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Part I

Mechanical Interaction of a Driven Roller (Wheel) on Soil Slopes

The Necessary Conditions for an Equilibrium-Velocity Solution

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PREFACE

The investigation documented in this report constitutes part of the lunar roving vehicle (LRV) research conducted by the Advanced Lunar Studies Team at the Jet Propulsion Laboratory. This study was performed to develop and provide a better understanding of mobility concepts on soft sloping terrains as applied to LRVs.
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CONTENTS

I. Summary ................................................................. 1
II. Background – Lunar Mobility ................................. 3
III. Solution Approach ..................................................... 4
IV. Soil-Roller Kinematics ............................................... 5
   A. Problem Statement ................................................. 5
   B. Soil-Roller Velocity and Boundary Conditions .......... 5
   C. Velocity Fields .................................................... 7
      1. Active zones .................................................. 7
      2. Transition and passive zones ............................ 10
V. Soil-Roller Equilibrium ............................................... 11
   A. Limiting Soil Stress Field .................................... 11
   B. Stresses Along Sliplines ...................................... 11
      1. Stress parameter \( p \) along straight sliplines ...... 12
      2. Stress parameter \( p \) along spiral sliplines ........ 13
      3. Transition and passive zones ............................ 15
   C. Limiting Slipline Directions ................................. 15
      1. Soil-roller interface – maximum stress obliquity .... 15
      2. Free soil surface – singular points (LO) and (NO) .. 17
      3. Summary of absolute admissible bounds for \( \theta_{LO}^l \), \( \theta_{LL}^r \), \( \theta_{NN}^r \), and \( \theta_{NO}^r \) .............. 17
   D. Equilibrium Equations ........................................... 18
   E. Soil Reactions ..................................................... 18
   F. Body Forces ........................................................ 22
   G. Moments ............................................................ 23
      1. Moments due to soil reactions ......................... 23
      2. Moments due to body forces ........................... 23
   H. Soil-Roller Interface Stresses ............................... 24
VI. Solution of Basic Equations

A. Basic Equations
B. Specific Energy Dissipation
C. Rigid Roller Sinkage
D. Mobility Safety Factors
E. Applications

VII. Conclusions

VIII. Recommendations

Appendix. Positive Rate of Dilation

A. Introduction
B. Trailing Zone ($\xi_i \leq \pi/2$)
C. Leading Zone ($\xi_i \geq \xi_M$)

Nomenclature

References

Tables

1.0. Soil-wheel input data for SWIP program application

1.1. Horizontal test for $\xi_M = 99$ and $s_k = 0.75$ (upper bound)

1.1a. Horizontal test for $\xi_M = 99$ and $s_k = 0.75$ (lower bound)

1.2. Horizontal test for $\xi_M = 101$ and $s_k = 0.75$ (upper bound)

1.2a. Horizontal test for $\xi_M = 101$ and $s_k = 0.75$ (lower bound)

1.3. Horizontal test for $\xi_M = 102$ and $s_k = 0.75$ (upper bound)

1.3a. Horizontal test for $\xi_M = 102$ and $s_k = 0.75$ (lower bound)

1.4. Horizontal test for $\xi_M = 104$ and $s_k = 0.75$ (upper bound)

1.4a. Horizontal test for $\xi_M = 104$ and $s_k = 0.75$ (lower bound)

2.1. Wheel performance on slope for $\xi_M = 110$ and $s_k = 0.55$ (upper bound)

2.1a. Wheel performance on slope for $\xi_M = 110$ and $s_k = 0.55$ (lower bound)

2.2. Wheel performance on slope for $\xi_M = 110$ and $s_k = 0.65$ (upper bound)

2.2a. Wheel performance on slope for $\xi_M = 110$ and $s_k = 0.65$ (lower bound)

2.3. Wheel performance on slope for $\xi_M = 105$ and $s_k = 0.65$ (upper bound)
2.3a. Wheel performance on slope for $\xi_M = 105$ and $s_k = 0.65$ (lower bound) ........................................... 54
2.4. Wheel performance on slope for $\xi_M = 105$ and $s_k = 0.70$ (upper bound) ........................................... 55
2.4a. Wheel performance on slope for $\xi_M = 105$ and $s_k = 0.70$ (lower bound) ........................................... 56
3.1. Wheel performance for Apollo LRV for $\xi_M = 105$ and $s_k = 0.85$ (upper bound) ........................................... 57
3.1a. Wheel performance for Apollo LRV for $\xi_M = 105$ and $s_k = 0.85$ (lower bound) ........................................... 58
3.2. Wheel performance for Apollo LRV for $\xi_M = 105$ and $s_k = 0.90$ (upper bound) ........................................... 59
3.2a. Wheel performance for Apollo LRV for $\xi_M = 105$ and $s_k = 0.90$ (lower bound) ........................................... 60
4.1. Wheel performance for Lunokhod-1 for $\xi_M = 112$ and $s_k = 0.80$ (upper bound) ........................................... 61
4.1a. Wheel performance for Lunokhod-1 for $\xi_M = 112$ and $s_k = 0.80$ (lower bound) ........................................... 62

Figures
1. Typical failure pattern for driven rigid wheel on a pack of aluminum rolls ........................................... 63
2. Roller motion on sloping soil ........................................... 63
3. Driven rigid roller 100% slip ($s_k = 0, V_C = 0$) on a pack of aluminum rolls ........................................... 64
4. Soil-roller plastic flow configuration ........................................... 64
5. Soil-roller rim-velocity boundary conditions (bifurcation of plastic zones) ........................................... 65
6. Limiting stress state ........................................... 66
7. Limit position of leading pole ($s_k = 0$) ........................................... 67
8. Locus of leading and trailing spiral poles ........................................... 68
9. Locus of trailing poles $l_1$ for varying $\xi_M$ and $s_k$ ........................................... 69
10. Soil-roller interface velocities ........................................... 70
11. Soil-roller free body equilibrium ........................................... 71
12. Stresses along sliplines (active zones) ........................................... 72
13. Limiting slipline directions at leading and trailing zones ........................................... 73
14. Roller sinkage ........................................... 73
ABSTRACT

A general solution is given to the mobility performance problem of a power-driven rigid cylindrical roller climbing a semi-infinite soft soil slope with uniform velocity. The roller axle is subjected to vertical and pull force components. A gravitating, cohesive-frictional soil is considered. Its application to lunar and planetary locomotion is emphasized. The mechanics of soil-roller interaction is described and solved, considering stresses and velocities, as a mixed boundary value problem. Kött's quasi-static equilibrium equations are connected to a plastic stress configuration satisfying Shield's velocity conditions along the characteristic lines. Solutions of the equilibrium equations yield the driving torque, slip, sinkage, and soil-roller interface stresses. Driving power requirements and thrust efficiency are determined.

A general concept of safety factor against immobilization is introduced. A computer program for the soil-wheel interaction performance (SWIP) was developed and limited applications of this theory to rigid wheel tests on horizontal terrains indicate very reasonable agreement. The method is also applied to the Apollo and Lunokhod-1 lunar roving vehicle wheels. Part I, as published, presents the basic and necessary conditions satisfying the limiting equilibrium and velocity equations. Part II, to be published separately, will provide the concepts of sufficiency asserting the completeness of a given solution and the computer program.
1. SUMMARY

The use of wheeled roving vehicles in future lunar and planetary explorations will require consideration of two technical problems: (1) the expense and difficulty of performing lunar soil surface tests to derive experimental coefficients which define soil-wheel performance, and (2) the lack of the basic fundamentals and experimental background to predict either on a theoretical or empirical basis the mechanics of lightly loaded rolling devices on soft horizontal or sloping terrains.

This study presents a method of solution to the two-dimensional problem of a power-driven long and rigid cylindrical roller moving up a generally sloping soil surface with uniform velocity. The roller axle is subjected to the combined action of a driving torque, a vertical load, and a pull force parallel to the terrain slope. All practical ranges of soil mechanical properties are considered for either lunar or earth-based soils, including soil friction, cohesion, and gravitational effects.

In principle, the soil-roller interaction study, as would one for a wheel, points to two basic aspects. One corresponds to the operational conditions of a roller whose mobility is always guaranteed, particularly for low contact pressures and reduced sinkage. In this case, the design objective for mobility is mainly concerned with an efficiency optimization problem. The other aspect relates to the particular limiting state, in which immobilization occurs when the soil thrust capacity has been reached due to the combined action of soil weight and applied roller loads. In this case, a margin of safety against immobilization, rather than efficiency, constitutes the constraint design factor.

This study indicates that the soil-roller solution represents, to a reasonable degree, a satisfactory basis for estimating the performance of rigid wheels under similar driving conditions. This is particularly true when the soil-wheel contact pressure level is low and the soil failure pattern develops mainly in the fore-aft direction rather than in a lateral mode. This characteristic is expected to occur predominantly for vehicle locomotion on the lunar surface, as evidenced from a photograph of wheel tracks left by the Soviet Lunokhod-1 (Fig. 3 of Ref. 1). Obviously, the soil-roller solution does not entail the solution of the wheel problem "per se" since the wheel is a finite representation of a roller. However, the roller approach is a necessary step which will permit the elucidation and disclosure of some important aspects applicable to the soil-wheel interaction problem. In particular, these aspects refer to:

1. The mechanics of slope climbing under various types of soil materials, and loading conditions of self-propulsion or pull forces.

2. The role of slip as a kinematic factor affecting the wheel mechanical performance and locomotion efficiency.

3. The basic principles concerning how soil thrust is generated and motion sustained as a continuous mechanical process.

If all these factors can be derived for a roller, the conclusions may be directly extended to wheels or otherwise approximated by appropriate modeling techniques. In this context, the roller analogy is more akin to the soil-wheel interaction problem than to using flat plate tests since in the former the mechanics of rolling is reflected at all slip levels as it actually occurs for a wheel.
The solution approach consists of selecting a soil-roller failure configuration in accordance with experimental evidence on horizontal terrains. This failure pattern is generalized to sloping surfaces. A compatible velocity field is defined that satisfies all velocity requirements. Then the governing stress equilibrium equations are solved along the velocity characteristics (stiplines) in connection with the remnant stress boundary conditions. The method of solution is implemented by a computer program which evaluates the basic soil-roller performance parameters as given by the torque, slip, sinkage, and interface stress. The concept of specific rolling energy dissipation is generalized to slopes, and the driving power requirement is determined under quite general conditions of load combinations and terrain slope. A general mobility safety factor definition is introduced and calculated for a soil-roller system. This definition applies also to wheels and covers the whole spectrum of rolling with slip up to and including immobilization.
II. BACKGROUND - LUNAR MOBILITY

The current status of soil-wheel interaction as a design tool for planetary vehicle exploration is rather unsuitable due to the lack of pertinent quantitative tests and design information that is normally available for on-earth vehicles. New theoretical concepts are required to predict vehicle performance requirements, especially with regard to safety and power needs when traversing lunar soft terrain and steep slopes. This context, a pertinent survey of the state-of-the-art on off-the-road vehicle design was made and has led to the following conclusions:

1. The present status of the theory of soil-wheel interaction applies exclusively to horizontal or gently sloping terrains. Wheel design parameters and vehicle performance are definitely connected with the soil mechanical properties and the nature of lunar topography. Consequently, lunar vehicle slope climbing and traversing capabilities are bound to constitute a controlling design factor in terms of mobility safety and locomotion energy requirements.

2. Current on-earth vehicle mobility experience refers mainly to high soil-wheel contact pressures and dictates that heavily loaded vehicles cannot practically negotiate soft slopes higher than approximately 25 deg. Instead, lunar roving vehicles will operate under comparatively low soil contact pressures (approximately 6.9 \times 10^3 \text{ N/m}^2, 1 \text{ psi}) and may require climbing slopes steeper than 25 deg. It is not known, using present mobility concepts, if a vehicle can safely operate under conditions of low pressure and reduced gravity on lunar crater slopes near limiting equilibrium where excessive sinkage and loss of soil stability support may be imminent.

3. Present approaches to derive basic performance parameters of soil-wheel interaction resort to separate sinkage and horizontal shear deformation plate tests performed on potential terrains of vehicle operation. On this basis, a comprehensive semi-empirical theory for off-the-road mobility was developed by Bekker (Ref. 2). The inherent uncertainty of this approach relates to the fact that the character and distribution of soil-wheel interface stresses derived from flat plate tests cannot be reliably translated into a roll on a sloping surface.

(4) There is insufficient evidence to what extent the combined or independent action of friction, cohesion, and gravitation influences soil-rolling thrust performance. Experiments and analysis by Costes, et al. (Ref. 3), indicate that the lunar soil surface mechanical properties relate to a frictional-cohesive behavior.

The use of wheeled roving vehicles in future lunar and planetary explorations will require consideration of two technical problems: (1) the expense and difficulty of performing lunar soil surface tests to derive experimental coefficients which define soil-wheel performance, and (2) the lack of the basic fundamentals and experimental background to predict either on a theoretical or empirical basis the mechanics of lightly loaded rolling devices on soft horizontal or sloping terrains.

The broad objective of this study is to establish a basis for a consistent and unified approach to the soil-wheel interaction problem to estimate the performance of lunar roving vehicles traversing generally sloping soft surfaces. To this end, basic soil mechanical concepts, applicable to the lunar environment, are incorporated in analytical expressions to derive torque, energy, interface stresses, operating slip, and sinkage values as may be applicable to lunar mobility requirements.

These analytical expressions can then be compared with the results of controlled experiments either on earth (Ref. 4) or on the moon (Refs. 1 and 5), using, for example, the Soviet Lunokhod-1 lunar roving vehicle mobility system.
III. SOLUTION APPROACH

The solution approach to the soil-roller interaction problem is based on the application of the theory of plasticity to soil mechanical problems. The governing differential equations of equilibrium (Ref. 6) and velocity compatibility along the slip-lines (Ref. 7) are solved as a mixed boundary value problem connecting stresses and velocities. This problem is concerned with a quasi-static, steady-state, two-dimensional plastic flow with soil assumed to behave as a rigid perfectly plastic material. The soil is granular in character with strength properties defined by the Mohr-Coulomb theory of failure, which depends on the soil angle of internal friction and cohesion. The strength properties are isotropic and homogeneous throughout the plastic domain up to and including the soil-roller interface. Soil self-weight is specifically considered.

The problem of rolling contact between a rigid towed cylinder on a plastic deforming horizontal half space was investigated theoretically by Marshall (Ref. 8) for a Tresca material, applying a perturbation method of solution. Dagan and Tulin (Ref. 9) studied the steady flow of a rigid plastic clay beneath a driven cylindrical roller on a horizontal half space, using the method of slip-lines. Experimentally, Boucherie (Ref. 10) obtained photographic flow patterns produced by towed and driven rigid wheels on a supporting bed of packed metal rolls (Fig. 1). Wong and Reece (Ref. 11) performed a series of tests on level sand surfaces, acted upon by rigid rollers and wheels for various loads, slip, and skid combinations. Using photographic techniques, they presented and commented in detail on the soil and patterns associated with soil failure due to rolling loads.

To date, all known complete solutions of uncontained rigid perfectly plastic flow problems consist of selecting an appropriate slip-line stress field and then verifying whether or not the boundary and the field velocities are also satisfied throughout. If not, a new stress slip-line pattern has to be tried (Ref. 12). Obviously, this procedure is rather limited regarding the boundary velocity functions which can be considered other than simple uniform ones.

Here, instead, an inverse procedure has been adopted. First, a compatible soil-roller velocity field of characteristic lines is generated which satisfies the roller boundary velocity conditions, then to this field is adjoined a stress domain that satisfies all the remnant boundary conditions. That this is possible is based on the fact that velocities and stress characteristic lines are coincident (Ref. 7) if the former derive from the theory of plastic potential (Ref. 13). To the author's knowledge, this is the first time a problem of mixed boundary values in the theory of plasticity is solved for both stresses and velocities by satisfying first the velocities rather than the stress conditions.

This method considerably facilitates the search for a complete solution. Also, this approach may be found useful in the study of metal forming and tooling operations and other soil mechanical problems which are accompanied by variable interface friction and complex moving plastic boundaries. There exist infinite stress field patterns which can satisfy equilibrium. In addition, completeness of a solution is defined only when (1) the kinematic compatibility conditions are also satisfied, (2) the dilation rate is positive throughout the plastic domain, and (3) at no point of the rigid domain do the stresses exceed yield. It is reasonable to expect that if the problem is well posed and a solution is found, the results obtained will coincide with a possible matching experiment.

The sequence of the study reported here is as follows: In Section IV, a velocity flow field pattern is selected and validated in accordance with experimental evidence of rolling tests on horizontal soil surfaces. These results are generalized to slopes. The kinematics of slip and velocity boundary conditions are analyzed as they relate to the soil-roller interface. These velocities are then propagated throughout the plastic domain. In Section V, a summary of the plane strain theory for rigid plastic solids is given. The slip-line fields defined in Section IV are used to determine soil limiting stresses. Soil reactions and moments are calculated by considering soil weight. The quasi-static equilibrium equations are defined and the soil-roller interface stresses are evaluated.

In Section VI, the results of Section V are synthesized into a system of equations whose solution is the basis for the complete solution of the soil-roller problem. The total torque, energy, and roller sinkage equations are formulated. A general definition of the factor of safety against immobilization is introduced. The computer program is applied to soil-rigid wheel test results which are numerically compared to illustrate the use of the method and to verify its prediction capabilities.

Finally, the theory is applied to the Apollo lunar roving vehicle (LRV) and the Soviet Lunokhod-1 wheels operating on the lunar surface to estimate their mobility performance.
IV. SOIL-ROLLER KINEMATICS

A. Problem Statement

The roller moves with uniform velocity $V_C$ parallel to the original undisturbed soil surface (Fig. 2). The surface slope angle $\alpha$ is measured positive in a counterclockwise sense from the positive $x$-axis. The rotational velocity $\omega$ (rad/s) is positive clockwise. An orthogonal Cartesian coordinate reference system $(x, z)$ is adopted with origin at the roller axle center $C$. The positive $z$-axis is oriented down, parallel to the local gravity vector. As a steady-state process, the kinematics of the roller is described by the position of its center of instantaneous rotation $I(x, z)$, located along a line passing through $C$ and normal to the original surface.

\[ x = -R_s k \sin \alpha \]  \hspace{1cm} (1)

\[ z = R_s k \cos \alpha \]  \hspace{1cm} (2)

The translational velocity of the roller center $C$ is

\[ V_C = \omega R_s k \]  \hspace{1cm} (3)

where $s_k = \text{slip factor}$ and $R_s k$ defines the position of point $I$ with respect to $C$. In terms of displacement, $s_k$ is an index number which, for a full wheel turn $(2\pi \text{ rad})$, indicates the axle displacement as a fraction of the developed wheel perimeter length. For a driven roller, $0 < s_k < 1.0$.

Limiting conditions are:

1. Pure rotation, $s_k = 0$ and $V_C = 0$.
2. Pure translation, $s_k = 1.0$ and $V_C = \omega R$.

The roller kinematics may also be evaluated, based on the knowledge of the rotational velocity $\omega$ and the translation velocity $V_C$, by following the definition of slip,

\[ s = \frac{V - V_C}{V} \times 100 \]  \hspace{1cm} (4)

where $V = \omega R$ = roller peripheral velocity. It is verified from Eq. (4) that, for driven rollers $s > 0$, the roller slips. For towed rollers, $s < 0$ and it is said that the roller skids. Based on Eq. (4), $s = 100\%$ slip for pure rotation and $s = 0\%$ slip for pure translation.

This type of motion description was used by Poletayev (Ref. 14) and by O'Nafeko and Reece (Ref. 15) in the study of rigid wheels on level terrain ($\alpha = 0$). Here, this motion concept is generalized to slopes where $\alpha \geq 0$. The problem is now posed as follows: Given a semifinite soil surface slope $\alpha$, possessing the unit volume weight $\gamma$ and strength properties defined by the cohesion $c$ and the friction angle $\phi$, loaded by an infinitely long rigid roller of radius $R$, carrying an axial weight $W$ and pulling a load $P^*$ per unit roller width $b$ parallel to the original slope, determine the

1. Operational slip factor $s_k$ which defines the limiting equilibrium condition that permits the roller center $C$ to move with uniform velocity $V_C$ (Eq. 3).
2. Driving torque $M$ capable of sustaining the velocity $V_C$.
3. Plastic failure pattern and state of stress that satisfy the kinematic, stress, and geometric boundary conditions.
4. Roller sinkage $z$ measured normal to the original surface.
5. Specific rolling energy required per unit surface normal load and unit distance of travel.
6. Safety factor with respect to roller immobilization, which occurs when the slip factor $s_k = 0$, for a given slope $\alpha$.

This study does not apply to the initial stages of rolling motion, and to conditions following roller immobilization. In the former case, inertial forces predominate and in the latter case, continued sinkage takes place. Both cases are associated with unsteady conditions. This study considers the problem of soil-roller interaction only under uniform operating velocity conditions up to and including immobilization.

B. Soil-Roller Velocity and Boundary Conditions

As mentioned, Wong and Reece (Ref. 11) and Boucherie (Ref. 10) have revealed the general nature of the soil failure pattern associated with rigid driven rollers and wheels moving on level soft surfaces at various slip and loading conditions. This photographic evidence indicates (Fig. 1):

1. Typical leading and trailing plastic regions are formed, each, respectively, moving fore and aft of the advancing roller.
2. Both flow regions tend to meet at a common point $M$ on the roller rim surface.
3. Soil sinkage increases with increasing loads and slip.
4. The roller leaves no rut due to local sinkage. This means, after the passage of the roller, the soil surface fully recovers due to backward soil transport produced by the advancing roller.
5. The relative sizes and pattern of the soil leading and trailing plastic regions depend on the kinematics of soil-roller interaction. At 100% slip, the leading plastic region disappears, leaving only the trailing plastic zone (Fig. 3).
The coordinates of a point on the roller rim are given by (Fig. 5)

\[ x_i = R \cos (\alpha + \xi_i) \]  
\[ z_i = R \sin (\alpha + \xi_i) \]

The corresponding absolute velocity components parallel to the coordinate axes \( x, z \) are, respectively,

\[ u_{x,i} = \omega (z_i - \bar{z}) \]  
\[ u_{z,i} = \omega (R - x_i) \]

The resultant velocity is

\[ V_i = \sqrt{u_{x,i}^2 + u_{z,i}^2} \]

The angular orientation of \( V_i \) is

\[ \beta_i = \tan^{-1} \left( \frac{u_{z,i}}{u_{x,i}} \right) \]

Next, the rim boundary velocities will be related to the soil flow in terms of the slipline velocity components along the soil-roller interface.

When the soil stress-strain law is derived, applying the concept of plastic potential, Drucker and Prager (Ref. 13) show that the relative particle velocity along a velocity discontinuity line is oriented at an angle \( \phi \) to the line. This concept will be used here to construct the plastic configuration.

At the bifurcation point \( M \), the corresponding trailing soil particle is subjected to a velocity component \( V_M^L \) tangent at \( M \) to the discontinuity slipline \( M(MN) \) and equal in magnitude and direction to the roller rim velocity \( V_M \). From Eqs. (9) and (10), for rim point \( i = M \) (Fig. 5),

\[ V_M = \left( u_{x,M}^2 + u_{z,M}^2 \right)^{1/2} = -V_M^L \]

The negative sign in Eq. (11) is consistent with the sign convention that a positive velocity component along the first slipline, when rotated through an angle \( (\pi/2) + \phi \), produces a positive velocity component along the second slipline (Ref. 7) (Fig. 6b). It is of interest to note that Wong (Ref. 16) postulated that the trajectory of the trailing soil particle at the bifurcation point \( M \) coincides with the direction \( \vec{b}_M \) of the trailing slipline at \( M \). Here, the adoption of a velocity component \( V_M^T \) instead of a resultant velocity as referred to by Wong, is due to the fact that the latter must be oriented at an angle \( \phi \) to the discontinuity line to comply with the requirements of the theory of plastic potential. Consequently, the trailing soil particle velocity resultant at \( M \) is

\[ V_M^T = \frac{V_M^L}{\cos \phi} \]

Continuity conditions at the soil-roller interface dictate that the radial velocity component of the leading soil particle at \( M \) must be equal to the radial velocity of the roller rim point \( M \). Then, the leading soil particle radial velocity component (Fig. 5) is

\[ V_{M,R} = V_M \cos (\omega + \xi_M - \vec{b}_M) = V_M \cos \Delta_M \]

where

\[ \Delta_M = \alpha + \xi_M - \vec{b}_M \]

and the corresponding velocity resultant is

\[ V_M^* = V_M R \sec \left[ \theta_M - (\omega + \xi_M + \phi) \right] \]

The velocity component along the first slipline at \( M \) is

\[ V_M^* = V_M^L \cos \phi \]

The angular orientation of the first slipline \( M(ML) \) at \( M \) is

\[ \theta_M - \vec{b}_M + \frac{\pi}{2} - \phi \]

Equations (11) to (18) represent the soil velocity boundary conditions at the plastic bifurcation point \( M \).

The validity of the theory of plastic potential to define the soil strain rates and the assumption
regarding the boundary velocity condition (Eqs. 11 and 12) have to be properly justified on an experimental basis. The former was adopted because of its rather simple application, and the latter appears to represent a reasonable fact.

C. Velocity Fields

The plastic failure pattern configuration is further postulated as follows (Fig. 4): The trailing region is divided into three plastic zones, M(MN)M, M(MN)A, and MAB, identified as the active, transition, and passive zones, respectively. The same applies to the leading plastic region for zones M(ML)L, L(ML)E, and L(EF), respectively.

1. Active zones. As mentioned previously, the sliplines M(MN)AB and M(ML)EF separate the rigid stationary boundary from the plastic deforming one. Since a boundary at rest must have a zero local normal velocity component, the velocity must be inclined at an angle \( \phi \) to the discontinuity line. Along M(ML)EF, the first slipline discontinuity, \( V' = 0 \), and along M(MN)AB, the second slipline discontinuity, \( V'' = 0 \). Since both these discontinuity lines start at rim point M, the resultant soil particle velocity at point M (trailing zone) is

\[
\beta - V_M \cos \phi = \omega a_M = \omega r_M \tag{19}
\]

where

\[
r_M = \frac{a_M}{\cos \phi} \tag{20}
\]

and

\[
a_M = \left[ (x_M - x) + (z_M - z)^2 \right]^{1/2} \tag{21}
\]

From Eq. (20), when \( \phi = 0, r_M = a_M \) and \( l \) coincides with \( I \) and the trailing spiral slipline transforms into circles centered at \( I \).

The resultant velocity of the leading soil particle at \( I \), based on Eqs. (14) and (15), is

\[
V_I = V_M \cos \Delta_M = \omega \Delta_M \cot \Delta_M = \omega \beta_M \tag{22}
\]

where, as shown in Fig. 5,

\[
\beta - \Delta_M = \frac{\beta_M}{\cos \Delta_M} \tag{23}
\]

Expressions (19) and (22) indicate the transformation of the rim velocity \( V_M \) into the corresponding soil particle velocity at the roller interface point \( M \). Particularly when

\[
s_k = 0, \quad \Delta_M = \phi + \beta_M - \beta_M = \frac{\pi}{2}
\]

then, from Eq. (23), \( \beta_M = 0 \), indicating that there is no leading failure zone, which is verified from tests (Fig. 3) and is schematically shown in Fig. 7. These velocity criteria produce, in general, a tangential soil velocity "jump" at \( M \), which may be obtained by projecting the velocities, Eqs. (19) and (22), along the roller tangential direction at \( M \).

Next, we discuss the nature of the remaining soil-roller interface velocity conditions and the plastic domain. The governing velocity equations that refer to the first and second characteristic lines, as determined by Shield (Ref. 7), are

\[
\frac{dV'_M}{V'_M} = (V' \tan \phi + V'' \sec \phi) \frac{d\theta}{0} = 0 \tag{24}
\]

\[
\frac{dV''}{V''} + (V' \tan \phi + V'' \sec \phi) \frac{d\theta}{0} = 0 \tag{25}
\]

Applying Eq. (25) to the trailing zone discontinuity line, with \( V'' = 0 \), yields

\[
\int V' + V' \tan \phi \frac{d\theta}{0} = 0 \tag{26}
\]

which, with Eq. (19), reduces to

\[
V' = V_M \exp \left[ (\theta_M - \phi) \tan \phi \right] \tag{27}
\]

Equations (26) and (27) indicate that the velocities along the discontinuity line M(MN) vary exponentially and that they relate to a logarithmic spiral function

\[
r = r_M \exp \left[ \tan \phi (\theta_M - 0) \right] \tag{28}
\]

The coordinate positions of the trailing spiral pole \( I \) \((x_I, z_I)\) are (Fig. 5)

\[
x_I = x_M - r_M \cos \theta_M \tag{29}
\]

\[
z_I = z_M - r_M \sin \theta_M \tag{30}
\]

with \((x_M, z_M)\) given by Eqs. (5) and (6) for \( i = M \) and \( \theta_M \) given by Eq. (18). Also, for the leading zone along the spiral velocity discontinuity line, M(ML) is from Eq. (24),

\[
V_M = V_M' \exp \left[ 0(\theta_M - 0) \tan \phi \right] \tag{31}
\]

JPL Technical Memorandum 33-477
From Eqs. (17) and (22), Eq. (31) reduces to

$$V^* - \omega \cos \phi r_M \exp \left[ i \theta - \theta_M \right]$$

which applies along

$$F = r_M \exp \left[ i \theta - \theta_M \right] \tan \phi$$

The coordinate positions of the leading spiral pole \(T_2(\bar{x}_2, \bar{y}_2)\) are

$$\bar{x}_2 = x_M - r_M \cos \theta_M$$

$$\bar{y}_2 = y_M - r_M \sin \theta_M$$

Equations (26) to (35) yield the geometry of the velocity distribution along the trailing and leading discontinuity lines. They indicate also that the velocities along these discontinuities correspond to rotations about the poles \(T_1\) and \(T_2\), respectively.

The geometrical character of the poles \(T_1\) and \(T_2\), in terms of the relative variations of \(s_k\) and \(\xi_M\), may be graphically described with reference to Fig. 8 as follows:

1. Given a point \(M\) on the roller rim with \(\xi_M = \pi/2\) and \(0 \leq s_k \leq 1, 0\), the locus of all corresponding trailing spiral poles \(T_1\) (Eqs. 29 and 30) defines a straight segment \(T_1T_1\) oriented at an angle \((-\pi/2) - \phi\) to the \(x\)-axis. The extreme points \(T_0\) and \(T_1\) of this segment correspond to \(s_k = 0\) and \(s_k = 1\), respectively. Any intermediate point between \(T_0\) and \(T_1\) pertains to \(s_k\) such as \(0 < s_k < 1\). For \(\xi_M = \pi/2\), the poles \(T_1\) and \(T_2\) are always located inside the roller periphery. The quadrant position of \(T_1\) is given by

$$\bar{p}_1 = \cos^{-1} \left( \frac{x_1}{\sqrt{x_1^2 + z_1^2}} \right)$$

A similar expression applies to \(\bar{p}_2\) for \(\xi_2\) (Fig. 5). In general, it may be verified that, for \(s_k < R \sin \xi_M\),

$$\bar{p}_1 = \frac{\pi}{2}$$

2. On the same basis as in (1) above, Fig. 8 indicates that the locus of all corresponding leading spiral poles \(L_2\) (Eqs. 33 and 34) defines a curve \(MM_1\) where \(M\) and \(M_1\) relate to \(s_k = 0\) and \(s_k = 1, 0\), respectively. In general, points \(M\) and \(M_1\) are always located on the roller rim. Any intermediate point between \(M\) and \(M_1\) corresponds to \(s_k\) such as \(0 < s_k < 1\).

Point \(M_1\) at \(\xi_M = \pi/4\) remains fixed under all conditions of rolling without slip \((s_k = 1, 0)\).

3. There is a conformal geometric relationship of \(\xi_M\) and the \(T_2T_1\) lines. When point \(M\) shifts along the roller periphery, the segment \(T_0T_1\) moves parallel to itself. Thus, for increasing (or decreasing) \(\xi_M\), the segment \(T_0T_1\) moves closer to (or farther away from) the roller center \(C\).

The locus of poles \(T_1\) for \(s_k = \text{constant}\) describes a series of circles with radius \(R_k = R \sin \phi\) (Fig. 9). The center of these circles are located along a line \(CO_1G_0\) oriented by the parameter \((-\pi/2) - \phi\) to the \(x\)-axis. Along this line, the center marked \(G_0 = G\) corresponds to the circle with \(s_k = 1, 0\) and the center \(G_0 = G\) to the circle for which \(s_k = 0\). The center of a generic circle corresponding to any \(0 < s_k < 1\) will be located proportionally between points \(CO_1G_0\) as shown in Fig. 9.

4. Given a fixed slip-factor \(s_k\), an increase (or reduction) of \(\xi_M\) correspondingly produces an increase (or reduction) of both spiral radial vectors \(r_M\) and \(\bar{p}_M\).

5. When \(\xi_M\) is fixed, an increase of \(s_k\) produces a reduction of \(r_M\) and \(\bar{p}_M\).

The geometrical implication of the above statements relates to the basic fact that the limiting soil stress and spiral orientations (characteristics) are controlled by the relative position of poles \(T_1\) and \(T_2\), as described by the parametric set \(\xi_M\) and \(s_k\). In Section V, it will be shown that once the equilibrium equations and stress boundary conditions are satisfied, the final magnitude of the soil stress will also relate directly to \(\xi_M\) and \(s_k\) in terms of \(r_M\) (Eq. 20) and \(\bar{p}_M\) (Eq. 21).

Next, consider the active plastic domains between the soil-roller interface and the discontinuity lines \(M(MN)\) and \(M(ML)\), which are fully described by a set of radial and logarithmic spiral characteristics with poles \(T_1\) and \(T_2\), respectively. First, it will be shown that the soil-roller velocity boundary conditions are also satisfied.

As mentioned at the beginning of this section, \(V^* = 0\) along the trailing velocity discontinuity slinline spiral \(M(MN)\). The corresponding intersecting first slinline are radial lines through the spiral pole \(T_1\), based on Eq. (25). \(V^* = 0\) throughout the active trailing zone. This implies that at a point \(i\) on the soil-roller rim interface (Fig. 10), the resultant soil particle velocity \(V_i\) must be normal to the radial slinline. Then, the velocity component along the spiral second slinline through point \(i\) is

$$V_i^* = -\frac{V_i}{\cos \phi} \cos \phi$$

and

$$V_i^* = 0$$
with $V_i$ obtained from Eq. (9) and $\theta_i$ from Eq. (10) and

$$ \theta_i = \tan^{-1} \left( \frac{x_j - x_i}{y_j - y_i} \right) $$

The velocity of any soil particle $(ij)$ located at a point $j$ along a generic spiral slipline passing through rim point $i$ within the trailing plastic domain (Fig. 10) is

$$ V_{ij}^1 = V_i \exp \left[ (\theta_i - \theta_{ij}) \tan \phi \right] $$

and

$$ V_{ij}^2 = 0 $$

Following the same continuity criteria set as for the leading soil particle at the bifurcation point $M$ (Eq. 14), the radial velocity component of the soil particle at any point $i$ along the leading soil-roller interface (ML), Fig. 10, is

$$ V_{i,R} = V_i \cos (\alpha + \xi_i - \bar{\theta}_i) $$

where

$$ V_i = \frac{V_{i,R} \cos \phi}{\cos (\theta_i - \alpha + \xi_i + \phi)} $$

Using Eq. (44) and $\theta_i + \phi = \theta' + \pi/2$, Eq. (45) reduces to

$$ V_{ij} = V_i \cos (\alpha + \xi_i - \bar{\theta}_i) $$

also,

$$ V_{ij}^2 = 0 $$

For a soil particle $(ij)$ located at point $j$ along a spiral slipline passing through rim point $i$.

$$ V_{ij}^2 = V_i^2 \exp \left[ \tan \phi (\theta_i - \theta_{ij}) \right] $$

and

$$ V_{ij}^1 = 0 $$

The active zone velocity expressions (42), (43), (48), and (49) extend up to and including the points along the radial lines $MN$ and $ML$. Thus far, the characteristic lines and velocities pertaining to the active leading and trailing zones have been completely defined. It has also been established that the velocity of each point on the roller rim can be transformed into an equivalent admissible velocity along the corresponding sliplines that intersect the point. In particular, the velocity orientation of any point on the soil-roller rim interface, with respect to its center of instantaneous rotation $L$, may also be defined in terms of the spiral pole positions $I_1(x_1, y_1)$ and $I_2(x_2, y_2)$ for points along the arcs $MN$ and $ML$, respectively.

The locations of poles $I_1$ and $I_2$ are functions of the velocity boundary conditions, described by $s_k$ and $s_M$. Instead the location of rim points $N$ and $L$, cannot only be obtained in connection with the solution of the stress equilibrium equations. These equations allow the determination of the extent and configuration of the plastic domain by specifying a final set of slipline directions consisting of (Fig. 4)

$$ \left\{ \theta_{i,LO}, \theta_{i,LL}, \theta_{i,M}, \theta_{NN}, \theta_{NO} \right\} $$

which will be calculated in Section VI-A.

The velocity field determined thus far satisfies the soil-roller velocity boundary conditions and the velocity field equations (Eqs. 24 and 25). Their detailed numerical evaluation is not a prerequisite for the solution of the limiting stress equations, but their existence is important to the correct statement of a "complete solution" within the context of the theory of plasticity (Ref. 12). Furthermore, the existence of an admissible velocity field must also satisfy the postulate of positive rate of dilation (Appendix).

Points $N$ and $L$ are singular points on which the velocities are multivalued. At these points, the soil-roller rim interface separates itself from the trailing and leading traction-free soil surfaces.

Rim points $(NN)$ and $(LL)$ correspond to sliplines $N(MN)$ and $L(ML)$, respectively, and as such both pertain to the soil-roller interface. Points $(NO)$ and $(LO)$ correspond to the characteristic lines $NA$ and $LE$ separating the transition from the passive zones and belong to the traction-free trailing and leading soil surfaces, respectively.

The geometry of the characteristic lines $MN$ and $ML$ are determined by

$$ r_N = \frac{x_{MN} - x_N}{\cos \theta_{NN}} $$

and

$$ r_L = \frac{x_{ML} - x_L}{\cos \theta_{LL}} $$

where

$$ x_{MN} = x_N + r_M \exp \left[ (\theta_M - \theta_{NN}) \tan \phi \right] \cos \theta_{NN} $$

JPL Technical Memorandum 33·477
\[ x_{ML} = x_L + F_x \exp \left[ (e^{0_{NN}} - 0_{LL}) \tan \phi \right] \cos \theta_{LL} \]

with coordinates \( x, z \) obtained from Eqs. (5) and (6) for \( i = N \) and \( i = L \).

\[ \theta_{NN} = \tan^{-1} \left( \frac{x_N - x_1}{x_N - x_1} \right) \]  

and

\[ \theta_{LL} = \tan^{-1} \left( \frac{x_L - x_2}{x_L - x_2} \right) \]

2. Transition and passive zones. The characteristics of the transition and passive zones (Fig. 4) are calculated after the soil-roller equilibrium conditions corresponding to the active zones are satisfied. This will be referred to again in Section V-B-3 and in Part II.
V. SOIL-ROLLER EQUILIBRIUM

A. Limiting Soil Stress Field

An admissible velocity characteristic field has been defined that satisfies both the velocity boundary conditions and the governing differential equations for the velocities. When the velocities are derived from the yield stress condition according to the concept of plastic potential, the stress characteristics coincide with the characteristics of the velocity (Ref. 7). The problem now is to associate an admissible stress field that satisfies the equilibrium conditions along the velocity characteristics.

The limiting state of stress at a point occurs when the Mohr circle of stress becomes tangent to the soil strength envelope line defined by the Coulomb formula $T = C + \frac{1}{2} \tan \phi$, where $C$ is the cohesion and $\phi$ the soil angle of internal friction. In terms of the point stresses $\sigma_x$, $\sigma_z$, and $\tau_{xz}$, considering compression stress as positive, the Coulomb yield criterion for soils is (Ref. 17) (Fig. 6a)

$$
\sigma_x = \sigma_y + \sin \phi \sin (2\theta - 2\Omega + \phi) - c \cot \phi \quad (59)
$$

$$
\sigma_z = \sigma_y + \sin \phi \sin (2\theta + \phi) - c \cot \phi \quad (60)
$$

$$
\tau_{xz} = p \sin \phi \cos (2\theta + \phi) \quad (61)
$$

where $p$ is the reduced mean stress parameter defined by

$$
p = \frac{\sigma_1 - \sigma_2}{2 \sin \phi} = \frac{1}{2} (\sigma_x + \sigma_z) + c \cot \phi \geq 0 \quad (62)
$$

To completely define a limiting state of stress at a point $(x, z)$, it is sufficient to know the stress parameters $p$ and $\theta$. Substituting Eqs. (59 - 61) in Eqs. (57) and (58) and adopting $s$ and $s'$, first and second slippelines, respectively, as a new set of curvilinear coordinates, the partial differential equations of limiting equilibrium are obtained along the characteristics (Refs. 6 and 19):

$$
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} - \gamma = 0 \quad (57)
$$

$$
\frac{\partial \sigma_z}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + \gamma = 0 \quad (58)
$$

Although during plastic deformation the soil density decreases (Appendix), it is assumed in Eqs. (63) and (64) that $\gamma$ remains constant.

B. Stresses Along Slippelines

Using the Mohr limiting stress circle, the normal stress $\sigma$ and shear stress $\tau$ at any point along a slippeline may be expressed by

$$
\sigma = p [1 + \sin \phi \sin (2\theta - 2\Omega + \phi)] - c \cot \phi \quad (65)
$$

$$
\tau = p \sin \phi \cos (2\theta - 2\Omega + \phi) \quad (66)
$$

where $\Omega$ is the direction of the slippeline normal through the point under consideration. In particular, along a first slippeline

$$
\Omega = \theta' - \phi \quad (67)
$$

and along a second slippeline

$$
\Omega = \theta + \phi \quad (68)
$$

Substituting Eq. (67) or (68) in Eqs. (65) and (66), the general state of stress along both families of characteristics are defined by

$$
\sigma = p \cos^2 \phi - c \cot \phi \quad (69)
$$

$$
\tau = p \sin \phi \cos \phi \quad (70)
$$

The plus sign in Eq. (70) indicates that the stress resultant at the point is rotated counterclockwise.
with reference to the local normal. In what follows, the corresponding \( p \) values in Eqs. (69) and (70) will be defined along the sliplines \( L(ML) \), \( M(MN) \), and \( N(NN) \), and the stress along these sliplines will be used to evaluate the corresponding soil reaction forces.

1. Stress parameter \( p \) along straight sliplines. Alternate equilibrium equations are obtained setting \( ds = dx/cos \theta \) and \( ds' = dx/cos \theta' \) in Eqs. (63) and (64):

\[
p + 2p \tan \phi \; dx = \gamma \; dx \; (\tan \theta + \tan \phi) \tag{71}
\]

\[
p - 2p \tan \phi \; dx = \gamma \; dx \; (\tan \theta' - \tan \phi) \tag{72}
\]

Expressions (71) and (72) will be used to determine the stresses along the straight sliplines \( N(MN) \) and \( L(ML) \), respectively. Since \( \theta_{LL} \) and \( \theta_{NN} \) are constant, \( \theta' = \theta' - \theta \), and Eqs. (71) and (72) reduce to

\[
dp = \gamma \; dx \; (\tan \theta + \tan \phi) \tag{73}
\]

\[
dp = \gamma \; dx \; (\tan \theta' - \tan \phi) \tag{74}
\]

Integration of Eqs. (73) and (74) gives, respectively,

\[
p - \gamma \tan \theta + \tan \phi)x + C \tag{75}
\]

\[
p = \gamma \tan \theta' - \tan \phi)x + C' \tag{76}
\]

To determine the constant of integration at rim point \( (LL) \), \( x = x_L, \theta' = \theta_{LL} \), and \( p = \rho_{LL} \) then

\[
C' = p_{LL} - \gamma \tan \theta_{NN} - \tan \phi)x_L \tag{77}
\]

For rim point \( (NN) \), \( x = x_N, \theta' = \theta_{NN} \), and \( p = \rho_{NN} \) then

\[
C = p_{NN} + \gamma \tan \theta_{NN} + \tan \phi)x_N \tag{78}
\]

For point \( (ML) \) along \( L(ML) \), \( x = x_{ML} \) and

\[
p_{ML} = \rho_{LL} + \gamma (\tan \theta'_{LL} - \tan \phi)(x_{ML} - x_L) \tag{79}
\]

\[
p = \rho_{LL} + \Delta p_{ML} \tag{80}
\]

where

\[
\Delta p_{ML} = \gamma \tan \theta'_{LL} - \tan \phi')(x_{ML} - x_L) \geq 0 \tag{81}
\]

Similarly, for trailing point \( (MN) \) along the slipline \( N(MN) \), \( x = x_{MN} \) and

\[
p_{MN} = p_{NN} + \gamma (\tan \theta'_{NN} + \tan \phi)(x_{MN} - x_N) \tag{82}
\]

\[
p = p_{NN} + \Delta p_{MN} \tag{83}
\]

where

\[
\Delta p_{MN} = \gamma \tan \theta'_{NN} + \tan \phi)(x_{MN} - x_N) \geq 0 \tag{84}
\]

Both points \( L \) and \( N \) constitute singular stress points around which the stress are multivalued, each representing geometrically a limiting first and second characteristic lines, respectively, having zero radius of curvature. When points \( L \) and \( N \) belong to the soil-rim interface side, they are identified as \( (LL) \) and \( (NN) \). If these points belong to the traction-free soil surface, they are identified as points \( (LO) \) and \( (NO) \). To completely determine Eqs. (77) and (79), it is required to define the value of \( \rho_{LL} \) and \( \rho_{NN} \). Applying Eq. (71) to point \( L \),

\[
p = -\gamma \tan \theta \; dx + \mu \tag{85}
\]

\[
\ln p = -2 \tan \theta \; dx + C \tag{86}
\]

For point \( (LO) \), with \( \theta = \theta_{LO} \) and \( p = p_{LO} \), we obtain

\[
C = \ln p_{LO} + \delta \tan \theta_{LO} \tag{87}
\]

For rim point \( (LL) \), \( x = x_{LL}, \theta = \theta_{LL} \), \( p = \rho_{LL} \) and

\[
\rho_{LL} = p_{LO} \exp \left( \theta_{LL} - \theta_{LO} \right) \tag{88}
\]

Similarly for rim point \( (NN) \),

\[
\rho_{NN} = p_{NO} \exp \left( \theta_{NN} - \theta_{NO} \right) \tag{89}
\]

The stress parameters \( p_{LO} \) and \( p_{NO} \) are determined, considering a passive failure condition. Since the soil surface is free of stresses, \( \sigma' = \tau_0 = 0 \), then from the Mout circle of stress.

\[
p_{LO} = p_{NO} = \frac{c \cos \phi}{1 - \sin \phi} \tag{90}
\]

According to Eqs. (77) and (79), the stress parameter \( p \) varies linearly with depth; consequently, we can operate with average stresses along \( L(ML) \) and \( N(MN) \) sliplines. The average stress
parameter $p_L$ along the leading slip line segment L(ML), using Eq. (77), is

$$p_L = p_{LL} + \frac{\Delta p_{ML}}{2} \quad (84)$$

where $\Delta p_{ML}$ is given by Eq. (78). The corresponding average normal and shear stresses within the second slip line segment L(ML), using Eqs. (69) and (70), are (Fig. 13)

$$\bar{\sigma}_L = p_L \cos^2 \phi - c \cot \phi \quad (85)$$

$$\bar{\tau}_L = p_L \sin \phi \cos \phi \quad (86)$$

The stress parameters $p_{MN}$ at (MN) must be the same when approaching point (MN) either along the slip line (M(MN)) or along the spiral M(MN). Thus, the average value of the stress parameter $p_{N}$ corresponding to the slip line segment N(MN) is

$$p_N = p_{MN} + \frac{\Delta p_{MN}}{2} \quad (87)$$

with $p_{MN}$ given by Eq. (79) and $\Delta p_{MN}$ by Eq. (80).

The corresponding average normal and shear stresses within the second slip line segment N(MN) are (Fig. 13)

$$\bar{\sigma}_N = p_N \cos^2 \phi - c \cot \phi \quad (88)$$

$$\bar{\tau}_N = p_N \sin \phi \cos \phi \quad (89)$$

2. Stress parameter $p$ along spiral slip lines. To determine the stresses along the spiral

$$p_{s} = -\frac{\bar{\tau}_{s}}{\cos \phi} \exp \left[ (2\theta + \theta_{s}) \tan \phi \right] \int_{\theta}^{\theta_{s}} \exp \left[ 3 \tan \phi \cos \phi \theta + \sin \phi \right] d\theta + C \quad (90)$$

Integrating,

$$p_{s} = -\frac{\bar{\tau}_{s}}{\cos \phi} \exp \left[ (2\theta + \theta_{s}) \tan \phi \right] \left\{ \exp \left[ 3 \tan \phi \cos \phi \theta + \sin \phi \right] \right\} \cos \theta + C \quad (91)$$

and, for the leading (MLM) spiral,

where $ds$ and $ds'$ are the elemental arc lengths along a first and second slip line, respectively.

To determine the state of stress along the spiral slip lines, adopt as positive directions of the characteristic spirals the ones which relate to decreasing values of $\theta$ and $\theta'$ (Fig. 11). Expressing $ds$ and $ds'$ in terms of the radius of curvatures $p$ and $p'$, for the trailing M(MN) spiral, we get

$$ds = -p \, d\theta = -\frac{r_M \exp \left[ (\theta_M - \theta) \tan \phi \right]}{\cos \phi} \, d\theta \quad (92)$$

and, for the leading (MLM) spiral,

$$ds' = -p' \, d\theta' = -\frac{r_M \exp \left[ (\theta_M - \theta) \tan \phi \right]}{\cos \phi} \, d\theta' \quad (93)$$

Replacing Eq. (92) in Eq. (90) yields the stress parameter $p_{s}^{L}$ along the leading spiral as a function of $\theta$:

$$p_{s}^{L} = -\frac{\bar{\tau}_{s}}{\cos \phi} \exp \left[ (2\theta + \theta_{s}) \tan \phi \right] \left\{ \exp \left[ 3 \tan \phi \cos \phi \theta + \sin \phi \right] \right\} \cos \theta + C \quad (94)$$

When $\theta' = \theta_{LL}$, $p_{s}^{L} = p_{ML}^{L}$, and the constant of integration is

$$C = p_{ML} + C_{s}^{L} \exp \left[ (2\theta_{LL} + \theta_{s}) \tan \phi \right] \left( \cos \phi \theta_{LL} + \sin \theta_{LL} \right) - p_{ML} + C_{s}^{L} \quad (95)$$

where

$$C_{s}^{L} = \frac{\bar{\tau}_{s}}{(9 \tan^2 \phi + 1) \cos^2 \phi} \quad (96)$$
Replacing Eq. (95) in Eq. (94) results in

$$p_s^* = p_{ML}^* + C_s^L - C_s^L \exp \left[ (\theta_M^* - \theta_M) \tan \phi \right] \left( 3 \tan \phi \cos \theta_M^* + \sin \theta_M^* \right)$$

(97)

To obtain $p_M^*$, setting in Eq. (97) $\theta' = \theta_M^*$ gives

$$p_M^* = p_{ML}^* + C_s^L - C_s^L \left( 3 \tan \phi \cos \theta_M^* + \sin \theta_M^* \right)$$

(98)

with

$$C_s^L = 3 \tan \phi \cos \theta_M^* + \sin \theta_M^*$$

(99)

and

$$F_3 = C_s^L - C_s^L C_s^L$$

(100)

Equation (98) reduces to

$$p_M = p_{ML} + F_3$$

(101)

Also, with Eq. (77),

$$p_M - p_{LL} + \Delta p_{ML} + F_3 = p_{LL} + F_4$$

(102)

where

$$F_4 = \Delta p_{ML} + F_3$$

(103)

Similarly, replacing Eq. (93) in Eq. (91), the stress parameter $p_s^T$ along the trailing spiral is

$$p_s^T = \frac{\gamma r_M}{\cos^2 \phi} \exp \left[ (\theta_M + \theta_M^*) \tan \phi \right] \int \exp \left[ -3 \theta \tan \phi \right] \cos \theta \ d\theta + C$$

(104)

Integrating,

$$p_s^T = \frac{\gamma r_M}{(9 \tan^2 \phi + 1) \cos^2 \phi} \exp \left[ (\theta_M + \theta_M^*) \tan \phi \right] \left\{ \exp \left[ -3 \theta \tan \phi \right] - 3 \tan \phi \cos \theta + \sin \theta \right\} + C$$

(105)

When $\theta = \theta_M^*$, $p_s^T = p_M^*$ and the constant of integration is

$$C = p_M - C_s^T (-3 \tan \phi \cos \theta_M + \sin \theta_M) = p_M - C_s^T$$

(106)

where

$$C_s^T = \frac{\gamma r_M}{(9 \tan^2 \phi + 1) \cos^2 \phi}$$

(107)

and

$$C_s^T = C_s^T (-3 \tan \phi \cos \theta_M + \sin \theta_M)$$

(108)

Replacing Eq. (106) in Eq. (105) results in

$$p_s^T = p_M - C_s^T + C_s^T \exp \left[ (\theta_M - \theta) \tan \phi \right] (-3 \tan \phi \cos \theta + \sin \theta)$$

(109)
To obtain $p_{MN}$, set $\theta = \theta_{NN}$ in Eq. (108):

$$p_{MN} = p_M - C^T_1 + C^T_s \exp \left[ (\theta_M - \theta_{NN}) \tan \phi \right] - 3 \tan \phi \cos \theta_{NN} + \sin \theta_{NN} = p_M + \Delta p_{MN}$$

where

$$\Delta p_{MN} = - C^T_1 + C^T_s \exp \left[ (\theta_M - \theta_{NN}) \tan \phi \right] - 3 \tan \phi \cos \theta_{NN} + \sin \theta_{NN}$$

With Eq. (102) and based on Eq. (109),

$$p_{MN} = p_{LL} + F_4 + \Delta p_{MN} - p_{LL} + F_{11} \geq 0$$

where $F_4$ is given by Eq. (103), and

$$F_{11} = F_4 + \Delta p_{MN}$$

The last condition in Expression (112) insures that point (MN) is at or below the ground surface.

The equations derived in this section will be utilized to determine the soil stresses and reactions that are considered in Sections V-C and E, and VI-A.

3. Transition and passive zones (Fig. 4).

The stress characteristic lines corresponding to both the transition and passive zones are determined by solving numerically the finite difference form of Eqs. (63) and (64), subject to their corresponding boundary stress conditions.

The boundary conditions of the transition zones relate to: (1) the stress along the characteristic lines L(ML) and N(MN), respectively, and (2) the state of stress around the singular points L and N as defined in Section V-B. These stresses are known from the solution of the soil-roller active zones equilibrium equations in connection with the stress compatibility conditions, described in Section VI-A. For the passive zones, the solution is obtained by extending the transition stress characteristics into the passive zones, considering the traction-free leading and trailing soil surfaces LF and NB. This solution also yields the final deformed and statically correct configuration of the soil surfaces LF and NB (Fig. 4).

The solution procedure and corresponding applications will be described in detail in Part II of this study.

C. Limiting Slipline Directions

There exist definite limitations regarding the values that the $\theta$ parameters may acquire along the soil-roller interface. It will be shown that the solution of the soil-roller problem entails, to a large extent, calculation of an admissible set of characteristic directions at points L, M, and N defined by

$$\left\{ \theta_L, \theta_L', \theta_M, \theta_{NN}, \theta_{NO} \right\}$$

For any solution to a given soil-roller problem, it must be verified that the values corresponding to Expression (114) are within acceptable bounds. Determination of these bounds is necessary not only to validate the solution itself, but also to systematically initiate and objectively search for solutions utilizing only admissible $\theta$ values. Obviously, the final set of Expression (114) is obtained only after satisfying equilibrium and boundary conditions. In general, the absolute bounds to Expression (114) result from considering the envelope of values originating from (1) the maximum obliquity of the stress resultant as related to the radial direction at the soil-roller interface, (2) the maximum free surface slope as dictated by either the local roller rim surface tangent or the soil natural angle of repose, and (3) the stress compatibility requirements along the transition slipline zone. Each of these cases are considered next.

1. Soil-roller interface - maximum stress obliquity. At any point along the soil-roller interface, the shear stress $\tau$ and the reduced stress $\sigma + c \cot \phi$ define a stress resultant

$$q_r = \left[ \tau^2 + (\sigma + c \cot \phi)^2 \right]^{1/2}$$

The obliquity angle $\delta$ of $q_r$ is measured positive for a clockwise rotation relative to the rim surface normal (radial direction) and is defined by

$$\delta = \tan^{-1} \left( \frac{\tau}{\sigma + c \cot \phi} \right) \leq \phi$$

When $\delta = \phi$, Eq. (116) indicates that one of the sliplines becomes tangent to the roller rim surface at the point under consideration. In this case, for the trailing point (NN), with

$$\xi_N < \frac{\pi}{2}$$

JPL Technical Memorandum 33-477
The corresponding slipline directions for points (NN) and (LL) are defined by

\[
\theta_{NN} = \alpha + \xi_N - \phi \quad (118)
\]

For the leading rim point (LL), with

\[
\xi_L > \xi_M > \frac{\pi}{2} \quad (119)
\]

\[
\theta_{LL} = \alpha + \xi_L + \phi \quad (120)
\]

The corresponding slipline directions for points (NN) and (LL) are defined by

\[
\tan \theta_{NN} = \frac{R \sin (\alpha + \xi_N) - x_1}{R \cos (\alpha + \xi_N) - x_1} \quad (121)
\]

or

\[
\theta_{NN} = \xi_N + \alpha - \sin^{-1} \left[ \frac{1}{R} (\xi_N \cos \theta_{NN} - x_1 \sin \theta_{NN}) \right] \quad (122)
\]

and

\[
\tan \theta_{LL} = \frac{R \sin (\alpha + \xi_L) - x_2}{R \cos (\alpha + \xi_L) - x_2} \quad (123)
\]

or

\[
\theta_{LL} = \xi_L + \alpha - \sin^{-1} \left[ \frac{1}{R} (\xi_L \cos \theta_{LL} - x_2 \sin \theta_{LL}) \right] \quad (124)
\]

Adopting Eq. (118) in connection with Eq. (121) and Eq. (120) with Eq. (123), the limiting \(\theta_{NN}\) and \(\theta_{LL}\) values for \(\phi = \phi\) are obtained from the following quadratic equations:

\[
\left( \frac{x_1^2 + x_1^2}{2} \right) \cos^2 \theta_{NN} + 2Rz_1 \sin \phi \cos \theta_{NN} \quad (125a)
\]

\[
+ \left( R^2 \sin^2 \phi - \frac{x_1^2}{2} \right) = 0
\]

\[
\left( \frac{x_2^2 + x_2^2}{2} \right) \cos^2 \theta_{LL} + 2Rz_2 \sin \phi \cos \theta_{LL} \quad (125b)
\]

\[
+ \left( R^2 \sin^2 \phi - \frac{x_2^2}{2} \right) = 0
\]

When the discriminant of Eqs. (125a, b)

\[
\begin{align*}
\frac{x_1^2 + x_1^2}{2} & \geq R \sin \phi = R \phi \\
\frac{x_2^2 + x_2^2}{2} & \geq R \sin \phi = R \phi
\end{align*}
\]
An admissible value of $\theta_{LL}$ must also satisfy the conditions of Eq. (78):

$$\Delta p_{ML} = \gamma (x_{ML} - x_L) (\tan \theta_{LL} - \tan \phi) \geq 0$$

(129)

When $\theta_{LL} < \pi/2$, $(x_{ML} - x_L) > 0$; therefore,

$$\min \theta_{LL} \geq \phi$$

(130)

When $\xi = 0$, $\xi_M = \xi_L$, and $x_{ML} = x_L$; then there is no leading plastic zone (Figs. 3 and 7). In this case, Eq. (78) reduces to $\Delta p_{ML} = 0$, and the spiral M(MN) is tangent to the roller rim at M:

$$\theta_{LO} - \theta_{LL} = \theta_{ML} - \phi + \xi_M - \frac{\pi}{2}$$

(131)

Also, Eq. (81) defines $p_{LL} = p_M = p_0$ as given by Eq. (83).

A similar analysis with reference to Eq. (83) yields, for $c \geq 0$,

$$\max \theta_{NN} \leq \pi - \phi$$

(132)

In general, $\xi_N \leq \pi/2$, which, from Eq. (121), corresponds to

$$\theta_{NN} = \tan^{-1} \left( \frac{R \cos \alpha - \xi_1}{-R \sin \alpha - \xi_1} \right)$$

(133)

2. Free soil surface – singular points (LO) and (NO). The limits of $\theta_{LO}$ and $\theta_{NO}$ are dictated by the condition that the free surface slopes at points L and N defined by $\alpha_{NO}$ and $\alpha_{NO}$ can at most be tangent to the roller rim surface. Thus,

$$\alpha_{LO} \leq \alpha + \xi_L + \frac{\pi}{2}$$

(134)

$$\alpha_{NO} \geq \alpha + \xi_N - \frac{\pi}{2}$$

(135)

Also when dealing with cohesionless soils ($c = 0$), the free surface maximum slope cannot exceed the soil natural angle of repose. Therefore,

$$\left| \alpha_{LO} \right| \leq \phi$$

(136)

$$\left| \alpha_{NO} \right| \leq \phi$$

(137)

Obviously, if $c > 0$, the angle of repose has no significance since the soil is stable up to and including vertical slopes as long as the critical height is not exceeded (Ref. 17, p. 152).

The transition plastic zone separating the active from the passive zones may, in the limit, disappear and allow the latter two zones to merge side by side; therefore,

$$\theta_{LO} \geq \theta_{LL}$$

(138)

$$\theta_{NO} \leq \theta_{NN}$$

(139)

In terms of slipline parameters in general, with $\mu = \pi/4 - \phi/2$, Eqs. (134) and (135) reduce to

$$\alpha_{LO} = \theta_{LO} + \mu$$

(140)

$$\alpha_{NO} = \theta_{NO} - \mu$$

(141)

Based on Eq. (141), Conditions (134) through (139) reduce, for cohesive soils ($c > 0$), to

$$\alpha + \xi_L + \frac{\pi}{2} + \mu \geq \theta_{LO} \geq \theta_{LL}$$

(142)

and, for cohesionless soils ($c = 0$), with Conditions (136) and (137),

$$\pi + \phi - \mu \geq \theta_{LO} \geq \pi - (\mu + \phi)$$

(144)

$$\mu - \phi \leq \theta_{NO} \leq \phi + \mu$$

(145)

3. Summary of absolute admissible bounds for $\theta_{LO}$, $\theta_{LL}$, $\theta_{NN}$, and $\theta_{NO}$. Recapitulating Subsections C-1 and C-2, the limiting slipline directions at points L and M are as follows:

For $c \geq 0$, from Expressions (118) and (130),

$$\phi \leq \theta_{LL} \leq \alpha + \xi_L + \phi$$

(146)

Based on Expressions (120), (132), and (133), we obtain

$$\left\{ \begin{array}{l} \pi - \phi \quad \text{if} \quad \phi - \theta_{NN} \\ \tan^{-1} \left( \frac{R \cos \alpha - \xi_1}{-R \sin \alpha - \xi_1} \right) \quad \text{if} \quad -\pi + \phi - \theta_{NN} \end{array} \right.$$
Expressions (146) to (148) apply to both cohesive and purely frictional soils.

Admissible values of $\theta_{\text{LO}}$ and $\theta_{\text{NO}}$ also depend on the soil characteristics. For $c > 0$,

$$\phi + \frac{\alpha}{2} + \frac{\tau}{2} - \mu \geq \theta_{\text{LO}} \geq \phi + \frac{\alpha}{2}.$$  \hspace{1cm} (149)$$

and

$$\phi + \frac{\alpha}{2} + \frac{\tau}{2} - \mu \geq \theta_{\text{NO}} \geq \phi + \frac{\alpha}{2}.$$  \hspace{1cm} (150)$$

And for $c = 0$, it can be proved that Conditions (149) and (150) also apply but subject to the following limitations:

$$\phi + \frac{\alpha}{2} - \mu \geq \theta_{\text{NO}} \geq \phi + \frac{\alpha}{2}.$$  \hspace{1cm} (151)$$

and

$$\mu - \phi \leq \theta_{\text{NO}} \leq \phi + \mu.$$  \hspace{1cm} (152)$$

For lunar locomotion ($c > 0$), then Expressions (149) and (150) will generally apply.

Expressions (146) to (152) are incorporated in the computer program to operate within a compatible range of stress parameters $p$ and $\theta$.

**D. Equilibrium Equations**

With the positive direction of forces the same as the positive $x$, $z$ coordinate directions, and with applied vertical axle loads $W$ and pull force $P$ parallel to the terrain slope, horizontal equilibrium conditions require (Fig. 11)

$$\sum X = H_L^k + H_T^k + P \cos \alpha = 0.$$  \hspace{1cm} (153)$$

and, for vertical equilibrium,

$$\sum Z = W_L^k + W_T^k + W + P \sin \alpha = 0.$$  \hspace{1cm} (154)$$

where $H_L^k$, $H_T^k$, $W_L^k$, $W_T^k$ are horizontal and vertical forces corresponding to leading ($L$) and trailing ($T$) plastic active zones adjoining the soil-roller interface arc $I$.$J$.$N$. Sub-index $k$ ($1, 2, \cdots$) identifies the possible existence of more than one soil-roller solution for a fixed slip value $sk$ and various $\delta_M$.

A driven roller moving over horizontal or sloping terrains operates under self-propulsion conditions when it transports only its axle weight $W$ (pull force $P = 0$). Under self-propulsion there is no net soil thrust development nor soil resistance to motion since the leading and trailing forces balance each other ($H_L^k + H_T^k = 0$). The net effect of the interplay of these self-equilibrated internal forces is to offset the position of the vertical soil reaction to accommodate the rolling torque $M = M_k$; instead, when $P > 0$ in Eq. (153), the leading and trailing forces make up for the difference in allowing for $P$ to become equilibrated.

Taking moments with respect to roller axle center $C$, with positive torques measured counterclockwise, moment equilibrium requires

$$\sum M = M_L^k + M_T^k + M_k = 0.$$  \hspace{1cm} (155)$$

where $M_k$ is the applied axle torque, and $M_L^k$ and $M_T^k$ correspond to leading ($L$) and trailing ($T$) moments produced by the mobilized soil strength and corresponding soil weight. The above forces and moments relate exclusively to the active zones $L$($ML$) and $T$($MN$). Expressions (153), (154), and (155) are further developed as follows:

$$\sum X = H_L^k, s + H_T^k, s + H_L^k, p + H_T^k, p + P \cos \alpha = 0.$$  \hspace{1cm} (156)$$

$$\sum Z = W_L^k, s + W_T^k, s + W_L^k, \gamma + W_T^k, \gamma + W_k, p + W_k, \gamma = 0.$$  \hspace{1cm} (157)$$

$$\sum M = M_L^k, s + M_L^k, p + M_T^k, \gamma + M_T^k, \gamma + M_k, p + M_k, \gamma = 0.$$  \hspace{1cm} (158)$$

The sub-index $s$ corresponds to forces (or moments) due to stresses along the spiral slipline $M$($ML$) and $M$($MN$). The sub-index $p$ indicates forces (or moments) due to stresses along the straight slipline $L$($ML$) and $M$($MN$), as derived from both the transition and passive zones. $W_L^k, \gamma$ and $W_T^k, \gamma$ are soil weights within the confines of the leading and trailing active zones $M$($ML$) and $M$($MN$), respectively. $M_L^k, \gamma$ and $M_T^k, \gamma$ are moments due to $W_L^k, \gamma$ and $W_T^k, \gamma$, respectively. To evaluate these forces and moments it is necessary to determine the nature and extent of both active plastic domains (Fig. 11), their associated stresses and corresponding soil weight participating with the roller motion.

**E. Soil Reactions**

The horizontal and vertical force components due to stresses acting along slipline $L$($ML$) are (Figs. 11 and 12).
\[ x_{ML} = \frac{\bar{x}_L + r_M \cos (\theta_L - \phi)}{\cos \theta_L} \left( \frac{x_{ML} - x_L}{b} \right) \]  
(159)

\[ w_{k,p} = -\left( \bar{\sigma}_L \cos \theta_L + \tau_L \sin \theta_L \right) \frac{x_{ML} - x_L}{\cos \theta_L} \left( \frac{b}{b} \right) \]  
(160)

where \( b \) is the roller width, and

\[ x_L = R \cos (\alpha + \xi_L) \]  
(162)

Replacing Eqs. (161) and (162) in Eqs. (159) and (160) and ordering terms results in

\[ H_{k,p}^L = C_{14} \bar{\sigma}_L \cos (\theta_L - \phi) + F_{12}(\theta_L) \]  
(163)

\[ w_{k,p}^L = -C_{14} \left( \bar{\sigma}_L \cos (\theta_L - \phi) - \frac{c}{\sin \phi} \cos \theta_L \right) \]  
(164)

where

\[ C_{14} = \frac{\cos \phi}{\cos \theta_L} (x_{ML} - x_L) \]  
(165)

\[ F_{12}(\theta_L) = C_{14} \left[ \frac{\Delta \tau_{ML}}{2} \sin (\theta_L - \phi) - \frac{c}{\sin \phi} \cos \theta_L \right] \]  
(166)

Derivation of the force components \( H_{k,p}^L, w_{k,p}^L \) on the trailing first sleipline \( N(MN) \) is basically similar to the procedure adopted for the leading sleipline \( L(ML) \). The horizontal and vertical force components from stress acting along \( N(MN) \) are (Fig. 11)

\[ dH_{k,s}^L = -b [\sigma^L \cos (\theta' - \phi) - \tau^L \sin (\theta' - \phi)] ds \]  
(175)

\[ dW_{k,s}^L = -b [\sigma^L \sin (\theta' - \phi) + \tau^L \cos (\theta' - \phi)] ds \]  
(176)

where \( \sigma^L \) and \( \tau^L \) correspond to \( \sigma \) and \( \tau \) as given by Eqs. (69) and (70) respectively, with \( p = p_{14}^L \) (Eq. 97).

Similarly, the horizontal and vertical components on the elemental arc \( ds' \) along the spiral \( M(MN) \) are

\[ x_{MN} = x_L + r_M \exp \left[ (\theta_M - \theta_{NN}) \tan \phi \right] \cos \theta_{NN} \]  
(169)

\[ x_N = R \cos (\alpha + \xi_N) \]  
(170)

Substituting Eqs. (169) and (170) in Eqs. (167) and (168) and ordering terms results in

\[ H_{k,p}^T = C_{15} \bar{\tau}_L \sin (\theta_{NN} + \phi) + F_{13}(\theta_{NN}) \]  
(171)

\[ w_{k,p}^T = -C_{15} \left[ \bar{\sigma}_N \cos (\theta_{NN} + \phi) - \frac{c}{\sin \phi} \cos \theta_{NN} \right] \]  
(172)
\[
\begin{aligned}
\text{d} \hat{H}_{k,s}^T &= b (\sigma_s^T \sin \theta' - r_s^T \cos \theta') \, \text{d}s', \\
\text{d} W_{k,s}^T &= -b (\sigma_s^T \cos \theta' + r_s^T \sin \theta') \, \text{d}s'.
\end{aligned}
\]

where \(\sigma_s^T\) and \(r_s^T\) correspond to \(\sigma\) and \(r\) as given by Eqs. (69) and (70), respectively, with \(p = p_1^T\) (Eq. 109).

The total force components are derived by replacing the \(\sigma, r\) stresses in Eqs. (175) to (178) and integrating within the corresponding spirals limits. Thus,

\[
\begin{aligned}
\hat{H}_{k,s}^L &= b \hat{F}_M \int_{\theta_M^{NL}}^{\theta_M} [p_s^L \cos (\theta' - 2\phi) - \frac{c}{\sin \phi} \cos (\theta' - \phi)] \exp \left( \theta' - \theta_M \right) \tan \phi \, \text{d}\theta', \\
W_{k,s}^L &= b \hat{F}_M \int_{\theta_M^{NL}}^{\theta_M} [p_s^L \sin (\theta' - 2\phi) - \frac{c}{\sin \phi} \sin (\theta' - \phi)] \exp \left( \theta' - \theta_M \right) \tan \phi \, \text{d}\theta'.
\end{aligned}
\]

Replacing Eq. (97) in Eqs. (179) and (180) and Eq. (108) in Eqs. (181) and (182), integrating, and arranging terms, the soil reactions \(H_{k,s}^L\) and \(W_{k,s}^L\) along the slip lines are obtained. For the leading zone, the solutions are

\[
\begin{aligned}
H_{k,s}^L &= C_S \left[ p_{M,L} F_1(\theta_M^{NL}, \theta_M) + F_1(\theta_M^{NL}, \theta_M) \right], \\
W_{k,s}^L &= C_S \left[ -p_{M,L} F_2(\theta_M^{NL}, \theta_M) + F_2(\theta_M^{NL}, \theta_M) \right],
\end{aligned}
\]

where

\[
C_S = b \hat{F}_M \cos \phi
\]

\[
\begin{aligned}
F_1 &= -\frac{c}{\sin \phi} \sin \theta_M^{NL} \exp \left( \theta_M^{NL} - \theta_M \tan \phi \right) \sin \theta_M^{NL}, \\
&+ C_s^L \left[ \sin (\theta_M^{NL} + \phi) - \exp \left( \theta_M^{NL} - \theta_M \tan \phi \right) \sin (\theta_M^{NL} + \phi) \right], \\
&- \frac{3}{4} \tan \phi \, C_s^L \left[ \frac{1}{\sin \phi} \sin (2\theta_M^{NL} + \phi) \right] + \frac{3}{4} \tan \phi \, C_s^L \exp \left( 2(\theta_M^{NL} - \theta_M^{NL}) \tan \phi \right), \\
&\times \left[ \frac{1}{\sin \phi} + \sin (2\theta_M^{NL} + \phi) \right] + \frac{C_s^L}{\sin \phi} \cos (2\theta_M^{NL} + \phi) - \exp \left( 2(\theta_M^{NL} - \theta_M^{NL}) \tan \phi \right) \\
&\times \cos (2\theta_M^{NL} + \phi)
\end{aligned}
\]
\[ F_2 = \sin (\theta_M + \phi) - \exp \left[ (\theta_{LL}^i - \theta_M^i) \tan \phi \right] \sin (\theta_{LL}^i + \phi) \]  
(187)

\[ F_7 = \cos (\theta_M + \phi) - \exp \left[ (\theta_{LL}^i - \theta_M^i) \tan \phi \right] \cos (\theta_{LL}^i + \phi) \]  
(188)

\[ F_8 = \frac{C}{\sin \phi} \left[ \cos \theta_M^i - \exp \left[ (\theta_{LL}^i - \theta_M^i) \tan \phi \right] \cos \theta_{LL}^i \right] - C^L \left[ \cos (\theta_M^i + \phi) ight. \\
- \exp \left[ (\theta_{LL}^i - \theta_M^i) \tan \phi \right] \cos (\theta_{LL}^i + \phi) \right] + \frac{3}{4} \tan \phi \frac{C^L}{4} \left[ \cos (2\theta_M^i + \phi) \\
- \exp \left[ 2(\theta_{LL}^i - \theta_M^i) \tan \phi \right] \cos (2\theta_{LL}^i + \phi) \right] - \frac{C}{4} \left[ \frac{1}{\sin \phi} - \sin (2\theta_M^i + \phi) \right] \\
+ \frac{C^L}{4} \exp \left[ 2(\theta_{LL}^i - \theta_M^i) \tan \phi \right] \left[ \frac{1}{\sin \phi} - \sin (2\theta_{LL}^i + \phi) \right] \]  
(189)

For the trailing zone, 

\[ H_{k,s}^T = C_7 \left[ \frac{P}{r} \cdot F_2 (\theta_N^i, \theta_M^i) + F_6 (\theta_N^i, \theta_M^i) \right] \]  
(190)

and 

\[ W_{k,s}^T = C_7 \left[ \frac{P}{r} \cdot F_2 (\theta_N^i, \theta_M^i) + F_10 (\theta_N^i, \theta_M^i) \right] \]  
(191)

where 

\[ C_7 = \frac{b}{r} \cos \phi \]  
(192)

\[ F_5 = \exp \left[ (\theta_M^i - \theta_N^i) \tan \phi \right] \sin (\theta_N^i - \phi) - \sin (\theta_M^i - \phi) \]  
(193)

\[ F_6 = - \frac{C}{\sin \phi} \left[ \exp \left[ (\theta_M^i - \theta_N^i) \tan \phi \right] \sin \theta_N^i - \sin \theta_M^i \right] \\
- C^L \left[ \exp \left[ (\theta_M^i - \theta_N^i) \tan \phi \right] \sin (\theta_N^i - \phi) - \sin (\theta_M^i - \phi) \right] \\
- \frac{1}{4} C^T \exp \left[ 2(\theta_M^i - \theta_N^i) \tan \phi \right] \left[ 3 \tan \phi \left[ - \frac{1}{\sin \phi} + \sin (2\theta_N^i - \phi) \right] + \cos (2\theta_N^i - \phi) \right] \\
+ \frac{1}{4} C^T \left[ 3 \tan \phi \left[ - \frac{1}{\sin \phi} + \sin (2\theta_M^i - \phi) \right] + \cos (2\theta_M^i - \phi) \right] \]  
(194)

\[ F_7 = \exp \left[ (\theta_M^i - \theta_N^i) \tan \phi \right] \cos (\theta_N^i - \phi) - \cos (\theta_M^i - \phi) \]  
(195)

\[ F_9 = - \frac{C}{\sin \phi} \left[ \exp \left[ (\theta_M^i - \theta_N^i) \tan \phi \right] \cos \theta_N^i - \cos \theta_M^i \right] \\
+ C^T \left[ \exp \left[ (\theta_M^i - \theta_N^i) \tan \phi \right] \cos (\theta_N^i - \phi) - \cos (\theta_M^i - \phi) \right] \\
+ \frac{1}{4} C^T \exp \left[ 2(\theta_M^i - \theta_N^i) \tan \phi \right] \left[ 3 \tan \phi \cos (2\theta_N^i - \phi) - \frac{1}{\sin \phi} + \sin (2\theta_N^i - \phi) \right] \\
- \frac{1}{4} C^T \left[ 3 \tan \phi \cos (2\theta_M^i - \phi) - \frac{1}{\sin \phi} + \sin (2\theta_M^i - \phi) \right] \]  
(196)
Expressions (183) to (196) will be incorporated in Eqs. (156), (157), and (158) for the detailed study of equilibrium in Section VI-A.

F. Body Forces

Soil body forces \( W_{k, Y} \) are obtained by integrating the soil volume contained within the leading and trailing active zone \( L(ML)M \) and \( M(MN)N \) as shown in Fig. 11.

For the leading zone,

\[
W_{k,Y}^L = \gamma b \left[ \text{area } \overline{T}_2(ML)M - \text{area circular sector } \overline{T}_2LM \right] \tag{197}
\]

\[
\text{Area } \overline{T}_2(ML)M = \int_{\theta_M}^{\theta'} r^2 \, d\theta' \tag{198}
\]

where \( \theta \) is given by Eq. (33).

For the trailing zone, \( M(MN)N \), the soil weight is (Fig. 11)

\[
W_{k,Y}^T = \gamma b \left[ \text{area } \overline{T}_1M(MN) - \text{area circular sector } \overline{T}_1MN \right] \tag{205}
\]

\[
\text{Area } \overline{T}_1M(MN) = \int_{\theta_M}^{\theta_N} \frac{1}{2} r^2 \, d\theta \tag{206}
\]

where \( r \) is given by Eq. (28).

\[
\text{Area } \overline{T}_1MN = (\text{area triangle } \overline{T}_1MN + \text{area circular segment with arc } MN) = C_T \text{ or } C_T^1
\]

\[
C_T = \frac{1}{2} r_M \sin (\theta_M - \theta_N) + A_c \tag{207}
\]

where

\[
A_c = \frac{1}{2} R^2 \left[ \xi_M - \xi_N - \sin (\xi_M - \xi_N) \right] \tag{208}
\]

and

\[
x_c = T^1N - R \cos (\alpha + \xi_N - \theta_N) \tag{209}
\]

Integrating Eq. (206), and with Eq. (207), Eq. (205) becomes

\[
W_{k,Y}^T = \gamma b \left\{ \frac{r_M^2}{4 \tan \phi} \left[ \exp \left[ 2(\theta_M - \theta_N) \tan \phi \right] - 1 \right] - C_T^1 \right\} \tag{213}
\]

where \( C_T^1 \) is given by Eq. (207).

Equations (204) and (213) will be used in Eqs. (156), (157), and (158) for the detailed study of
soil-roller equilibrium in Section VI-A and also to determine the driving torque \( M \) in Section V-G.

**G. Moments**

1. **Moments due to soil reactions.** The resultant force originating from reduced stresses along a spiral slipline intercepts the spiral pole (Fig. 12). The moments due to these spiral stresses relative to the roller axes \( C(x = z = 0) \) depend exclusively on the position of the spiral poles \( I_1(x_1, z_1) \); \( I_2(x_2, z_2) \). Positive moments tend to produce a counterclockwise rotation. The trailing \( (T) \) and leading \( (L) \) moments due to stresses along the spirals \( M(MN) \) and \( M(ML) \), respectively, are

\[
M_k, s^T = \frac{1}{2} \exp \left( \frac{2(\Theta_{MN} - \Theta_M) \tan \phi}{2 \tan \phi} \right) - 1 \]  
\[
M_k, s^L = \frac{1}{2} \exp \left( \frac{2(\Theta_{LL} - \Theta_M) \tan \phi}{2 \tan \phi} \right) - 1
\]

with \( W_k, s \) and \( H_k, s \) forces given by Eqs. (183) to (189). \( M_k, s^T \) and \( M_k, s^L \) are the moments due to cohesion stresses along the spirals \( M(MN) \) and \( M(ML) \) with respect to poles \( I_1 \) and \( I_2 \), respectively:

\[
M_k, s^T = W_k, s^T - H_k, s^T + M_k, s^T
\]
\[
M_k, s^L = W_k, s^L - H_k, s^L + M_k, s^L
\]

Regarding the moments produced by the stresses along the sliplines \( L(ML) \) and \( N(MN) \),

\[
M_k, s^L = W_k, s^L - H_k, s^L + M_k, s^L
\]
\[
M_k, s^T = W_k, s^T - H_k, s^T + M_k, s^T
\]

where \( W_k, s \) and \( H_k, s \) are forces given by Eqs. (163), (164), (171), and (172), and

\[
\overline{M}_c = \frac{1}{5} \overline{r}_c \sin (\Theta_L - \Theta_M)(z_M + z_L + z_c) + \overline{A}_c \overline{v}_c
\]

with \( \overline{A}_c \) given by Eq. (200) and

\[
\overline{M}_c = \frac{1}{5} \overline{r}_c \sin \left( \frac{2}{3} \frac{r_M - r_L}{r_M} \cos \left( \frac{r_L + r_M}{2} \right) \right) \left( \frac{r_L + r_M}{2} \right)
\]

Integrating Eq. (222) results in

\[
\overline{M}_c = \overline{r}_c \left[ \frac{1}{3} \overline{r}_c \sin \left( \frac{2}{3} \frac{r_M - r_L}{r_M} \cos \left( \frac{r_L + r_M}{2} \right) \right) \left( \frac{r_L + r_M}{2} \right) \right]
\]

For the leading zone \( L(ML)M \),

\[
M_k, s^T = \frac{1}{2} \exp \left( \frac{2(\Theta_{MN} - \Theta_M) \tan \phi}{2 \tan \phi} \right) - 1 \]  
\[
M_k, s^L = \frac{1}{2} \exp \left( \frac{2(\Theta_{LL} - \Theta_M) \tan \phi}{2 \tan \phi} \right) - 1
\]

where \( W_k, s \) and \( H_k, s \) are forces given by Eqs. (183) to (189). \( M_k, s^T \) and \( M_k, s^L \) are the moments due to cohesion stresses along the spirals \( M(MN) \) and \( M(ML) \) with respect to poles \( I_1 \) and \( I_2 \), respectively:

\[
M_k, s^T = W_k, s^T - H_k, s^T + M_k, s^T
\]
\[
M_k, s^L = W_k, s^L - H_k, s^L + M_k, s^L
\]

where \( W_k, s \) and \( H_k, s \) are forces given by Eqs. (163), (164), (171), and (172), and

\[
\overline{M}_c = \frac{1}{5} \overline{r}_c \sin (\Theta_L - \Theta_M)(z_M + z_L + z_c) + \overline{A}_c \overline{v}_c
\]

with \( \overline{A}_c \) given by Eq. (200) and

\[
\overline{M}_c = \frac{1}{5} \overline{r}_c \sin \left( \frac{2}{3} \frac{r_M - r_L}{r_M} \cos \left( \frac{r_L + r_M}{2} \right) \right) \left( \frac{r_L + r_M}{2} \right)
\]

Integrating Eq. (222) results in

\[
\overline{M}_c = \overline{r}_c \left[ \frac{1}{3} \overline{r}_c \sin \left( \frac{2}{3} \frac{r_M - r_L}{r_M} \cos \left( \frac{r_L + r_M}{2} \right) \right) \left( \frac{r_L + r_M}{2} \right) \right]
\]
For the trailing zone $M(MN)$,

$$M_{k,y}^T = \gamma b \left( \int_0^{\theta_M} \left[ \frac{1}{2} r^2 \left( \frac{K}{2} + \frac{2}{3} r \cos \theta \right) \right] d\theta - M_c \right)$$

(224)

where $r$ is given by Eq. (28), and

$$M_c = \frac{1}{6} M_c r \sin (\theta_M - \theta_{NN}) (x_1 + x_N + \pi) + A_c x_c$$

$A_c$ is given by Eq. (208), and

$$x_c = \frac{4}{3} R \sin \left( \frac{\delta_M - \delta_N}{2} \right) \cos \left( \frac{\delta_M^* - \delta_N^*}{2} \right)$$

Integrating Eq. (224),

$$M_{k,y}^T = \gamma b \left[ \frac{2}{3} \tan \phi \right] \exp \left[ 2\left( \theta_M - \theta_{NN} \right) \tan \phi \right] - \frac{1}{2} M_c$$

$$- \left( \frac{3}{3(2\tan \phi + 1)} \right) \exp \left[ \frac{1}{2} \left( \theta_M - \theta_{NN} \right) \tan \phi \right] \left( 3 \tan \phi \cos \theta_{NN} - \sin \theta_{NN} \right)$$

(225)

The driving torque will be used in Section VI-B to determine the roller driving power requirements.

H. Soil-Roller Interface Stresses

Once the pattern and extent of the active plastic domain zone are derived, the soil-roller interface stress at a generic rim point $i$ are (Eqs. 65 and 66)

$$\sigma_i = p_i \left( 1 + \sin \phi \sin \psi \right) - c \cot \phi$$

(226)

$$\tau_i = p_i \sin \phi \cos \psi$$

(227)

where

$$\psi_i = 2\theta_i - 2\Omega_i + \phi$$

(228)

with

$$\Omega_i = \alpha + \delta_i$$

(229)

For the leading zone,

$$\theta_i = \alpha + \delta_i - \sin^{-1} \left( \frac{1}{R} \left[ \pi_1 \cos \theta_i - \pi_1 \sin \theta_i \right] \right)$$

$$+ \frac{\pi}{2} - \phi$$

(230)

and for the trailing zone,

$$\theta_i = \alpha + \delta_i - \sin^{-1} \left( \frac{1}{R} \left[ \pi_1 \cos \theta_i - \pi_1 \sin \theta_i \right] \right) + \pi - \phi$$

(231)

The parameter $p$ along the slipline $M(ML)$ is derived from Eq. (97), and for the slipline $M(MN)$ from Eq. (108). On the leading zone, along a radial slipline which intersects the roller rim at a point $i (\delta_i \leq \delta_M)$ and also the slipline $M(ML)$ at point $(M_1)$ is

$$y_i^L = p_{M_1}^L + \Delta p_i^L$$

(232)

where $p_{M_1}^L$ corresponds to Eq. (97) for $\theta_i^* = \theta_i$, and

$$\Delta p_i^L = \gamma (x_i - x_{M_1}) (\tan \theta_i^* + \tan \phi)$$

(233)
with

\[ x_{Mi} = \pi_2 + \pi_M \exp \left[ (\theta_i^1 - \theta_i^M) \tan \phi \right] \cos \theta_i^1 \]  

(234)

and \( x_i \) from Eq. (5) and \( \pi_M \) from Eq. (23).

Similar criteria apply to the trailing zone \( (\xi_t < \pi/2) \). Along the radial slipline intersecting the roller rim at point \( t \) and the slipline \( M(MN) \) at point \( iM \),

\[ T_{p_1} = T_{p_{1M}} + \Delta T_{p_1} \]  

(235)

where \( T_{p_{1M}} \) corresponds to Eq. (108) for \( \theta = \theta_i \), and

\[ \Delta T_{p_1} = \gamma(x_i - x_{iM})(\tan \theta_i - \tan \phi) \]  

(236)

with

\[ x_{iM} = \pi_i + r_M \exp \left[ (\theta_i^M - \theta_i^1) \tan \phi \right] \cos \theta_i \]  

(237)

and \( x_i \) from Eq. (5) and \( r_M \) from Eq. (20).

The orientation \( \delta_i \) of the soil-roller interface stress resultant \( q_r \) is given by Eq. (122) in connection with Eqs. (226) and (227).
A. Basic Equations

It was shown in Section IV-C that the geometry and extent of the active and transition zones relate exclusively to the slipline directions at points L, M, and N, as defined by Expression (114) Fig. 4). Also, if the set (Expression (114) is known, a velocity slipline pattern can be unambiguously built that satisfies the roller velocity conditions. To initiate a solution of a given soil-roller problem, as postulated in Section IV-A, a set of parameters $s_k$ and $s_M$ is selected first. These parameters define unique values of $\theta_M$ (Eq. 12) and $\theta_M$ (Eq. 18). Also determine the position of poles $\theta_1$ (Eqs. 29 and 30) and $\theta_2$ (Eqs. 34 and 35).

To evaluate the four remaining unknown parameters of Expression (114), there are available four basic simultaneous equations. Two of them originate from satisfying the horizontal and vertical equilibrium conditions, as given by Eqs. (156) and (157). For completeness, Eqs. (156) and (157) are repeated as Eqs. (238) and (239):

$$\sum X = H_{k,s}^L + H_{k,p}^T + H_{k,s}^L + H_{k,p}^T + P \cos \phi = 0$$

$$\sum Z = W_{k,s}^L + W_{k,p}^L + W_{k,s}^T + W_{k,p}^T + W_{k,y}^T + W_{k,y}^T + P \sin \phi = 0$$

(Eq. 238)

(Eq. 239)

Substituting in Eq. (238) the corresponding force components given by Eqs. (163), (171), (183), and (190), after a short transformation, results in

$$P_{LL} = \frac{G_2}{G_1} = F_L (0_{LL}, 0_{NN}) = p_0$$

(Eq. 240a)

where

$$G_1 = -C_6 (F_2 + \Delta p_{M1} + F_1) - F_{12}$$

$$G_2 = C_6 (F_2 + C_{14} \sin (0_{LL} - \phi) + C_7 \cdot F_5 + C_{15} \sin (0_{NN} + \phi)$$

Since, in Eq. (240a), $p_{LL} > p_0 > 0$, $G_1$ and $G_2$ must have the same sign.

From Eqs. (81) and (83),

$$p_{LL} = p_0 \exp \left[ (\theta_{LL} - \theta_{NN}) \tan \phi \right]$$

(Eq. 240b)

After equating (240a) and (240b),

$$\theta_{LL} = \frac{\theta_{NN} - \theta_{NO}}{2 \tan \phi} \ln \frac{G_1}{p_0 G_2}$$

(Eq. 241)

Also, it is known from Eqs. (82) and (83) that

$$p_{NN} = p_0 \exp \left[ (\theta_{NN} - \theta_{NO}) \tan \phi \right]$$

(Eq. 242)

According to Eq. (79),

$$p_{NN} = p_{MN} - \Delta p_{MN}$$

(Eq. 243)

After equating (242) and (243),

$$\theta_{NO} = \theta_{NN} - \frac{1}{2 \tan \phi} \ln \frac{p_{MN} - \Delta p_{MN}}{p_0}$$

(Eq. 244)

Thus, we arrive at a system of four equations (Eqs. 238, 240a, 241, and 244) in the unknown parameters $\theta_{LL}$, $\theta_{NN}$, $\theta_{NO}$. These equations are solved by iteration using a computer program. Admissible values of $\theta_{LL}$ are first selected within the corresponding bounds stated in Section V-C. Subsequently, $\theta_{NN}$ is determined from Eq. (239). Substituting $\theta_{LL}$ and $\theta_{NN}$ in Eq. (241), a unique $\theta_{NO}$ value is determined which satisfies simultaneously the horizontal and vertical equilibrium equations. With $\theta_{LL}$, $\theta_{NN}$, and $\theta_{NO}$ known, $\theta_{NO}$ is determined from Eq. (244). During the solution process, the computer program verifies the fact that the determined angular parameters $\theta$ comprise only admissible values; otherwise, a new $\theta_{LL}$ is selected or a new case $\theta_{M}$ is initiated, as required. Equations (238) and (239) express the necessary conditions under which an
Equilibrium solution is admissible with regard to the soil-roller active plastic domain. It is of significance to point out that the existence of the set of values, Expression (114), which satisfies the equilibrium and the stress compatibility equations, does not necessarily imply that a solution to the problem has been found, unless the following conditions are also simultaneously met:

1. The rate of dilation, as mentioned in Section IV-C must be positive throughout the plastic domain. (This is verified in the Appendix.)

2. At no point outside the plastic regions shall the stress exceed yield.

3. The free-surface points F and B (Fig. 4), which belong to both the rigid and plastic domains, must also be aligned with the original surface slope $\alpha$; thus,

$$
\tan \alpha = \frac{x_F - x_B}{y_F - y_B} \tag{243}
$$

which assumes that, after the roller passage, the original trailing surface slope is not significantly altered. This was verified experimentally by Wong and Reece (Ref. 11) for roller tests on level surfaces. Here, this condition is assumed to prevail also for slopes.

Although conditions (1) and (2) do not bear directly on the solution process of the equations, they are fundamental to describing the requirements for the completeness of a solution. Condition (2) is not specifically verified, but may be considered satisfied if no sharp corners of rigid material develop within the rigid plastic boundary. In fact, it may be proved that the soil state of stress at point M does not exceed yield and, in general, it may be assumed that the rigid material can support the plastic deforming body. The method of extending the plastic stress field into the rigid domain, as studied by Shield (Ref. 20) and by Cox, et al. (Ref. 21), can also be applied to this problem.

Regarding Eq. (245), the coordinates of F and B are obtained, satisfying the boundary conditions on the leading and trailing traction-free surface slope. The points of positions F and B are satisfactorily approximated based on the following analysis. It was noted in Sections V-D and V-E that the horizontal and vertical stresses along $L(ML)$ (Fig. 11) determine the reaction forces $H_{k,p}$ and $W_{k,p}$, respectively. Similarly, the stresses along $M(MN)$ define the force components $H_{k,p}$ and $W_{k,p}$. These leading and trailing forces have a special significance with regard to the validity of any solution of Eqs. (238) and (239). If a solution is found, it has to be verified that the mentioned forces $H_{k,p}$ and $W_{k,p}$ can be sustained at the corresponding transition and passive plastic domains. In other words, it must be verified that all plastic zones satisfy simultaneously the basic requirements of local and overall equilibrium. In this context, a solution to Eqs. (238) and (239) represents only a necessary condition. Sufficiency is proven when, in addition, the solution of the two active zones can be appropriately coupled to the neighboring transition and passive fields, which satisfy Eqs. (63) and (64) and the remaining boundary conditions. Here, we advance the fact that the active plastic field solution can be extended up to and including the traction-free surfaces LF and NB (Fig. 4), if the input data is consistent. In Part II it will be shown that the completion of the plastic field also yields the statically correct deformed free surface and that Condition (245) can, in general, be satisfied.

B. Specific Energy Dissipation

The roller moves parallel to the original soil surface with uniform velocity $V_0 = \omega R_s k$, where $R_e$ is the effective rolling radius. For a rigid cylindrical roller $R_e = R$. The axle traverses a distance 1 per unit time.

$$
L = \omega R_s k \tag{246}
$$

The total soil thrust parallel to the original surface is

$$
T = P \times W \sin \alpha \tag{247}
$$

with $P$ = the drawbar pull force and $W = \text{the applied axle load}$. The load component normal to the original surface is

$$
N = W \cos \alpha \tag{248}
$$

The total soil thrust parallel to the original surface is

$$
F_{SM} = \sum_{T} + F_{S/R} \tag{249}
$$

where

$$
F_{SM} = M \omega \tag{250}
$$

and

$$
E_T = TL \tag{251}
$$

In general, the soil energy dissipation per unit travel length Eq. (216) and per unit normal load Eq. (248) will be defined as the soil-roller specific energy dissipation coefficient:

$$
E = \frac{E_{S/R}}{NL} = \frac{M}{\omega R_s k \cos \alpha} \left( \frac{P}{W \cos \alpha + \tan \alpha} \right) \tag{252}
$$

Equation (252) accounts for the soil and tire deformation and soil-wheel interface friction energy losses due to slip. It represents a general energy expression and defines the performance of any rolling.
power-driven device; the equation applies to both rigid and flexible rollers or wheels. If the roller is flexible, \( E \) will express the specific energy dissipation produced by both the soil and the rolling device. Care must be taken to properly measure or calculate the effective turning radius \( R_e \). For the rigid roller, the values of \( M \) and \( sk \) in Eq. (252) are determined from the problem solution (Section VI-A).

When \( a = 0 \), Eq. (252) reduces to

\[
F_0 = \frac{M}{WR e^a_k} \times \frac{P}{W} \tag{253}
\]

Equation (253) was used by Leflaive (Ref. 22) to analyze test results of driven rigid and flexible wheels on horizontal soil surfaces. The specific energy (Eq. 252) shows two terms. One is the specific torque energy input at the axle per unit normal load and unit travel distance.

\[
E_M = \frac{M}{WR e^a_k \cos \alpha} \tag{254}
\]

The other is the specific thrust energy output per unit normal load and unit travel distance.

\[
E_T = \frac{P}{W \cos \alpha} \times \tan \alpha \tag{255}
\]

Only when \( a = 0 \) does the specific thrust energy equal the pull/load ratio \( E_T = P/W \); otherwise, for \( a > 0 \), Eq. (255) refers to the specific thrust energy. Equations (254) and (255) can be used to evaluate the performance of power-driven vehicles, providing the relative wheel slip factors and axle load distribution are known. The wheel thrust efficiency is given in general by

\[
n\% = \frac{P + \sin \alpha}{W} \times \frac{M}{WR e^a_k} \times 100 \tag{256}
\]

In practice, the specific power consumption per kilometer of travel along a straight line on a slope \( a \geq 0 \) is given by

\[
P_w = E_M \times W \times \left( \frac{1}{3.6} \right) \text{watt-hour/km} \tag{256a}
\]

where \( E_M \) is defined by Eq. (254) for \( sk > 0 \) and \( W \) is the applied axle load in Newtons.

C. Rigid Roller Sinkage

The roller sinkage \( z \) is measured perpendicularly to the original surface. Once the leading and trailing points \( F \) and \( B \) are determined, satisfying the basic equilibrium equations, it is verified if points \( F \) and \( B \) are aligned on a slope \( a \) (Fig. 14). To this effect, the coordinates of \( F \) and \( B \) yield

\[
\tan a' = \frac{z_B - z_F}{x_B - x_F}
\]

where \( a' \) is the direction \( BF \) for a trial solution. Only when \( a' = a \) does the solution found correspond to the given problem. Then, with

\[
t = z_B + x_B \tan a
\]

and

\[
n = -\cos a
\]

the sinkage is

\[
z = R \times n \tag{257}
\]

D. Mobility Safety Factors

It was shown in Section VI-A that the soil-roller mobility problem can be solved satisfying the velocity and limiting equilibrium equations that are subjected to the corresponding boundary conditions. The solution determines the operational slip factor \( sk \) and torque \( M \) for a given axle loading \( P \) and \( W \). It was also shown (Section I) that, when \( sk \approx 0 \), the roller becomes immobilized \( (V_C \approx 0) \). From a mobility safety standpoint, given the soil conditions \( (c, \phi, Y, a) \) and a fixed set of operational loads, pull \( P_0 \) and weight \( W_0 \), it is required to determine the corresponding maximum load \( W_{\text{max}} \) (or \( P_{\text{max}} \)) which, in combination with \( P_0 \) and \( W_0 \), produces immobilization of the roller. Thus, there exist two basic loading conditions capable of immobilizing the roller: (1) increasing only the pull force from \( P_0 \) to \( P_{\text{max}} \), and (2) increasing only the axle weight from \( W_0 \) to \( W_{\text{max}} \). Consequently, two types of mobility safety factors \( (SF) \) can be defined, depending on the ultimate cause that stops the roller.

The first definition of \( SF \) is

\[
SF = \frac{P_{\text{max}} + W_{\text{max}} \sin \alpha}{P_0 + W_0 \sin \alpha} - \frac{T_{\text{max}}}{T_0} \tag{258}
\]

where the numerator indicates that the maximum soil thrust \( T_{\text{max}} \) is reached by incrementing \( P_0 \), pull load, to \( P_{\text{max}} \), setting \( sk = 0 \). The denominator \( T_0 \) corresponds to the roller operating thrust for \( 0 < sk < 1.0 \). In this case, when \( P_{\text{max}} = P_0 \), \( SF = 1 \) and the roller would stop due to excessive pull.

The second definition of \( SF \) corresponds to

\[
\overline{SF} = \frac{P_0 + W_{\text{max}} \sin \alpha}{P_0 + W_0 \sin \alpha} - \frac{T_{\text{max}}}{T_0} \tag{259}
\]

JPL Technical Memorandum 33-477
where, as in Eq. (258), the $T_{\text{max}}$ corresponds to $s_k = 0$ by incrementing the axle weight $W_0$ to $W_{\text{max}}$. When $W_{\text{max}} = W_0$, $SF = 1$ and the roller stops due to excessive weight. In essence, both Eqs. (258) and (259) relate to the soil maximum thrust capacity developed for $s_k = 0$.

For the general case of self-propulsion; $P_0 = 0$, the roller propels its own weight $W_0$. Under this condition:

1. If $\alpha > 0$, Eq. (258) reduces to

$$SF = \frac{P_{\text{max}} + W_0 \sin \alpha}{W_0 \sin \alpha} \quad (260)$$

In Eq. (260), if for $s_k = 0$, it is determined that $P_{\text{max}} = 0$, then $SF = 1$. This condition defines a limiting roller slope angle climbing capability, $\alpha = \alpha_{\text{max}}$, for self-propulsion.

2. If $\alpha = 0$, then $T_{\text{max}} = 0$, and Eq. (259) has no practical significance, since on a level terrain self-propulsion is unrelated to soil thrust. This conclusion derives from the fact that, under the action of a vertical load, there is no net soil thrust mobilized. Under this condition, the leading and trailing soil reactions are balanced (Eq. 153) (Fig. 11):

$$H_L + T = 0$$

Therefore, in this case, the SF refers exclusively to the maximum vertical soil load capacity for $s_k = 0$. Thus, from Eq. (259),

$$SF = \frac{W_{\text{max}}}{W_0} \quad (261)$$

Equation (261) is the factor of safety versus immobilization valid only for self-propulsion on level terrains ($\alpha = 0$).

For $P_0 > 0$ and $\alpha > 0$, the applicable SF definition corresponds to Eqs. (258) and (259), as specified.

Given $s_k = 0$ and $\xi_M$, the computer program determines $W_{\text{max}}$ connected with an operational pull load $P_0$. On the same basis, given an operational load $W_0$, it is possible to determine the corresponding $P_{\text{max}}$, which immobilizes the roller.

E. Applications

The foregoing soil-roller analysis was programmed in Fortran II for use with the IBM 1620 computer. In the following, it is assumed that the soil-roller model developed also applies to a finite-width roller (wheel) as long as the predominant soil failure mode occurs in the fore-aft direction rather than in the lateral direction. The soil-wheel interaction performance (SWIP) program input data is: wheel axle loads $(W, P)$, surface slope $\alpha$, soil properties $(Y_c, \rho, c)$, and wheel dimensions $(R, B)$. Two additional input parameters, $\xi_M$, slip factor $s_k$, are also required and are entered by means of the computer's teleprinter. The user, with a minimum of experience and iterating on $\xi_M$ and $s_k$ values, can determine a number of possible admissible solutions. In this report, results obtained satisfy the equilibrium Eqs. (238) and (239) (active zones) and the limiting conditions stated in V-C-3. It is expected that from all solutions which are found at least one will satisfy Condition (245). (Refer to Section VI-A.) It is also assumed that if more than one solution satisfies Condition (245), then the one with the minimum torque energy (Eq. 254) would represent the actual wheel performance.

Thus far, Condition (245) is not incorporated in the SWIP program. This will be done only after the plastic field is extended into the transition and passive regions, utilizing a program subroutine (Part II).

The SWIP program outputs the following:

1. Stress parameters $p, \theta$ and soil-wheel interface normal and tangential (shear) stresses (psi) at points $L, M,$ and $N$ (Fig. 4).

2. Geometric parameters. Coordinates of the center of instantaneous rotation $I$, the leading and trailing spiral poles $I_1$ and $I_2$, and the radial angular directions $\xi_L$ and $\xi_N$ where the soil detaches from the wheel rim at the leading and trailing edges, respectively.

3. The total and partial, vertical and horizontal soil reaction forces considering the effect of soil weight on both the leading and trailing regions (to an accuracy of 0.5%). The partial moments and total required driving torque. The specific energy input $E_M$ (Eq. 254) and specific thrust energy $E_T$ (Eq. 255).

The program outputs the nature of any incompatibility which may arise from either the equilibrium Eqs. (238) and (239) or from the limiting conditions stated in Section V-C-3. By this means, the user can, on an interactive basis, select appropriate new $\xi_M$, $s_k$ values to bring about a solution.

In what follows, the SWIP program is applied to estimate:

1. Driven rigid wheel performance tests carried out on horizontal terrains under controlled slip. The wheel axle is subjected to a vertical load $W_0$ and a pull force $P_0$ parallel to the undisturbed soil surface.

2. Driven rigid wheel slope climbing performance. The wheel axis is subjected to a total vertical load $W = (W_0 + P_0)/2$ acting on a slope angle defined by $\alpha = \tan^{-1}(P_0/W_0)$, where $W_0$ and $P_0$ are loads corresponding to the horizontal test conditions defined in (1), above. The purpose of this slope climbing calculation is to verify if there is theoretically any performance difference when equivalent
wheel normal and pull loads act either on a horizontal or a sloping terrain.\(^2\)

(3) Lunar roving vehicles (LRVs) on the assumption of driven rigid wheels rolling on a level lunar surface. This is applied particularly to the Apollo and Lunokhod-I vehicles.

Table 1.0 provides a summary of the above-mentioned applications indicating typical wheel loads, dimensions, and soil properties to be used in connection with the given application.

Test case 1, Table 1.0, was performed by the Waterways Experiment Station (WES), Vicksburg, Mississippi, under controlled 25% slip \((\delta_k = 0.75)\) (Ref. 18). The SWIP program applied to this test condition produced the results shown in Tables 1.1 through 1.3a, which correspond to \(\xi_M = 99, 101,\) and 102 deg. For intermediate values of \(\xi_M\), such as 99 deg \(\leq \xi_M \leq 102\) deg, there exists an infinity of solutions satisfying Eqs. (238) and (239). The indicated results correspond only to the bounding values pertaining to a given set of \(\xi_M\) and \(\delta_k = 0.75\). Intermediate solutions were also obtained but unfortunately lack of space precludes their inclusion. As mentioned, these results have to also satisfy Condition (245). This condition will eliminate all cases which do not meet the boundary requirements referred to in Section VI-A. It was also found that for \(\delta_k = 0.75\) and \(\xi_M = 98\) deg and \(\xi_M = 107\) deg, there are no other compatible solutions, thus indicating the fact that the operational range on \(\xi_M\) is bounded and if a solution exists satisfying Condition (245), it must lie within the results given in Tables 1.1 through 1.3a.

The measured torque was \(M = 600\) lb-in., and the results indicate that this value is appropriately bounded by the program output as shown. Additional rigid soil wheel tests were also checked using the SWIP program. Particularly, in the case of test M = 720 lb-in. and also was satisfactorily approximated and bounded.

In general, concerning the mobilizable soil strength, it is typically assumed that the same \(c, n\), soil parameters apply to both the trailing and leading regions. Any divergence between the leading and trailing limiting values of \(c\) and \(\phi\) to be used is concerned with the question of how the soil parameters \(c\) and \(\phi\) are modified due to the disturbance produced by the passage of the wheel's leading edge or other wheels along the same track. For instance, with reference to the total torque, a difference in \(\phi\) values does not appear to be very sensitive, as seen in Tables 1.1 - 1.3a for \(\phi = 42, 3\) deg and Tables 1.4 - 1.4a for \(\phi = 35\) deg. However, in order to satisfy Condition (245) and thereby arriving at a complete solution, it may be necessary to resort to different \(c, \phi\) values for the leading and trailing zones.

Regarding the wheel slope performance, Table 1.0, case 2, the computer results are shown in Tables 2.1 through 2.4a. First, it was found that there are no compatible slope solutions for a 25% slip as it occurs for \(\phi = 0\) test. This indicates that there is no analogy which relates equivalent loading between horizontal and sloping tests. Second, the slip performance for self-propulsion \((P = 0)\) is a minimum when \(\phi = 0\) and slip increases for increasing slope angle. When both \(\delta_k\) and \(\xi_M\) are varied, as shown, solutions will be found between the values indicated. For self-propulsion conditions, the horizontal leading and trailing soil reactions are equal and of opposite direction. The slope climbing energy requirements are generally higher than for \(\phi = 0\) due to the combination of larger torques and wheel slippage.

A hypothetical application of the SWIP program to the Apollo LRV flexible wheels is given in case 3, Table 1.0, on the assumption of rigid wheel behavior. Results shown in Tables 3.1 to 3.2a represent self-propulsion conditions on a level lunar soil surface. Results indicate that the wheel operating range for this case is within 10% to 15% slip \((\delta_k = 0.90\) to 0.85). Calculations also indicate that the wheels could not operate at 20% slip for \(\phi = 0\). The soil-wheel interface stress level is rather low \((<3/4\) psi), and energy requirements are not unlike the expected mobility performance for on-earth operation.

Case 4 (Table 1.0) represents an application of the SWIP program to the Lunokhod-I to investigate its mobility performance for \(\phi = 0\). To this effect, lunar soil properties similar to those applied to the Apollo LRV (case 3, Table 1.0) are considered. Under self-propulsion, it is assumed the wheel load is approximately \(W = 35\) lb \((15.6\) kg) (Ref. 25). The wheel radius scales roughly \(R = 10.0\) in. \((25.4\) cm) and width \(B = 6\) in. \((15.35\) cm). The wheel is assumed to operate as a rigid finite-width roller. A pattern of grousers and a metal mesh covers the wheel rim which, on ground contact, confines a soil layer of an approximate thickness equivalent to the projecting grouser lugs. This condition insures the soil-wheel interface mechanical properties are at least equivalent to the lunar soil strength, thus eliminating any uncertainty connected with the soil-wheel interface adhesion. Since the Lunokhod-I mobility performance is independent of the wheel's surface material, appropriate correlations of lunar soil properties can be made utilizing its mobility performance records in a lunar traverse.

The operational performance of the Lunokhod-I is shown in Tables 4.1 and 4.1a. Results refer to \(\delta_k = 112\) deg for 20% slip \((\delta_k = 0.80)\). It is noted that no compatible solutions were found for \(\xi_M = 110\) deg and \(\delta_k = 0.80\).

A review of the results shown for the Apollo and Lunokhod-I vehicles indicates that after extending the plastic field up to the traction-free surfaces, Condition (245) may not be satisfied and a complete solution defined. Further applications of this program are planned to estimate the limiting wheel slope climbing performance conditions after incorporating Condition (245) (Part II).
(1) A comprehensive theory for the solution of the soil-roller interaction problem has been presented. This solution is applicable to power-driven rollers moving on horizontal or sloping soft soil surfaces under quite general conditions of terrain slope angles, soil properties (cohesion, friction), and loading conditions including gravitational effects.

(2) In this study, Part I, the method of solution satisfies both the roller velocity (slip conditions) and equilibrium requirements within the active zones (Fig. 4). The solution was programmed for computer use. The nature of the developed soil-wheel interaction performance (SWIP) program is that it only outputs bounding values of wheel performance parameters. In Part II, it will be shown that these bounds can be narrowed further and that, from the point of view of the theory of plasticity, complete solutions can be obtained which satisfy overall equilibrium, velocity, and boundary conditions, which include both the transition and passive zones (Fig. 4).

(3) It is considered that a finite-width roller also represents, on a first approximation, the performance of a rigid wheel, which must be verified by tests. Limited application of the theory to rigid wheel tests on level terrains indicates that experimental results compare favorably with theoretical predictions. Experiments have to be performed considering mobility on level and sloping soil surfaces, taking into consideration the underlying concepts of the theory, as formulated, particularly with regard to (Fig. 4):

(a) Soil-wheel failure pattern on slopes.
(b) The existence and shifting of the bifurcation point M separating the leading from the trailing plastic zone along the soil-wheel interface.
(c) The position of the leading and trailing edge detachment points L and N, respectively.

(4) The limiting soil-roller interface radial and tangential (shear) stresses were defined and it was found that the obliquity angle of the resultant interface stresses with respect to the radial directions varies along the roller rim.

(5) A general normalized energy expression (2.52) was derived which is of practical use for evaluating and correlating wheel (vehicle) test results for slopes ($\theta \geq 0$). The nature of expression (2.52) and limited application of the theory (Tables 1.1-1.3a and Tables 2.1-2.4a) indicate that the wheel thrust, torque, and efficiency performances relate to the particular slope $\theta$. This result points to the fact that wheel tests using equivalent normal and pull forces on horizontal and sloping terrains do not represent similar loading systems since the state of stress and limiting equilibrium conditions of a soil slope and a level terrain are different. Thus, horizontal wheel tests based on equivalent loadings cannot be used to predict wheel slope climbing performance.

(6) A safety factor (SF) concept against wheel immobilization is introduced which is applicable to any driven rigid or flexible wheel for varying loads and slopes. This SF concept sets the framework for the study of mobility as a basic mechanical process whereby a safety number can be assigned to each of the commonly used wheel efficiency performance parameters.

(7) Regarding the validity of the theory, particularly its reliability, the proposed method of solution has to be more extensively evaluated by applying the computer program to a wider range of mobility conditions, slope angles, load combinations, soil properties, and wheel slip values.
It is recommended that:

(1) The theory and computer program developed for driven rigid rollers (wheels) on soil slopes be extended to also include towed rigid rollers (wheels).

(2) Both the driven and towed rigid wheel solutions, referred to in (1) above, be generalized to consider flexible driven and towed wheels on soil slopes, thus covering the whole spectrum of potential wheel operations as may be applicable to different mobility modes on planetary surfaces.

(3) The solutions mentioned in (1) and (2) above for single wheels be coupled to consider the mechanical interaction between the wheels of a vehicle. Since each wheel of a vehicle system is subjected to varying loads, wheel slips, torque, terrain slopes, and soil properties, prediction of vehicle performance requires knowledge of coupled wheel mechanical behavior. This program will assist in (a) modeling vehicle-terrain interaction: vehicle design configuration, safety factor against immobilization, and power requirements; (b) defining planetary vehicle operation modes: route selection, decision risks, and data rate requirements; and (c) interpreting mobility operations and test results.

(4) The results of the theory be verified and validated by implementing a comprehensive soil-wheel interaction testing program.
APPENDIX

POSITIVE RATE OF DILATION

A. Introduction

Drucker and Prager (Ref. 13) applied the concept of plastic potential to Eq. (56) and derived the stress-strain laws connected with the rigid perfectly plastic material. On this basis, the axial plane strain rates are

\[ i_x = \frac{\partial u_z}{\partial x} = \frac{\lambda}{2} [\sin \phi - \sin (2\theta + \phi)] \] (A-1)

\[ i_z = \frac{\partial u_z}{\partial z} = \frac{\lambda}{2} [\sin \phi + \sin (2\theta + \phi)] \] (A-2)

where \( \lambda \) is a positive factor of proportionability, in general a function of time and position. For steady state \( \lambda = \lambda(x, z) \). The rate of dilation based on Eqs. (A-1) and (A-2) is

\[ \dot{\Delta} = \dot{i}_x + \dot{i}_z = \lambda \sin \phi \geq 0 \] (A-3)

From Eq. (A-3), for \( \phi > 0, \dot{\Delta} > 0 \). Shield (Ref. 7) expressed the velocity components \( u_x, u_z \) of a point at failure in terms of the slip line velocity components \( V^0 \) and \( V' \) as follows:

\[ u_x = \frac{V^0}{\cos \phi} (\cos \phi + V' \sin \theta) \] (A-4)

\[ u_z = \frac{V^0}{\cos \phi} (\sin \phi + V' \cos \theta) \] (A-5)

Next Eq. (A-2) will be determined in connection with the strain rates derived from Eqs. (A-4) and (A-5).

B. Trailing Zone (\( \xi_i = \pi/2 \))

For a generic trailing point \((ij)\) (Fig. 10), setting \( \theta_{ij} = \theta \) in Eqs. (42) and (43),

\[ \dot{V}_{ij} = V' = V'_1 \exp \left[ (\theta_{ij} - \theta) \tan \phi \right] \] (A-6)

\[ \dot{V}^0_{ij} = 0 \] (A-7)

and substituting Eqs. (A-6) and (A-7) in Eqs. (A-4) and (A-5) results in

\[ u_x = -V' \sin \theta \cos \phi = -V_i \frac{\cos \phi}{\sin \phi} \exp \left[ (\theta_{ij} - \theta) \tan \phi \right] \] (A-8)

\[ u_z = V' \cos \theta = -V'_i \frac{\cos \phi}{\sin \phi} \exp \left[ (\theta_{ij} - \theta) \tan \phi \right] \] (A-9)

With \( x, z \) coordinates of point \((ij), \theta_{ij} = \theta \): \( \theta = \tan^{-1} \left( \frac{x - x_i}{z - z_i} \right) \) (A-10)
\[
\frac{\partial u_x}{\partial x} = \frac{V_i}{\cos \phi} \exp \left[ (\theta_0 - \theta) \tan \phi \right] (\tan \phi \sin \theta - \cos \theta) \frac{\partial \theta}{\partial x}
\]
(A-11)

\[
\frac{\partial u_z}{\partial z} = \frac{V_i}{\cos \phi} \exp \left[ (\theta_0 - \theta) \tan \phi \right] (\tan \phi \cos \theta + \sin \theta) \frac{\partial \theta}{\partial z}
\]
(A-12)

Substituting Eqs. (A-13) and (A-14) in Eqs. (A-11) and (A-12), respectively, Eq. (A-2) reduces to

\[
\Delta_{ij} = -\frac{V_i}{r_i \cos \phi} \tan \phi .
\]
(A-15)

Introducing Eq. (38) in Eq. (A-10), and with Eq. (40),

\[
\Delta_{ij} = \frac{V_i}{r_i \cos \phi} \tan \phi = \frac{V_i \tan \phi}{r_i \cos (\vec{r}_i - \theta_0 + \phi)} - \Delta_i
\]
(A-16)

or

\[
\Delta_i = \frac{V_i \tan \phi}{r_i \sin (\theta_0 - \vec{r}_i)} \geq 0
\]
(A-17)

Equations (A-15), (A-16), and (A-17) indicate that under steady state conditions the dilation rate \( \Delta_{ij} \) at a point (ij) reduces to the dilation \( \Delta_i \) of a point on the soil-roller interface. Also for \( \phi = 0, \Delta_{ij} = 0 \), representing the incompressibility condition of a Tresca material with \( c = 0 \) shear yield stress.

To satisfy Eq. (A-17), \( \sin (\theta_0 - \vec{r}_i) \) must be greater than zero; this condition in general holds true as may be verified graphically or analytically in most practical cases.

C. Leading Zone (\( \theta_i \geq \theta_M \))

For a generic leading point (ij) (Fig. 10), setting \( \theta_{ij} = \theta \) in Eqs. (48) and (49),

\[
V_{i}^{\theta} = V_{i} = V_{i}^{\theta} \exp \left[ (\theta - \theta_i) \tan \phi \right]
\]
(A-18)

\[
V_{i}^{\theta} = V_{i}^{\theta} - \theta
\]
(A-19)

Replacing Eqs. (A-18) and (A-19) in Eqs. (A-4) and (A-5), with \( \theta + \phi = \theta' + \pi/2 \),

\[
\frac{u}{\cos \phi} \cos \theta = V_{i}^{\theta} \sin \theta' - \exp \left[ (\theta - \theta_i) \tan \phi \right]
\]
(A-20)

\[
u = \frac{V_{i}^{\theta}}{\cos \phi} \sin (\theta + \phi)
\]
(A-21)

With \((x, z)\) coordinates of (ij),

\[
0 = \tan^{-1} \left( \frac{x - \vec{x}}{z - \vec{z}} \right)
\]
(A-22)

From Eqs. (A-23) and (A-24) with \( x - \vec{x} = \tau_1 \exp \left[ (\theta - \theta_i) \tan \phi \right] \cos \theta' \),

\[
\frac{\partial \theta}{\partial x} = -\frac{\sin \theta'}{r_i} \exp \left[ (\theta - \theta_0) \tan \phi \right]
\]
(A-25)
\[ \frac{\partial \theta}{\partial z} = \frac{\cos \theta'}{r_i} \exp \left[ \left( \theta_i - \theta \right) \tan \phi \right] \]  
(A-26)

In particular, for rim point \( i = M \), \( \theta_M = \theta_{M}^* \)
and Eqs. (A-16) and (A-28) yield, with Eqs. (20) and (23),

\[ \Delta_T = \frac{V_M \tan \phi}{a_M} \]
(A-29)

\[ \Delta_L = \frac{V_M \cot \Delta \tan \phi}{a_M} = \frac{V_M \tan \phi}{a_M} \]
(A-30)

Consequently, as expected, since the state of stress along the soil-roller interface is uniform and continuous, the soil dilation rate at point M is the same when approaching M along \((ML)M\) or along \((MN)M\) (Fig. 4). Also, since \( \Delta_i \) is proportional to \( \dot{\theta}_i \) (rad/s), it would be of interest to verify to what extent the theory of plastic potential (Ref. 13) is applicable to the prediction of soil deformation under conditions of steady-state motion. It is known that for continued straining under unsteady conditions the dilation predictions far exceed the measured increments (Ref. 23).
### NOMENCLATURE

- \( a_M \): velocity arm
- \( B \) or \( b \): roller width
- \( C \): roller center
- \( c \): cohesion
- \( ds, ds' \): elemental arc lengths along first and second sliplines
- \( E \): soil-roller specific energy dissipation coefficient
- \( E_M \): energy input per unit time due to moment \( M \) at roller axis
- \( E_S/R \): soil-roller energy dissipation
- \( E_P \) or \( E_T \): specific thrust force energy output
- \( H \): horizontal force soil reaction
- \( I \): center of instantaneous rotation
- \( i, j, k \): leading and trailing spiral poles
- \( i, j \): point on roller rim
- \( i, j \): point of intersection of \( i \) and \( j \)
- \( j \): point along slipline
- \( L \): distance parallel to slope \( \alpha \)
- \( M \): moment
- \( N \): normal load
- \( P_r \): roller load per unit width
- \( P \): pull force on roller axle, parallel to slope \( (P = bp^2) \)
- \( P_w \): power consumption per kilometer
- \( \rho \): stress parameter
- \( q_r \): stress resultant (soil-roller interface)
- \( R \): roller radius
- \( R_e \): effective rolling radius
- \( R_\phi \): radius of \( \phi \) circle (Fig. 13)
- \( \bar{r}_i, \bar{r}_j \): leading and trailing spiral radial vectors of point \( i \)
- \( SF \): safety factor related to \( P_{max} \)
- \( \bar{SF} \): safety factor related to \( W_{max} \)
- \( s \): slip %
- \( s, s' \): first and second slipline curvilinear coordinates
- \( s_k \): slip factor \( = 1 - s \)
- \( T \): thrust force
- \( u_x, u_z \): velocity components of point \( i \) in \( x \) and \( z \) directions
- \( v_c \): translational velocity of roller center \( C \)
- \( V_i \): absolute velocity of point \( j \)
- \( \bar{V}_i^x, \bar{V}_i^z \): velocity components of point \( i \) along first and second sliplines
- \( W \): roller axle weight, vertical reaction
- \( x, z \): Cartesian coordinates
- \( z \): roller sinkage
- \( \alpha \): terrain slope
- \( \bar{p}_T, \bar{p}_L \): angular orientation of \( \bar{1}_T, \bar{1}_L \) with reference to \( x \)-axis (Fig. 5)
- \( \gamma \): soil unit volume weight
- \( \Delta \): increment
- \( \Delta' \): rate of dilation
- \( \Delta M \): angle as defined in Eq. (15)
- \( \delta \): obliquity angle of \( q_r \) with respect to a radial roller direction
- \( \iota_{x, z} \): axial strain rates, \( x \) and \( z \) directions
- \( \theta \): stress parameter
\( \theta_{i,0} \) angular orientations of first and second sliplines at point \( i \)

\( \lambda \) positive factor of proportionability

\( \mu = \pi/4 - \phi/2 \)

\( \xi_i \) angular orientation of rim point \( i \) (Fig. 2)

\( \xi_M \) angular orientation of rim point \( M \) (Bifurcation point) (Fig. 4), deg

\( \rho, \rho' \) spiral radii of curvature for first and second sliplines

\( \phi_i \) angular orientation of velocity \( \mathbf{V}_i \) of rim point \( i \)

\( \sigma \) normal stress

\( \tau \) shear stress

\( \phi \) soil friction angle

\( \eta \) thrust efficiency

\( \Omega \) angular orientation of a vector normal to a surface

\( \omega \) angular velocity, rad/s

**Superscripts**

- \( L \) leading
- \( T \) trailing

**Subscripts**

- \( k \) see Section V-D
- \( p \) passive
- \( s \) spiral
- \( T \) transition
REFERENCES


24. Private communication from D. R. Freitag, Waterways Experiment Station (WES), Vicksburg, Mississippi, Nov. 9, 1969.

<table>
<thead>
<tr>
<th>Case</th>
<th>Application</th>
<th>Loads</th>
<th>Wheel</th>
<th>Soil</th>
<th>Slip s, %</th>
<th>Torque M, lb-in.</th>
<th>Remarks</th>
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<td></td>
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<td>P, lb</td>
<td>R, in.</td>
<td>B, in.</td>
<td>a, deg</td>
<td>y, 1b/in.³</td>
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<td>35.0</td>
<td>13.95</td>
<td>12.0</td>
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<td>0.0584 b</td>
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<td>35.0</td>
<td>13.95</td>
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<td>10.00</td>
<td>6.5</td>
<td>0</td>
<td>0.01</td>
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a Reference 18.
b Reference 24.
Table 1.1. Horizontal test for \( \lambda_m = 99 \) and \( s_k = 0.75 \) (upper bound)

<table>
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<tr>
<th>Parameter</th>
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<td>( \lambda_m )</td>
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<tr>
<td>( s_k )</td>
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<td>( \Phi_1 )</td>
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<td>( \Phi_2 )</td>
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<td>( \Phi_4 )</td>
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---

**Vertical Reactions**

\( W_{KL} = -92.04 \) \( W_{KT} = 4.52 \) \( W_{KL} = 59.92 \) \( W_{KT} = 7.00 \) \( W_{KL} = 5.99 \) \( W_{KT} = 0.98 \)

---

**Horizontal Reactions**

\( H_{KL} = 27.04 \) \( H_{KT} = -14.59 \) \( H_{KL} = 14.19 \) \( H_{KT} = -34.44 \)

---

**reactions**

\( H_{KL} = -14.27 \) \( H_{KT} = 276.71 \) \( H_{KL} = 5.82 \) \( H_{KT} = 1.00 \) \( H_{KL} = 63.50 \) \( H_{KT} = 0.07 \)

---

**total torques**

\( T_{KL} = 554.48 \) \( T_{KT} = 78.72 \) \( T_{KL} = 554.48 \) \( T_{KT} = 78.72 \)
Table 1.1a. Horizontal test for $\xi_M = 99$ and $s_k = 0.75$ (lower bound)

<table>
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<tr>
<td>$\xi_M$</td>
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<tr>
<td>$s_k$</td>
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### Table 1.1a Data

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<td>$T_3$</td>
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<table>
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<td>$T_3$</td>
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Additional notes and calculations...
Table 1.2. Horizontal test for $\varepsilon_M = 101$ and $\varepsilon_R = 0.75$ (upper bound)

### Angular Parameters

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<td>45.0000</td>
<td>100.9999</td>
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<td>$\alpha$</td>
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### Stress Parameters

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### Normal Stresses at $t$, $n$:

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### Tangential Stresses at $t$, $n$, $e$:

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### Geometric Parameters

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### Vertical Reactions

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<td>$-25.3242$</td>
<td>$W_{KL}$</td>
<td>$3.1253$</td>
<td>$W_{KL}$</td>
</tr>
<tr>
<td>$W_{KT}$</td>
<td>$-7.1561$</td>
<td>$W_{KT}$</td>
<td>$5.5013$</td>
<td>$W_{KT}$</td>
</tr>
</tbody>
</table>

### Horizontal Reactions

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{KL}$</td>
<td>$29.1000$</td>
<td>$W_{KL}$</td>
<td>$-11.6943$</td>
<td>$W_{KL}$</td>
</tr>
<tr>
<td>$W_{KT}$</td>
<td>$-47.4108$</td>
<td>$W_{KT}$</td>
<td>$-47.4108$</td>
<td>$W_{KT}$</td>
</tr>
</tbody>
</table>

### Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{KL}$</td>
<td>$-15.9624$</td>
<td>$M_{KL}$</td>
<td>$284.7600$</td>
<td>$M_{KL}$</td>
</tr>
<tr>
<td>$M_{KT}$</td>
<td>$4.4709$</td>
<td>$M_{KT}$</td>
<td>$311.4613$</td>
<td>$M_{KT}$</td>
</tr>
</tbody>
</table>

### Total Torque

$T_{TOTAL} = 623.2243$ lb-inches $= 56.1640$ kg-meters

$F_{MIP} = 0.3336, 0.373$
### Table 1.2a. Horizontal test for $\xi_{M} = 101$ and $s_{R} = 0.75$ (lower bound)

#### Structural Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_{M}$</td>
<td>101</td>
<td>101</td>
<td>101</td>
<td>101</td>
<td>101</td>
</tr>
<tr>
<td>$s_{R}$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>

#### Stress Parameters

<table>
<thead>
<tr>
<th>Stress Component</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{H}$</td>
<td>$\sigma_{L}$</td>
<td>$\sigma_{M}$</td>
<td>$\sigma_{N}$</td>
<td>$\sigma_{P}$</td>
<td>$\sigma_{Q}$</td>
</tr>
<tr>
<td>$\tau_{H}$</td>
<td>$\tau_{L}$</td>
<td>$\tau_{M}$</td>
<td>$\tau_{N}$</td>
<td>$\tau_{P}$</td>
<td>$\tau_{Q}$</td>
</tr>
</tbody>
</table>

#### Geometric Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>100</td>
</tr>
<tr>
<td>$y$</td>
<td>100</td>
</tr>
</tbody>
</table>

### Vertical Reactions

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{K}$</td>
<td>$W_{T}$</td>
<td>$W_{M}$</td>
<td>$W_{S}$</td>
<td>$W_{N}$</td>
<td>$W_{P}$</td>
</tr>
<tr>
<td>$H_{K}$</td>
<td>$H_{T}$</td>
<td>$H_{M}$</td>
<td>$H_{S}$</td>
<td>$H_{N}$</td>
<td>$H_{P}$</td>
</tr>
</tbody>
</table>

### Moments

<table>
<thead>
<tr>
<th>Moment Component</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{K}$</td>
<td>$M_{T}$</td>
<td>$M_{M}$</td>
<td>$M_{S}$</td>
<td>$M_{N}$</td>
<td>$M_{P}$</td>
</tr>
</tbody>
</table>

**Total Torque** = $581,467$ lb-inches = $10,390$ KF-meters

**JPL Technical Memorandum 33-477**
Table 1.3. Horizontal test for $\xi_M = 102$ and $s_k = 0.75$ (upper bound)

<table>
<thead>
<tr>
<th>INPUT PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_0, X_I, s_k =$</td>
</tr>
<tr>
<td>$R, k =$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRESS PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{100}, \eta_{110}, \eta_{110} =$</td>
</tr>
<tr>
<td>$R, k =$</td>
</tr>
</tbody>
</table>

| NORMAL STRESSES AT $1, k, n =$ | 1.7474 | 1.7474 |
| TANGENTIAL STRESSES AT $1, k, n =$ | -1.0714 |

<table>
<thead>
<tr>
<th>GEOMETRIC PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTER OF INSTANTANEOUS REFERENCE</td>
</tr>
<tr>
<td>TRAILING SPINAL REFERENCE</td>
</tr>
<tr>
<td>LEADING SPINAL REFERENCE</td>
</tr>
<tr>
<td>LEADING SPINAL COORDINATES</td>
</tr>
<tr>
<td>TRAILING SPINAL COORDINATES</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VERTICAL REACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{KP} =$</td>
</tr>
<tr>
<td>$W_{KL} =$</td>
</tr>
<tr>
<td>$W_{KL} =$</td>
</tr>
<tr>
<td>$W_{KL} =$</td>
</tr>
<tr>
<td>$W_{KL} =$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HORIZONTAL REACTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{KP} =$</td>
</tr>
<tr>
<td>$W_{KL} =$</td>
</tr>
<tr>
<td>$W_{KL} =$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MOMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{KL} =$</td>
</tr>
<tr>
<td>$M_{KL} =$</td>
</tr>
<tr>
<td>$M_{KL} =$</td>
</tr>
</tbody>
</table>

| TOTAL TORSION | 649.870UGE | 649.870UGE |
| $P, Q =$ | 1.4724 |

JPL Technical Memorandum 33-477
Table 1. 3a. Horizontal test for $S_M = 102$ and $s_K = 0.75$ (lower bound)

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
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</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
<th>$x_0$, $y_0$, $z_0$, $S_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
<td>$x_0$, $y_0$, $z_0$, $S_M$</td>
</tr>
</tbody>
</table>
Table 1.4. Horizontal test for $\xi_M = 104$ and $\alpha_L = 0.75$ (upper bound)

### INPUT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>0.346060</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>103.4944</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.7500</td>
</tr>
<tr>
<td>$\alpha$, $\gamma$, $\phi$, $\epsilon$</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha$, $\gamma$, $\phi$, $\epsilon$</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$, $\theta_2$, $\theta_3$, $\theta_4$</td>
<td>147.0000</td>
</tr>
<tr>
<td>$\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$</td>
<td>102.6787</td>
</tr>
<tr>
<td>$\phi_5$, $\phi_6$, $\phi_7$, $\phi_8$</td>
<td>38.3496</td>
</tr>
</tbody>
</table>

### GEOMETRIC PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$, $y_1$, $z_1$</td>
<td>-11.1730</td>
</tr>
</tbody>
</table>

### VERTICAL REACTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{KL}$</td>
<td>-17.3331</td>
</tr>
<tr>
<td>$W_{SL}$</td>
<td>-31.6086</td>
</tr>
<tr>
<td>$W_{ST}$</td>
<td>2.2742</td>
</tr>
</tbody>
</table>

### HORIZONTAL REACTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{KL}$</td>
<td>23.1116</td>
</tr>
<tr>
<td>$H_{ST}$</td>
<td>-15.6464</td>
</tr>
</tbody>
</table>

### MOMENTS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{KL}$</td>
<td>22.3400</td>
</tr>
<tr>
<td>$M_{ST}$</td>
<td>244.9668</td>
</tr>
</tbody>
</table>

TOTAL TORSION = 613.5141 LB-INCHES = 84.8215 KG-METERS

THRUST EFFICIENCY = 0.6238

JPL Technical Memorandum 33-477
Table 1.4a. Horizontal test for $M = 104$ and $K = 0.75$ (lower bound)

### INPUT PARAMETERS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H, P, X(M), SK$</td>
<td>94,000</td>
</tr>
<tr>
<td>$\alpha, \gamma, \phi, C$</td>
<td>0,000</td>
</tr>
<tr>
<td>$R, N$</td>
<td>13,450</td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS

| $\theta_{LNP}$ | 144.4447 |
| $\theta_{LMP}$ | 48.3343 |
| $\theta_{LMP}$ | 47.6747 |
| $P_{LL}$ | 2,000 |
| $P_{PM}$ | 0.2009 |

### CENTRAL INSTANTANEOUS ROTATION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X = 0.0000$</td>
<td>$Z = 10.2425$</td>
</tr>
<tr>
<td>$X = 2.1519$</td>
<td>$Z = 11.1037$</td>
</tr>
</tbody>
</table>

### LEADING SPIRAL POLE

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X = -3.3746$</td>
<td>$Z = 13.5356$</td>
</tr>
<tr>
<td>$X = -6.0313$</td>
<td>$Z = 16.7657$</td>
</tr>
</tbody>
</table>

### TRAILING SPIRAL COORDINATES

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X = 5.3013$</td>
<td>$Z = 12.9493$</td>
</tr>
<tr>
<td>$X = 8.1656$</td>
<td>$Z = 15.1846$</td>
</tr>
</tbody>
</table>

### CENTER OF INSTANTANEOUS ROTATION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(LNP)$</td>
<td>113.6174</td>
</tr>
<tr>
<td>$X(LNP)$</td>
<td>67.6336</td>
</tr>
</tbody>
</table>

### VERTICAL REACTION

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{KL}$</td>
<td>-16.4021</td>
</tr>
<tr>
<td>$H_{KL}$</td>
<td>4.0320</td>
</tr>
<tr>
<td>$W_{SL}$</td>
<td>-34.5912</td>
</tr>
<tr>
<td>$W_{SL}$</td>
<td>-65.5958</td>
</tr>
</tbody>
</table>

### TOTAL LEADING

- Leading: -46.9713
- Trailing: -46.9671

### MOMENT

- $H = 22.6438$
- $M = 34.2253$
- $H = 277.4944$
- $M = 297.7944$

### TOTAL TRAILING

- Leading: -53.1443
- Trailing: -53.1443

### TOTAL TORQUE

- 601.0575 
- 83.0993 
- 83.0993
Table 2.1. Wheel performance on slope for $\xi_M = 110$ and $s_k = 0.55$ (upper bound)

**INPUT PARAMETERS***

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p, \xi_M, s_k$</td>
<td>100.3000</td>
<td>0.00000</td>
<td>109.9999</td>
</tr>
<tr>
<td>$\alpha, \gamma, \psi, \chi, c$</td>
<td>20.4199</td>
<td>0.0545</td>
<td>43.1999</td>
</tr>
<tr>
<td>$R, R'$</td>
<td>13.9500</td>
<td>12.0000</td>
<td></td>
</tr>
</tbody>
</table>

**STRESS PARAMETERS***

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_{L,M,$}</td>
<td>135.4994</td>
<td>0.4135</td>
<td>9.1125</td>
</tr>
<tr>
<td>$\Theta_{L,$}</td>
<td>108.4994</td>
<td>81.7609</td>
<td>49.3804</td>
</tr>
<tr>
<td>$\Phi_{L,M,$}</td>
<td>2.0284</td>
<td>0.9474</td>
<td>1.0565</td>
</tr>
<tr>
<td>$\Phi_{L,$}</td>
<td>1.0314</td>
<td>0.9135</td>
<td>1.0111</td>
</tr>
</tbody>
</table>

NORMAL STRESSES AT L,M,N = 0.4372, 0.4994, 0.5715
TANGENTIAL STRESSES AT L,M,N = -0.464, 0.7064, 0.474

**GEOMETRIC PARAMETERS***

CENTER OF INSTANTANEOUS ROTATION --- $X_R = -2.6764$, $Z_M = 7.1905$
TRAILING SPIRAL POLE --- $X_P^1 = -5.8977$, $Z_P^1 = 1.2107$
LEADING SPIRAL POLE --- $X_P^2 = -10.3741$, $Z_P^2 = 8.1417$
BIFURCATION POINT --- $X_M = -9.0442$, $Z_M = 10.6209$
LEADING EDGE --- $X_L = -10.4454$, $Z_L = 4.2793$
LEADING SPIRAL COORDINATES --- $X_M = -10.5774$, $Z_M = 12.7547$
TRAILING SPIRAL COORDINATES --- $X_M = -3.5423$, $Z_M = 16.5337$

$X_M(N) = 117.8875$, $X_L(N) = 46.3746$

**VERTICAL REACTION***

$W_{PT} = -13.2699$, $W_{GT} = -6.1382$, $W_{ST} = -54.3763$, TOTAL TRAILING = -61.4665

**HORIZONTAL REACTION***

$H_{PL} = 19.0691$, $H_{KL} = -3.2536$, TOTAL LEADING = 15.8154
$H_{PT} = -16.2901$, $H_{KT} = -0.474$, TOTAL TRAILING = -15.8154

**MOMENTS***

$M_{GL} = -21.1603$, $M_{SL} = 261.8861$, $M_{PL} = -31.9469$, TOTAL LEADING = 230.7748
$M_{GT} = -38.5212$, $M_{ST} = 244.5758$, $M_{PT} = 294.5256$, TOTAL TRAILING = 500.5633

TOTAL Thrust = 731.3572 Lb-INCHES = 101.134 kg-METERS
Efficiency = 1.0140, 0.3722, 0.3670

JPL Technical Memorandum 33-477
Table 2.1a. Wheel performance on slope for $s_m = 110$ and $s_k = 0.55$ (lower bound)

### INPUT PARAMETERS

<table>
<thead>
<tr>
<th>$w$, $x$, $y$, $z$, $s_k$</th>
<th>100, 3000</th>
<th>0, 0000</th>
<th>110, 4400</th>
<th>0, 5500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$, $\gamma$, $\phi$, $c$</td>
<td>20, 4199</td>
<td>0, 0546</td>
<td>43, 1494</td>
<td>0, 0600</td>
</tr>
<tr>
<td>$r$, $\theta$</td>
<td>13, 9900</td>
<td>12, 0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS

<table>
<thead>
<tr>
<th>$\theta_{fall}$</th>
<th>137.2756</th>
<th>$\theta_{fall}$</th>
<th>100.7264</th>
<th>$\theta_{fall}$</th>
<th>51.6843</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{tan}$</td>
<td>100.4843</td>
<td>$\theta_{tan}$</td>
<td>74.0657</td>
<td>$\theta_{tan}$</td>
<td>51.6840</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>4.7025</th>
<th>$p_2$</th>
<th>4.7064</th>
<th>$p_3$</th>
<th>4.7064</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>4.1411</td>
<td>$d_2$</td>
<td>7.3377</td>
<td>$d_3$</td>
<td>4.207</td>
</tr>
</tbody>
</table>

### NORMAL STRESSES AT $L$, $M$, $N$ | 0.2419 | 0.6670 | 0.3174 |
| TANGENTIAL STRESSES AT $L$, $M$, $N$ | -0.2747 | 0.6284 | 0.2853 |

### GEOMETRIC PARAMETERS

| CENTER OF INSTANTANEOUS ROTATION | $x$ | 2.6764 | $z$ | 7.1905 |
| TRAILING SPIRAL POLE | $x_p$ | -9.8977 | $z_p$ | 1.2107 |
| LEADING SPIRAL POLE | $x_p$ | -10.3741 | $z_p$ | 8.4517 |
| HINGE FIXATION POINT | $x_m$ | -4.0442 | $z_m$ | 10.6209 |
| LEADING EDGE | $x_l$ | -10.5624 | $z_l$ | 9.1113 |
| LEADING SPIRAL COORDINATES | $x_m$ | -11.4775 | $z_m$ | 13.3860 |
| TRAILING EDGE | $x_n$ | -2.3159 | $z_n$ | 13.7564 |
| TRAILING SPIRAL COORDINATES | $x_m$ | -1.1089 | $z_m$ | 17.9838 |

### VERTICAL REACTION

| $w_{pl}$ | -17.3385 | $w_{pl}$ | 3.1863 | $w_{pl}$ | -27.3460 |
| $w_{pt}$ | -11.6645 | $w_{pt}$ | 11.0070 | $w_{pt}$ | -58.6578 |
| TOTAL LEADING | -41.7393 |
| TOTAL TRAILING | -68.6563 |

### HORIZONTAL REACTION

| $w_{pl}$ | 22.4985 | $w_{pt}$ | -1.2492 | TOTAL LEADING | 21.7343 |
| TOTAL TRAILING | -21.7343 |

### MOMENTS

| $m_{pl}$ | -34.0184 | $m_{pl}$ | 367.4669 | $m_{pl}$ | -70.4631 |
| $m_{pt}$ | -96.1932 | $m_{pt}$ | 267.1864 | $m_{pt}$ | 283.7736 |
| TOTAL LEADING | 202.4853 |
| TOTAL TRAILING | -689.3271 |

TOTAL TORQUE = 971.8124 LB-INECHES = 92.8815 KG-METERS

$F_M$, $F_T$, THRUST EFFICIENCY = 0.315, 1.3722, 0.3495
Table 2.2. Wheel performance on slope for $\xi_M = 110$ and $\xi_K = 0.65$ (upper bound)

### INPUT PARAMETERS###

<table>
<thead>
<tr>
<th>$W, P, X(M), SK$</th>
<th>$\Phi, \Phi$</th>
<th>$\Gamma, R, K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.9, 30.000</td>
<td>0.0, 0.000</td>
<td>109.6, 999, 550</td>
</tr>
<tr>
<td>$\alpha, \gamma, \phi, \epsilon$</td>
<td>$20.6, 159$</td>
<td>$.0584$</td>
</tr>
<tr>
<td>$\alpha, \gamma, \phi, \epsilon$</td>
<td>$43.1, 950$</td>
<td>$.0580$</td>
</tr>
<tr>
<td>$\alpha, \gamma, \phi, \epsilon$</td>
<td>$13.9, 950$</td>
<td>$12.0, 000$</td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS###

<table>
<thead>
<tr>
<th>$\theta_T(A) =$</th>
<th>$\theta_T(A) =$</th>
<th>$\theta_T(A) =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>134.7457</td>
<td>116.9512</td>
<td>134.9507</td>
</tr>
<tr>
<td>$\theta_T(A) =$</td>
<td>$\theta_T(A) =$</td>
<td>$\theta_T(A) =$</td>
</tr>
<tr>
<td>70.1512</td>
<td>44.6100</td>
<td>70.6100</td>
</tr>
<tr>
<td>$\nu =$</td>
<td>$\nu =$</td>
<td>$\nu =$</td>
</tr>
<tr>
<td>.2025</td>
<td>.10312</td>
<td>.11457</td>
</tr>
<tr>
<td>$\nu =$</td>
<td>$\nu =$</td>
<td>$\nu =$</td>
</tr>
<tr>
<td>1.0318</td>
<td>1.0572</td>
<td>.9348</td>
</tr>
</tbody>
</table>

### NORMAL STRESSES AT L,M,N ###

<table>
<thead>
<tr>
<th>$\sigma_L =$</th>
<th>$\sigma_M =$</th>
<th>$\sigma_N =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.2025</td>
<td>1.0312</td>
<td>1.1457</td>
</tr>
<tr>
<td>$\sigma_L =$</td>
<td>$\sigma_M =$</td>
<td>$\sigma_N =$</td>
</tr>
<tr>
<td>1.0318</td>
<td>1.0572</td>
<td>.9348</td>
</tr>
</tbody>
</table>

### TANGENTIAL STRESSES AT L,M,N ###

<table>
<thead>
<tr>
<th>$\tau_L =$</th>
<th>$\tau_M =$</th>
<th>$\tau_N =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.7401</td>
<td>1.2450</td>
<td>.7461</td>
</tr>
<tr>
<td>$\tau_L =$</td>
<td>$\tau_M =$</td>
<td>$\tau_N =$</td>
</tr>
<tr>
<td>1.2450</td>
<td>.7461</td>
<td>.7461</td>
</tr>
</tbody>
</table>

### GEOMETRIC PARAMETERS###

<table>
<thead>
<tr>
<th>CENTER OF INSTANTANEOUS ROTATION</th>
<th>$X_R =$</th>
<th>$Z_R =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAILING SPIRAL POLAR</td>
<td>$X_P =$</td>
<td>$Z_P =$</td>
</tr>
<tr>
<td>LEADING SPIRAL POLAR</td>
<td>$X_P =$</td>
<td>$Z_P =$</td>
</tr>
<tr>
<td>RIFLE POINT</td>
<td>$X_M =$</td>
<td>$Z_M =$</td>
</tr>
<tr>
<td>LEADING EDGE</td>
<td>$X_L =$</td>
<td>$Z_L =$</td>
</tr>
<tr>
<td>LEADING SPIRAL COORDINATES</td>
<td>$X_M =$</td>
<td>$Z_M =$</td>
</tr>
<tr>
<td>TRAILING SPIRAL COORDINATES</td>
<td>$X_M =$</td>
<td>$Z_M =$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_R =$</th>
<th>$X_L =$</th>
<th>$X_M =$</th>
</tr>
</thead>
<tbody>
<tr>
<td>115.6725</td>
<td>99,9106</td>
<td></td>
</tr>
</tbody>
</table>

### VERTICAL REACTION ###

<table>
<thead>
<tr>
<th>$W_KPL =$</th>
<th>$W_KGL =$</th>
<th>$W_KPT =$</th>
<th>TOTAL LEADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.15,000</td>
<td>-.7114</td>
<td>-.13,4206</td>
<td>-27.7392</td>
</tr>
<tr>
<td>$W_KPT =$</td>
<td>$W_KGL =$</td>
<td>$W_KPL =$</td>
<td>TOTAL TRAILING</td>
</tr>
</tbody>
</table>

### TOTAL MOMENTS ###

<table>
<thead>
<tr>
<th>$M_KPL =$</th>
<th>$M_KGL =$</th>
<th>$M_KPT =$</th>
<th>TOTAL LEADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.14,730</td>
<td>-2.0915</td>
<td>9.8874</td>
<td>4.8821</td>
</tr>
<tr>
<td>$M_KPT =$</td>
<td>$M_KGL =$</td>
<td>$M_KPL =$</td>
<td>TOTAL TRAILING</td>
</tr>
<tr>
<td>-19.7401</td>
<td>9.8874</td>
<td>-4.8821</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M_KPT =$</th>
<th>$M_KGL =$</th>
<th>$M_KPL =$</th>
<th>TOTAL LEADING</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8.5330</td>
<td>-20.8120</td>
<td>169.4144</td>
<td></td>
</tr>
<tr>
<td>$M_KGL =$</td>
<td>$M_KPL =$</td>
<td>$M_KPT =$</td>
<td>TOTAL TRAILING</td>
</tr>
<tr>
<td>214.5340</td>
<td>391.0730</td>
<td>562.8018</td>
<td></td>
</tr>
</tbody>
</table>

| TOTAL TURBO = | 732.2163 | 101,2137 |
| EFF, THRUST EFFICIENCY = | .8490 | .3722 | .4332 |

JPL Technical Memorandum 33-477
Table 2.2a. Wheel performance on slope for \( \delta_M = 110 \) and \( s_K = 0.65 \) (lower bound)

### INPUT PARAMETERS

<table>
<thead>
<tr>
<th>( W, P, x I M )</th>
<th>100.000</th>
<th>0.0000</th>
<th>100.9999</th>
<th>.6500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X, K )</td>
<td>13.95000</td>
<td>12.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS

<table>
<thead>
<tr>
<th>( \Theta(UP) )</th>
<th>144.5818</th>
<th>( \Theta(UP) )</th>
<th>99.0428</th>
<th>( \Theta(MP) )</th>
<th>70.1512</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Theta(M) )</td>
<td>116.9512</td>
<td>( \Theta(NMN) )</td>
<td>64.0370</td>
<td>( \Theta(ND) )</td>
<td>48.1827</td>
</tr>
</tbody>
</table>

### GEOMETRIC PARAMETERS

| CENTER OF INSTANTANEOUS ROTATION | \( X_0 = -3.1630 \) | \( Z_0 = 8.4479 \) |
| TRAILING SPIRAL POLE | \( X_P = -5.1520 \) | \( Z_P = 2.9751 \) |
| LEADING SPIRAL POLE | \( X_P = -10.2389 \) | \( Z_P = 10.1299 \) |
| TRANSITION POINT | \( X_M = -9.4442 \) | \( Z_M = 9.4442 \) |
| LEADING SPIRAL COORDINATES | \( X_L = -10.4442 \) | \( Z_L = 14.1299 \) |
| TRAILING SPIRAL COORDINATES | \( X_T = -4.2429 \) | \( Z_T = 13.2891 \) |

### VERTICAL REACTION

| \( W_{KL} \) | -15.6411 | \( W_{KLN} = 1.1163 \) | \( W_{KST} = 8.2754 \) |
| \( W_{KST} \) | -18.3141 | \( W_{KST} = 6.3445 \) | \( W_{KST} = -54.6836 \) |

TOTAL LEADING = \(-30.5710\)
TOTAL TRAILING = \(-69.7263\)

### HORIZONTAL REACTION

| \( H_{KL} \) | 14.1199 | \( H_{KLN} = -1.7498 \) |
| \( H_{KST} \) | -19.8063 | \( H_{KST} = 7.4752 \) |

TOTAL LEADING = \(12.3300\)
TOTAL TRAILING = \(-12.3300\)

### MOMENTS

| \( M_L \) | -17.6101 | \( M_{LST} = 143.1972 \) |
| \( M_{ST} \) | -52.2301 | \( M_{ST} = 212.8244 \) |

TOTAL LEADING = \(176.7316\)
TOTAL TRAILING = \(541.0335\)

TOTAL TORDUS = \(711.7451\) LB-INCHES = \(99.2347\) KG-METERS

FM, FT, THRUST EFFICIENCY = \(.8421\) \(.3722\) \(.4420\)
Table 2.3. Wheel performance on slope for $\xi_M = 105$ and $s_K = 0.65$ (upper bound)

<table>
<thead>
<tr>
<th>INPUT PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$W = 105,000$</td>
<td>$s = 0.65$</td>
</tr>
<tr>
<td>$\alpha = 0.01$</td>
<td>$\gamma = 0.01$</td>
</tr>
<tr>
<td>$\phi = 0.01$</td>
<td>$\epsilon = 0.01$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STRESS PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{T1}$</td>
<td>140.4762</td>
</tr>
<tr>
<td>$\theta_{T2}$</td>
<td>105.5416</td>
</tr>
<tr>
<td>$\theta_{T3}$</td>
<td>106.6502</td>
</tr>
<tr>
<td>$\theta_{T4}$</td>
<td>37.2357</td>
</tr>
<tr>
<td>$P_L$</td>
<td>2.027</td>
</tr>
<tr>
<td>$P_P$</td>
<td>1.0562</td>
</tr>
<tr>
<td>$P_M$</td>
<td>1.0401</td>
</tr>
<tr>
<td>$P_N$</td>
<td>0.7313</td>
</tr>
</tbody>
</table>

| NORMAL STRESSES AT L,M,N = | 2492 | 1.0410 |
| TANGENTIAL STRESSES AT L,M,N = | 4.413 |

<table>
<thead>
<tr>
<th>GEOMETRIC PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTER OF INSTANTANEOUS ROTATION</td>
<td></td>
</tr>
<tr>
<td>$X_R$ = -3.1630</td>
<td>$Z_R$ = 8.4779</td>
</tr>
<tr>
<td>TRAILING SPiral POLE</td>
<td></td>
</tr>
<tr>
<td>$X_P1$ = -4.3914</td>
<td>$Z_P1$ = 4.1461</td>
</tr>
<tr>
<td>LEADING SPiral POLE</td>
<td></td>
</tr>
<tr>
<td>$X_P2$ = -4.3914</td>
<td>$Z_P2$ = 4.1461</td>
</tr>
<tr>
<td>RIGIFICATION POINT</td>
<td></td>
</tr>
<tr>
<td>$X_M$ = -8.1084</td>
<td>$Z_M$ = 11.2687</td>
</tr>
<tr>
<td>LEADING EDGE</td>
<td></td>
</tr>
<tr>
<td>$X_L$ = -9.5076</td>
<td>$Z_L$ = 10.2080</td>
</tr>
<tr>
<td>LEADING SPiral COORDINATES</td>
<td></td>
</tr>
<tr>
<td>$X_M1$ = -4.9371</td>
<td>$Z_M1$ = 13.4351</td>
</tr>
<tr>
<td>TRAILING SPiral COORDINATES</td>
<td></td>
</tr>
<tr>
<td>$X_MN$ = -2.9136</td>
<td>$Z_MN$ = 14.2883</td>
</tr>
<tr>
<td>$XI_L$ = 12.5401</td>
<td>$XI_M$ = 14.4513</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VERTICAL REACTIONS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{KPL}$ = -14.7940</td>
<td>$W_{KGL}$ = 2.3175</td>
</tr>
<tr>
<td>$W_{KPT}$ = -11.0238</td>
<td>$W_{KG1}$ = 4.4975</td>
</tr>
<tr>
<td>$W_{KST}$ = -26.9440</td>
<td>$W_{KST}$ = -51.2613</td>
</tr>
<tr>
<td>TOTAL LEADING</td>
<td>-53.4354</td>
</tr>
<tr>
<td>TOTAL TRAILING</td>
<td>-57.1245</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HORIZONTAL REACTIONS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{KPL}$ = 22.2113</td>
<td>$M_{KPL}$ = -3.0039</td>
</tr>
<tr>
<td>$H_{KPT}$ = -16.1493</td>
<td>$M_{KPT}$ = -3.0380</td>
</tr>
<tr>
<td>TOTAL LEADING</td>
<td>14.2073</td>
</tr>
<tr>
<td>TOTAL TRAILING</td>
<td>-19.2073</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MMoments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_L$ = -22.9015</td>
<td>$M_1 = 23.7918$</td>
</tr>
<tr>
<td>$M_2$ = -37.5753</td>
<td>$M_3 = 54.6997$</td>
</tr>
<tr>
<td>TOTAL LEADING</td>
<td>130.0012</td>
</tr>
<tr>
<td>TOTAL TRAILING</td>
<td>507.9349</td>
</tr>
<tr>
<td>TOTAL TORSION</td>
<td>688.0229 KI/G-METERS</td>
</tr>
<tr>
<td>FMI, FTC, THRUST EFFICIENCY</td>
<td>0.4072</td>
</tr>
</tbody>
</table>

JPL Technical Memorandum 33-477
Table 2.3a. Wheel performance on slope for $s_M = 105$ and $s_k = 0.65$ (lower bound)

### INPUT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha,\gamma,\phi,\xi, s_k$</td>
<td>100.3400</td>
<td>0.0000</td>
<td>104.9494</td>
<td>0.6500</td>
<td></td>
</tr>
<tr>
<td>$\rho_r, \rho_l$</td>
<td>13.4500</td>
<td>12.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_I, \theta_P, \theta_M, \theta_N$</td>
<td>144.6045</td>
<td>111.9612</td>
<td>59.7410</td>
<td>50.5730</td>
<td></td>
</tr>
<tr>
<td>$\rho_u, \rho_v, \rho_w$</td>
<td>0.2029</td>
<td>0.9805</td>
<td>0.9421</td>
<td>0.8924</td>
<td>0.6214</td>
</tr>
<tr>
<td>$\rho_n, \rho_m,$</td>
<td>0.1742</td>
<td>0.2313</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NORMAL STRESSES AT $L, M, N$</td>
<td>-1.238</td>
<td>0.8868</td>
<td>1.220</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TANGENTIAL STRESSES AT $L, M, N$</td>
<td>-0.0364</td>
<td>0.0092</td>
<td>0.152</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### GEOMETRIC PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAILING SPIRAL POLE</td>
<td>$X_H$</td>
<td>$-3.1630$</td>
<td>$-5.8401$</td>
<td>$-11.3648$</td>
<td>$-9.7267$</td>
</tr>
<tr>
<td>LEADING SPIRAL COORDINATES</td>
<td>$X_L$</td>
<td>$-4.7268$</td>
<td>$-4.7268$</td>
<td>$14.7055$</td>
<td>$13.6948$</td>
</tr>
<tr>
<td>TRAILING SPIRAL END</td>
<td>$X_T$</td>
<td>$-7.2333$</td>
<td>$13.6948$</td>
<td>$18.3139$</td>
<td></td>
</tr>
<tr>
<td>TRAILING SPIRAL COORDINATES</td>
<td>$X_T$</td>
<td>$2.2950$</td>
<td>$18.3139$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_H(I)$</td>
<td>113.7920</td>
<td>$X_T(N)$</td>
<td>70.9424</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### VERTICAL REACTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{KL}$</td>
<td>-14.2141</td>
<td>$W_{KL}$</td>
<td>4.9177</td>
<td>$W_{SL}$</td>
<td>-36.9447</td>
</tr>
<tr>
<td>$W_{KL}$</td>
<td>-6.3939</td>
<td>$W_{KL}$</td>
<td>14.5754</td>
<td>$W_{SL}$</td>
<td>-82.3644</td>
</tr>
<tr>
<td>TOTAL LEADING</td>
<td>-46.2371</td>
<td>TOTAL TRAILING</td>
<td>-54.1829</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### HORIZONTAL REACTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{KL}$</td>
<td>79.1734</td>
<td>$M_{KL}$</td>
<td>7.5920</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{KL}$</td>
<td>-14.7801</td>
<td>$M_{KL}$</td>
<td>-16.9292</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL LEADING</td>
<td>31.7094</td>
<td>TOTAL TRAILING</td>
<td>-31.7094</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### MOMENTS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
<th>Value 4</th>
<th>Value 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{KL}$</td>
<td>-44.7388</td>
<td>$M_{KL}$</td>
<td>338.9845</td>
<td>$M_{KL}$</td>
<td>-213.3103</td>
</tr>
<tr>
<td>$M_{KL}$</td>
<td>-45.3722</td>
<td>$M_{KL}$</td>
<td>317.9378</td>
<td>$M_{KL}$</td>
<td>275.8248</td>
</tr>
<tr>
<td>TOTAL TRACTIVE</td>
<td>589.2861</td>
<td>LAW-INGCES</td>
<td>81.47lb</td>
<td>KG-METERS</td>
<td></td>
</tr>
<tr>
<td>$F_M, F_T, THRUST EFFICIENCY$</td>
<td>0.6913</td>
<td>0.3722</td>
<td>0.5383</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.4. Wheel performance on slope for $\xi_M = 105$ and $s_k = 0.70$ (upper bound)

### INPUT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w, p, x$</td>
<td>114.1</td>
</tr>
<tr>
<td>$s_k$</td>
<td>0.70</td>
</tr>
<tr>
<td>$\alpha, \gamma, \phi, c$</td>
<td>20.4159</td>
</tr>
<tr>
<td>$r, h$</td>
<td>10.4949</td>
</tr>
</tbody>
</table>

### STRESS PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{input}}$</td>
<td>140.1015</td>
</tr>
<tr>
<td>$\theta_{\text{out}}$</td>
<td>95.0272</td>
</tr>
</tbody>
</table>

### GEOMETRIC PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{\text{L}} (N)$</td>
<td>85.4203</td>
</tr>
</tbody>
</table>

### VERTICAL REACTION

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{\text{KL}}$</td>
<td>-17.4271</td>
</tr>
<tr>
<td>$w_{\text{KT}}$</td>
<td>-17.4271</td>
</tr>
</tbody>
</table>

### MOMENTS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\text{KL}}$</td>
<td>-248.9197</td>
</tr>
<tr>
<td>$m_{\text{KT}}$</td>
<td>-317.2593</td>
</tr>
</tbody>
</table>

TOTAL TORQUE = 688.7750 LN-INCHES = 95.2267 KG-METERS

$F_m, f_t, \text{THRUST EFFICIENCY}$ = .7503 .3722 .4940
Table 2.4a. Wheel performance on slope for $\xi_M = 105$ and $s_R = 0.70$ (lower bound)

**INPUT PARAMETERS***

\[
\begin{align*}
W, P, X, Y, Z, S_k &= 100.0000, 106.9999, 7.0000 \\
\alpha, \gamma, \phi, \xi, \theta &= 20.4159, 0.594, 43.1999, 216.0000 \\
R, \xi &= 13.9500, 12.2000
\end{align*}
\]

**STRESS PARAMETERS***

\[
\begin{align*}
\theta_{L(1)} &= 142.3620 \\
\theta_{L(2)} &= 106.4128 \\
\theta_{L(3)} &= 64.4997 \\
\theta_{N(1)} &= 111.4400 \\
\theta_{N(2)} &= 67.2300 \\
\theta_{N(3)} &= 50.4761
\end{align*}
\]

\[
\begin{align*}
\rho &= 0.2025 \\
\pi &= 0.2025 \\
\omega &= 0.9012 \\
\beta &= 0.6938 \\
\gamma &= 0.3456
\end{align*}
\]

**NORMA L STRESSES AT L, M, N =**

\[
\begin{align*}
\sigma_L &= 1.1004 \\
\sigma_M &= 1.0101 \\
\sigma_N &= 1.2044
\end{align*}
\]

**TANGENTIAL STRESSES AT L, M, N =**

\[
\begin{align*}
\tau_L &= 1.0111 \\
\tau_M &= 0.2292 \\
\tau_N &= 0.2292
\end{align*}
\]

**CENTRAL OF INSTANTANEOUS ROTATION --- XR = -3.4063 ZR = 9.1316
TRAILING SPIRAL MILE --- XPL = -5.4434 ZPL = 4.7588
TRAILING SINE --- XNL = -4.3745 ZNL = 4.7588
LEADING EDGE --- XL = -9.6941 ZL = 10.0258
LEADING SPIRAL COORDINATES --- XML = -10.9419 ZML = 14.7607
TRAILING EDGE --- XN = -1.6794 ZN = 13.9393
TRAILING SPIRAL COORDINATES --- XMN = -1.6794 ZMN = 14.7607

\[
\begin{align*}
X_{L(1)} &= 113.6349 \\
X_{N(1)} &= 78.4715
\end{align*}
\]

**TOTAL LEADING = -42.5390
TOTAL TRAILING = -57.3926
TOTAL LEADING = 26.0273
TOTAL TRAILING = -26.0273
TOTAL LEADING = 26.0273
TOTAL TRAILING = -26.0273

**MOMENTS***

\[
\begin{align*}
M_L &= -33.9926 \\
M_S &= -29.4108 \\
M_P &= -144.4642 \\
M_T &= -68.4105 \\
M_E &= 295.0673 \\
M_F &= 284.6905 \\
M_H &= 525.7422
\end{align*}
\]

**TOTAL TORSION = 635.7212 LB-INCHES = 7.4217 KG-METERS

**FM, FT, THRUST EFFICIENCY =**

\[
\begin{align*}
&0.6725 \\
&0.3722 \\
&0.5374
\end{align*}
\]
Table 3.1. Wheel performance for Apollo LRV for $\frac{F}{M} > 105$ and $s_k = 0.85$ (upper bound)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k, m$</td>
<td>$60,000$</td>
</tr>
<tr>
<td>$104,9999$</td>
<td>$9559$</td>
</tr>
<tr>
<td>$\alpha, \gamma, \phi, C$</td>
<td>$0,0000$</td>
</tr>
<tr>
<td>$\beta, \delta, \tau$</td>
<td>$37,9999$</td>
</tr>
<tr>
<td>$10,5000$</td>
<td>$10,0000$</td>
</tr>
</tbody>
</table>

**Stress Parameters**

- $\theta_{F} = 152.0067$  $\theta_{T} = 107.7177$  $\theta_{A} = 64.0427$  $\theta_{I} = 29.3706$
- $\delta_{P} = 0.1690$  $\delta_{L} = 0.6664$  $\delta_{M} = 0.5734$
- $\delta_{N} = 0.5057$  $\delta_{Q} = 0.4771$  $\delta_{W} = 0.4197$

**Normal Stresses at $L$, $M$, $N$**

- $\sigma_{L} = 0.1417$  $\sigma_{M} = 0.1068$
- $\sigma_{N} = 0.0998$  $\sigma_{W} = 0.0741$

**Tangential Stresses at $L$, $M$, $N$**

- $\tau_{L} = 0.1647$  $\tau_{M} = 0.1064$
- $\tau_{N} = 0.0999$  $\tau_{W} = 0.0740$

**Geometric Parameters**

- $x_{F} = 0.0000$  $\phi_{F} = 13.4017$
- $x_{M} = -6.4471$  $\phi_{M} = 10.0462$
- $x_{N} = -4.1411$  $\phi_{N} = 14.6064$
- $x_{L} = -7.5771$  $\phi_{L} = 18.0917$
- $x_{V} = 1.7097$  $\phi_{V} = 15.9562$
- $x_{I} = 3.3771$  $\phi_{I} = 20.3762$

**Vertical Reactions**

- $w_{P} = -6.4797$  $w_{S} = 1.1179$  $w_{SL} = -214.9963$  Total Leaning = $-246.3623$
- $w_{K} = -4.0141$  $w_{G} = 1.7411$  $w_{ST} = -24.9735$  Total Trailling = $-34.1455$

**Moments**

- $m_{L} = -7.2449$  $m_{S} = 158.9629$  $m_{PL} = -214.5682$  Total Leaning = $-64.2412$
- $m_{M} = -4.9226$  $m_{G} = 10.2999$  $m_{KT} = 261.6051$  Total Trailling = $741.4481$

**Total Torque**

- $204.7998$  $lb-$inches = $29,000.5$  $kg$-meters
- $r_{f}, r_{p} = 0.2871$  $0.0000$
Table 3.1a. Wheel performance for Apollo LRV for $\delta_M = 105$ and $s_K = 0.85$ (lower bound)

**INPUT PARAMETERS**

<table>
<thead>
<tr>
<th>$\delta_M$ (deg)</th>
<th>$s_K$</th>
<th>$\phi_0$ (deg)</th>
<th>$\phi_1$ (deg)</th>
<th>$\phi_2$ (deg)</th>
<th>$\phi_3$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150.441</td>
<td>0.85</td>
<td>112.561</td>
<td>22.755</td>
<td>5.421</td>
<td>1.074</td>
</tr>
<tr>
<td>127.472</td>
<td>12.757</td>
<td>112.461</td>
<td>22.755</td>
<td>5.421</td>
<td>1.074</td>
</tr>
</tbody>
</table>

**STRESS PARAMETERS**

<table>
<thead>
<tr>
<th>$\theta_1$ (deg)</th>
<th>$\theta_2$ (deg)</th>
<th>$\theta_3$ (deg)</th>
<th>$\theta_4$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140.441</td>
<td>122.461</td>
<td>112.461</td>
<td>22.755</td>
</tr>
</tbody>
</table>

**NORMAL STRESSES AT L,M,N**

<table>
<thead>
<tr>
<th>$\sigma_{LMN}$</th>
<th>$\sigma_{LMN}$</th>
<th>$\sigma_{LMN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.545</td>
<td>2.545</td>
<td>2.545</td>
</tr>
</tbody>
</table>

**TANGENTIAL STRESSES AT L,M,N**

<table>
<thead>
<tr>
<th>$\tau_{LMN}$</th>
<th>$\tau_{LMN}$</th>
<th>$\tau_{LMN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.545</td>
<td>2.545</td>
<td>2.545</td>
</tr>
</tbody>
</table>

**GEOMETRIC PARAMETERS**

<table>
<thead>
<tr>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>127.428</td>
<td>77.0746</td>
</tr>
</tbody>
</table>

**VERTICAL REACTIONS**

<table>
<thead>
<tr>
<th>$W_{KL}$</th>
<th>$W_{KL}$</th>
<th>$W_{KL}$</th>
<th>$W_{KL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0464</td>
<td>2.0464</td>
<td>2.0464</td>
<td>2.0464</td>
</tr>
</tbody>
</table>

**TOTAL REACTIONS**

<table>
<thead>
<tr>
<th>$W_{KL}$</th>
<th>$W_{KL}$</th>
<th>$W_{KL}$</th>
<th>$W_{KL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0464</td>
<td>2.0464</td>
<td>2.0464</td>
<td>2.0464</td>
</tr>
</tbody>
</table>

**REMARKS**

- $W_{KL} = 2.0464$, $W_{KL} = 2.0464$, $W_{KL} = 2.0464$, $W_{KL} = 2.0464$.
Table 3.2. Wheel performance for Apollo LRV for \( \xi_M = 105 \) and \( s_K = 0.90 \) (upper bound)

### INPUT PARAMETERS

\[
\begin{align*}
W, P, X1(M), SK &= 60.0000, 0.0000, 104.4449, 0.4800 \\
\alpha, \gamma, \phi_1, \phi_2 &= 0.0000, 0.0100, 32.9949, 0.0500 \\
K, K &= 16.0000, 10.0000
\end{align*}
\]

### STRESS PARAMETERS

\[
\begin{align*}
\theta_{T}(L_{OP}) &= 149.3241 \\
\theta_{T}(M) &= 132.7096 \\
\theta_{T}(MNN) &= 74.7521 \\
\theta_{T}(N) &= 31.3747 \\
P_0 &= 1.0000 \\
P_L &= 0.9649 \\
P_M &= 0.9857 \\
P_N &= 0.9506
\end{align*}
\]

- NORMAL STRESSES AT \( L, M, N \) = 0.2041, 0.8044, 0.6419
- TANGENTIAL STRESSES AT \( L, M, N \) = -0.1360, 0.0086, 0.2614

### GEOMETRIC PARAMETERS

\[
\begin{align*}
&\text{CENTER OF INSTANTANEOUS ROTATION--- } X_H = 0.0000 \\
&\text{LEADING SPIRAL POLE--- } X_{P1} = -4.6949, ZP1 = 11.7107 \\
&\text{HORIZONTAL POINT--- } X_M = -4.1411, ZM = 14.5494 \\
&\text{LEADING EDGE--- } X_L = -7.2542, ZL = 17.7234 \\
&\text{TRAILING SPIRAL COORDINATES--- } X_{MN} = 4.5571, ZMN = 20.5449 \\
&X_{(L)} = 115.3086, X_{(N)} = 39.5742
\end{align*}
\]

### VERTICAL REACTION

\[
\begin{align*}
W_{KL} &= -7.1048, W_{GL} = 0.6949, W_{SL} = -16.7480 \\
W_{PT} &= -9.9000, W_{KT} = 1.3934, W_{ST} = -29.5018 \\
&\text{TOTAL LEADING} = -22.8440 \\
&\text{TOTAL TRAILING} = 47.1166
\end{align*}
\]

### HORIZONTAL REACTION

\[
\begin{align*}
W_{KL} &= 11.5436, W_{KL} = 7.5015 \\
W_{PT} &= -16.5940, H_{KT} = 4.2949 \\
&\text{TOTAL LEADING} = 12.5441 \\
&\text{TOTAL TRAILING} = -12.5441
\end{align*}
\]

### MOMENTS

\[
\begin{align*}
M_{GL} &= -3.6166, M_{SL} = 120.1469, M_{PL} = -146.5716 \\
M_{PT} &= -1.7554, M_{ST} = -75.9949, M_{PT} = 301.4446 \\
&\text{TOTAL TRAILING} = 223.7546
\end{align*}
\]

TOTAL TORQUE = 203.7541 LBS-INCHES = 78,1700 KG-METERS

\[
FM, EP, = 0.2358, 0.0000
\]
Table 3.2a. Wheel performance for Apollo LRV for $r_M = 105$ and $\phi_K = 0.90$ (lower bound)

**INPUT PARAMETERS**

| $W, P, X, X_H, K$ | 0.0000 | 0.0000 | 106.9994 | 0.9000 |
| $\alpha, \gamma, \phi, R$ | 0.0000 | 0.0000 | 12.9999 | 0.0300 |
| $K, R$ | 10.0000 | 10.0000 |

**STRESS PARAMETERS**

| $\Theta (MP) =$ | 156.1713 | $\Theta (MP) =$ | 114.7695 | $\Theta (MP) =$ | 75.7096 |
| $\Theta (MP) =$ | 156.1713 | $\Theta (MP) =$ | 114.7695 | $\Theta (MP) =$ | 75.7096 |

| $\nu =$ | 0.1690 | $\Delta =$ | 3.357 | $\nu =$ | 6.662 |
| $\Delta =$ | 0.476 | $\Delta =$ | 3.175 | $\Delta =$ | 2.770 |

NORMAL STRESSES AT $L, M, N = 0.9384$, $5.937$, $12.01$
TANGENTIAL STRESSES AT $L, M, N = 0.0010$, $0.0056$, $0.1201$

**GEOMETRIC PARAMETERS**

CENTER OF INFINTE DUS ROJATION --- $X_L = 0.0000$, $Z_K = 16.0000$
TRAILING SPIRAL POINT --- $X_{PL} = 11.1141$, $Z_{PL} = 11.1141$
MAINTENANCE POINT --- $X_{PM} = -4.1141$, $Z_{PM} = 4.1141$
LEADING EDGE --- $X_L = -8.9400$, $Z_L = 11.9400$
LEADING SPIRAL COORDINATES --- $X_{LM} = 12.2540$, $Z_{LM} = 12.2540$
TRAILING SPIRAL COORDINATES --- $X_{NM} = 4.7059$, $Z_{NM} = 4.7059$

$X(I) = 274.1486$, $X(I) = 7.014890$

VERTICAL REACTION

$W_{PL} = -3.8413$, $W_{GL} = 2.1753$, $W_{SL} = 26.1374$, $W_{PL} = -27.8200$
$W_{PT} = -1.5527$, $W_{GT} = 4.4927$, $W_{ST} = -4.6118$, $W_{PT} = -32.0114$

HORIZONTAL REACTION

$W_{PL} = 29.0737$, $W_{ML} = 2.3534$, $W_{PT} = -17.0587$, $W_{HT} = -17.3634$

TOTAL LEADING = 27.4322
TOTAL TRAILING = 27.4322

COMMENTS

$MGL = -6.6617$, $MML = 16.5952$, $MPL = -27.8200$, $MGL = -12.4904$
$NGL = 9.4000$, $NML = 12.9054$, $NPL = 31.2016$, $NGL = 45.3017$

TOTAL TORSION = 244.5937, LA/INCHES = 33,516, KI-NEERS
$W_{EP} = 0.0000$, $0.2830$
Table 4.1. Wheel performance for Lunokhod-1 for \( \xi_M = 112 \) and \( \xi_K = 0.80 \) (upper bound)

### INPUT PARAMETERS

\[
\begin{align*}
\psi, \alpha, \gamma, \phi, \xi, \kappa &= 35.0000, 0.0000, 111.9999, 8000 \\
\alpha, \gamma, \phi, \xi, \kappa &= 0.0000, 0.0100, 32.9449, 0.0100 \\
\end{align*}
\]

### STRESS PARAMETERS

\[
\begin{align*}
\Theta_{1}(\xi_{11}) &= 155.6436 \\
\Theta_{1}(\xi_{11}) &= 107.6283 \\
\Theta_{1}(\xi_{11}) &= 71.2486 \\
\Theta_{1}(\xi_{11}) &= 24.4682 \\
\end{align*}
\]

\[
\begin{align*}
\psi &= 1.1690 \\
\psi &= 0.5943 \\
\phi &= 0.5484 \\
\end{align*}
\]

NORMAL STRESSES AT \( L, M, N = \)

\[
\begin{align*}
1.7224, 0.3443 \\
\end{align*}
\]

TANGENTIAL STRESSES AT \( L, M, N = \)

\[
\begin{align*}
-1.1447, 1.1207, 0.2502 \\
\end{align*}
\]

### GEOMETRIC PARAMETERS

\[
\begin{align*}
\xi_{11} &= 178.0258 \\
\xi_{11} &= 41.4463 \\
\end{align*}
\]

### VERTICAL REACTION

\[
\begin{align*}
W_{KPL} &= 3.4942 \\
W_{KPL} &= 0.5896 \\
W_{KPL} &= -10.6441 \\
W_{KPL} &= -10.6441 \\
\end{align*}
\]

\[
\begin{align*}
W_{KPT} &= 3.2561 \\
W_{KPT} &= 0.176 \\
W_{KPT} &= -10.6441 \\
W_{KPT} &= -10.6441 \\
\end{align*}
\]

### TOTAL REACTION

\[
\begin{align*}
MK_{KPL} &= -0.7884 \\
MK_{KPT} &= -9.7117 \\
\end{align*}
\]

horizontal reactions:

\[
\begin{align*}
\end{align*}
\]

### MOMENTS

\[
\begin{align*}
W_{ML} &= -1.9440 \\
W_{ML} &= -9.7117 \\
\end{align*}
\]

\[
\begin{align*}
H_{ML} &= -0.4661 \\
H_{ML} &= 112.0715 \\
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]

### TOTAL TORQUE

\[
\begin{align*}
\text{ft-lbs} &= 11.9371 \\
\end{align*}
\]

\[
\begin{align*}
\text{ft-lbs} &= 11.9371 \\
\end{align*}
\]
Table 4.1a. Wheel performance for Lunokhod-1 for $\delta_M = 112$ and $s_R = 0.80$ (lower bound)

**INPUT PARAMETERS**

<table>
<thead>
<tr>
<th>W,P,X(M),S,K</th>
<th>35.0000</th>
<th>0.0000</th>
<th>111.9949</th>
<th>0.0000</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA,GAMMA,PHI,C</td>
<td>0.0000</td>
<td>0.0000</td>
<td>57.9949</td>
<td>0.0000</td>
</tr>
<tr>
<td>R,P</td>
<td>10.0000</td>
<td>6.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***STRESS PARAMETERS***

| THETA(L) | 169.4939 |
| THETA(M) | 128.4864 |
| THETA(N) | 23.3997 |
| THETA(N) | -2.5815 |

| PM | 1.6500 |
| PL | 0.6440 |
| PL | 0.5367 |

NORMAL STRESSES AT L,M,N = 0.1446 0.6426 0.1379

TANGENTIAL STRESSES AT L,M,N = 0.0873 0.1087 0.1343

***GEOMETRIC PARAMETERS***

| CENTER OF INSTANTANEOUS ROTATION--- | XH = 0.0000 | LH = 0.0000 |
| TRAILING SPIRAL POLE--- | XP1 = -4.2959 |
| LEADING SPIRAL POLE--- | XP2 = -5.2219 |
| MIFICATION POINT--- | XM = -3.7960 |
| TRAILING EDGE--- | XL = -7.3514 |
| LEADING SPIRAL COORDINATES--- | XML = -14.8494 |
| TRAILING EDGE--- | XM = 5.4549 |
| TRAILING SPIRAL COORDINATES--- | XMN = 13.3810 |

| X(M) | 139.0378 |
| X(M) | 56.3384 |

***VERTICAL REACTION***

| WKPL | 4.6079 |
| WKGL | 1.8160 |
| HSKL = -20.7413 |
| TOTAL LEADING = -14.3173 |
| WKPT | 5.3207 |
| WKST = -29.5777 |
| TOTAL LEADING = -20.4144 |

***HORIZONTAL REACTION***

| HKPL | 17.6385 |
| HKLT | 12.3711 |
| TOTAL LEADING = 24.4046 |
| HKPT | -11.7633 |
| HKTT = -17.6443 |
| TOTAL LEADING = -29.4046 |

***MOMENTS***

| MGL | -3.1823 |
| MLS = 117.5277 |
| MPL = -200.4954 |
| TOTAL LEADING = -86.3134 |
| MGT | 10.9818 |
| MSR = -44.7006 |
| MRT = 166.9795 |
| TOTAL LEADING = 141.1207 |

TOTAL TORQUE = 55.1293 LB-INCHES = 7.6219 KG-METERS

FM,FT, = 0.1994 0.0000
Fig. 1. Typical failure pattern for driven rigid wheel on a pack of aluminum rolls

Fig. 2. Roller motion on sloping soil
Fig. 3. Driven rigid roller 100% slip ($v_k = 0$, $V_C = 0$) on a pack of aluminum rolls

Fig. 4. Soil-roller plastic flow configuration
Fig. 5. Soil-roller rim-velocity boundary conditions (bifurcation of plastic zones)
Fig. 6. Limiting stress state
Fig. 7. Limit position of leading pole ($v_k = 0$)
Fig. 8. Locus of leading and trailing spiral poles
Fig. 9. Locus of trailing poles $\vec{I}_1$ for varying $\xi_M$ and $s_K$
Fig. 10. Soil-roller interface velocities
Fig. 11. Soil-roller free body equilibrium
Fig. 12. Stresses along sliplines (active zones)
Fig. 13. Limiting slipline directions at leading and trailing zones

Fig. 14. Roller sinkage