM1)-2.*PSI(NOPTY,NPTXM2)+PSI(NOPTY,NPTYM1)
G )
)
YTERM(NOPTY,NOPTX)= DY*(CON5*(ZETA(NOPTY,NOPTX)-ZETA(NOPTY,NPTXM1
A )+ZETA(NPTYM1,NOPTX)-ZETA(NOPTY,NPTXM1))-CON2*(ZETA(NOPTY,NOPTX)
B )*(U(NOPTY,NOPTX)-ZETA(NPTYM1,NOPTX)*U(NPTYM1,NOPTX)-(U(NOPTY,
C NOPTX)*(PSI(NPTYM2,NOPTX)-2.*PSI(NPTYM1,NOPTX)+PSI(NOPTY,NOPTX))+
D U(NPTYM1,NOPTX)*(PSI(NPTYM3,NOPTX)-2.*PSI(NPTYM2,NOPTX)+PSI(NPTYM1,
E NOPTX)))/CON6*CON7*(V(NOPTY,NOPTX)*DIF1+V(NPTYM1,NOPTX)*
F (PSI(NPTYM2,NOPTX)-PSI(NPTYM2,NPTXM1)+PSI(NPTYM1,NPTXM1)-PSI(NPTYM1,
G NPTYM1))
)
C COMMENT--INITIALIZE PRESSURE FIELD.
C
DO 200 J=1,NOPTX
DO 200 I=1,NPTYM1
200 P(I,J)= 1.0
 ITKONT= 0
202 ITKONT= ITKONT+1
 JAIL= .FALSE.
 DO 210 I=1,NPTYM2
 DO 210 J=1,NOPTX
204 IF (J .GT. 2) GO TO 205
 PX= P(I,J+1)-XTERM(I,J)
 GO TO 207
205 PX= P(I,J-1)+XTERM(I,J)
207 PNEW= 0.5*(PX+P(I+1,J)+YTERM(I,J))
 IF (JAIL) GO TO 209
 IF (ABS((PNEW-P(I,J))/PNEW) .GT. PCT) JAIL = .TRUE.
209 P(I,J) = PNEW
210 CONTINUE
 DO 220 I=NPTYM1,NOPTY
 DO 220 J=1,NOPTX
214 IF (J .GT. 2) GO TO 215
 PX = P(I,J+1)-XTERM(I,J)
 GO TO 217
215 PX = P(I,J-1)+XTERM(I,J)
217 PNEW = 0.5*(PX+P(I-1,J)-YTERM(I,J))
 IF (JAIL) GO TO 219
 IF (ABS((PNEW-P(I,J))/PNEW) .GT. PCT) JAIL = .TRUE.
219 P(I,J) = PNEW
220 CONTINUE
 IF (.NOT. JAIL) GO TO 229
 IF(ITKONT .LT. 500) GO TO 202
 WRITE (6,225)
225 FORMAT(1H14X,5OH****NO CONVERGED SOLUTION IN 500 ITERATIONS.****)
 A)
229 WRITE (6,230)
230 FORMAT(1H1H8X,18HPRESSURE FIELD, P*)
 IF(RE .LE. 1.) WRITE (6,231)
231 FORMAT(1H1H26X,1H*)
 WRITE (6,232) ITKONT
36
232 FORMAT(1H08X, 19H NO. OF ITERATIONS = I4/1X)
DO 235 I = 1, NOPTY, YINDX
235 WRITE (6, 106) (P(I, J), J = 1, NOPTX, XINDX)
106 FORMAT(9X, 15F8.4)
IF (JAIL) GO TO 202
DO 240 J = 1, NOPTX
DO 240 I = 1, NOPTY
240 PNORM(I, J) = P(I, J) - PREF(J, JREF)
WRITE (6, 241)
241 FORMAT(1H18X, 26H NORMALIZED PRESSURE FIELD, /1X)
DO 245 I = 1, NOPTX, YINDX
245 WRITE (6, 106) (PNORM(I, J), J = 1, NOPTX, XINDX)
DX03 = DX / 3.
NPTXPI = NOPTX + 1
DO 250 I = 1, NOPTX
II = NPTXPI - I
IF (PNORM(I, II) .LT. 0.) GO TO 252
250 CONTINUE
252 II = II + 1
LEREG = 0.
IF (MOD(II, 2) .NE. 0) GO TO 255
LEREG = (PNORM(1, II) + PNORM(1, II + 1)) * DX / 2.
II = II + 1
255 IF (II .EQ. NOPTX) GO TO 263
DO 260 J = 1, NPTXM2, 2
260 LEREG = (PNORM(1, J) + 4. * PNORM(1, J + 1) + PNORM(1, J + 2)) * DX03 + LEREG
263 WHSURF = 0.
DO 265 J = 1, NPTXM2, 2
265 WHSURF = WHSURF + (PNORM(1, J) + 4. * PNORM(1, J + 1) + PNORM(1, J + 2)) * DX03
WRITE (6, 505) LEREG, WHSURF
505 FORMAT(1HK8X, 35H NET FORCE FOR LEADING EDGE REGION = 1PE13.5/23X, 15H
AWHOLE SURFACE = E13.5)
IF (NOPLT .EQ. 0) RETURN
DO 333 J = 1, NOPTX
333 CALL BCDUMP(PNORM(1, J), PNORM(NOPTY, J), 1)
RETURN
END
DIMENSION PSI(65,65),W(65,65),DPSIDY(65,65),DPSIDX(65,65)
A WPCT(65,65)
REAL LAMBDA
INTEGER XINDX,YINDX
LOGICAL INDKT
READ (5,3) LAMBDA,RE,OMEGA,PCT,NOPTY,NOPTX,NOLINY,NOLINX
3 FORMAT (4F6.0,4I3)
NPTXM1 = NOPTX-1
NPTYM1 = NOPTY-1
XINDX = NPTXM1/(NOLINX-1)
YINDX = NPTYM1/(NOLINY-1)
BSQDSQ= 1./LAMBDA**2
DX = 1./FLOAT(NPTXM1)
DY = 1./FLOAT(NPTYM1)
DXSQ= DX*DX
DYSQ= DY*DY
TWODX= DX+DX
TWODY= DY+DY
WPF = 2.*((DXSQ + DYSQ)/(OMOWPF*OMOWPF))/RE
ONMOM = 1. - OMEGA
OMOWPF= OMEGA/WPF

COMMENT -- Set up stream function distribution
READ (5,5) ((PSI(I,J) ,I=1,NOPTY),J=1,NOPTX)
5 FORMAT (9F8.0)
DO 8 J=1,NOPTX
DO 8 I=1,NOPTY
8 PSI(I,J)= PSI(I,J)-1.E-4
DO 12 J=2,NPTXM1
DO 12 I=2,NPTYM1
DPSIDY(I,J)= (PSI(I-1,J)-PSI(I+1,J))/TWODY
12 DPSIDX(I,J)= (PSI(I,J+1)-PSI(I,J-1))/TWODX

C
COMMENT -- INITIALIZE W* FIELD
C
     DO 14 J=2,NPTXM1
     W(NOPTY,J) = 0.0
     DO 14 I=1,NPTYM1
    14 W(I,J) = -1.0
     W(1,1) = -1.0
     W(I,NOPTX) = -1.0
     DO 16 I=2,NOPTY
     W(I,1) = 0.0
    16 W(I,NOPTX) = 0.0
    17 WRITE (6,28)
    28 FORMAT(1H1)
    29 READ (5,181 DPDZ
    18 FORMAT( F8.0)
     ITKONT = 0
C
COMMENT -- ITERATE W* FIELD FOR GIVEN DP/DZ VALUE.
C
20 INDKT = .FALSE.
     ITKONT = ITKONT + 1
     DO 22 I=2,NPTYM1
     DO 22 J=2,NPTXM1
     WF = DPSIDX(I,J)*(W(I,J+1)-W(I,J-1))/TWDX + DPSIDX(I,J)*(W(I-1,J)
A=W(I+1,J))/TWODY+DPDZ-((W(I,J+1)+W(I,J-1))/DXSQ+BSQDSQ*(W(I+1,J)+
BW(I-1,J)))/DYSQ)/RE
     WNEW = ONMOM*W(I,J) - OMOWPF*WF
     WPCT(I,J) = (W(I,J)-WNEW)/WNEW
     IF (INDKT) GO TO 22
    25 IF (ABS(WPCT(I,J)) .GT. PCT) INDKT = .TRUE.
    22 W(I,J) = WNEW
     IF (INDKT) GO TO 20
C
COMMENT -- CALCULATE Q,NET FROM CONVERGED W* FIELD.
C
QNET = 0.
     DO 60 I=2,NPTYM1,2
     DO 60 J=2,NPTXM1,2
A+W(I-1,J-1)+W(I-1,J+1)+W(I+1,J-1)+W(I+1,J+1)
     QNET = QNET*DX*DY/9.
C
COMMENT -- OUTPUT
C
     WRITE (6,26) LAMBDA,RE,OMEGA,PCT,ITKONT,NPTXM1,NPTYM1,DPDZ,QNET
    26 FORMAT( 9X,8HLAMBDA =F6.3,5X,4HRE =F7.1,5X,19HRELAXATION FACTOR
A=F5.2/9X,24HMAXIMUM RELATIVE ERROR =F6.3,5X,22HNUMBER OF ITERATION
BS=F5.2/9X,37HNUMBER OF INCREMENTS IN X-DIRECTION =I4,5X,16HIN Y-DIR
CECTION =I4/9X,9HDOP/DZ* =F7.3,6X,6HQ*,Z = 1PE13.5/1H08X,20HVELO
DCITY PROFILE,1X)
     DO 30 I=1,NOPTY,YINDEX
    30 WRITE (6,33) (W(I,J),J=1,NOPTX,XINDEX)
    33 FORMAT(1X,15F8.4)
     GO TO 17
END
$IBFTC$ GRAPH LIST, DECK
COMM LAMBA, NOPTX, NOPLS, NOYPLS, DY, DX, PNORM
COMMON /SKAF, /XSCALF, YSCALF, ZSCALF
DIMENSION Z(13000), PNORM(65, 65)
EXTERNAL PSURF
REAL LAMBDA
READ (5, 4) LAMBDA, NOPTX, NOYPLS, NOPLS, PNORM
4 FORMAT (F6.0, 4I3, 3F6.0)
ZSCALF = PNORM
DY = 1./FLOAT(NOPTY - 1)
DX = 1./FLOAT(NOPTX - 1)
NOPTS = 3.(NOPTX*NOYPLS+NOPTY*NOXPLS)
DO 10 J = 1, NOPTX
10 CALL BCREAD(PNORM(I, J), PNORM(NOPTY, J))
CALL PLOT3D (0., 0., LAMBDA, Z, NOPTX, NOYPLS, NOXPLS, NOPTY, PSURF,
A, TRUE.)
CALL ROTATE (0., 0., 45., .FALSE.)
CALL ROTATE (0., 35., 0., TRUE.)
STOP
END

$IBFTC$ SURFAC LIST, DECK
FUNCTION PSURF(I, J)
COMMON LAMBA, NOPTX, NOYPLS, NOPLS, DX, PNORM(65, 65)
II = NOPTY + 1 – I
PSURF = PNORM(I, J)
RETURN
END

$IBFTC$ SURFAC LIST, DECK
FUNCTION PSURF(I, J)
COMMON LAMBA, NOPTX, NOYPLS, NOPLS, DX, PNORM(65, 65)
II = NOPTY + 1 – I
PSURF = PNORM(I, J)
RETURN
END

ENTRY BCREAD
BCREAD SAVE 1, 4
CLA 3, 4
LQ 5, 4
TLQ **2
XCA
STQ TEMP
ADD SYSONE
STA STO
STA TEMP
STA IX1
ANC UN5-3, 4
STA SYSLOC, 4
CALL ..RCRD
READ TSX, ..FLOC, 4
TSX ..FTCK, 4
IX1 AXT **1, 4
IX4 AXT **1, 4
SST CLA ..4POLB, 4
STO STO **1
TXI TXI **1, 1, -1
$1842  TUX  STD-1,4,-22  CR. REC COUNT
  TPA  READ  NO SAVE REMAINING COUNT
  LSTC  TXL  DONE,1,0  ANY MORE WORDS
  AXT  DONE,4  YES STORE TO EXIT NEXT
  SXA  LASTC-1,4  TIME
  SCO  CKIP4,1  SET REC CNT = NO WORDS LEFT
  TRA  IX4  GO PROCESS RECORD
  AXT  READ,4  RESTORE EXIT
  SXA  LASTC-1,4  RESTORE REC CNT
  AXT  ??,4  ADD OF UNIT 5
  SXA  CKIP4,4
  RETURN  BCREAD
  PZE  JUMP.
  #LOAD  END

$1842  BCRWO
ENTRY  ...PROC
ENTRY  ...BCRD
ENTRY  ...GCDW
SIZE  SET  ?9  RECORD SIZE
...RRC  RLA  PTH  READ ENTRY FOR BCRREAD
  TPA  #+2  GET CORRECT ARG FOR .FINDS
...357D  CAL  PTH  WRITE ENTRY FOR BCDUMP
  SXA  LK,DR,4  SAVE IR4
  CALL  ...FIND(SFL)  SET UP READ OR WRITE
  ORA  #=-1
  SEL  FORT  ...APOB,SIZE  I/O COMMAND
  LXA  LK,DR,4  RESTORF 4
  TRA  1,4
  P7N  P7N  0,0
  DT4  DT4  0,0
  ...APOB  BSS  SIZE  I/O BUFFER
  LK,DR  LOAD  END
APPENDIX E

SAMPLE PROBLEM

A sample problem, which was solved on the Lewis IBM 7044-7094 direct-couple system, is included to show the user how to set up the input data cards and to show what output can be expected. For this problem LAMBDA = 1 (square groove), RE = 100, OMEGA = 1.3, PCT = 0.001, NOPTY and NOPTX = 29, and NOLINY and NOLINX = 15. The reference mesh point for the normalized pressure field calculations is (29, 15), and the punched output for the w*-velocity calculations and normalized pressure field plot is required. The control words PROBNO and FLUFLG are zero in this problem. Results for this case are discussed in reference 1. The input data card for the ISGPSM program should be punched as

```
1 8|9 16|17 24|25 32|35 38|41 44|47 50|53|56|59|62|63|72|73|80
| 1.000 100.0 1.30 0.001 29 29 15|15 29 15 1 1 0 0
```

The printed output for this problem is shown in table I. The number of iterations printed in table I(a) is the number of iterations required for convergence of the stream function to within the designated relative change. It should be noted that the values of the stream function have been multiplied by $10^4$ before printout to facilitate writing the output format of the stream function and to present more significant figures in the individual values. In table I(b) the values of the vorticity at the four corners are those calculated from the equations for the vertical stationary walls. If the values at the four corners based on the equations for horizontal walls are desired, they are $\xi^* = 0$ for the lower stationary wall and $\xi^* = -2/\lambda \Delta y^*$ for the upper moving wall. The static pressure field is printed in table I(e) along with the number of iterations to convergence to within the designated relative change. The relative change for convergence of the pressure field is equal to that of the stream function. The normalizing factor for this particular problem is 1.0025 and is the entry found in the eighth column of the bottom row. This number is used to calculate the normalized pressure field distribution (table I(f)). Also shown in table I(f) is the leading edge force and net force acting on the moving surface.

Using the punched output of the stream function from the ISGPSM program, the w*-field and $Q_z^*$ can now be calculated using the WFIELD program. The additional data
### TABLE I - SAMPLE PROBLEM OUTPUT

(a) Stream function

<table>
<thead>
<tr>
<th>I</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

(b) Vorticity

<table>
<thead>
<tr>
<th>I</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

(c) U-Velocity profile

<table>
<thead>
<tr>
<th>I</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE I. SAMPLE PROBLEM OUTPUT

#### (d) v*-Velocity profile

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

#### (e) Pressure field

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

#### (f) Normalized pressure field

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
required are that $\text{LAMBDA} = 1$, $\text{RE} = 100$, $\text{PCT} = 1.30$, $\text{NOPTY} = 29$, $\text{NOPTX} = 29$, $\text{NOLINY} = 15$, $\text{NOLINX} = 15$, and $\partial \text{P}^*/\partial z^* = -0.115$. The first \textit{WFIELD} program input data card is punched as

```
1  6  7  12 13  18  19  24  25  27  28  30  31  33  34  36  37
1.000 100.0 1.30  .001  29  29  15  15
```

This card is followed by the deck of cards containing the stream function values of \textit{ISGPSM} and the card with the $\partial \text{P}^*/\partial z^*$ value.

If more $\partial \text{P}^*/\partial z^*$ values are to follow for a given problem, they must be punched one to a card according to this last format.

Table II contains the one page of output per $\partial \text{P}^*/\partial z^*$ value from the \textit{WFIELD} program. The number of iterations printed is the number of iterations of the $w^*$-velocity profile from initial value to converged value within the prescribed maximum relative error. The net volume flow $Q_z^*$ associated with the $\partial \text{P}^*/\partial z^*$ value of -0.115 in this problem is 0.0003128 and is printed on the same line as the $\partial \text{P}^*/\partial z^*$ value.

The computer execution time on the Lewis Computer for this sample problem was approximately a half a minute for each program.

### Table II - Sample Program Output for WFIELD

<table>
<thead>
<tr>
<th>$\partial \text{P}^<em>/\partial z^</em>$</th>
<th>$-1.311$</th>
<th>$-1.313$</th>
<th>$-1.315$</th>
<th>$-1.317$</th>
<th>$-1.319$</th>
<th>$-1.321$</th>
<th>$-1.323$</th>
<th>$-1.325$</th>
<th>$-1.327$</th>
<th>$-1.329$</th>
<th>$-1.331$</th>
<th>$-1.333$</th>
<th>$-1.335$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$-0.335$</td>
<td>$-0.337$</td>
<td>$-0.339$</td>
<td>$-0.341$</td>
<td>$-0.343$</td>
<td>$-0.345$</td>
<td>$-0.347$</td>
<td>$-0.349$</td>
<td>$-0.351$</td>
<td>$-0.353$</td>
<td>$-0.355$</td>
<td>$-0.357$</td>
<td>$-0.359$</td>
</tr>
<tr>
<td>0.3</td>
<td>$-1.511$</td>
<td>$-1.517$</td>
<td>$-1.523$</td>
<td>$-1.529$</td>
<td>$-1.535$</td>
<td>$-1.541$</td>
<td>$-1.547$</td>
<td>$-1.553$</td>
<td>$-1.559$</td>
<td>$-1.565$</td>
<td>$-1.571$</td>
<td>$-1.577$</td>
<td>$-1.583$</td>
</tr>
<tr>
<td>0.7</td>
<td>$-3.881$</td>
<td>$-4.004$</td>
<td>$-4.128$</td>
<td>$-4.252$</td>
<td>$-4.376$</td>
<td>$-4.500$</td>
<td>$-4.624$</td>
<td>$-4.748$</td>
<td>$-4.872$</td>
<td>$-4.996$</td>
<td>$-5.120$</td>
<td>$-5.244$</td>
<td>$-5.368$</td>
</tr>
<tr>
<td>0.9</td>
<td>$-5.065$</td>
<td>$-5.189$</td>
<td>$-5.313$</td>
<td>$-5.437$</td>
<td>$-5.561$</td>
<td>$-5.685$</td>
<td>$-5.809$</td>
<td>$-5.933$</td>
<td>$-6.057$</td>
<td>$-6.181$</td>
<td>$-6.305$</td>
<td>$-6.429$</td>
<td>$-6.553$</td>
</tr>
</tbody>
</table>

46
The three-dimensional plot of the normalized pressure distribution is obtained from program GRAPH using the column binary punched output of the pressure distribution from the ISGPSM program. The additional information required is that LAMBDA = 1; NOPTY = 29, NOPTX = 29, NOYPLS = 15, NOXPLS = 15, and XCALF, YSCALF, PSCALF each equal 6.0. The first program input data card is punched as

```
<table>
<thead>
<tr>
<th>1   6</th>
<th>7   9</th>
<th>10</th>
<th>12</th>
<th>13</th>
<th>15</th>
<th>16</th>
<th>18</th>
<th>19</th>
<th>24</th>
<th>25</th>
<th>30</th>
<th>31</th>
<th>36</th>
<th>37</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>29</td>
<td>29</td>
<td>15</td>
<td>15</td>
<td>6.00</td>
<td>6.00</td>
<td>6.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

After this is the deck of cards containing the pressure distribution values. The output for this particular problem is the graph found in figure 15. The maximum value of $P^* = 1.2501$ is found at the corner where the moving wall meets the leading edge and the minimum value of $P^* = -0.3304$ is near the corner formed by the moving wall and the trailing edge. Note also the shallow pressure drop or relative minimum in the vicinity of the vortex center.
REFERENCES


Figure 1. - Spiral groove pumping seal model for limiting case of land clearance.

Figure 2. - Streamlines in groove cross flow plane. \( i \) designates vortex center. 

\[
\psi = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = +1
\]

\[
(0,1) \quad (1,1) \quad (i,j-1) \quad (i,j+1) \quad (i+1,j)
\]

\[
\psi = 0, \quad \frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0
\]

Figure 3. - Mesh point representation of \( x - y \) plane flow field.

Figure 4. - MAIN program.

- Call SVCALC
- Calculate stream function, vorticity, and \( u^* \) and \( v^* \) velocity profiles

- FLUFLG - 0?
  - No
  - Call PRESS
    - Pressure calculations
  - Yes

\[ 49 \]
Figure 5. - Stream function and vorticity calculations.
Figure 5. - Stream function initial distribution.

Figure 6. - Vorticity boundary wall values.

Figure 7. - Vorticity initial distribution.

Figure 9. - u and v velocity profiles.
Subroutine PNESS

Calculate constant terms

Coefficient terms CON1,2, CON3, CON4, CON6, CON7 for RE > 1

Yes

RE > 1?

No

Coefficients CON1 and CON5 for RE > 1

Adjust CO12, CO13, CON4, CON6, CON7 for RE ≤ 1

Correct CON1 and CON5 for horizontal boundary walls

Vorticity at upper corners for moving wall values

Correct CON1 and CON5 for horizontal boundary walls

Partial ∂P/∂x and ∂P/∂y constant terms at upper corners

Vorticity at upper corners for stationary wall values

Complete ∂P/∂x and ∂P/∂y constant terms at upper corners

No

PROBNO = 0?

Yes

∂P/∂x and ∂P/∂y constant terms at lower corners

Initialize P'' - FIELD = 1, Iteration counter (ITKONT) = 0

ITKONT = ITKONT + 1, J:IL = .FALSE.,

For all mesh points (i, j)

P'' - V-VALUE (PNEW at each mesh point

J:IL = .TRUE.

Yes

No

PNEW(1, J) > PCT?

Yes

JAIL = .TRUE.

No

PNENW(1, J) = PNEW

PNENW(1, J) = PNEW

JAIL = .TRUE.

No

ITKONT < 500?

Yes

No

Figure 10. - Pressure field.
Figure 10. - Concluded.
Figure 11. - $w^*$-velocity profile and $Q_{ind}$ calculation.
GetAPH program

Read in program variables

Calculate program constants

CALL SCREAP

Read in normalized pressure field

Call PLOT3D

Initialize three-dimensional plotting routine

CALL ROTATE

Rotate data 45° about P-axis

CALL ROTATE

Rotate data 35° about y-axis and draw figure

STOP

PSURF function

Calculate P°-value as function of x, y, i, j position

RETURN

Figure 12. - Three-dimension pressure plot.
Figure 13. - Finite difference approximation of the boundary points.

Figure 14. - Overlay structure of spiral grooved pumping seal model programs.
Figure 15. Calcomp plot of normalized static pressure distribution due to cross flow in a square groove. Reynolds number, $Re = 100$. 

$P_{\text{MIN}} = -0.33$

$P_{\text{MAX}} = 1.25$
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— National Aeronautics and Space Act of 1958

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