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X-641-71-90
PREPRINT

NASA TM X-65466

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MARCH 1971

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NASA STI FACILITY
INPUT BRANCH
MAR 1971

GSFC

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

FACILITY FORM 602

N71-20468 (ACCESSION NUMBER)	(THRU)
9 (PAGES)	63 (CODE)
TMX 65466 (NASA CR OR TMX OR AD NUMBER)	29 (CATEGORY)

Shell Splitting Due to Electric Fields

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Drift shells of trapped particles starting from a single field line in the earth's magnetic field may become separated due to an electric field. Using the formalism of the shell splitting function, this separation is calculated for an arbitrary curl-free time-independent electric field stisfying $\underline{E}\cdot\underline{B} = 0$, with no limitation on the particle's longitudinal excursion. The effect of non-dipole field components is also taken into account to the lowest order.

In a dipole field trapped particles starting from a common field line with different values of the mirror-point field intensity B_m will, in general, share the same drift shell. If the field suffers an asymmetric perturbation, this no longer occurs and the drift shells of particles with different B_m will separate as the distance from the starting field line increases. This effect is commonly known as shell splitting . It is of interest, since in conjunction with pitch-angle scattering it may transport particles across field lines.

Electric fields may cause shell splitting; in a recent article by Roederer and Schulz [1971] , shell splitting due to a certain choice of electric field was derived, for particles with small longitudinal excursions (i.e. near-equatorial particles) in a dipole magnetic field.

Roederer and Schulz derived their result more or less from first principles, but a systematic approach also exists, based on the concept of the shell splitting function [Stern, 1968, henceforth referred to as A] . This approach will be used here to generalize the preceding result and to derive shell splitting due to a more general electric field. The calculation is not restricted to near-equatorial orbits and can be readily generalized for magnetic fields that deviate somewhat from the dipole configuration.

SHELLS IN A DIPOLE FIELD

Consider first only the dipole component $\underline{B}_0$ of the earth's magnetic field; quantities pertaining to this component will henceforth be distinguished by subscript zero. This field may be described by Euler potentials (α_0, β_0)

$$\underline{B}_0 = \nabla\alpha_0 \times \nabla\beta_0 \qquad (1)$$

[Stern, 1970, and references listed there], with

$$\alpha_0 = a\, g_1^0\, (a/r)\ \sin^2\theta \qquad (2)$$

$$\beta_0 = a\varphi \qquad (3)$$

Here a is the earth's radius, g_1^0 is the axial dipole coefficient of the scalar magnetic potential and (r, θ, φ) are spherical dipole coordinates [Chapman, 1963], so that g_1^0 describes the entire dipole component of the field.

In the absence of electric fields the momentum p is conserved and the longitudinal invariant J becomes

$$J = 2\,p\,I$$

$$= 2\,p \int_{B'_m}^{B_m} \left\{ 1 - B_0/B_m \right\}^{\frac{1}{2}} ds \qquad (4)$$

with the integration carried along a field line. To facilitate this integration in a dipole field one may express the field intensity as

$$B_0 = B_0(\alpha_0, \theta) \qquad (5)$$

(β_0 does not enter due to axial symmetry). Then I is given by a function $I_0(\alpha_0, B_m)$, defined through

$$I_0(\alpha_0, B_m) = \int_{\pi-\theta_m}^{\theta_m} \left\{ 1 - B_0(\alpha_0, \theta)/B_m \right\}^{\frac{1}{2}} (\partial s/\partial\theta) \, d\theta \qquad (6)$$

where s is differentiated with α_0 held constant and where θ_m is the colatitude associated with B_m (for the actual use of I_0 and similar functions, see Stern, 1967a). The equation of the drift shell associated with a given pair of values of (I, B_m) is then

$$I = I_0(\alpha_0, B_m) \qquad (7)$$

and this may be inverted to

$$\alpha_0 = G_0(I, B_m) \qquad (8)$$

Note that this is a particular case of the general equation

$$f(\alpha_0, \beta_0, I, B_m) = 0 \qquad (9)$$

of a two-parameter family of surfaces that are everywhere orthogonal to $\underline{B}_0$ (β_0 is absent in a dipole field due to symmetry). Instead of using the parameters (I, B_m) one can characterize this family by any two independent functions of these parameters. In particular, if G_0 is one of these functions, the shell equation depends only on one parameter, namely G_0 . Thus the function G_0 is closely related to McIlwain's shell parameter L McIlwain, 1961 and in fact

$$L(I, B_m) = a\, g_1^0 / G_0 \qquad (10)$$

SHELL SPLITTING BY AN ELECTRIC FIELD

Let now a static electric field be added

$$\underline{E} = - \nabla \phi \tag{11}$$

with no time dependence and no component parallel to $\underline{B}_0$. This last condition implies that

$$\phi = \phi (\alpha_0, \beta_0) \tag{12}$$

Let it furthermore be assumed that this field is relatively weak, so that the motion of particles moving in the vicinity of the dipole is only slightly modified. For particles trapped in the combined electromagnetic field J and B_m are still conserved, but I is not, since p is no longer a constant. One may nevertheless resolve J into a product involving I

$$J = p I \tag{13}$$

If at a particular instant the field line followed by the particle is characterized by Euler potentials (α_0, β_0), then I is expressed by the function I_0 of eq. (6) while p is given by

$$p^2 c^2 = W^2 - m^2c^4$$

$$= (W_s + e [\phi (\alpha_0, \beta_0) - \phi_s])^2 - m^2c^4 \tag{14}$$

Here W is the energy and the subscript s refers to some starting field

line to which all energies are referred (obvious choices would be field lines in the noon meridian or in the midnight meridian). Neglecting second order corrections (as will henceforth <u>always</u> be assumed)

$$p^2c^2 = p_s^2 c^2 + 2 e W_s \left[\phi(\alpha_0, \beta_0) - \phi_s\right] \quad (15)$$

$$p = p_s \left\{1 + (e W_s / p_s^2 c^2) \left[\phi(\alpha_0, \beta_0) - \phi_s\right]\right\} \quad (16)$$

Let a constant I_s be defined, equal to I on the initial field line

$$I_s = J / p_s \quad (17)$$

Substituting (16) in (13) then gives as the shell equation to lowest order

$$I_0(\alpha_0, B_m) = (1 - \delta)\, I_s \quad (18)$$

where

$$\delta(p_s, \alpha_0, \beta_0) = (e W_s / p_s^2 c^2) \left[\phi(\alpha_0, \beta_0) - \phi_s\right] \quad (19)$$

As in the transition between equations (7) and (8), this may be inverted to give the shell equation as

$$\alpha_0 = G_0\left[I_s(1 - \delta), B_m\right]$$

$$\cong G_0(I_s, B_m) - \delta(\partial G_0 / \partial I) \quad (20)$$

The second term on the right is what was defined in reference A as the "shell splitting function" and it distinguishes the shells in the present case from those given by eq. (8) for the case when the electric field is absent. To evaluate this function one notes that, since (8) is the inverse of (6)

$$I \equiv I_0 \left[G_0(I, B_m), B_m \right] \tag{21}$$

Differentiation yields

$$1 = (\partial I_0 / \partial \alpha_0)(\partial G_0 / \partial I) \tag{22}$$

and hence

$$(\partial G_0 / \partial I) = (\partial I_0 / \partial \alpha_0)^{-1} \tag{23}$$

It may be shown [Stern, 1967a] that

$$(\partial I_0 / \partial \alpha_0) = -(a^2 g_1^0 / 2 \alpha_0^2) K_4 \tag{24}$$

where K_4 is an integral defined by Pennington [1961]. The expression given in (24) does depend on α_0, but since α_0 varies only slightly around a drift shell, we are justified (at least to lowest order) in replacing it with its initial value α_s, and a similar substitution may be performed in ϕ. This gives the shell equation to lowest order as

$$\alpha_0 = G_0(I_s, B_m) + (2 \alpha_s^2 \cdot W_s / a^2 g_1^0 K_4 c^2 p^2)[\phi(\alpha_s, \beta_0) - \phi_s] \tag{25}$$

The second term in (25) gives explicitely the effect of the electric field on the shape of drift shells. Since it depends on both W_s and β_0, it indeed leads to energy-dependent shell splitting.

SHELL SPLITTING IN NEAR - DIPOLE FIELDS

In slightly asymmetrical fields the equations of drift shells are further modified in the manner described in reference A . To the first order the two modifications may be independently applied, leading to a first-order shell equation

$$\alpha_0 + \alpha_1 = G_0(I_s, B_m) - G_1(\alpha_s, \beta_0, B_m)$$
$$+ (2\alpha_s^2 e W_s / a^2 g_1^0 K_4 c^2 p_s^2)\left[\phi(\alpha_s, \beta_0) - \phi_s\right] \tag{26}$$

where α_1 is the first-order correction to the Euler potential [Stern, 1967b] and where G_1 is the shell splitting function for near-dipole magnetic fields, as given in A and also as implicit in the work of Pennington [1961] .The last two terms both lead to shell splitting, since both contain β_0, but only the latter one includes dependence on the initial energy W_s .

ACKNOWLEDGMENT

The author is grateful to Dr. Thomas Birmingham for his critical evaluation of this work and for his helpful comments.

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