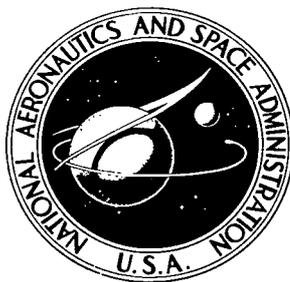


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METHOD FOR ESTIMATING
MINIMUM REQUIRED EJECTION VELOCITY
FOR PARACHUTE DEPLOYMENT

by Earle K. Huckins III and Lamont R. Poole

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16. Abstract An approximate method for estimating the minimum required ejection velocity for parachute deployment was developed. The approach used was to obtain a first integral to the equation for the linear motion of an inelastic parachute deployed in the lines-first mode. A criterion was developed which limits the theory to forced ejection deployments. The theory was extended for application to static vertical ground tests and for cases in which nonzero bag-strip velocity is desired. Results obtained using the new approximation closely agreed with results obtained by numerical integration for a series of hypothetical sample cases.			
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METHOD FOR ESTIMATING MINIMUM REQUIRED EJECTION VELOCITY FOR PARACHUTE DEPLOYMENT

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SUMMARY

A technique was developed for estimating, for preliminary design purposes, the minimum required ejection velocity for forced ejection parachute deployments. The approach used was to obtain a first integral to the equation for the linear motion of an inelastic parachute deployed in the lines-first mode. A criterion was developed which limits the theory to forced ejection deployments by constraining the ratio of acceleration of the deployment bag to acceleration of the vehicle at bag strip. In order to perform the integration, the acceleration of the deployment bag relative to the vehicle was approximated by a function of deployed distance. The theory was extended for application to static vertical ground tests and cases in which the bag-strip velocity is constrained to be a specified percentage of the ejection velocity. The results obtained by using the present approximation closely agreed with results obtained by numerical integration for a series of hypothetical sample cases.

INTRODUCTION

A method of lines-first (ref. 1) parachute deployment, which is particularly useful for parachute deployment in the wake of a bluff body having a low ballistic coefficient, is forceful ejection of the parachute pack from the vehicle. A fundamental design parameter for the associated deployment system is the pack ejection velocity. In the past, selection of the design ejection velocity has been based primarily on previous experience with parachutes of similar size and weight.

If, as in many cases, the time interval of deployment is not constrained, minimizing the design ejection velocity is usually desirable. A minimum ejection velocity results in minimum reaction force acting on the vehicle, since the reaction load for an ideal deployment system with a fixed stroke is directly proportional to the square of the ejection velocity (ref. 2). In addition, a reduction in the reaction load is usually accompanied by a reduction in system weight, both of which are desirable system characteristics.

A definition of the minimum required ejection velocity must be based on some arbitrary criterion for successful deployment. The simplest definition, and the one which is

used in the present analysis, is that the minimum required ejection velocity is the lowest ejection velocity for which the parachute canopy will be completely extracted ("stripped") from the deployment bag. Under this criterion, the deployment will be satisfactory in the sense that the parachute will be completely extended and exposed to the flow field. However, satisfactory inflation and proper functioning of the parachute depend on numerous additional factors. No experimental evidence is presently available to verify or refute the assumption that complete bag strip corresponds, in itself, to successful deployment. However, defining the required ejection velocity as that required to strip the deployment bag completely, produces some results which are quite useful in preliminary system design.

The appropriate pack ejection velocity for any particular set of constraints can be accurately determined by numerical integration of the equations for the motion of the deployment bag relative to the vehicle (ref. 2). However, numerical integration techniques are not always practical during the preliminary design phase. In the present analysis, a closed-form approximation to the minimum required ejection velocity (as defined above) is developed which accounts for the dissipative influences of vehicle drag and bag friction, in addition to the effect of deployment-bag drag in an arbitrary wake profile. Criteria are developed which bound the range of validity for the approximate result.

SYMBOLS

A	reference area, meters ²
C _D	drag coefficient, $\frac{\text{Drag}}{q_{\infty} A}$
D ₀	nominal diameter of parachute, meters
F _{re}	unfurling resistance force, newtons
g	acceleration due to gravity, meters/second ²
K	integral parameters defined by equations (11) to (13), meters/kilogram
l	extended length of segment of parachute, meters
m	mass, kilograms
m'	linear mass density of parachute exiting deployment-bag mouth, kilograms/meter

q_{∞}	free-stream dynamic pressure, newtons/meter ²
T_B	tension at mouth of deployment bag, newtons
t	time, seconds
V	velocity vector, meters/second
x	deployed distance, meters (see fig. 1)
β	ballistic coefficient, kilograms/meter ²
γ	vehicle flight-path angle, degrees
ϵ	ratio of bag-strip velocity to ejection velocity
η	wake parameter (ratio of drag coefficient of deployment bag in vehicle wake to that in free stream)
$(\Delta V)_{req}$	required ejection velocity corresponding to a specified velocity at bag strip, meters/second

Subscripts:

b	deployment bag
bs	bag strip
c	parachute canopy
cs	canopy skirt
e	deployment bag plus contents
p	parachute
sl	suspension lines
T	total parachute and deployment bag

u	unfurled portion of parachute
v	vehicle
w	vehicle wake
1	interval of deployed distance from $x = 0$ to $x = l_{S1}$, meters
2	interval of deployed distance from $x = l_{S1}$ to $x = l_{S1} + l_C$, meters

Dots and bars over symbols denote time derivatives and average values, respectively.

ANALYSIS

In the analysis to follow, an approximate first integral (relative energy integral) to the equation which governs the acceleration of the deployment bag relative to the vehicle is obtained. This integral can be used to determine the ejection velocity which corresponds to a specified relative velocity at bag strip.

Equation of Motion

A typical vehicle-parachute configuration for a deployment of the lines-first type is shown in figure 1. The forces which influence the dynamics of this type of deployment process are shown in figure 2. An equation for the acceleration of the deployment bag relative to the vehicle was derived in reference 2 under the following assumptions:

- (1) The motion is linear.
- (2) The parachute is inelastic.
- (3) The partially unfurled parachute is in tension during the deployment process.
- (4) The aerodynamic drag of the partially unfurled parachute is much less than the drag of the vehicle.
- (5) The deployment rate is much less than the velocity of the vehicle. This assumption allows the dependence of the dynamic pressure at the deployment bag on the deployment rate to be neglected.

Under these assumptions, the governing equation of relative motion was derived in reference 2 and can be rewritten in the following form:

$$\ddot{x} = \frac{\eta(C_{DA})_b q_\infty - F_{re}}{m_e} - \frac{(C_{DA})_v q_\infty + T_B}{m_v + m_u} \quad (1)$$

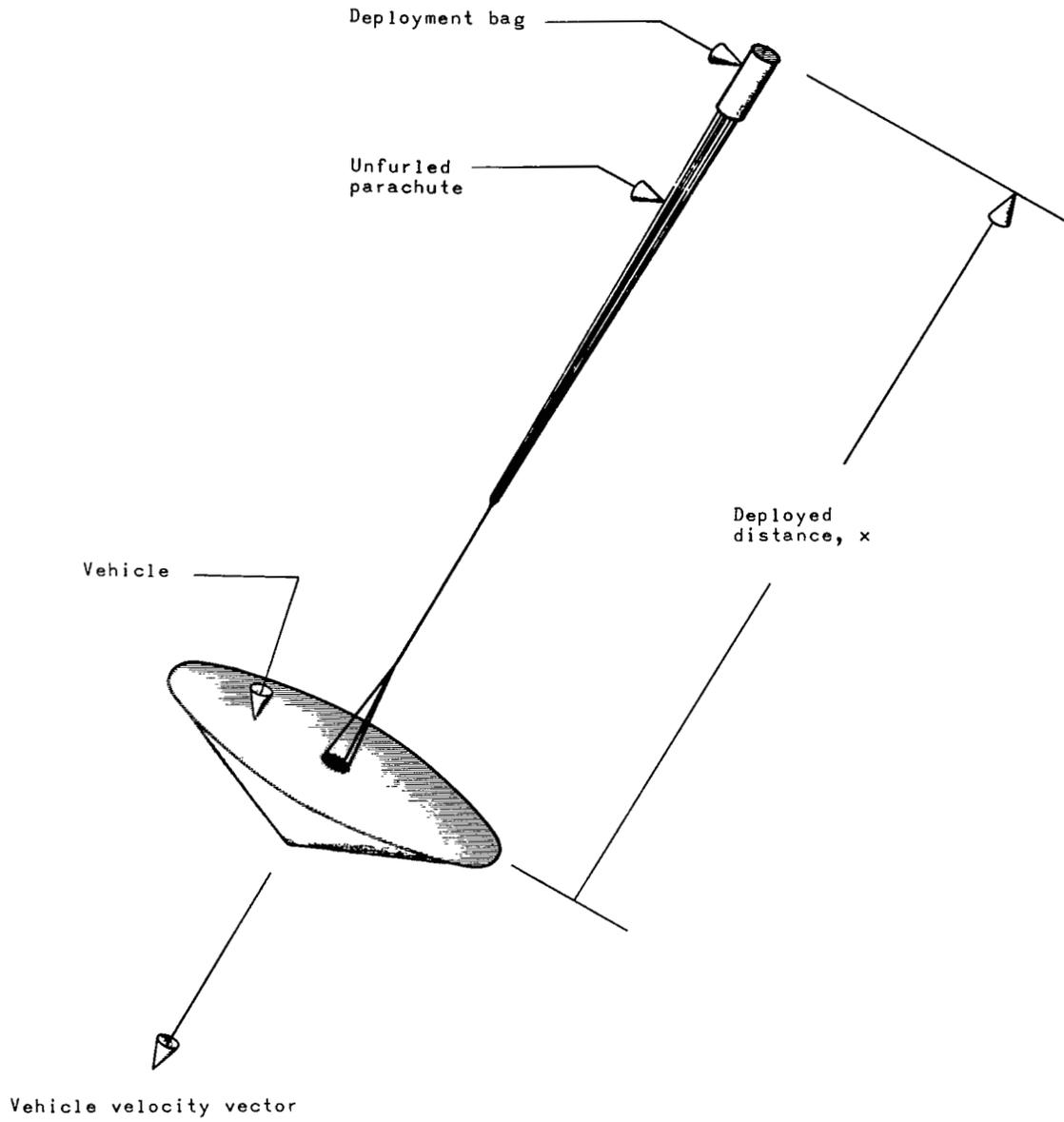


Figure 1.- Sketch of deployment configuration.

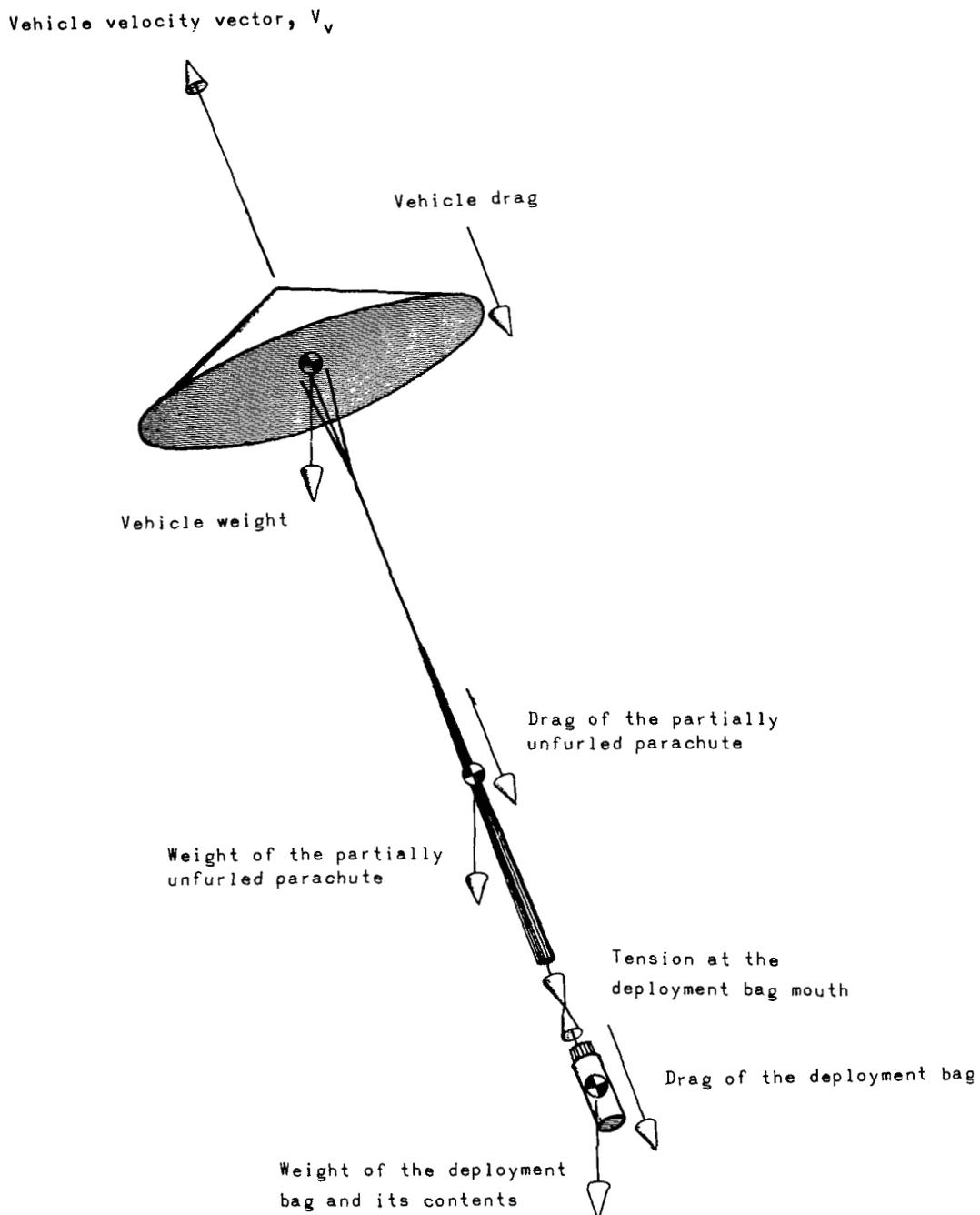


Figure 2.- Forces affecting dynamics of parachute deployment.

where

\ddot{x}	deployment acceleration
η	wake parameter
C_D	drag coefficient
A	reference area
q_∞	free-stream dynamic pressure
F_{re}	unfurling resistance force
m	mass
T_B	tension at mouth of deployment bag, $m'\dot{x}^2 + F_{re}$
m'	linear mass density of parachute exiting deployment-bag mouth
\dot{x}	deployment velocity

and subscripts:

b	deployment bag
e	deployment bag plus contents
v	vehicle
u	unfurled portion of parachute

In deriving the governing equation, the effect of premature canopy inflation on the motion of the deployment bag has been neglected. Limited canopy inflation prior to bag strip results in a wedging effect on the bag mouth which tends to assist stripping of the deployment bag. However, this process is quite difficult to model mathematically and strongly depends on factors, such as packing irregularities, that are random in nature. The effect of neglecting this influence is to generate results which are conservative by a small but indeterminate amount.

Approximate First Integral

The velocity of the deployment bag relative to the vehicle can be determined as a function of deployed distance by using the procedure given below. The relative acceleration can be expanded by the chain rule for differentiation as

$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx} \frac{dx}{dt} = \dot{x} \frac{d\dot{x}}{dx} \quad (2)$$

Therefore,

$$\ddot{x} dx = \dot{x} d\dot{x} \quad (3)$$

which can be integrated to give

$$\int_{x=0}^{x=l_{s1}+l_c} \ddot{x} dx = \frac{1}{2} (\dot{x}_{bs}^2 - \dot{x}_{x=0}^2) \quad (4)$$

where

l_{s1} extended length of suspension lines

l_c extended length of canopy

\dot{x}_{bs} relative velocity at bag strip (i.e., at $x = l_{s1} + l_c$)

The above integration assumes implicitly that the deployed distance is a monotonically increasing function of time. For a zero relative velocity at bag strip, a necessary condition for monotonic increase in the deployed distance is a nonpositive acceleration of the deployment bag at bag strip. By use of equation (1), this condition can be expressed as follows:

$$\frac{\frac{\eta(C_{DA})_b q_\infty - F_{re}}{m_b}}{\frac{(C_{DA})_v q_\infty + F_{re}}{m_v + m_p}} \leq 1 \quad (5)$$

where m_p is the total mass of the parachute and all parameters are evaluated at the time of bag strip. This criterion limits the theory to forced ejection deployments. For cases in which the criterion is violated, the deployment process approaches that of a drogue deployment and the bag-strip velocity is always greater than zero.

The required ejection velocity is equal to the deployment velocity evaluated at the lower limit ($x = 0$). Solving equation (4) for the required ejection velocity gives

$$(\Delta V)_{\text{req}} = \left(\dot{x}_{\text{bs}}^2 - 2 \int_{x=0}^{x=l_{\text{sl}}+l_{\text{c}}} \ddot{x} \, dx \right)^{1/2} \quad (6)$$

where

$$\int_{x=0}^{x=l_{\text{sl}}+l_{\text{c}}} \ddot{x} \, dx = \int_{x=0}^{x=l_{\text{sl}}+l_{\text{c}}} \left[\frac{\eta(C_{\text{DA}})_{\text{b}} q_{\infty} - F_{\text{re}}}{m_{\text{e}}} - \frac{(C_{\text{DA}})_{\text{v}} q_{\infty} + T_{\text{B}}}{m_{\text{v}} + m_{\text{u}}} \right] dx \quad (7)$$

In order to obtain the integral of the relative acceleration, the relative acceleration must be approximated by a function of the deployed distance only. This approximation should be conservative with regard to ultimate determination of the required ejection velocity. A convenient approximate expression for the relative acceleration can be obtained by introducing the following assumptions:

(1) The wake parameter and the unfurling resistance are functions of the deployed distance. In addition, the unfurling resistance has a constant value during deployment of the suspension lines and another constant value during deployment of the canopy.

(2) Drag coefficients are constant.

(3) The mass of the unfurled portion of the parachute is negligible in comparison with the mass of the vehicle.

(4) The tension at the deployment-bag mouth can be conservatively approximated by assuming a constant deployment rate equal to the ejection velocity and by neglecting the unfurling resistance. These assumptions are used only in approximating the tension. For a case in which the predicted required ejection velocity is relatively high, this assumption makes the result highly conservative.

(5) The free-stream dynamic pressure is constant during the deployment process. This assumption is conservative for a negative dynamic pressure gradient. For a constant free-stream density during the deployment process, a negative dynamic pressure gradient results under the following condition:

$$\frac{-g \sin \gamma}{\frac{m \dot{x}^2 + F_{\text{re}}}{m_{\text{v}}} + \frac{q_{\infty}}{\beta_{\text{v}}}} \leq 1 \quad (8)$$

where β_{v} is the ballistic coefficient of the vehicle. Integrations for cases which violate this criterion are valid but the results must be considered nonconservative.

Under the above assumptions, the integral of the relative acceleration (eq. (7)) is given by

$$\begin{aligned}
\int_{x=0}^{x=l_{s1}+l_c} \ddot{x} \, dx &= (C_{DA})_b q_\infty \int_{x=0}^{x=l_{s1}+l_c} \frac{\eta}{m_e} \, dx \\
&- (F_{re})_1 \int_{x=0}^{x=l_{s1}} \frac{1}{m_e} \, dx - (F_{re})_2 \int_{x=l_{s1}}^{x=l_{s1}+l_c} \frac{1}{m_e} \, dx \\
&- \frac{q_\infty}{\beta_v} (l_{s1} + l_c) - \frac{m_p}{m_v} (\dot{x}_{x=0})^2
\end{aligned} \tag{9}$$

where

$(F_{re})_1$ unfurling resistance over interval of deployed distance from $x = 0$ to $x = l_{s1}$

$(F_{re})_2$ unfurling resistance over interval of deployed distance from $x = l_{s1}$ to $x = l_{s1} + l_c$

An expression for estimating the required ejection velocity can now be generated by substituting equation (9) into equation (6). The final result can be written as

$$(\Delta V)_{req} = \left(\frac{1}{1 - 2 \frac{m_p}{m_v}} \right)^{1/2} \left\{ \dot{x}_{bs}^2 + 2 \left[\frac{q_\infty}{\beta_v} (l_{s1} + l_c) + K_1 (F_{re})_1 + K_2 (F_{re})_2 - K_w (C_{DA})_b q_\infty \right] \right\}^{1/2} \tag{10}$$

where

$$K_1 = \int_{x=0}^{x=l_{s1}} \frac{1}{m_e} \, dx \tag{11}$$

$$K_2 = \int_{x=l_{s1}}^{x=l_{s1}+l_c} \frac{1}{m_e} \, dx \tag{12}$$

$$K_w = \int_{x=0}^{x=l_{s1}+l_c} \frac{\eta}{m_e} \, dx \tag{13}$$

The integral parameters defined by equations (11) to (13) can be evaluated numerically, analytically, or graphically for a particular parachute and vehicle wake profile.

For some applications, it may be useful to determine the ejection velocity which corresponds to a bag-strip velocity that is a specified fraction of the ejection velocity.

That is,

$$\dot{x}_{bs} = \epsilon(\Delta V)_{req} \quad (14)$$

Under this criterion, equation (10) reduces to

$$(\Delta V)_{req} \Big|_{\dot{x}_{bs}=\epsilon(\Delta V)_{req}} = \left(\frac{1}{1 - \epsilon^2 \frac{1}{1 - 2 \frac{m_p}{m_v}}} \right)^{1/2} (\Delta V)_{req} \Big|_{\dot{x}_{bs}=0} \quad (15)$$

For small values of ϵ and the ratio of parachute mass to vehicle mass, equation (15) can be closely approximated by

$$(\Delta V)_{req} \Big|_{\dot{x}_{bs}=\epsilon(\Delta V)_{req}} = \left(1 + \frac{1}{2} \epsilon^2 \right) (\Delta V)_{req} \Big|_{\dot{x}_{bs}=0} \quad (16)$$

The significance of this equation is to indicate the high sensitivity of bag-strip velocity to ejection velocity for ejection velocities close to the minimum required.

An expression for the required ejection velocity for a static vertical ground test, which corresponds to equation (10) for an in-flight deployment, can be derived in a similar manner using equation (1). In a ground environment the effective deceleration of the vehicle, due to drag and tension (the second term of eq. (1)), is the local acceleration due to gravity. The deployment-bag drag acts in a direction opposite to the relative deployment motion. The wake integral parameter is equal to the sum of the other two integral parameters. The free-stream dynamic pressure varies considerably between pack ejection and bag strip. Therefore, an average dynamic pressure must be estimated in order to perform the closed-form integration. For these conditions, the required ejection velocity is given by

$$(\Delta V)_{req} = \left\{ \dot{x}_{bs}^2 + 2 \left[g(l_{s1} + l_c) + K_1(Fre)_1 + K_2(Fre)_2 + K_1(C_{DA})_b(\bar{q}_\infty)_1 + K_2(C_{DA})_b(\bar{q}_\infty)_2 \right] \right\}^{1/2} \quad (17)$$

where \bar{q}_∞ is the average dynamic pressure over the appropriate interval of the unfurling process.

Evaluation of Integral Parameters

The integral parameters defined by equations (11) to (13) express essentially the sensitivity of the required ejection velocity to the mass distribution of the parachute.

These integrals can be evaluated graphically or numerically for a particular parachute mass distribution and an experimentally determined wake profile. In many cases, the parachute mass distribution can be closely approximated by that of an equivalent flat circular parachute. Closed-form integrations for an ideal flat circular parachute are developed in the appendix and can be expressed as

$$K_1 = -\frac{l_{sl}}{m_{sl}} \ln\left(1 - \frac{m_{sl}}{m_T}\right) \quad (18)$$

and

$$K_2 = \frac{D_o}{2\sqrt{m_c m_b}} \tan^{-1}\left(\frac{m_c}{m_b}\right)^{1/2} \quad (19)$$

where

m mass

D_o nominal diameter of parachute canopy

and subscripts:

sl suspension line

T total parachute and deployment bag

b deployment bag

c parachute canopy

The form of the integral parameter K_2 given in equation (19) can be used to study the effects of variations in bag weight. Increasing the bag weight by ballasting minimizes the relative deceleration due to unfurling resistance and decreases the relative acceleration due to bag drag. However, ballasting may in some cases be undesirable because of the associated weight penalty.

The integral defined by equation (13) can be approximated by

$$K_w \approx \bar{\eta}_1 K_1 + \bar{\eta}_2 K_2 \quad (20)$$

where $\bar{\eta}$ is an average value of the wake parameter over the appropriate interval.

RESULTS AND DISCUSSION

In order to study the accuracy obtained by using the present approximation to the required ejection velocity, a series of hypothetical parachute-deployment situations were studied. These sample cases were chosen such that the criteria given by equation (5) and equation (8) were satisfied. An ideal flat circular parachute was assumed. The effects of variations in parachute size and weight, ballistic coefficient of the vehicle, and free-stream dynamic pressure on prediction accuracy were studied. Parameters for the selected test cases are given in table I. The required ejection velocity was calculated from equation (10); it was determined for the sample cases by numerical integration of equation (1) with a digital computer. Results obtained by using both techniques are given in table II. Agreement between the two methods of calculation is good. The largest conservative error (11%) is found in case 3. This particular example postulated a relatively large parachute deployed from a vehicle having a low ballistic coefficient at a high dynamic pressure. For these conditions, a relatively large required ejection velocity would be expected. The assumption of tension based on a constant deployment rate results in an overprediction when equation (10) is used. However, the result is not considered to be overly conservative and could be used for engineering design purposes.

CONCLUDING REMARKS

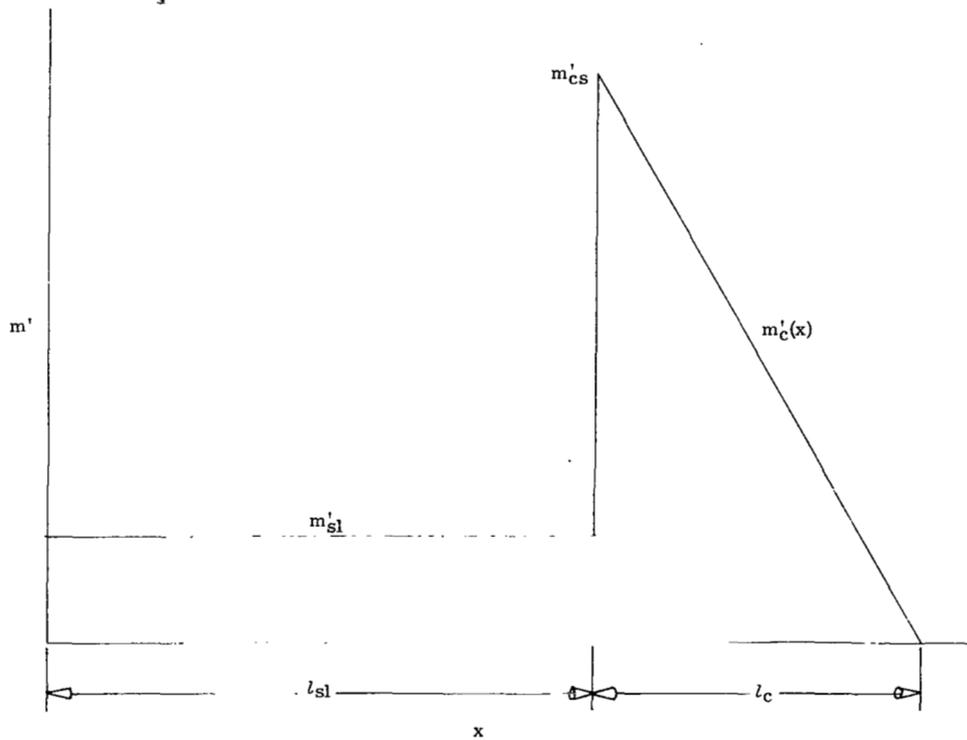
An analysis has been made to determine the minimum required ejection velocity for parachute deployment by forced ejection. An approximate first integral to the equations for the inelastic linear motion characteristic of a deployment of the lines-first type was obtained. Criteria were developed which insure conservative results and limit the theory to forced ejection deployments. Results obtained by using the approximate closed-form integration closely agreed with results obtained by numerical integration.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., March 30, 1971.

APPENDIX

CALCULATION OF APPROXIMATE INTEGRALS

Approximate values of the integrals describing the effects of unfurling resistance and the wake of the vehicle on the required ejection velocity are derived in this appendix. In the derivation, it is assumed that the parachute consists of an ideal flat circular canopy and uniform suspension lines. A mass distribution for this type of parachute is shown in the following sketch:



Sketch 1

where

m' linear mass density

l unfurled length

and subscripts:

sl suspension lines

APPENDIX – Continued

- c canopy
cs canopy skirt

The unfurling resistance parameter for the suspension lines can be written as follows:

$$K_1 = \int_0^{l_{sl}} \frac{1}{m_e(x)} dx \quad (A1)$$

where

- x displacement of deployment bag relative to towing vehicle
 $m_e(x)$ mass of deployment bag and its instantaneous contents as a function of deployed distance

In general, $m_e(x)$ can be represented by

$$m_e(x) = m_T - \int_0^x m' dx \quad (A2)$$

where m_T is the mass of the entire parachute and the deployment bag. In the range of the suspension lines, equation (A2) can be written as

$$m_e(x) = m_T - m'_{sl} x \quad (A3)$$

By substituting equation (A3) into equation (A1) and integrating, the parameter K_1 can be written as

$$K_1 = - \frac{l_{sl}}{m'_{sl}} \ln \left(1 - \frac{m'_{sl} l_{sl}}{m_T} \right) \quad (A4)$$

where m'_{sl} is the mass of the suspension lines.

The unfurling resistance parameter for the canopy can be written in general form as

$$K_2 = \int_{l_{sl}}^{l_{sl}+l_c} \frac{1}{m_e(x)} dx \quad (A5)$$

By defining $x_c = x - l_{sl}$ the parameter can be written as

$$K_2 = \int_0^{l_c} \frac{1}{m_e(x_c)} dx_c \quad (A6)$$

APPENDIX – Concluded

In general, $m_e(x_c)$ can be written as

$$m_e(x_c) = m_T - m_{s1} - \int_0^{x_c} m'_c dx_c \quad (A7)$$

By referring to sketch 1, $m'_c(x_c)$ can be written as

$$m'_c(x_c) = m'_{cs} \left(1 - \frac{x_c}{l_c}\right) \quad (A8)$$

By substituting equation (A8) into equation (A7) and integrating,

$$m_e(x_c) = m_T - m_{s1} - m'_{cs} x_c \left(1 - \frac{x_c}{D_0}\right) \quad (A9)$$

where D_0 is the nominal diameter and is equal to $2l_c$. Through substitution of equation (A9) into equation (A6) and integration, there results

$$K_2 = \frac{D_0}{2\sqrt{m_c m_b}} \tan^{-1} \left(\sqrt{\frac{m_c}{m_b}} \right) \quad (A10)$$

where

m_c mass of parachute canopy

m_b mass of deployment bag

The integral parameter which describes vehicle wake sensitivity can be written in the following form:

$$K_w = \int_0^{l_{s1} + l_c} \frac{\eta}{m_e(x)} dx \quad (A11)$$

where η is a ratio of the deployment-bag drag in the vehicle wake to the deployment-bag drag in the free stream.

The wake integral parameter can be approximated by

$$K_w \approx \bar{\eta}_1 K_1 + \bar{\eta}_2 K_2 \quad (A12)$$

where $\bar{\eta}$ is the average value of η over the appropriate interval of deployed distance.

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TABLE I.- PARAMETERS USED IN SAMPLE CALCULATIONS

Vehicle characteristics:

Weight = 5000 newtons

Drag coefficient = 0.5

Wake parameters:

$$\bar{\eta}_1 = 0.4; \bar{\eta}_2 = 0.8$$

Parachute characteristics:

Ideal flat circular canopy

$$l_{sl} = D_0$$

$$l_c = D_0/2$$

$$m_{sl} = (m_T - m_b)/3$$

$$(Fre)_1 = 50 \text{ newtons}$$

$$(Fre)_2 = 200 \text{ newtons}$$

Bag weight = 10 newtons

$$(CDA)_b = 0.1 \text{ meter}^2$$

Conditions at parachute pack ejection:

Altitude = 20 kilometers

Flight path range = -90°

Case	Parachute weight, N	$l_{sl} + l_c,$ m	$\left(\frac{m}{CDA}\right)_v,$ kg/m ²	$q_\infty,$ N/m ²
1	200	15	50	3000
2	200	15	200	2000
3	500	30	50	3000
4	500	30	200	2000

TABLE II.- RESULTS OF SAMPLE CALCULATIONS

Case	Integral parameters, m/kg			$(\Delta V)_{req}$, m/sec	
	K_1	K_2	K_w	Approximate method (eq. (10))	Numerical integration (eq. (1))
1	0.5619	1.7500	1.6248	41.46	40.33
2	.5619	1.7500	1.6248	21.01	21.64
3	.4656	2.3764	2.0873	64.66	58.16
4	.4656	2.3764	2.0873	30.86	30.27