EXPERIMENTAL INVESTIGATION OF
AN AXISYMMETRIC FULLY DEVELOPED
LAMINAR FREE JET

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SUMMARY

An experimental investigation was conducted to determine dynamic characteristics of a circular, fully developed, laminar free jet. The jet exited from a nozzle having the external shape of a tube and positioned parallel to the gravity vector. Complete velocity profiles are presented at Reynolds numbers of 437 and 1839 at 0, 3, 6, 10, 15, and 25 diameters from the nozzle exit. Centerline velocity decay was obtained over a range of Reynolds numbers from 255 to 1839. Also presented are the angles of spread associated with the diffusion of the jet downstream of the nozzle. Where possible, results are compared with those obtained for laminar jets having an initially flat or uniform velocity profile at the nozzle exit.

INTRODUCTION

A thorough knowledge of the dynamics of various types of low-speed jet flow is required for many engineering applications. Some of these include the extensive area of fluidic amplifiers, liquid inflow and vapor impingement in weightlessness and near weightlessness, and the laminar gas jet diffusion flame. The prediction of the interaction of a jet with any object necessitates a basic understanding of the dynamic characteristics of the jet itself. The area of incompressible low-speed jets is further complicated by the fact that the jet behavior varies in different regions as a function of the jet Reynolds number. Also, the jet characteristics are dependent on such factors as the shape of the nozzle from which the jet exits, the region or environment into which the jet enters, the initial velocity profile of the jet at the nozzle exit, the orientation of the jet with respect to the gravity vector, and even the external shape of the nozzle. A survey of the available literature on jet flow indicates that many of these factors (Reynolds number, internal and external nozzle shape, orientation, and environment) have not been specified. A knowledge of the mass or volumetric flow rate through a nozzle alone is
insufficient to fully describe the subsequent downstream jet behavior.

The only known analytical result for a three-dimensional laminar free jet is that attributed to Schlichting (ref. 1). A similarity solution for the velocity profile downstream of the nozzle exit was obtained by employing the three-dimensional Navier-Stokes equations, taking symmetry into account, and using boundary-layer assumptions. The major disadvantage with Schlichting's analytical solution is that the jet is assumed to exit from an infinitesimally small aperture and, thus, cannot be expected to be adequate except at very large distances from the nozzle. Experimental results for both laminar and turbulent free jets exiting from a circular aperture can be found in references 2 to 9, and an excellent summary of the experimental studies of both laminar and turbulent free jets can be found in reference 7. Each of these works is discussed in more detail in the section entitled BACKGROUND INFORMATION, but to the authors' knowledge, the behavior of the fully developed laminar jet has not been investigated.

The purpose of this report is to present the results of an experimental investigation conducted at the NASA Lewis Research Center to study the dynamic characteristics of a circular, fully developed laminar free jet exiting into a constant-pressure region. Helium was employed both as the jet and as the environment into which the jet exited in order to maximize the difference between total and static pressure at any given Reynolds number. The external shape of the experiment nozzle was that of a tube, and the axis of the nozzle was oriented parallel to the gravity vector (vertical). The complete velocity profiles were measured at 0, 3, 6, 10, 15, and 25 nozzle diameters downstream at two specific Reynolds numbers. The centerline velocity decay and the angle of jet spreading were obtained at various Reynolds numbers between 255 to 1839. Where possible, comparisons of the centerline velocity decay and the angle of spread of the jet were made with results obtained for laminar jets having an initially flat velocity profile at the nozzle exit.

**SYMBOLS**

- \( D \) internal diameter of nozzle, cm
- \( H \) axial distance from nozzle, cm
- \( P \) pressure, N/cm\(^2\)
- \( \Delta P \) change in pressure, N/cm\(^2\)
- \( P_D \) dynamic pressure, N/cm\(^2\)
- \( P_s \) static pressure, N/cm\(^2\)
- \( P_t \) total pressure, N/cm\(^2\)
The general category of jet flow is extensive, and a variety of reports and several books have been written on the subject. However, the bulk of the available information is concerned with turbulent jets. Both analytical and experimental studies have also been conducted with jets impinging on solid surfaces, liquid surfaces, and each other. However, relatively few papers have treated, either theoretically or experimentally, the laminar jet.

A number of previous investigators (refs. 2, 5, and 8) have considered a jet exiting from a circular-cross-section nozzle and having an initially uniform velocity profile at the nozzle exit. A representative study (ref. 2) defines two distinct regions for a jet of this nature. The first region is termed the region of flow establishment and extends from the nozzle exit to the apex of the potential core. (The potential core is the region of the jet in which the centerline velocity remains essentially constant and equal to the velocity at the nozzle exit.) The second region is termed the region of established flow. This region begins at the end of the region of flow establishment (end of the potential core region) and is characterized by some gradual dissipation of the jet centerline velocity. Each of these regions is further characterized by a corresponding angle of spread (defined by the nominal limits of the diffusing jet). For the region of flow establishment, this angle of spread is approximately $6^\circ$ to $8^\circ$ on either side of the jet center-
line, while for the region of established flow the angle of spread is approximately $10^\circ$ to $12^\circ$ on either side of the jet centerline. The lowest Reynolds number investigated in reference 2 was about 1500, and the length of the potential core was found to be about 6.2 nozzle diameters (independent of Reynolds number). A report by Hrycak, Lee, Gauntner, and Livingood (ref. 8) examined a jet having the same uniform initial velocity profile but extended the lowest value of the Reynolds number to 600. They found that the potential core length was a function of the Reynolds number for laminar flow but was independent of Reynolds number for a turbulent jet. They did not examine the angle of spread extensively, but did show fair agreement with the results of reference 2 at high Reynolds numbers.

To the authors' knowledge, there have been no studies of laminar jets in which the initial velocity profile was shown to be or was defined as parabolic. Several investigators have made some visual observations (no profiles measured) of low Reynolds number jets (refs. 3, 4, and 6); however, in none of these studies was the initial velocity profile of the jet either defined or discussed, nor were all of the other system parameters which could have affected the jet development specified. As a result, it is impossible to make a meaningful comparison of the findings of these studies.

**APPARATUS**

**Selection of Experiment Nozzle and Gas**

Since the objective of this work was to determine the dynamic characteristics of a circular, fully developed laminar jet, it was essential that the jet Reynolds number (based on nozzle diameter) not exceed a value of about 2300. It was further desired to investigate some lower Reynolds numbers, as low as 200 to 500, and obtain the velocity profiles by measuring the dynamic pressure of the jet (a function of jet velocity). Thus, an examination of the parameters in the Reynolds number $\rho U_{av}D/\mu$ revealed that, for a constant jet Reynolds number, the largest velocity (and, hence, the largest dynamic pressure) would be obtained by minimizing the nozzle diameter and maximizing the kinematic viscosity $\mu/\rho$. These, of course, must be chosen with some practicality, for if the nozzle diameter was chosen too small in relation to the pressure probe, it would be impossible to obtain enough data to define a velocity profile. As a result of these considerations, helium was chosen as the working gas, and a nozzle diameter of 0.254 centimeter (0.1 in.) was selected.
Experiment Hardware

A photograph of the experiment apparatus employed in this study is shown in figure 1. The major components of this system are described in the following sections.

Test chamber. - As shown in figure 1, the test chamber consisted of a plastic section having a hemispherical top, a cylindrical section, and a stainless-steel base plate. The plastic section was fabricated as one unit and had three ports, circular in shape, with flanges attached so as to be compatible with the neoprene gloves and accordion sleeves which were used for access to the components within the chamber. This dome was fastened to the base plate at six locations by the use of knurled nuts, and a positive seal was obtained by means of an O-ring. The dome was optically clear so that visual observation of the apparatus contained within the dome was possible.

Base plate and supporting structure. - The stainless-steel base plate was supported by three cylindrical legs which were in turn securely anchored to a concrete floor to eliminate the disturbing influences of vibrations which could be transmitted to the jet. The legs were equipped with leveling jacks, and the base plate could thus be aligned using a precision level capable of measuring deflections as small as 0.0083 centimeter per meter (0.001 in./ft). Hermetically sealed small circular ports in the base plate permitted connection of gas lines, vacuum lines, and pressure lines to the test chamber.

Actuation device. - A photograph showing the actuation device for varying the position of the pressure probe is shown in figure 2. This slide assembly consisted of three mutually perpendicular machined surfaces and thus permitted movement in three directions. Slide travel in the plane normal to the nozzle axis was precisely measured by two dial indicators accurate to within 0.0025 centimeter (0.001 in.), and vertical travel was measured to within 0.00508 centimeter (0.002 in.). The actuation device was attached to the stainless-steel base plate by four screws, and each machined surface was leveled with respect to the base plate and to each of the other two surfaces.

Pressure readout device. - The precision pressure readout unit is shown mounted in the panel in figure 1. In order to obtain the dynamic pressure, a capsule acting as a pressure transducer and contained within the unit had two inlet ports, one connected to the total head pressure probe and the other connected to a reference port located at the base plate. A gage, through the internal quartz Bourdon tube capsule, indicated the difference between the total and static pressures to provide a measure of the dynamic pressure of the jet. Through a procedure discussed in the section DATA ANALYSIS, the dynamic pressure was converted to a velocity measurement. Dynamic pressures as low as \(1.15 \times 10^{-1}\) newton per square meter (0.000017 psi) could be read with this device. This pressure reading corresponds to the axial velocity of 1.18 centimeters per second, which is, thus, the lowest value of velocity which could be resolved with the apparatus. It should be noted that the accuracy of the calculated velocity is a function of the reading...
of the pressure readout device, and the corresponding errors involved become insignifi-
cant as the magnitude of the reading increases. It is estimated that the maximum error
obtained in this study was about 6 percent, and this occurred at the positive readings
obtained near the periphery of the jet.

Nozzle. - A drawing of the brass nozzle used in this investigation is shown in fig-
ure 3. The length-diameter ratio of the nozzle was chosen so that fully developed lami-
nar flow would be obtained at the nozzle exit, even at the highest flow rates expected
(see ref. 5). The nozzle, which was machined to very close tolerances, was fastened to
a supporting structure on the base plate. This supporting structure had also been pre-
cisely machined and aligned so as to ensure that the nozzle would be oriented normal to
the base plate. This assured that the nozzle axis would be parallel to the gravity vector.

Pressure probe. - Pressure and thus velocity measurements of the helium jet were
obtained with a total head pressure probe fabricated from stainless steel (see fig. 4).
This probe was rigidly attached to the actuation device to assure that the axis of the
probe was parallel to the axis of the nozzle. The opening at the end of the probe was
circular and had a diameter of 0.015 centimeter (0.006 in.). The probe was sized to
yield a reasonable time constant and was connected to a port located on the base plate
by small-diameter plastic tubing.

OPERATING PROCEDURE

The flow schematic is shown in figure 5. The gas in the experiment test chamber
was initially evacuated by means of the vacuum pump (shown in fig. 1) to a pressure on
the order of 20 newtons per square meter (0.15 torr). In order to prevent the gloves
from imploding, both sides of the gloves were evacuated by placing cover plates over
the glove ports. The system was then filled with helium and again evacuated. This
procedure was repeated four times in order to ensure that the environment in the test
chamber was essentially pure helium before any measurements of the velocity profile
were taken.

After the fourth pumpdown, helium was introduced into the test chamber until a
positive pressure of $0.138 \times 10^4$ newtons per square meter (0.2 psi) above atmosphere
was obtained in the chamber and the cover plates on the glove ports were removed.
This pressure was maintained through the entire test by a pressure regulator and relief
valve system and served to eliminate the danger of air leakage into the chamber.

Helium was then introduced into the chamber through the test nozzle by activating a
series of pressure regulators and valves in series with one of the two available roto-
meters. The rotometers were chosen so that one would be used to measure higher mass
flow rates and the other, lower mass flow rates. The temperature and the pressure of
the helium were measured on gages located upstream of the rotometer. Thus, any necessary corrections due to these system variables would be made. The flow rate could be determined to an accuracy of 96 percent by this system.

The determination of the jet centerline was made by a pressure survey across the jet at the nozzle exit. The highest pressure obtained indicated the location of the jet centerline. Once the centerline position was determined, surveys were taken across the jet using this point as a reference. Profiles were taken at distances of 0, 3, 6, 10, 15, and 25 nozzle diameters from the exit.

**DATA ANALYSIS**

The velocity distribution downstream of the nozzle exit was obtained from the measurement of the dynamic pressure as obtained from the precision pressure readout instrument. This instrument measured the difference between the total pressure associated with the incoming gas jet entering the probe and the static pressure as measured by means of a pressure tap located on the base plate. By employing Bernoulli's equation, conservation of energy along a streamline, between the point at which the jet velocity is desired and the point at which the gas jet comes to rest (i.e., the stagnation point), the total pressure can be calculated:

\[ P_s + \frac{\rho U_x^2}{2} = P_t \]  

(1)

The pressure readout instrument measures the difference between the total and static pressure, or the dynamic pressure:

\[ P_D = \frac{\rho U_x^2}{2} \]  

(2)

From the manufacturer of the readout instrument, a chart of pressure and gage counter reading was furnished, and recalibration verified that the values given were correct. An equation for the pressure gage counter reading was developed and solved simultaneously with the dynamic-pressure - velocity expression (eq. (2)). The simultaneous solution of these two equations, therefore, yielded the appropriate functional relation between the gas jet velocity and the gage counter reading.
RESULTS AND DISCUSSION

Comparison of Initial Velocity Profile With Theory

If we consider flow through a tube having a constant circular cross section and assume that the flow has come to equilibrium (constant pressure gradient), it is then possible to write the following equation for fully developed laminar flow:

\[ u_x = \frac{2q}{\pi R^2} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \]  

(3)

The form of equation (3) is that of a paraboloidal surface; thus, fully developed laminar flow has a parabolic velocity profile. Equation (3) can be simplified by substituting \( \bar{u}_a \) for \( q/\pi R^2 \):

\[ u_x = 2 \bar{u}_a \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \]  

(4)

Nondimensionalizing equation (4) with respect to \( U_{\text{max}} \), we obtain

\[ \frac{u_x}{U_{\text{max}}} = \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \]  

(5)

The initial velocity profile was obtained and compared with the theoretical expression of equation (5) for two Reynolds numbers. The experimental data at a Reynolds number of 437 are plotted in figure 6, which shows dimensionless axial velocity as a function of dimensionless radial position. As shown, the experimental data agree with the theory.

In figure 7, a similar plot is obtained for data taken at a Reynolds number of 1839. Note that the agreement appears to be good. However, it should be noted that points of zero velocity (and, hence, zero dynamic pressure) were not obtained at a nondimensional radial position of 1. This apparent discrepancy may be explained by the fact that the experiment probe used had an inside diameter of 0.015 centimeter (0.006 in.) and, as such, at a dimensionless radial position of 1, half of the probe, 0.0076 centimeter (0.003 in.), was actually positioned within the flow. The entire probe would have been positioned external to the flow at a dimensionless radial position of 1.06; however, no reading was taken at that position. This was also true for the data shown in figure 6; however, the lower Reynolds number gave rise to a lower velocity which was, in fact, small enough
so that its contribution was not of sufficient magnitude to indicate a pressure. Thus, both profiles agreed quite well with the theoretical curve, and the desired initial condition of fully developed laminar flow was obtained.

Velocity Profiles Downstream of Nozzle

Velocity profiles at downstream distances \( H/D \) of 0, 3, 6, 10, 15, and 25 diameters from the nozzle exit are shown in figure 8 for a jet Reynolds number of 437 and in figure 9 for a jet Reynolds number of 1839.

An initial examination of the profiles indicated that a fair approximation of the data could be obtained by employing the Gaussian distribution, which can be written in the following form:

\[
\frac{U_x}{U_{max}} = \frac{U_{c1}}{U_{max}} \exp \left( \frac{-r^2}{2\sigma^2} \right)
\]

where \( \sigma \) is the value of \( r \) at which \( U_x = 0.605 U_{c1} \). It should be emphasized that equation (6) has no theoretical basis and is employed only as a curve fit.

Values of \( \sigma \) were determined from each survey of axial velocity, and the values varied from 0.032 to 0.042 for these experimental data. The value of \( \sigma \) was larger at a given \( H/D \) for lower Reynolds numbers.

The solid lines shown in figures 8 and 9 represent this calculated curve. Note that the agreement is adequate except at the outer limits of the diffusing jet (the experimental points having lowest velocities and located farthest from the jet axis).

The profiles at a constant value of \( H/D \) appear to differ only slightly in shape as a function of Reynolds numbers and tend to be more peaked at higher Reynolds numbers. For example, a comparison of the profiles at an \( H/D \) of 25 shows that the axial velocity at a Reynolds number of 1839 has a value of 0.885 \( U_{max} \), while at a Reynolds number of 437 the axial velocity is 0.660 \( U_{max} \). Furthermore, for the same profiles, if we compare the axial velocity at a dimensionless radial position of 1, the two profiles have nearly identical values of about 0.325 \( U_{max} \).

Centerline Velocity Decay as Function of Reynolds Number

Measurements of centerline velocity decay were taken at distances of 3, 6, 10, 15, and 25 diameters from the nozzle exit. The results for jet Reynolds numbers of 255, 437, 925, 1255, and 1839 are plotted in figure 10. Note that the centerline velocity for
the lowest Reynolds number (255) decreased very rapidly after exiting the nozzle. For all other Reynolds numbers, the centerline velocity decreased slightly with distance from the nozzle, with the magnitude of the decrease generally being larger for lower Reynolds numbers.

It should be noted that the centerline velocity had decreased in all tests at a location 3 nozzle diameters from the exit, and any potential core which may have existed would, of necessity, be shorter than this length. Therefore, the results of this study differ from the results obtained in reference 8 for jets having similar Reynolds numbers. In that study, a potential core length of approximately 15 nozzle diameters was observed at a Reynolds number of 620. The length of the potential core then increased with Reynolds number and reached a maximum of approximately 21 nozzle diameters at a Reynolds number of 1000; it then decreased to approximately 6 nozzle diameters as the Reynolds number approached 2000. In that study, the initial velocity profile of the jet was uniform. In conclusion, the initial velocity profile of the jet has an effect on the centerline velocity decay and the potential core length.

Jet Spreading

An indication of the magnitude of jet spreading can be obtained from an examination of the velocity profiles. The radial distance at which the dynamic pressure (and, hence, axial velocity) decreased to zero indicates the nominal boundary of the spreading jet.

Jet radius at which the dynamic pressure decreases to zero is plotted as a function of the distance from the nozzle exit for jet Reynolds numbers of 437, 925, and 1839 in figures 11, 12, and 13. At some values of H/D, the point of zero dynamic pressure was found at different values of the dimensionless radial position on each side of the jet centerline. For these cases, two points appear on the curve. From a line faired through the data, a half-angle of spread \( \theta \) can be calculated. Values of this half-angle obtained are indicated, and their value was found to be approximately 2\(^\circ\) to 3\(^\circ\). Furthermore, the magnitude of the angle did not seem to be significantly affected by jet Reynolds number for laminar flow over the range of Reynolds numbers investigated. Reference 2 investigated jet Reynolds numbers as low as 1500 and found that the half-angle of spread for a jet having an initially flat velocity profile was approximately 6\(^\circ\) to 8\(^\circ\) in the region of flow establishment (the potential core region) and reached a value of 12\(^\circ\) to 14\(^\circ\) in the region of established flow. Thus, the angle of spread associated with the fully developed laminar flow nozzle exit appears to be considerably different from that associated with a uniform velocity profile at the nozzle exit.
SUMMARY OF RESULTS

An experimental investigation was conducted to determine the dynamic characteristics of a fully developed laminar free jet. The jet exited from a nozzle having the shape of a thin-walled tube and positioned parallel to the gravity vector. Jet Reynolds numbers (based on nozzle diameter) ranged from 255 to 1839 and measurements were taken at 0, 3, 6, 10, 15, and 25 diameters from the nozzle. In all tests, helium was employed as the working fluid. The following results were obtained:

1. At three nozzle diameters from the nozzle exit, the centerline velocity was found to have decayed. Therefore, no definable potential core was observed.

2. The centerline velocity decay at downstream distances greater than approximately 4 nozzle diameters was found to be larger for lower values of jet Reynolds number.

3. A half-angle of spread of about 2° to 3° was determined for jet Reynolds numbers of 437, 925, and 1839.

4. The dynamic behavior of the jet was found to be dependent on the initial velocity profile of the jet.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, January 5, 1971,
124-08.

REFERENCES


Figure 1. - Experiment apparatus.

Figure 2. - Actuation device.

Figure 3. - Experiment nozzle (dimensions in centimeters).
Figure 4. - Experiment pressure probe (dimensions in centimeters).

Figure 5. - Flow schematic.
Figure 6. - Comparison of initial velocity profile with theory. Reynolds number, 437.

Figure 7. - Comparison of initial velocity profile with theory. Reynolds number, 1839.
Figure 8. - Velocity profiles downstream of nozzle. Reynolds number, 437.

Figure 9. - Velocity profiles downstream of nozzle. Reynolds number, 1839.
Figure 10. - Dependence of centerline velocity decay on jet Reynolds number.

Figure 11. - Jet radius as function of distance from nozzle exit. Reynolds number, 437; half-angle of spread $\theta$, approximately $2^\circ$; tan $\theta$, approximately 0.034.
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—National Aeronautics and Space Act of 1958

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