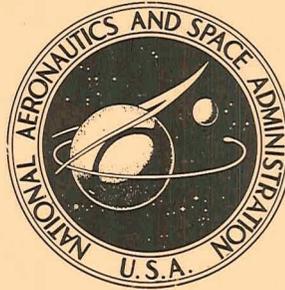


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FIXED-ANGLE TRANSLUNAR
GUIDANCE PROCEDURES USING
ONBOARD OPTICAL MEASUREMENTS

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FIXED-ANGLE TRANSLUNAR GUIDANCE PROCEDURES USING ONBOARD OPTICAL MEASUREMENTS

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SUMMARY

Onboard procedures requiring only a few optical measurements and simple calculations have been developed for midcourse and approach guidance for translunar trajectories. The midcourse-guidance procedure is based on an optical range measurement to earth. The approach-guidance procedure requires a star-to-body measurement and may require a range measurement, depending on the distance to the moon. This procedure is developed whereby either one or two approach-guidance maneuvers are applied, depending on the accuracy desired as well as the distance to the moon. In both the midcourse and approach procedures, the direction of the velocity correction is predetermined – hence the expression "fixed-angle guidance."

Inasmuch as the range is the critical measurement for the midcourse guidance, a method was established for updating range information from measurements made close to the earth where the accuracy is greatest. This technique triples the accuracy of the guidance measurement at the time of the midcourse maneuver.

An error analysis with several one-sigma magnitudes assumed for midcourse measurement error showed that the onboard procedures were adequate for simplified control of translunar trajectories. The analysis showed that perilune radius can be controlled to a one-sigma accuracy of about 30 km. The approximations made for the midcourse procedure are the dominant error source in controlling perilune radius; the effects of measurement error and maneuvering error are also discussed. For the bulk of the error analysis, perilune was selected as the midcourse aim point. It was determined that accuracy of perilune radius could be improved by guiding to an aim point at the lunar sphere of influence and incorporating a second midcourse correction.

INTRODUCTION

At present, manned lunar missions are planned to terminate in 1972. It seems inevitable, however, that future generations will conceive manned missions to study and exploit the moon on an ever increasing scale. Whenever such intricate missions are flown, problems can develop in guidance and control of the spacecraft, such as failure in

the ground-based radar or loss of communications. Hence, it is desirable to have emergency onboard guidance procedures capable of guiding the spacecraft safely to its destination. Extremely simple procedures are presented in this paper for application to earth-moon trajectories. It is possible that the procedures could be adapted to control the trajectory completely from translunar injection to perilune.

The results reported herein are based on trajectory data and procedures developed in references 1 and 2. In reference 1 an onboard midcourse procedure was devised which determines the magnitude of the guidance velocity as well as its three-component direction, by measuring several different star-to-body angles. Reference 2 developed an approach-guidance procedure which can be applied within the lunar sphere of influence to correct the errors incurred at midcourse.

The midcourse procedure described herein requires only a range measurement to predict the midcourse-guidance correction. "Fixed-angle guidance" signifies that the velocity-correction vector is applied in the same inertial direction for all perturbed trajectories. The approach procedure is essentially the same as that of reference 2, except provision is made for incorporating a second approach-guidance correction. As in the case of the midcourse guidance, the direction of the approach velocity-correction vector is inertially fixed for all trajectories.

The accuracy characteristics of the method are examined by means of a Monte Carlo error analysis. The analysis includes the effects of measurement error, velocity-cutoff error, and approximation error caused by assumptions and simplifications made in developing the procedures. The results were obtained by use of the Jet Propulsion Laboratory n-body trajectory program (see ref. 3).

SYMBOLS

D position deviation in direction of specified star

ΔD increment in D at a given time

$$\delta D = K D_1 - D_2$$

\bar{h} vector perpendicular to instantaneous earth-moon-vehicle plane, $\bar{r} \times \bar{r}_l$

K desired ratio of D_2 to D_1

r range to earth center (geocentric distance)

- r_l range to moon center (selenocentric distance)
 r_p perilune radius
 Δr incremental geocentric range, $r_a - r_n$
 Δr_p incremental perilune radius, $r_{p,a} - r_{p,n}$
 s position deviation from nominal trajectory, $(\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}$
 T time from translunar injection
 T_p time to nominal perilune time
 T_{pf} time of midcourse position fix
 u velocity deviation from nominal trajectory, $(\Delta \dot{x}^2 + \Delta \dot{y}^2 + \Delta \dot{z}^2)^{1/2}$
 V geocentric velocity
 V_l selenocentric velocity
 V_p perilune velocity
 ΔV velocity correction (that is, "guidance velocity")
 ΔV_{add} additional approach-guidance velocity to account for second midcourse maneuver
 x,y,z position coordinates in Cartesian axis system in which X-axis is toward the vernal equinox, XY-plane is parallel to earth equatorial plane, and Z-axis is in direction of north celestial pole
 $\Delta x, \Delta y, \Delta z$ off-nominal position component in direction of X-, Y-, and Z-axis, respectively – for example, $\Delta x = x_a - x_n$

$\overline{\Delta x}$ the vector $\begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix}$

- $\dot{x}, \dot{y}, \dot{z}$ velocity coordinates in Cartesian axis system
- $\Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z}$ off-nominal velocity component in direction of X-, Y-, and Z-axis, respectively – for example, $\Delta\dot{x} = \dot{x}_a - \dot{x}_n$
- $\overline{\Delta\dot{x}}$ the vector $\begin{Bmatrix} \Delta\dot{x} \\ \Delta\dot{y} \\ \Delta\dot{z} \end{Bmatrix}$
- β in-plane midcourse-guidance angle (fig. 2(a))
- γ flight-path angle
- δ out-of-plane midcourse-guidance angle (fig. 2(b))
- ϵ eccentricity of orbit
- θ included angle between star and moon center
- λ guidance pointing angle (angle between \overline{V}_l and approach $\overline{\Delta V}$)
- μ product of universal gravitational constant and mass of moon
- σ standard deviation or root-mean-square (rms) error
- σ_s standard deviation or rms value of s
- σ_u standard deviation or rms value of u
- $[\Phi]$ state-transition matrix between times T_F and T_S
- $[\Phi_1], [\Phi_2], [\Phi_3], [\Phi_4]$ 3×3 submatrices in state-transition matrix
- ψ in-plane angle between $\overline{\Delta V}_S$ and \overline{V}_n

Subscripts:

- a actual value
- D position deviation

F	first midcourse maneuver
m	measured value
n	nominal value
r	range to earth center
r,mc	range to earth center at time of midcourse position fix
r,p	perilune radius
S	second midcourse maneuver
T	time from translunar injection
T,F	time of first midcourse maneuver
T,S	time of second midcourse maneuver (aim-point time)
T,pf	time of midcourse position fix
ΔV	guidance velocity
α	semisubtended angle of earth (used for range measurement)
θ	star-to-body angle
1,2	first and second approach-guidance correction times

Notation:

$||$ absolute value

A bar over a symbol denotes a vector.

BASIC METHOD

Synopsis

The guidance procedures presented herein are designed to accomplish two tasks: first, a midcourse-guidance procedure is applied to correct the trajectory to a point on the nominal trajectory near the moon; second, the errors incurred at midcourse are corrected by a lunar approach-guidance procedure to control the perilune distance. Both the midcourse method and the approach method rely heavily on precomputed data on the nominal trajectory as well as on characteristics of trajectories randomly perturbed about the nominal. This preflight computation results in rapid and simple onboard determination of guidance requirements. The n-body trajectory program (ref. 3) is used throughout the paper in developing the procedures and in performing the error analyses.

Midcourse guidance.- In the present midcourse-guidance procedure, the only measurement required for determining the midcourse correction is range. The inertial direction of the midcourse velocity correction is the same for all perturbed trajectories considered - hence the expression "fixed-angle guidance." Fixed-angle guidance is an outgrowth of the onboard midcourse-guidance procedure developed in reference 1. A midcourse maneuver is required to correct the trajectory in order to remove perturbations due to errors attributed to injection and other sources. A fixed-time-of-arrival law is used for the guidance equations. When a given point within the lunar sphere of influence is used as the aim point, the first midcourse maneuver corrects only for the position error at this point; a second midcourse maneuver is normally required at the aim point to correct the spacecraft-velocity vector back to the nominal vector. The present analysis makes use of the nominal and perturbed trajectories of reference 1; the first midcourse maneuver is simulated at 10 hours from injection (about one-third the distance to the moon for a 70-hour translunar trajectory). The guidance measurement is simply a range determination and is performed one-half hour before the maneuver. For much of the analysis, the nominal perilune is selected as the aim point and a second midcourse correction is not included.

Approach guidance.- In the approach-guidance procedure, most of the calculations are preflight calculations. As described in reference 2, the guidance velocity requirements are developed from two-body relationships, wherein a closed-form expression relates perilune and upstream conditions. The upstream conditions can be related to deviations from the nominal trajectory, and these deviations can be determined by simple onboard optical angular measurements. At relatively large distances from the moon, only a single star-to-body measurement is required; close to the moon an additional subtended-angle measurement is required to determine the range.

Results obtained from Monte Carlo samples of trajectories perturbed at translunar injection, as well as at first midcourse, are used to show that within the lunar sphere of influence, the position deviation in a certain direction predicts the perilune radius and perilune velocity with relatively high accuracy. The variation of the deviation with the guidance velocity required to correct the perilune distance is then derived. From this precalculated variation the navigator determines the approach-guidance correction. The only onboard calculation required is the simple computation of the deviation by using optical measurements and their nominal values. In the procedure, one or two maneuvers may be required, depending on the desired accuracy and the time selected for initiating the approach guidance.

As in the case of the midcourse guidance, the direction of the approach-guidance velocity vector is inertially fixed. For most distances from the moon, this selected direction is essentially optimum with regard to the fuel requirement.

Errors considered in guidance procedures.- All major error sources were investigated. Random perturbations after first midcourse were assumed to be caused by onboard measurement error; the effect of maneuvering errors (velocity cutoff and pointing direction) was found negligible. Also considered were errors due to the method of approximating the magnitude and direction of the first midcourse ΔV from the range measurement. Only measurement error was considered for the second midcourse maneuver inasmuch as this maneuver is derived from the first-midcourse-maneuver measurements. No other errors were considered for this maneuver because it can be combined with the approach-guidance maneuver. In the approach guidance, the following types of errors were considered: measurement errors, velocity-cutoff errors, and approximation errors associated with the procedure. The effect of error in the pointing direction was found negligible. (See ref. 2.)

Thrust Assumptions

In the guidance procedure, the thrust is considered to be impulsive; that is, the burning time is negligible relative to the trajectory time scale. Each impulsive velocity correction is assumed to be applied in a constant predetermined direction at initiation of the thrust maneuver. The approach velocity correction is in the nominal selenocentric orbital plane and is perpendicular to the nominal velocity vector. Except for the effect of velocity-cutoff error, the guidance correction is assumed to be perfectly executed. These assumptions are all appropriate inasmuch as their effect on the overall results is negligible.

Midcourse-Guidance Procedure

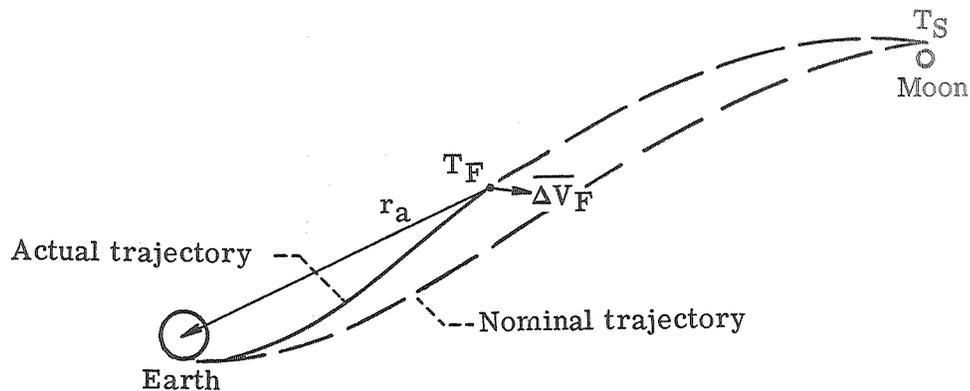
General considerations.- The results in figures 1 and 2 pertain to trajectories requiring approximately 70 hours to reach the moon (ref. 1) and are representative of many types of translunar trajectories. Each data point signifies a trajectory which was perturbed at injection. The injection errors were essentially spherically distributed, having 1σ values of approximately 3 km in position and 3 m/sec in velocity. These perturbations are larger than would normally be incurred (ref. 4) and yield relatively large values for Δr . The data in figures 1 and 2 correspond to the midcourse-guidance equations of reference 1 in which the magnitude and direction of ΔV at $T = 10$ hours were determined by onboard measurements to three stars at $T = 9.5$ hours. The aim point was selected at nominal perilune. As is subsequently discussed, this choice of aim point leads to some error because it eliminates the second midcourse maneuver.

The fixed-angle procedure is based on the phenomenon that perturbed trajectories generally yield a strong correlation between first midcourse ΔV and range deviation Δr , as shown in figure 1. (Because of lower injection velocity at the moon, this correlation does not exist for moon-to-earth trajectories; hence, the fixed-angle procedure would not apply.) Further, the required direction of the velocity correction does not vary greatly for the perturbed trajectories, as shown in figure 2. This figure gives the exact in-plane and out-of-plane angles of the guidance velocity vector required for different trajectories of reference 1. The angles are shown with respect to the nominal velocity vector of the spacecraft. The relatively small dispersions about the average, especially for the more highly perturbed trajectories (large Δr), suggest the use of one inertial direction for correcting all trajectories. Justification for this method is given in figure 3. Each data point represents a perturbed trajectory which has been corrected by a fixed-angle midcourse maneuver and for which no measurement or maneuver-execution errors were assumed. The large errors in perilune radius are essentially predictable by D and hence correctable by approach guidance. The quantity D is the deviation in a certain direction from the nominal trajectory and is determined from onboard measurements, as discussed in a subsequent section. Results are shown in figures 3(a) and 3(b) for two directions chosen for the midcourse velocity-correction vector. In figure 3(a) the direction corresponds to the value noted in figure 2 as that principally used for the analysis. For figure 3(b) another direction was arbitrarily selected for comparison. The 1σ value of the scatter of the data is indicative of the final accuracy. The small scatter error in relation to the total error, together with the fact that the 1σ values are essentially the same in both plots, justifies the use of a compromise guidance pointing angle λ . Using the compromise pointing angle with respect to \bar{V} means that the midcourse ΔV is always applied in a fixed inertial direction. This direction, as well as the variation of ΔV with Δr , can be determined from a preflight analysis.

Limitations.- In order to apply the fixed-angle guidance technique, good correlation between Δr and ΔV is required. As shown in figure 4, the magnitude of injection error affects the correlation between Δr and ΔV . The 1σ values for position error and velocity error are approximately 26 km and 5 m/sec, respectively. The position error at injection is the critical error because it can lead to the condition at midcourse where $\Delta r \neq s$. Good correlation is obtained only when the position-error vector and the range vector at midcourse lie in the same general direction; that is, $\Delta r \approx s$. Examination of the data in figure 4 indicates that good correlation between Δr and ΔV is obtained when $s < 10$ km; the correlation is marginal when $10 \text{ km} < s < 20 \text{ km}$ and is unacceptable when $20 \text{ km} < s < 69.3 \text{ km}$. Reference 4 indicates that for normal operation, the injection errors are well below the unacceptable range. The data in figure 4 correspond to an error analysis performed on a 90-hour translunar trajectory for which a variable-time-of-arrival law was used; results for fixed-time-of-arrival guidance are similar.

Midcourse-guidance equations.- The guidance equations used to calculate the data of figures 1 and 2 were derived from those of reference 1, which employed onboard measurements to determine the three-component trajectory deviations. In the equations which follow, the measurements are replaced by the deviations obtained directly from precomputed trajectory data.

The equations are developed according to the following sketch:



The deviations from the nominal trajectory at the aim point (time T_S) after a midcourse correction at T_F are

$$\begin{Bmatrix} \overline{\Delta x_{T,S}} \\ \overline{\Delta \dot{x}_{T,S}} \end{Bmatrix} = \begin{bmatrix} \overline{\Phi_1} & \overline{\Phi_2} \\ \overline{\Phi_3} & \overline{\Phi_4} \end{bmatrix} \begin{Bmatrix} \overline{\Delta x_{T,F}} \\ \overline{\Delta \dot{x}'_{T,F}} \end{Bmatrix} \quad (1)$$

where the transition matrix maps from time T_F to time T_S and where the prime denotes the velocity deviation immediately after the guidance maneuver. Since, for a

fixed-time-of-arrival guidance law, the objective is to arrive at the aim point on the nominal trajectory without regard for the final velocity deviation $\{\overline{\Delta\dot{x}_{T,S}}\}$, then

$$\{\overline{\Delta x_{T,S}}\} = [\Phi_1]\{\overline{\Delta x_{T,F}}\} + [\Phi_2]\{\overline{\Delta\dot{x}_{T,F}}\} = \{0\}$$

and

$$\{\overline{\Delta\dot{x}_{T,F}}\} = -[\Phi_2]^{-1}[\Phi_1]\{\overline{\Delta x_{T,F}}\} \quad (2)$$

Next, the velocity deviation immediately before the instantaneous guidance maneuver $\{\overline{\Delta\dot{x}_{T,F}}\}$ is obtained directly from precomputed perturbed-trajectory data. The required first-midcourse-maneuver velocity-correction vector, obtained by subtracting equation (2) from $\{\overline{\Delta\dot{x}_{T,F}}\}$, is therefore

$$\overline{\Delta V}_F = \{\overline{\Delta\dot{x}_{T,F}}\} + [\Phi_2]^{-1}[\Phi_1]\{\overline{\Delta x_{T,F}}\}$$

In deriving the equations for the first midcourse maneuver, no provision is made for controlling the velocity vector at the aim point. Hence, a second midcourse maneuver must correct the velocity error induced at the aim point by the derivation of the first midcourse correction. From equation (1), the velocity deviation at the aim point is

$$\{\overline{\Delta\dot{x}_{T,S}}\} = [\Phi_3]\{\overline{\Delta x_{T,F}}\} + [\Phi_4]\{\overline{\Delta\dot{x}_{T,F}}\}$$

Substituting from equation (2) gives the required second-midcourse-maneuver velocity-correction vector as

$$\overline{\Delta V}_S = \{\overline{\Delta\dot{x}_{T,S}}\} = [\Phi_3]\{\overline{\Delta x_{T,F}}\} - [\Phi_4][\Phi_2]^{-1}[\Phi_1]\{\overline{\Delta x_{T,F}}\}$$

By use of the inversion property of the transition matrix (ref. 5) which is

$$[\Phi]^{-1} = \begin{bmatrix} [\Phi_4]^T & -[\Phi_2]^T \\ -[\Phi_3]^T & [\Phi_1]^T \end{bmatrix}$$

it can be shown that the expression for $\overline{\Delta V}_S$ reduces to

$$\overline{\Delta V}_S = -[\Phi_2]^{-1}[\Phi_1]^T\{\overline{\Delta x_{T,F}}\}$$

It should be emphasized that these equations are not required onboard the spacecraft; they are used for preflight analysis on a number of perturbed trajectories to determine the variation of ΔV_F (and ΔV_S) with Δr .

Range determination.- A summary of the accuracy characteristics of existing methods for determining range from onboard measurements (ref. 1) is presented in figure 5. It is apparent that at T_{pf} (that is, at $T = 9.5$ hours), the time of the midcourse measurement, a minimum error of about 40 km is obtained with the method which uses the one-star measurement. (It is shown subsequently that range-determination errors of this magnitude can be tolerated in the fixed-angle midcourse procedure.) The method based on the one-star measurement constrains the star to lie within 10 or 20 arc seconds of the nominal instantaneous earth-moon-vehicle plane. (See ref. 1.) The two-star method is more practical in that the locations of the stars are not as limited. For this method, however, the range-determination error is essentially doubled.

Examination of figure 5 shows that at the time of the position fix ($T = 9.5$ hours) σ_r may vary from 40 to 100 km, depending on the method used. These values can be substantially reduced as shown in figures 6 and 7. The characteristics of perturbed trajectories permit the range deviation from the nominal value at a given time to be predicted from a range measurement at an earlier time. This prediction can be made from data such as are presented in figure 6, which shows the average factor by which the range deviation at T_{pf} increases from that at any given prior time. Figure 7 shows the 1σ dispersion of the ratio $\Delta r_{T,pf}/\Delta r_T$, which is an indication of the accuracy of the range prediction. As an example, a subtended-angle measurement for range at $T = 4$ hours, where this type of measurement is relatively accurate (fig. 5), gives a range-determination error σ_r of 29 km. Propagating information on the range to $T = 9.5$ hours (fig. 6) results in a prediction error of 6 percent (fig. 7) leading to a value of $\sigma_{r,mc}$ of 33 km. This value (33 km) was calculated from

$$\sigma_{r,mc} = \left[(29)^2 + (0.06 \times 250)^2 \right]^{1/2}$$

where 250 km is the average magnitude of Δr at $T = 4$ hours for the perturbed trajectories considered herein.

In order to apply the fixed-angle midcourse-guidance procedure, the Δr values of perturbed trajectories should be distinguishable above the known range-measurement noise level. If preflight analysis of the perturbed-trajectory characteristics indicates that this condition generally does not exist at the desired time for the midcourse correction, two alternatives are suggested:

- (1) Omit the midcourse maneuver and apply only the approach-guidance procedure near the moon. (This procedure would be adequate when only small perturbations are expected.)
- (2) Select the time of the first midcourse maneuver when the Δr magnitude has increased to a measurable quantity. (Figure 8 shows the increase in Δr relative to

its value at $T = 9.5$ hours for 70-hour translunar trajectories. Note that the magnitude of Δr doubles after 6 hours and redoubles after 16 hours.)

Approach-Guidance Procedure

General considerations.- Because of the approximations used, the fixed-angle midcourse procedure leads to relatively large errors at the aim point. An example of the aim-point errors for the midcourse procedure of reference 1 is shown in figure 9. By the method of reference 1, no approximations are made in the magnitude or direction of ΔV ; the guidance error is caused only by measurement error. The data in figure 9 are shown for $\sigma_{r,mc} = 22$ km; this error at midcourse has the dominant effect on the aim-point accuracy. Aim points were considered along the nominal trajectory from entrance into the lunar sphere of influence to perilune.

For the fixed-angle procedure, the aim-point errors are roughly three times as large as those shown in figure 9. The approach-guidance procedure of reference 2, however, can be applied to correct these errors, but two approach maneuvers may be required.

In reference 2, it was shown that for maximum accuracy the aim point must be chosen at or before the approach-guidance measurement time and the second midcourse correction must be taken into account. The second midcourse correction, however, is relatively unimportant in the fixed-angle midcourse procedure. Except where otherwise stated, results presented in this report pertain to the aim point chosen at perilune, with the second midcourse correction omitted.

The approach-guidance procedure is illustrated in figure 10. The method normally employs a range measurement to the moon and an angular measurement to a preselected star. Error analyses (ref. 2) have shown that the star must lie near the orbital plane (say, within $\pm 30^\circ$) and approximately 90° from the nominal range vector at the lunar sphere of influence. This requirement is essential even if the measurements for the approach-guidance maneuver are made within several hours of reaching perilune, where the nominal range vector has rotated 10° or more. The measurements determine the quantity D , which in turn is used to predict the magnitude of the approach ΔV required to correct perilune radius. The direction of ΔV is taken perpendicular to the nominal velocity vector, because this direction is near optimum regardless of the distance from the moon.

For approach-guidance measurements at or near the lunar sphere of influence,

$$\theta_n \approx 90^\circ$$

in which case the effect of error in range is negligible and

$$D \approx r_{l,n}(\cos \theta_m - \cos \theta_n)$$

Hence, a range measurement is not required.

Number of maneuvers required.- Data in figures 11 to 14 illustrate effects of using one or two approach-guidance maneuvers. Shown for various conditions are the variations of r_p , V_p , and ΔV with deviation D . As previously noted, each data point represents a perturbed trajectory corrected by a fixed-angle midcourse maneuver. The variation of ΔV with D is the essential information required on board the spacecraft. The variation of r_p with D is shown inasmuch as the scatter of these data is a good measure of the guidance accuracy. Data on V_p and r_p are required in the preflight calculations for ΔV . (The scatter in V_p does not affect the guidance accuracy appreciably.)

The equation for approach ΔV , derived in reference 2, is

$$\Delta V = \frac{V_l \left[r_l^2 \cos \gamma \cos(\gamma + \lambda) - r_{p,n}^2 \cos \lambda \right]}{r_{p,n}^2 - r_l^2 \cos^2(\gamma + \lambda)}$$

$$\pm \frac{r_{p,n} \left\{ \left(r_l^2 - r_{p,n}^2 \right) V_l^2 \sin^2 \lambda + \left[r_l^2 \cos^2(\gamma + \lambda) - r_{p,n}^2 \right] \left(\frac{2\mu}{r_{p,n}} - \frac{2\mu}{r_l} \right) \right\}^{1/2}}{r_{p,n}^2 - r_l^2 \cos^2(\gamma + \lambda)}$$

where

$$\cos \gamma = \frac{r_p V_p}{r_l V_l}$$

The values derived with the alternate signs of the second term in the equation correspond to correcting to either side of the moon. The lesser magnitude of ΔV would ordinarily be chosen to assure posigrade trajectory motion. The relatively smooth ΔV data in figures 11(c), 12(c), and 13(c) were determined from the faired curves for r_p and V_p .

Ordinarily, an approach-guidance maneuver should be made at the lunar sphere of influence in order to take advantage of the low fuel requirements. A second approach-guidance maneuver may be required because of the inability of the first maneuver to keep perilune-radius error $\sigma_{r,p}$ below about 56 km. This 56-km error is equivalent to the 1σ dispersion of the data shown in figure 11(a) and represents the effect of midcourse approximation error only, because zero midcourse measurement error is assumed. The faired line in figure 11(a) represents the value of perilune radius, predicted from approach-guidance measurements at the sphere of influence, which is used in the calculation of the approach ΔV . Hence the distance from each point to the line is an indicator of the error in controlling perilune radius for the corresponding trajectory. In figure 12

the data include midcourse measurement errors, with $\sigma_{r,mc} = 44$ km. Comparison of figure 12(a) with figure 11(a) indicates only a small effect of $\sigma_{r,mc}$ on scatter.

If the approach-guidance measurements are postponed to a time closer to the moon, only one maneuver is necessary because the effect of approximation error decreases as time to the moon decreases. (See fig. 13.) However, the fuel requirements (that is, velocity corrections ΔV) increase sharply as illustrated by figure 15. Note also in figure 15 that the optimum guidance pointing angle λ , with regard to fuel required, is essentially 90° . It can be seen that close to the moon ($T_p = 2.617$ hours), the optimum value of λ is about 80° ; however, the change in the value of ΔV from that at $\lambda = 90^\circ$ is negligible. Furthermore, the nearly constant values shown for ΔV at the larger values of λ signify that extremely large errors in the pointing direction can be tolerated.

The perilune-radius error $\sigma_{r,p}$ illustrated in figure 11(a) can be halved by the use of a small second approach-guidance correction at a time near the moon. This correction is determined by assuming that for corrected trajectories which will yield the desired r_p , the deviation D_2 bears a constant ratio to D_1 ; if this ratio is not constant, the desired r_p will not be attained. Therefore, a second approach ΔV is required. The desired ratio (K) was determined as follows: trajectories represented by the data points in figures 11(a) and 12(a) which are on (or very near) the faired line were assumed to be on course for the desired r_p after the first maneuver. The ratio of D_2 to D_1 for each of these trajectories was calculated by using the same measurement star; the average value of this ratio (K) for each set of data was used to calculate the second approach ΔV . The average values for the two sets (figs. 11(a) and 12(a)) are shown in figure 14 to be 0.29 and 0.4.

Data are shown in figure 14(a) for no error in the midcourse range measurements and no error in implementing the first approach-guidance maneuver; the data of figure 14(b) incorporate errors in both. The scatter of the data points in figure 14 gives an index to the perilune-radius error from the two approach-guidance maneuvers. As in the case for the first approach-guidance maneuver (figs. 11 and 12), the effect of midcourse range-measurement error is seen to be small. Also, the velocity-cutoff error in the first approach maneuver is negligible. The error in r_p was determined from

$$\sigma_{r,p} = \frac{\partial r_p}{\partial \Delta V} \sigma_{\Delta V}$$

where $\sigma_{\Delta V}$ is the one-sigma dispersion of the data points and $\frac{\partial r_p}{\partial \Delta V} \approx 17.4$, as determined from cross plotting data such as those in figures 13(a) and 13(c). The ratio $\frac{\partial r_p}{\partial \Delta V}$ can also be determined from nominal values by the equation (ref. 2)

$$\frac{\partial r_p}{\partial \Delta V} = \frac{(r_l^2 - r_p^2)(1 + \epsilon)}{r_l V_p \epsilon} \quad (3)$$

Combining second midcourse maneuver with approach maneuver. - The foregoing figures correspond to the case in which the aim point for the midcourse guidance is selected at nominal perilune and no second midcourse correction is made. Analysis of trajectories for which the aim point (time of second midcourse maneuver) is selected at the lunar sphere of influence has shown that the accuracy of the approach-guidance procedure can be improved. The second midcourse maneuver can be conveniently combined with the approach-guidance maneuver, inasmuch as ΔV_S is approximately linear with $\Delta r_{T,pf}$ and is in the same general direction for any injection error, as shown in figures 16 and 17, respectively. As in figures 1 and 2, each test-point symbol represents a different perturbed trajectory due to injection error. It is of interest to note that the difference between the first-midcourse-maneuver velocity requirements for the two aim points is only about 2 percent. (See figs. 1 and 16.) Figure 16 shows that ΔV_S can be predicted by $\Delta r_{T,pf}$, the range measurement at the time of the midcourse position fix. In figure 17 the precise angles of the second midcourse-guidance correction vector are shown. The angles are essentially in the orbital plane of the spacecraft. Note that the directions do not differ greatly, especially at the larger values of $\Delta r_{T,pf}$ where the magnitude of $\overline{\Delta V}_S$ is significant. The dispersions in the out-of-plane direction are even less than those shown in figure 17.

The linear results of figure 18 were determined by applying to each of the perturbed trajectories the faired values of ΔV_S from figure 16 at a constant angle ψ of 15° . This value of the angle ψ was selected near the average angle corresponding to the higher values of $\Delta r_{T,pf}$. (See fig. 17.) Figure 18, in effect, shows the change in perilune radius due to the second midcourse correction $\overline{\Delta V}_S$. Even though ΔV_S would ordinarily be applied at angles ranging from about 10° to 35° (fig. 17), the small amount of scatter shown in figure 18 indicates that the perilune radius can be effectively corrected when ΔV_S is applied at a constant angle.

It follows that $\overline{\Delta V}_S$ can be converted to an equivalent vector in a direction perpendicular to \overline{V}_n with small loss in accuracy and that its magnitude can then be added to the approach ΔV magnitude. For this purpose, the following equation is used:

$$\frac{\Delta V_{add}}{\Delta r_{T,pf}} = \left(\frac{\Delta r_p}{\Delta r_{T,pf}} \right) \left(\frac{\partial \Delta V}{\partial r_p} \right)$$

where $\Delta r_p / \Delta r_{T,pf}$ is obtained from figure 18 and $\partial \Delta V / \partial r_p$ from equation (3). In this case,

$$\Delta V_{add} = (-0.065)(0.0200)\Delta r_{T,pf}$$

It should be emphasized that if ΔV_G is applied in this manner, a separate maneuver prior to the approach-guidance correction is eliminated.

GUIDANCE ACCURACY CHARACTERISTICS

In this section the errors associated with the guidance procedures are defined and analyzed and their effect on the accuracy of controlling perilune radius is determined. In general, the analysis spans the region for performing the approach-guidance correction from near the lunar sphere of influence to within several hours of perilune passage. For comparison with those of the present procedure, figure 19 presents accuracy characteristics of the onboard methods of references 1 and 2. These results are based on exact midcourse-guidance pointing angles.

The present perilune-radius accuracy characteristics and guidance velocity requirements are summarized in table I for both the one-maneuver and two-maneuver approach-guidance procedures. Results are shown for single maneuvers at $T_p = 14.6$ hours and $T_p = 4.6$ hours. Results which include a second midcourse correction are given in parentheses and the aim point is at the lunar sphere of influence. For the other results in this table, the aim point is at perilune.

The subject method includes no provision for controlling the position or the velocity of the spacecraft at perilune. The inherent errors in these quantities are presented in table II.

Most of the error shown in tables I and II is contributed by the approximation made in using a faired midcourse ΔV magnitude applied in a fixed direction. The effect of approach measurement error is omitted, as figure 20 indicates that this error is negligible. In table I the effect of velocity-cutoff error (fig. 21), though small, is included in maneuvers made at $T_p = 14.6$ hours. This effect is negligible in table II.

The rms approximation error was determined both by the method described in connection with figure 14 and from a Monte Carlo analysis wherein for each perturbed trajectory the approach $\overline{\Delta V}$ predicted by D was added to the corresponding \overline{V} and the trajectory propagated to perilune. The two methods produced similar results.

The following facts are evident from the results presented in table I:

- (1) The application of one approach maneuver only, near the lunar sphere of influence ($T_p = 14.6$ hours), leads to relatively high inaccuracy in the perilune radius.
- (2) The application of one approach maneuver near the moon ($T_p = 4.6$ hours) or two approach maneuvers ($T_p = 14.6$ hours and 4.6 hours) gives equal accuracy in r_p , but the latter requires half as much fuel.
- (3) Although the fixed-angle method is crude, the inaccuracy is only twice that of the more precise procedure (fig. 19).

(4) Comparison of the data for the three values of $\sigma_{r,mc}$ in table I shows that midcourse range-measurement error has little effect on the perilune-radius accuracy and fuel requirements.

(5) Most of the results pertain to selecting the midcourse aim point at perilune and omitting the second midcourse correction. Table I shows, however, that guiding to an aim point at the lunar sphere of influence ($T_p = 14.6$ hours) and applying the second midcourse correction there produces accuracy comparable to that resulting from the use of two approach maneuvers. As an example, for $\sigma_{r,mc} = 22$ km, the perilune-radius rms error decreased from 58 km to 36 km, with no increase in the approach ΔV requirement. (The effect of including ΔV_S is negligible when the aim point is close to the moon or when two approach maneuvers are used.)

CONCLUDING REMARKS

Midcourse and approach fixed-angle guidance procedures have been developed which require only a few optical measurements and simple calculations. The inertially fixed direction of the velocity-correction vector is determined through preflight analysis. These procedures are capable of controlling perilune radius to a one-sigma accuracy of about 30 km.

The midcourse-guidance method corrects translunar trajectories provided there is a measurable deviation in range at the time of the intended maneuver. If the range deviation is below the onboard measurement noise level at this time, the midcourse correction can be delayed until the range deviation increases to a measurable value. Because the midcourse procedure relies solely on the range measurement, a simple method has been devised for updating range information from measurements made close to the earth where the accuracy is greatest.

The approach-guidance procedure can be applied either with or without inclusion of a second midcourse maneuver. An error analysis showed, however, that the perilune-radius accuracy is improved by using a second midcourse correction, which can be conveniently combined with the approach-guidance correction, with no increase in fuel requirements. As an example, the guidance accuracy for one approach-guidance correction from the lunar sphere of influence with the added second midcourse correction was found comparable to that resulting from the use of two approach-guidance corrections.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., August 12, 1971.

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TABLE I.- CHARACTERISTICS OF FIXED-ANGLE GUIDANCE

Approach maneuver(s) at -	One-sigma error in controlling r_p , km			Average approach $ \Delta V $, m/sec		
	$\sigma_{r,mc} = 0$ km	$\sigma_{r,mc} = 22$ km	$\sigma_{r,mc} = 44$ km	$\sigma_{r,mc} = 0$ km	$\sigma_{r,mc} = 22$ km	$\sigma_{r,mc} = 44$ km
$T_p = 14.6$ hr	56 b(26)	^a 58 a,b(36)	^a 67 a,b(57)	4.3 b(4.3)	5.3 b(5.3)	7.1 b(7.5)
$T_p = 4.6$ hr	25	27	32	13.4	16.2	22.1
$T_p = 14.6$ hr and 4.6 hr	22	^a 28	^a 30	6.9	8.1	10.1

^aIncludes effect of one-sigma velocity-cutoff error of 0.2 m/sec.

^bIncludes second midcourse correction.

TABLE II.- FIXED-ANGLE-GUIDANCE rms ERRORS AT PERILUNE

$[\sigma_{r,mc} = 22 \text{ km}]$

Parameter	One approach maneuver at $T_p = 14.6$ hr (a)	Two approach maneuvers at $T_p = 14.6$ and 4.6 hr (b)
s, km	110	106
V_p , m/sec	6.7	7.8

^aAim point at lunar sphere of influence; ΔV_S included with approach maneuver.

^bAim point at perilune; hence, no ΔV_S included.

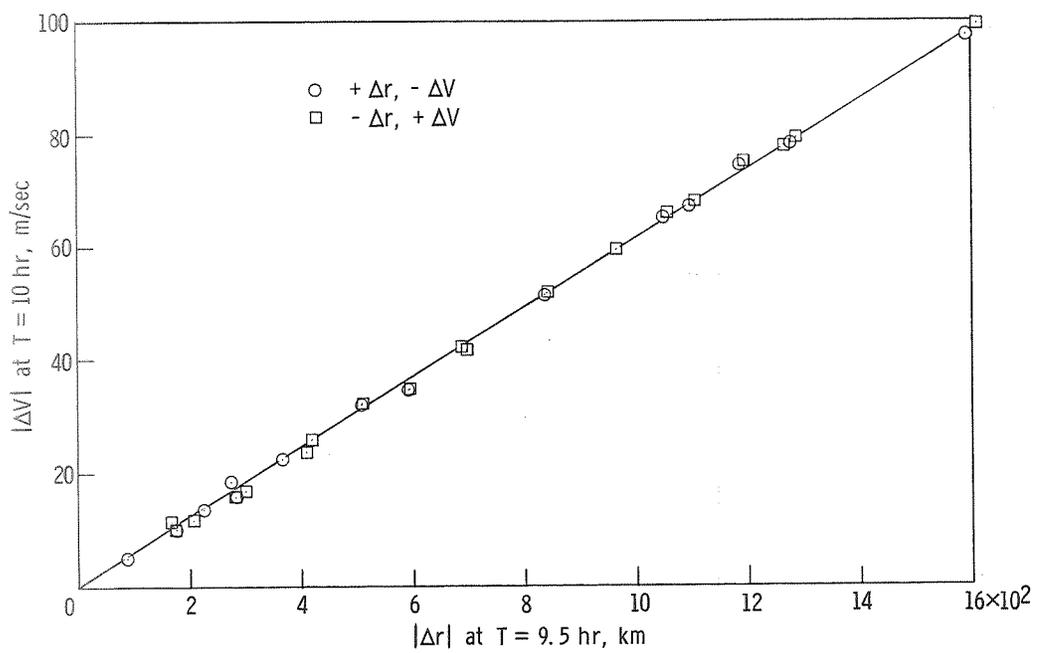
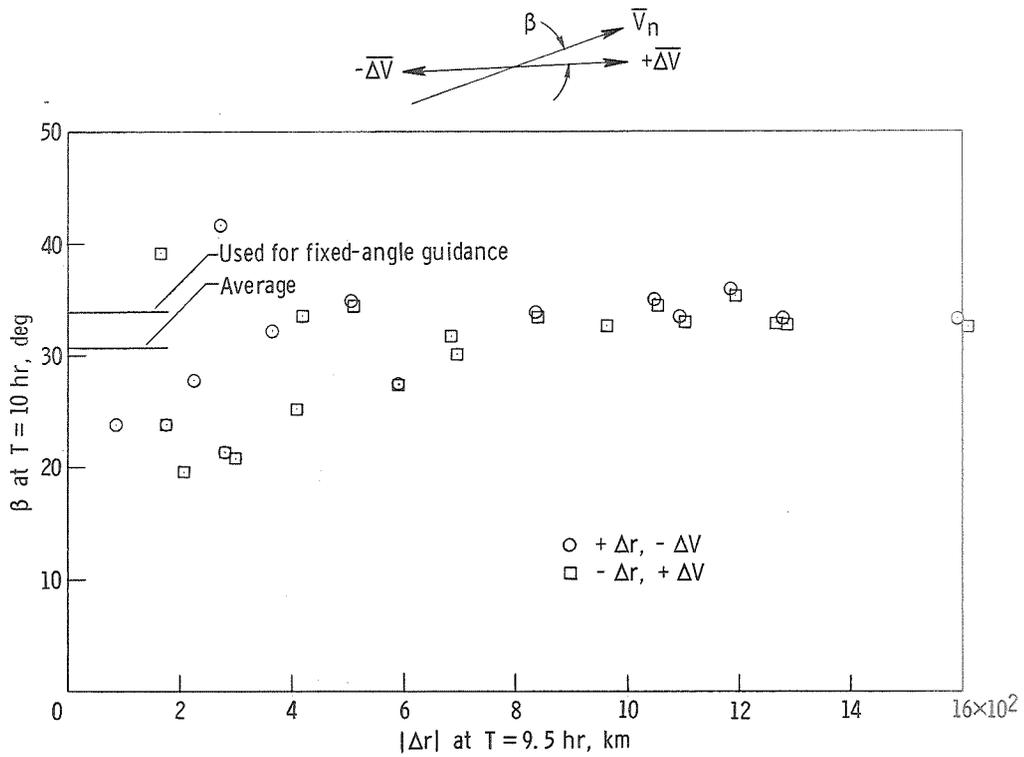
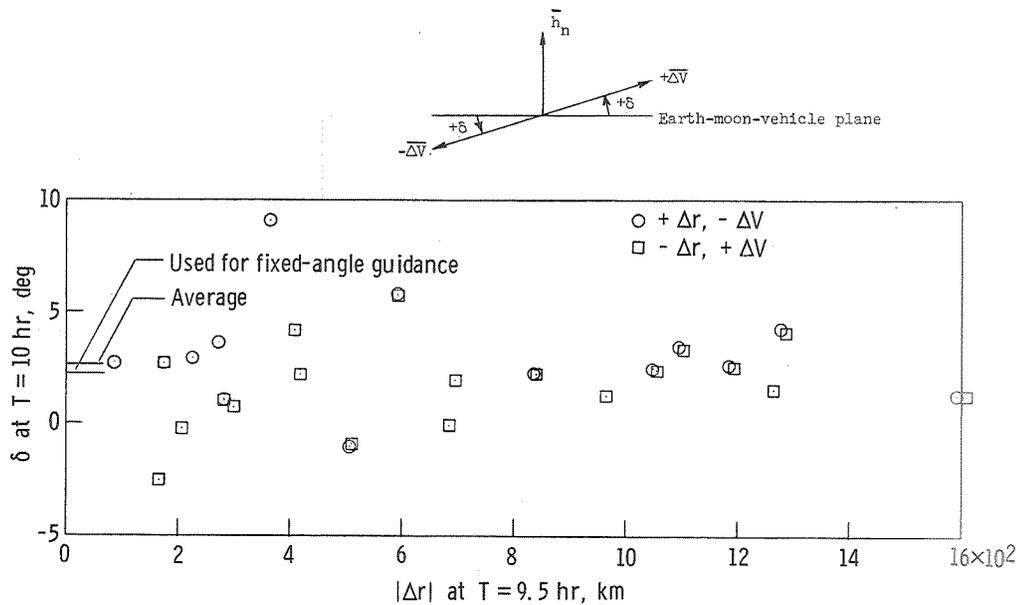


Figure 1.- First-midcourse-maneuver velocity requirement as a function of range deviation. (Aim point selected at perilune.)

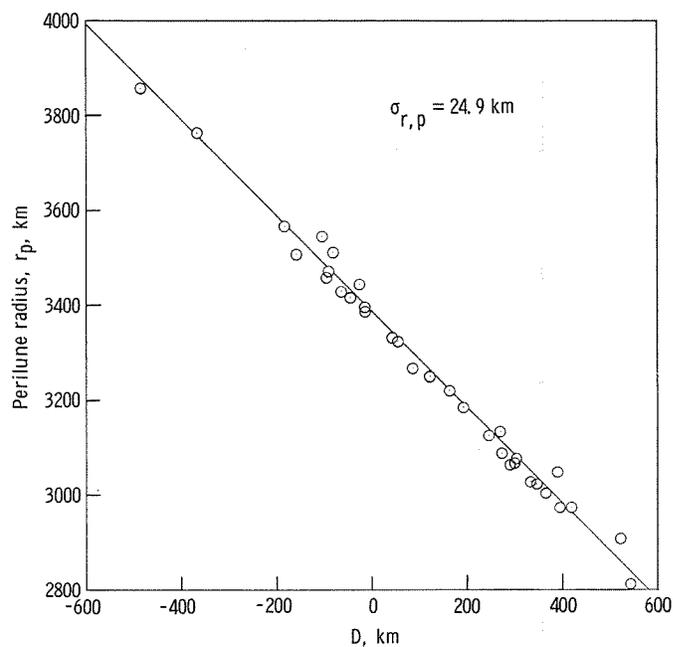


(a) In-plane angle.

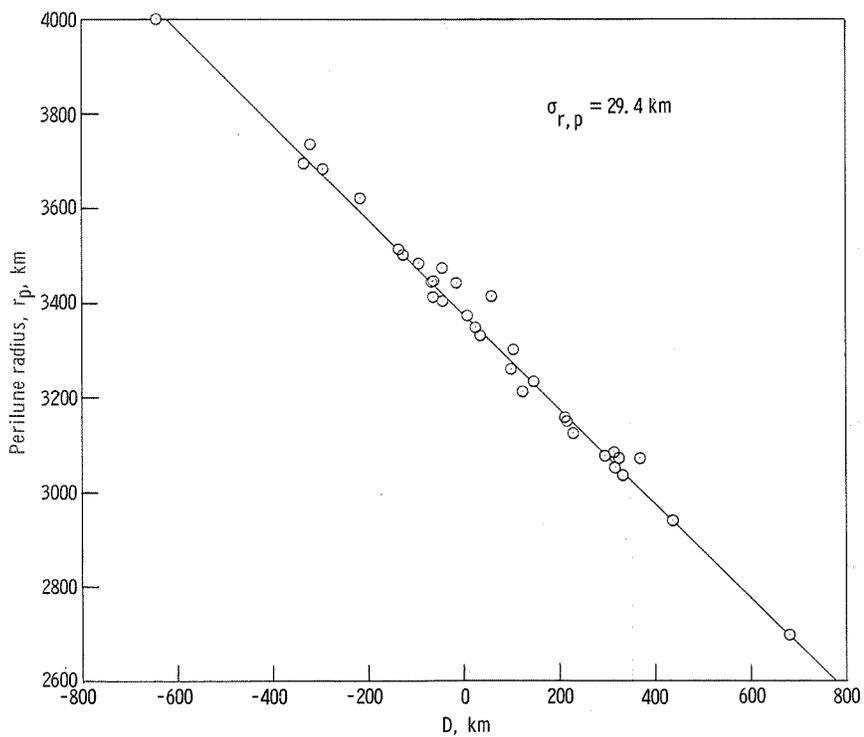


(b) Out-of-plane angle.

Figure 2.- Variation of direction of first-midcourse-maneuver velocity-correction vector with range deviation.



(a) $\beta = 33.809^\circ$; $\delta = 2.232^\circ$.



(b) $\beta = 32.464^\circ$; $\delta = 1.207^\circ$.

Figure 3.- Capability of predicting perilune radius from approach-guidance measurements made at $T_p = 4.6$ hours. $\sigma_{r,mc} = 0$.

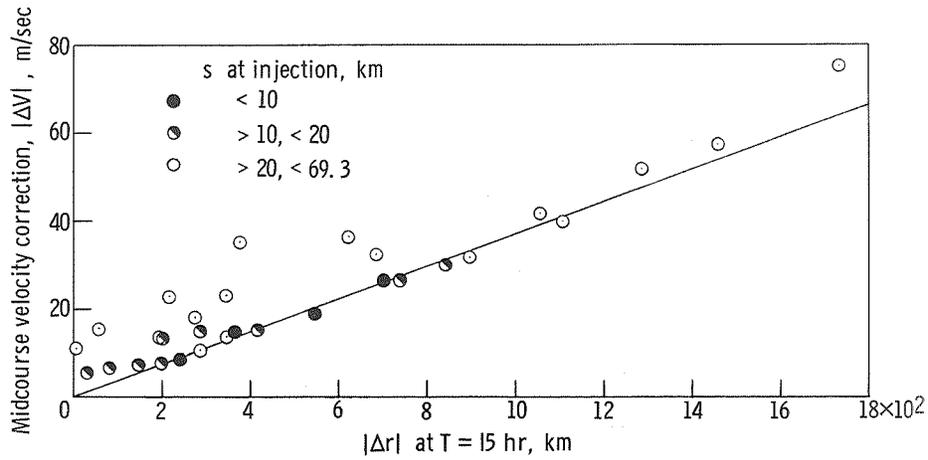


Figure 4.- Example of poor correlation between ΔV and Δr due to excessive position error at injection.

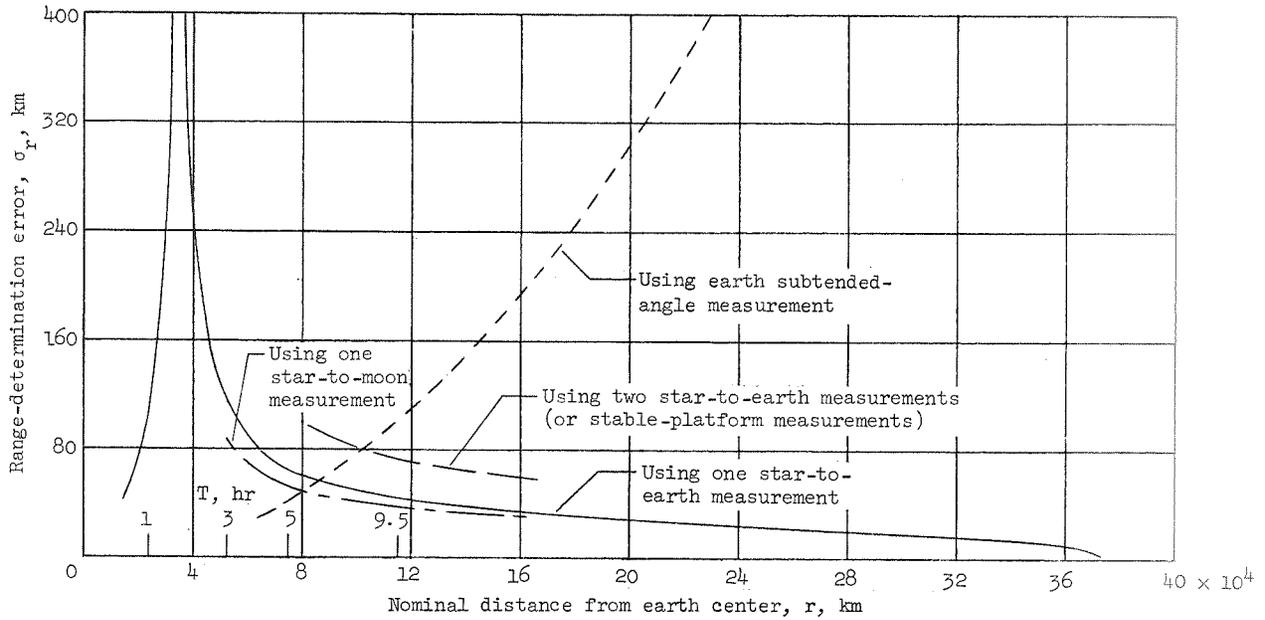


Figure 5.- Accuracy characteristics of various range-determination methods. (One-sigma angular-measurement error is 10 arc seconds; methods involving star-to-body measurements also include angular measurement between earth and moon.)

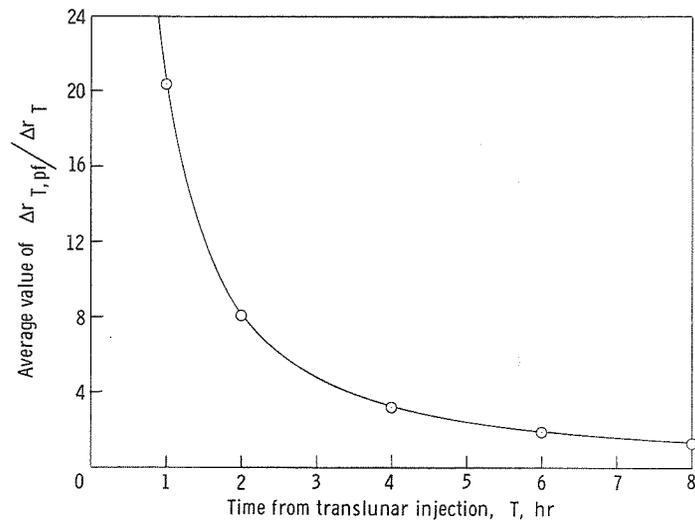


Figure 6.- Range deviation at T_{pf} (that is, at $T = 9.5$ hours) as a function of range deviation at any prior time.

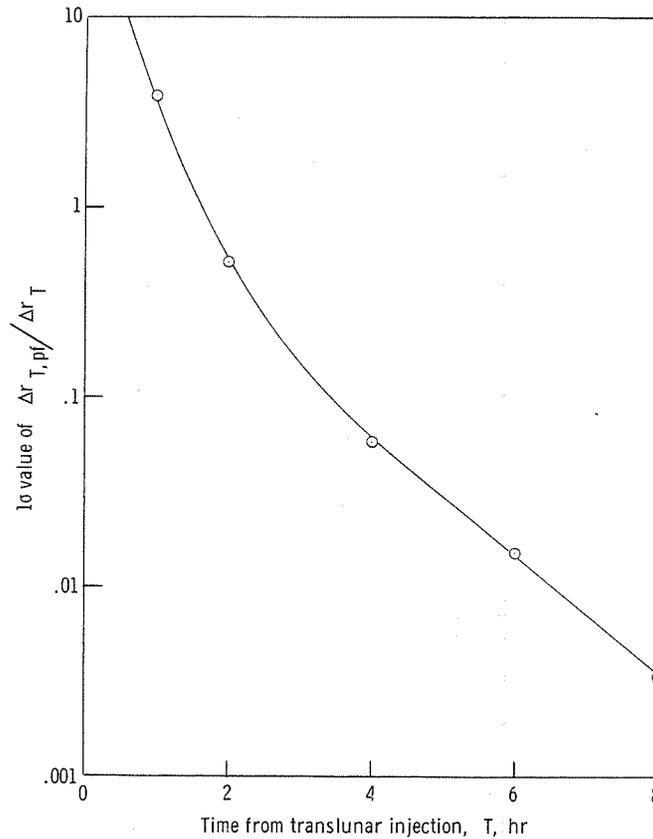


Figure 7.- One-sigma error in approximating range deviation at T_{pf} (that is, at $T = 9.5$ hours) from measurement of range deviation at any prior time.

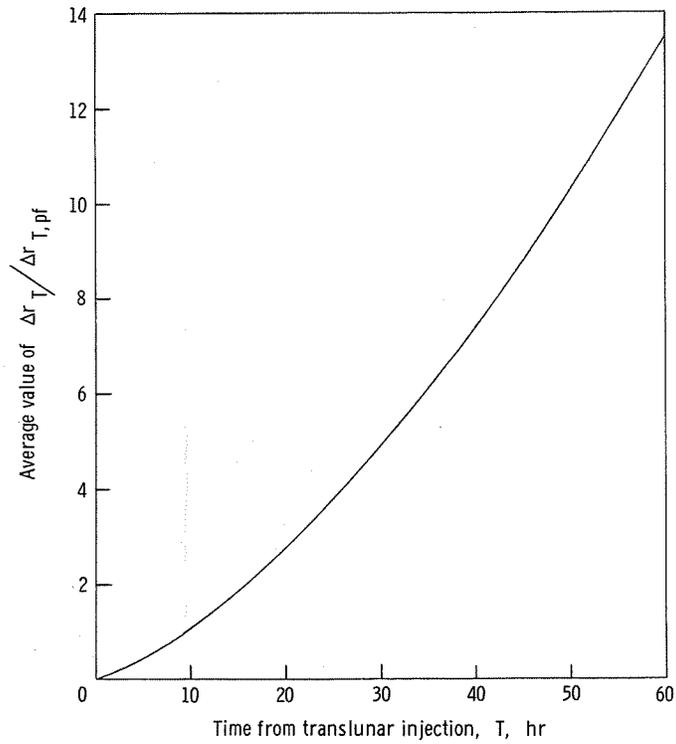


Figure 8.- Range deviation at any given time as a function of range deviation at T_{pf} (that is, at $T = 9.5$ hours).

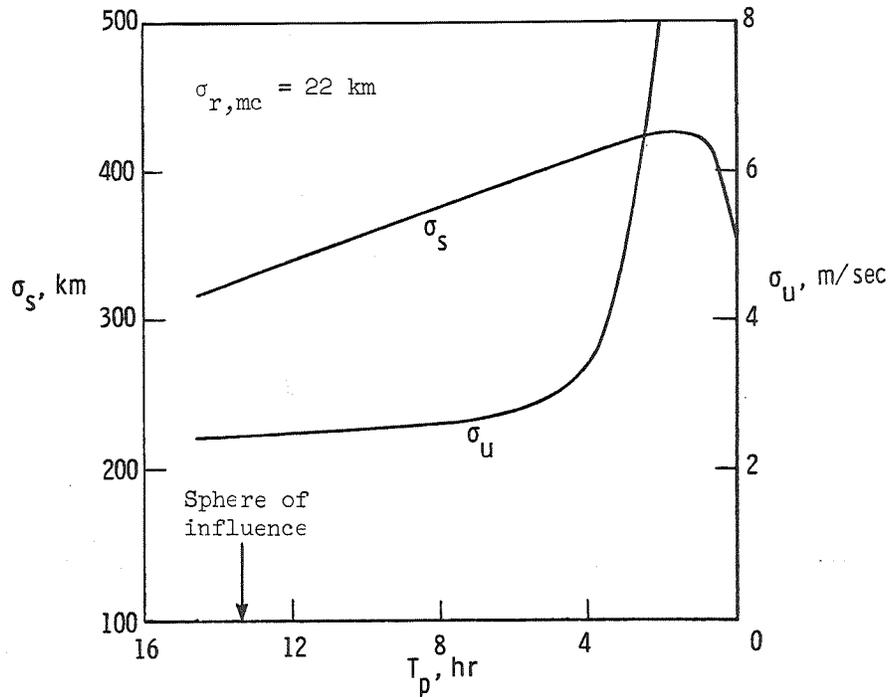
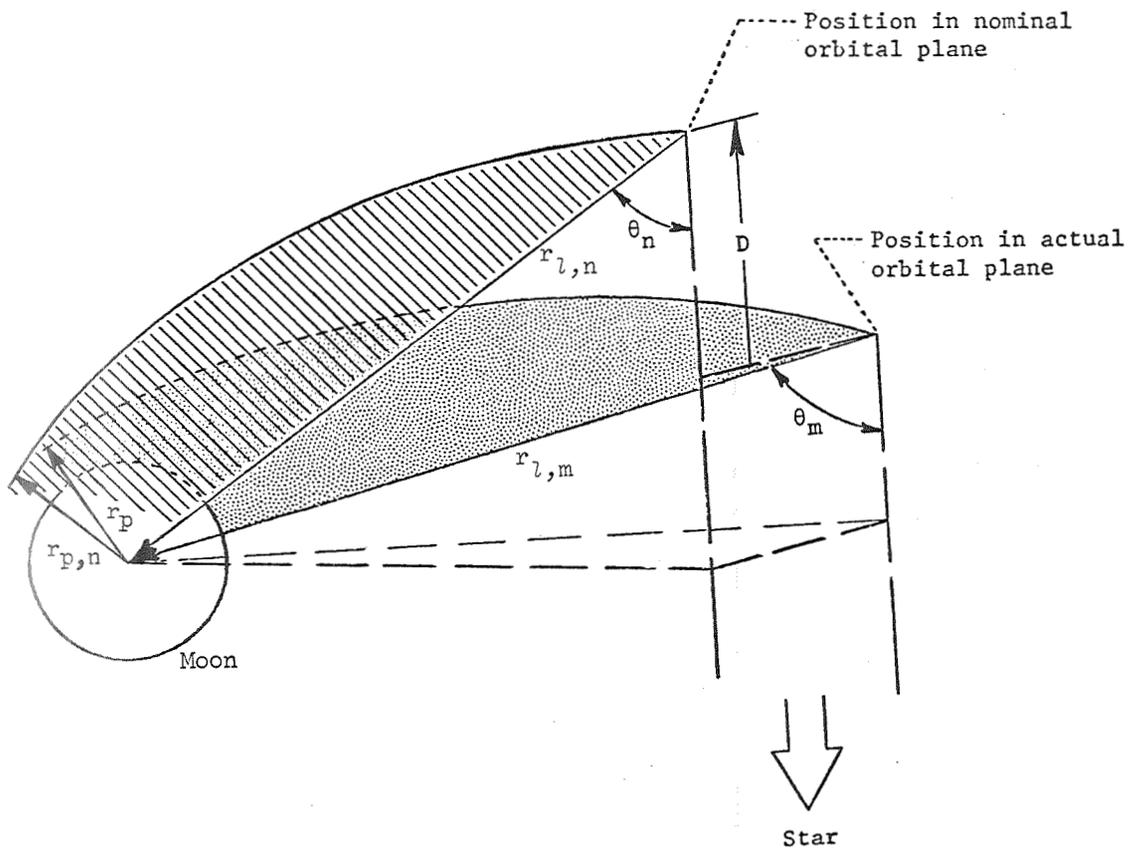
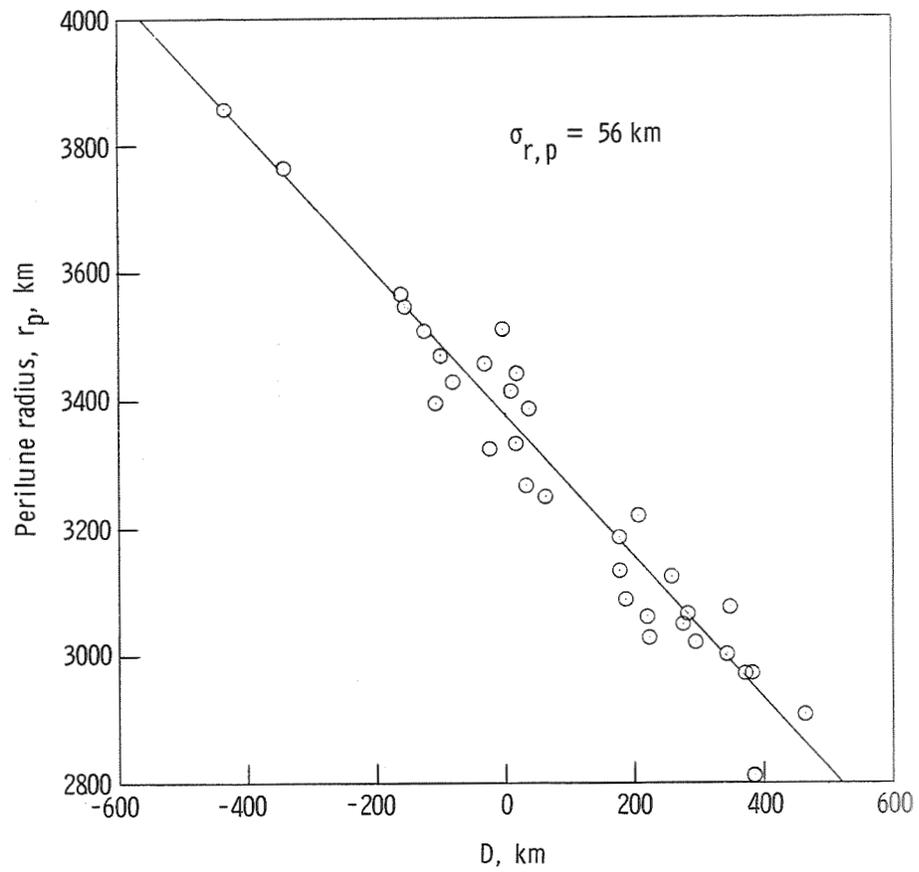


Figure 9.- Aim-point errors resulting from midcourse range-measurement error for method of reference 1.



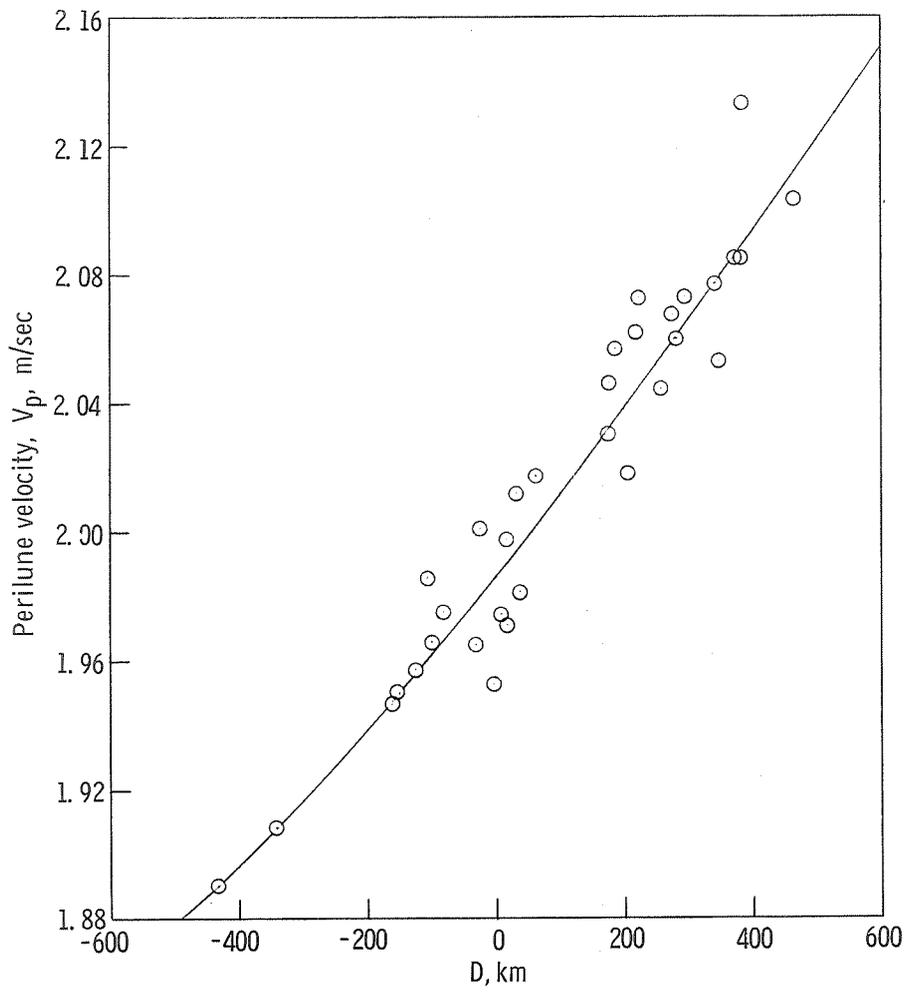
$$D = r_{l,m} \cos \theta_m - r_{l,n} \cos \theta_n$$

Figure 10.- Approach-guidance geometry.



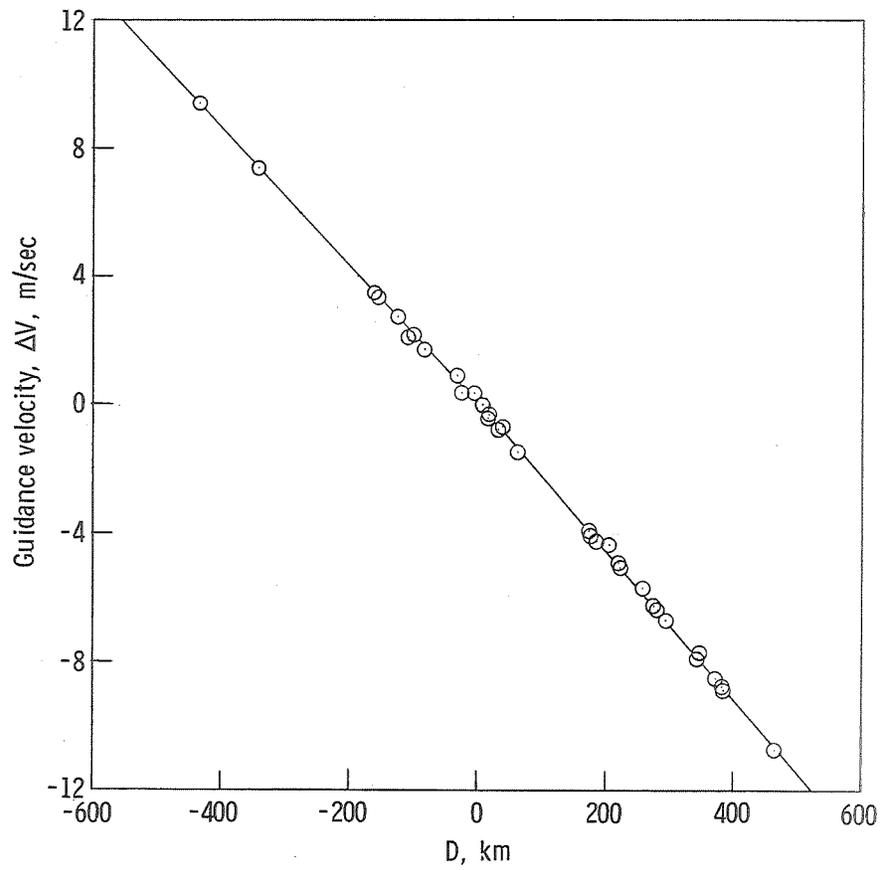
(a) Perilune-radius variation.

Figure 11.- Variation of approach-trajectory characteristics with deviation D at lunar sphere of influence ($T_p = 14.6$ hours). $\sigma_{r,mc} = 0$.



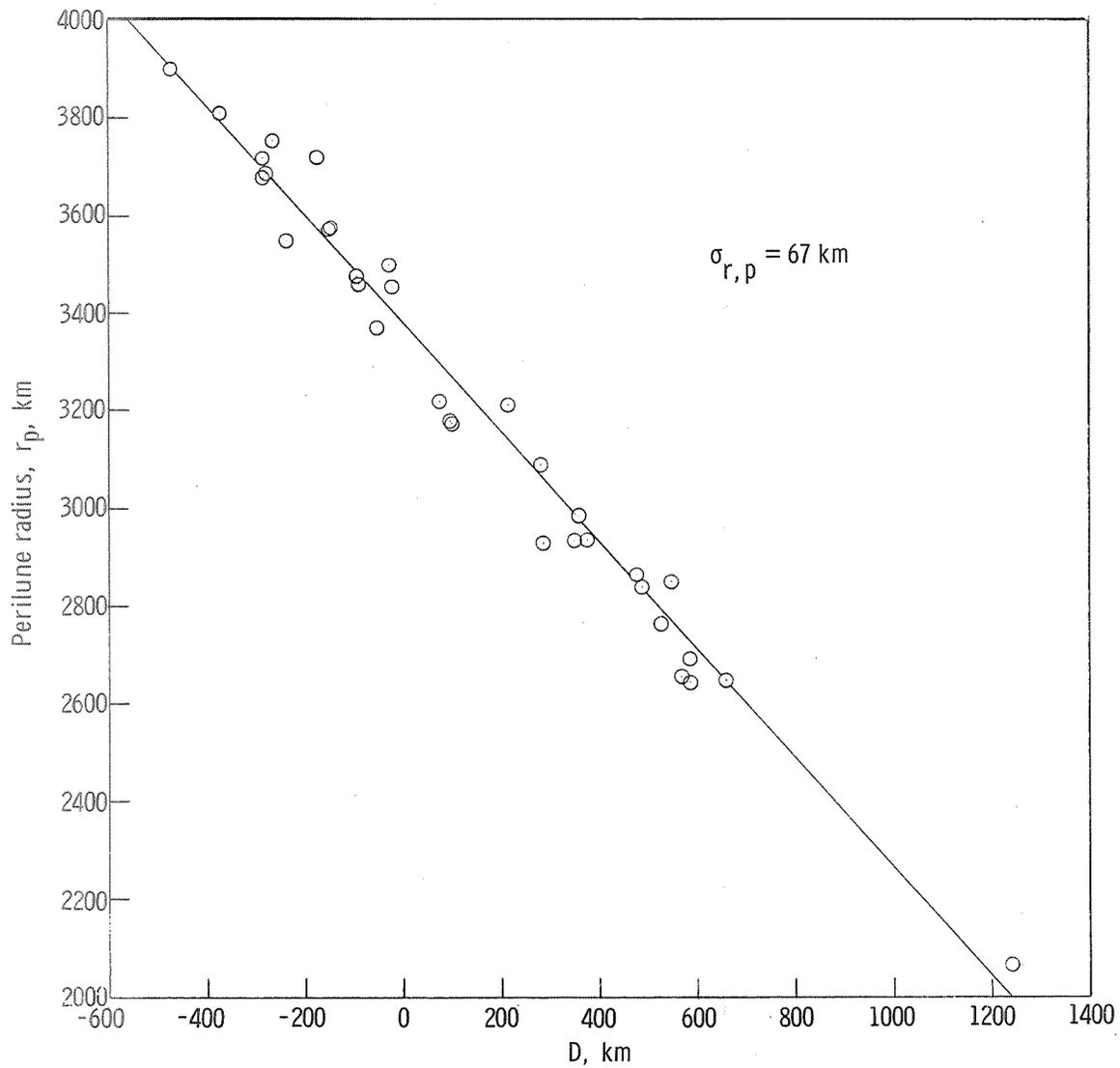
(b) Perilune-velocity variation.

Figure 11.- Continued.



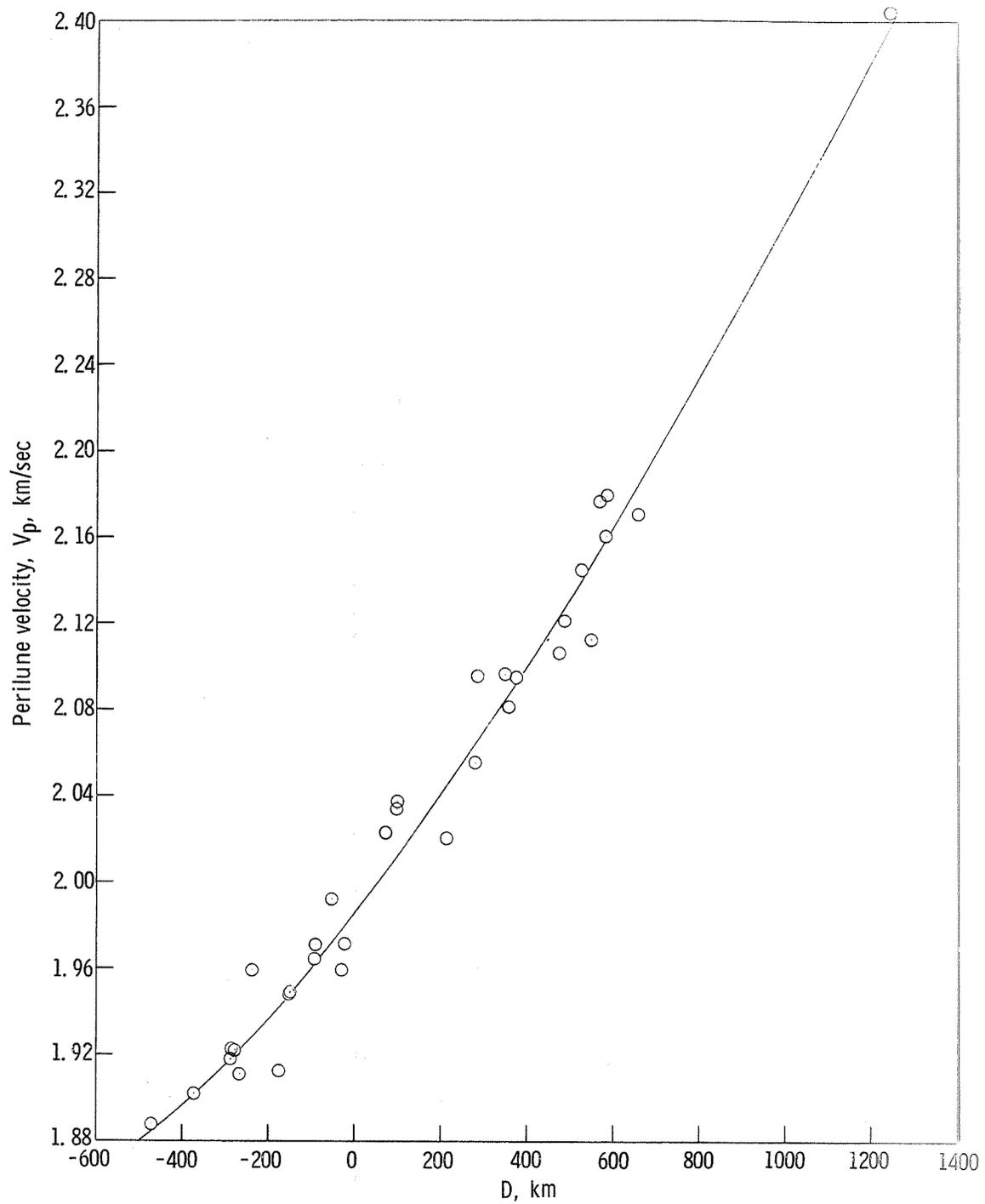
(c) Variation of approach-guidance velocity.

Figure 11.- Concluded.



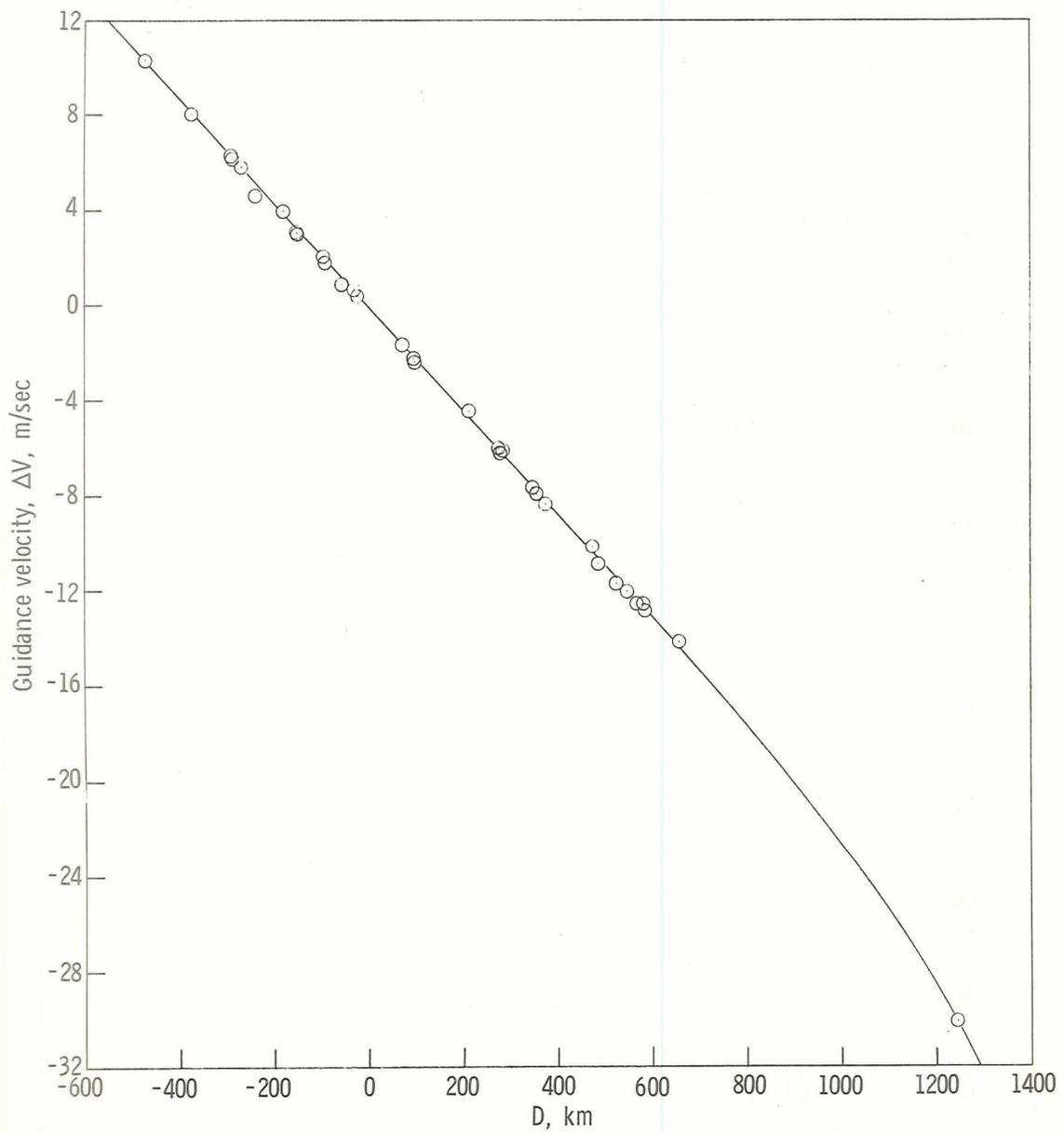
(a) Perilune-radius variation.

Figure 12.- Variation of approach-trajectory characteristics with deviation D at lunar sphere of influence ($T_p = 14.6$ hours). $\sigma_{r,mc} = 44$ km.



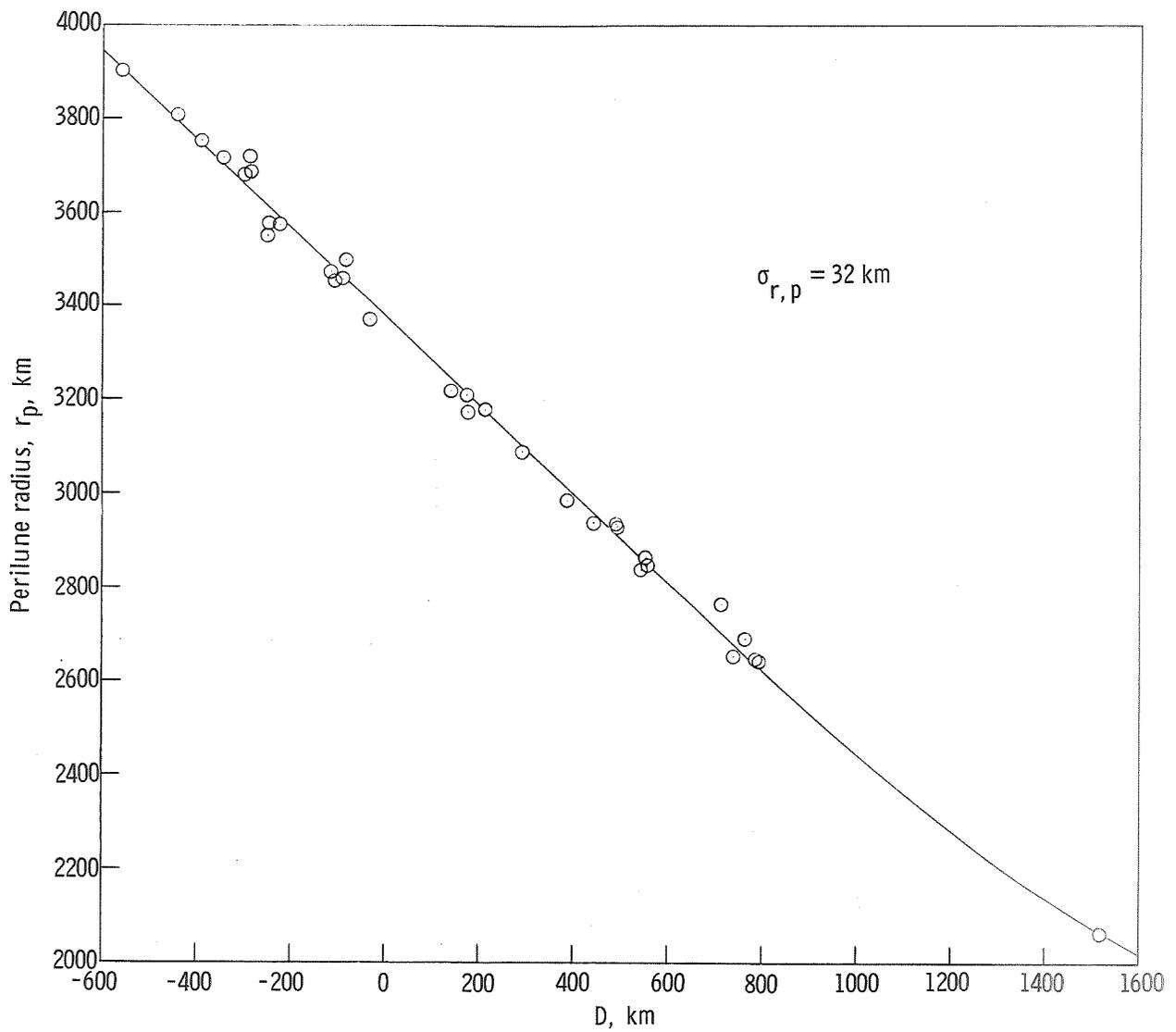
(b) Perilune-velocity variation.

Figure 12.- Continued.



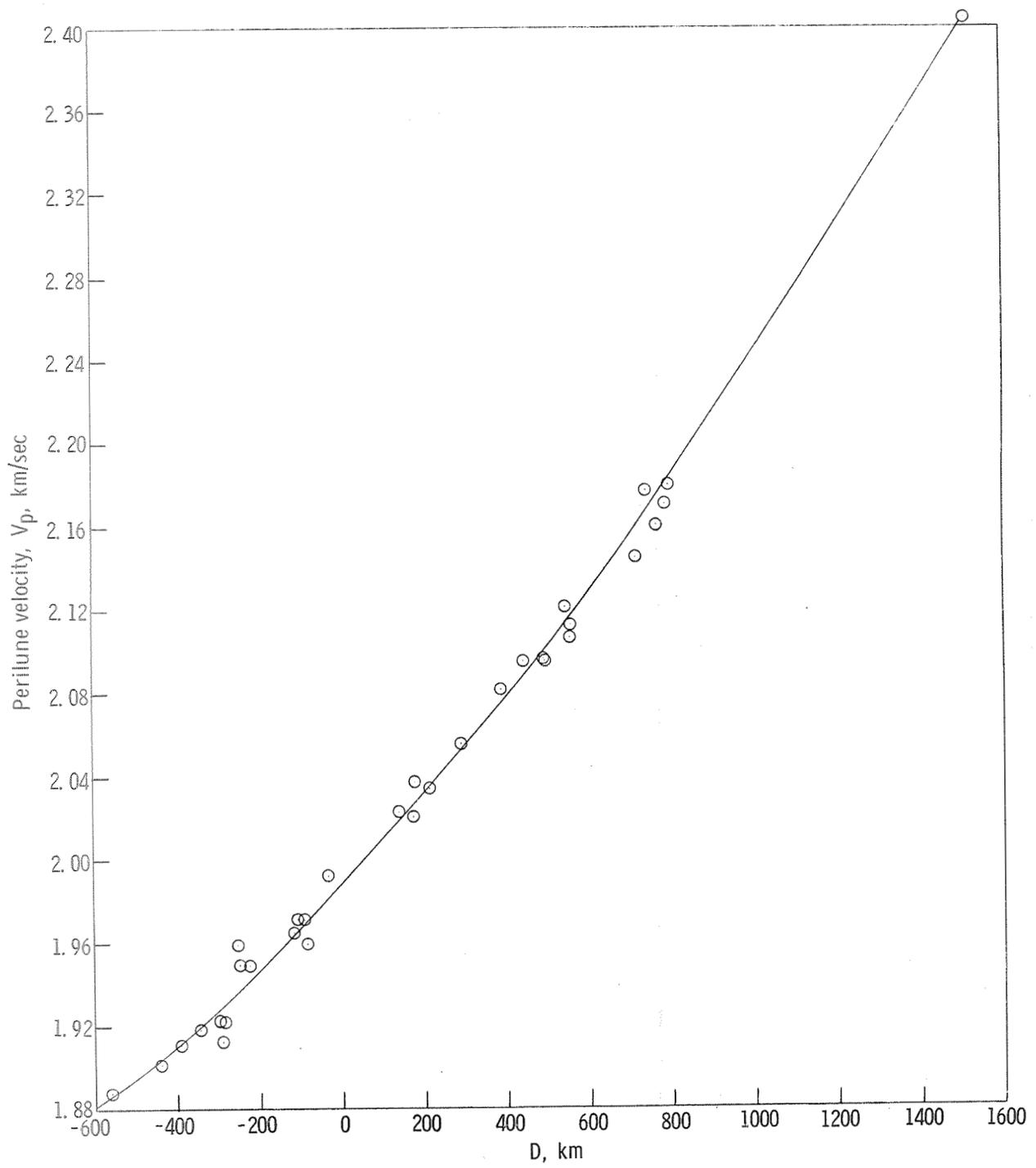
(c) Variation of approach-guidance velocity.

Figure 12.- Concluded.



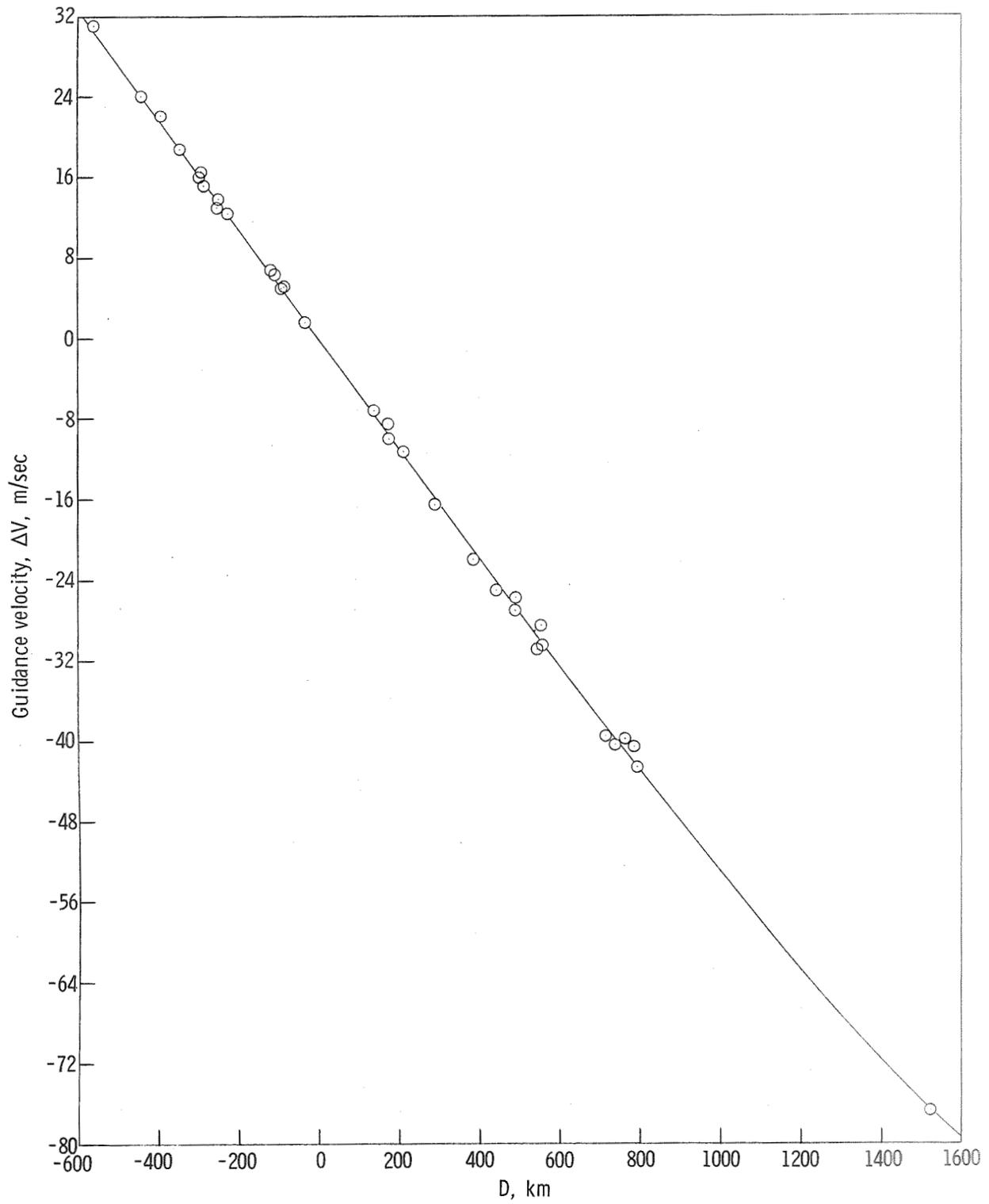
(a) Perilune-radius variation.

Figure 13.- Variation of approach-trajectory characteristics with deviation D at $T_p = 4.6$ hours. $\sigma_{r,mc} = 44$ km; no prior approach-guidance maneuver.



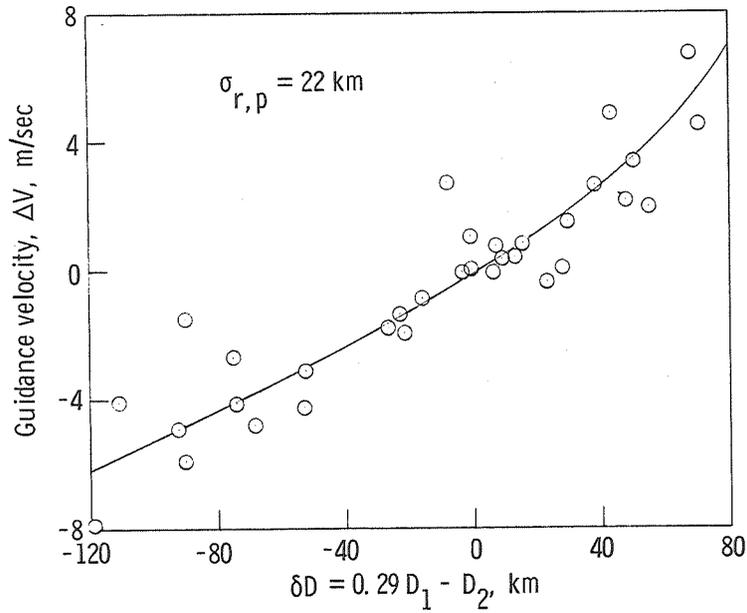
(b) Perilune-velocity variation.

Figure 13.- Continued.

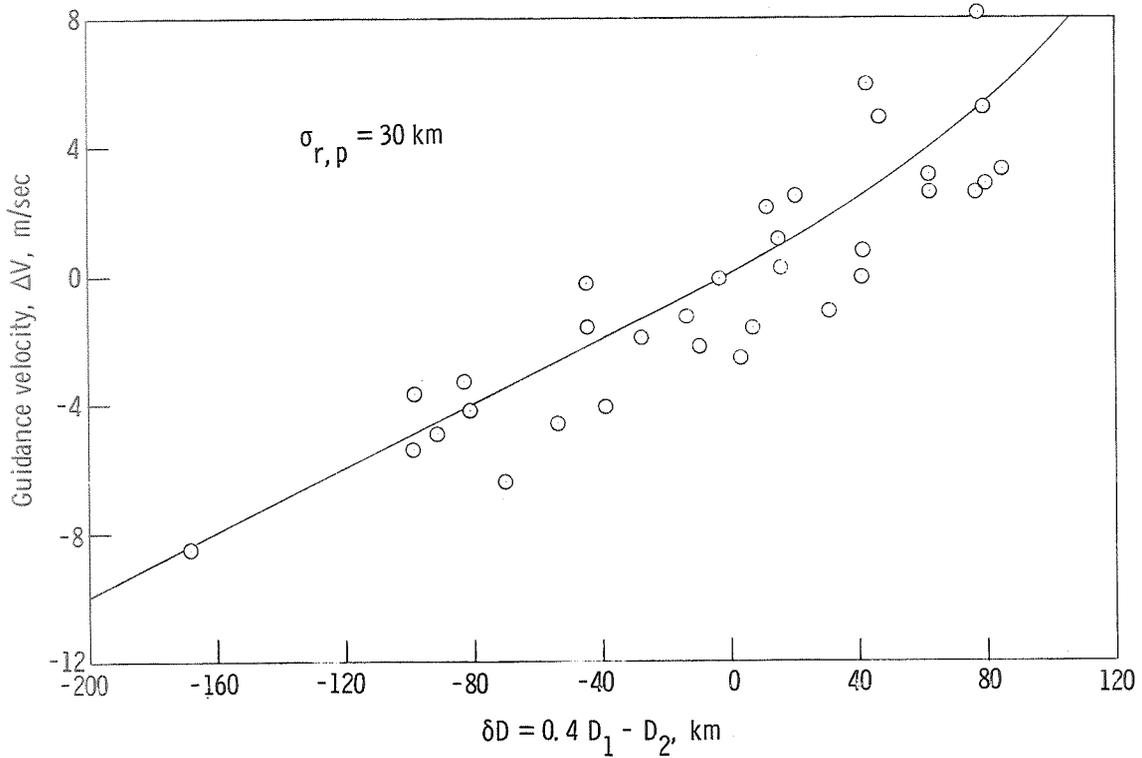


(c) Variation of approach-guidance velocity.

Figure 13.- Concluded.



(a) $\sigma_{r,mc} = 0$; $(\sigma_{\Delta V})_{\text{cutoff}} = 0$ in first approach-guidance maneuver.



(b) $\sigma_{r,mc} = 44$ km; $(\sigma_{\Delta V})_{\text{cutoff}} = 0.2$ m/sec in first approach-guidance maneuver.

Figure 14.- Variation of guidance velocity for second approach-guidance maneuver at $T_p = 4.6$ hours. (First approach-guidance maneuver at lunar sphere of influence.)

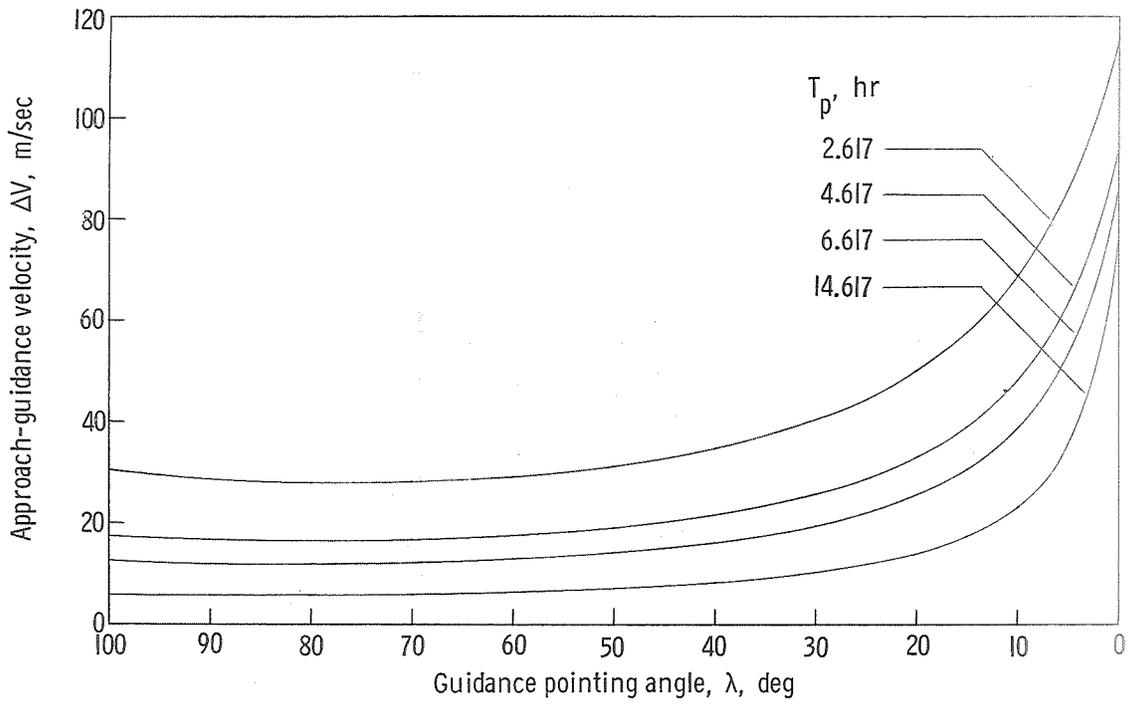


Figure 15.- Example of approach-guidance velocity requirements for a typical perturbed trajectory.

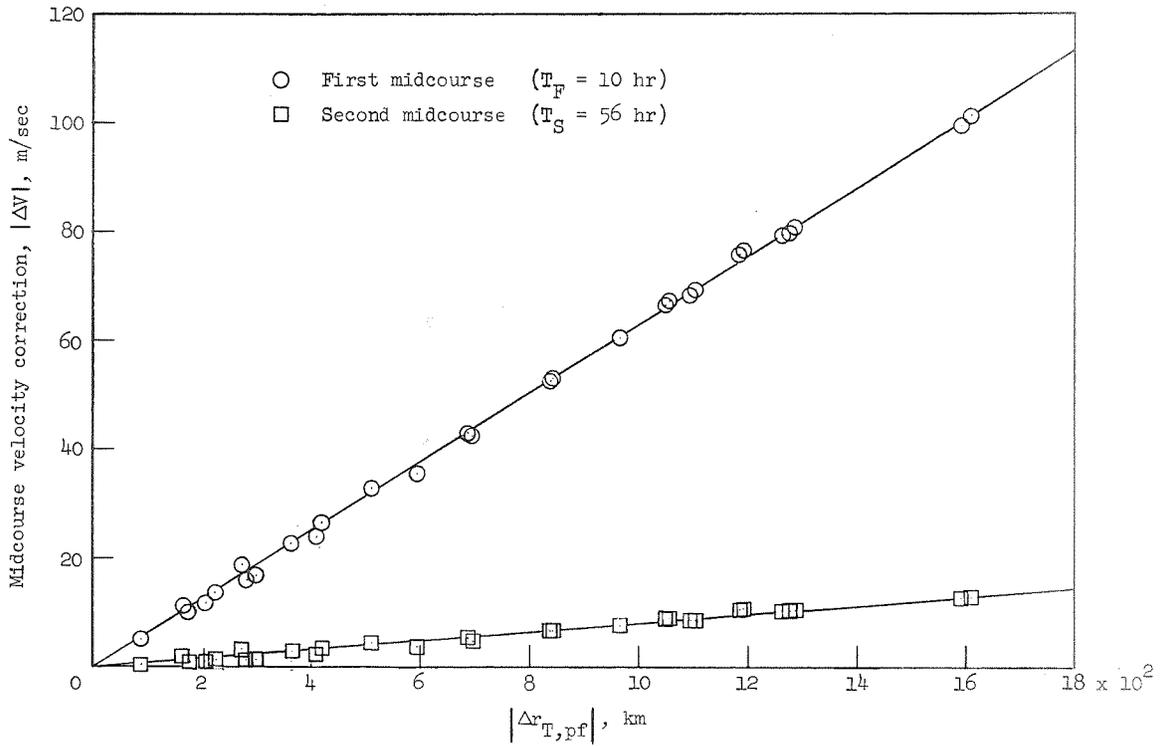


Figure 16.- Midcourse-maneuver velocity requirements. $T_{pf} = 9.5$ hours. (Aim point selected at lunar sphere of influence ($T_p = 14.6$ hours)).

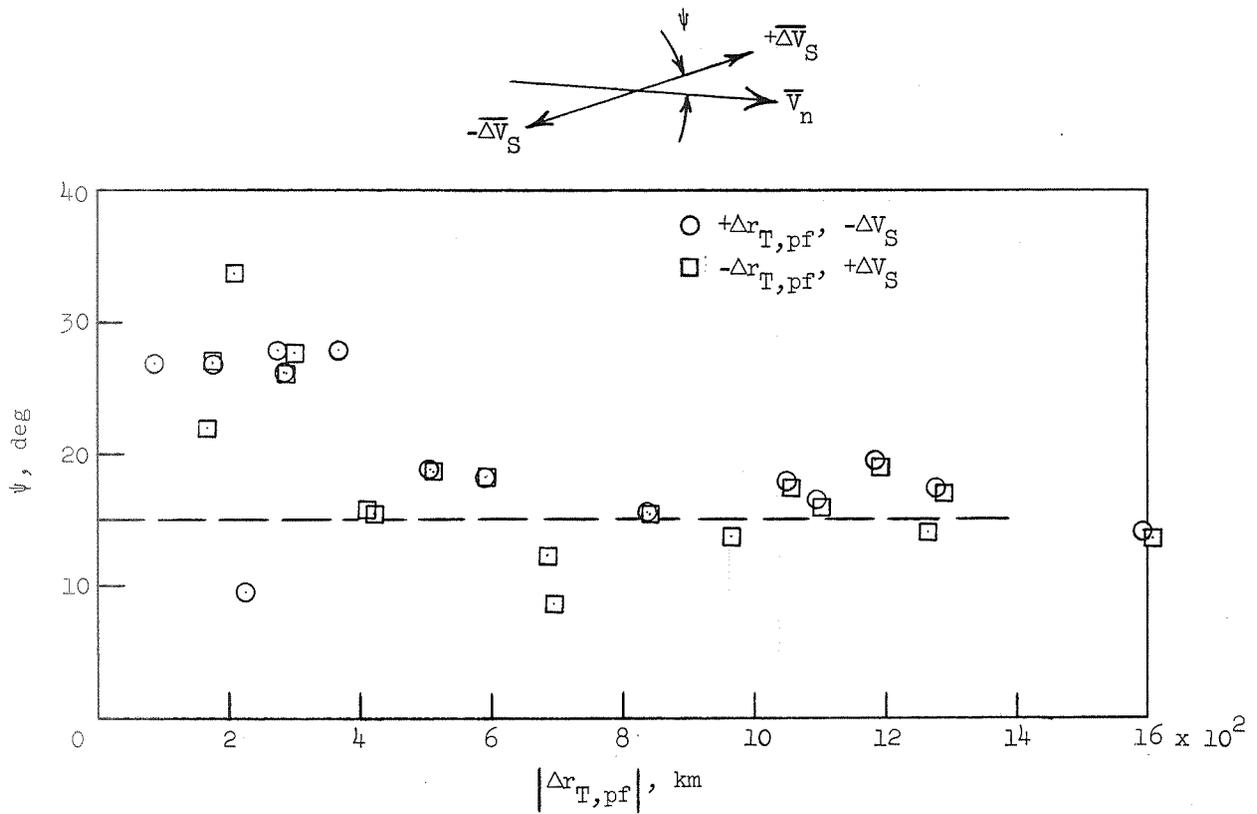


Figure 17.- Variation in direction of second-midcourse-maneuver velocity-correction vector. $T_{pf} = 9.5$ hours. (Dashed line represents value used to derive figure 18.)

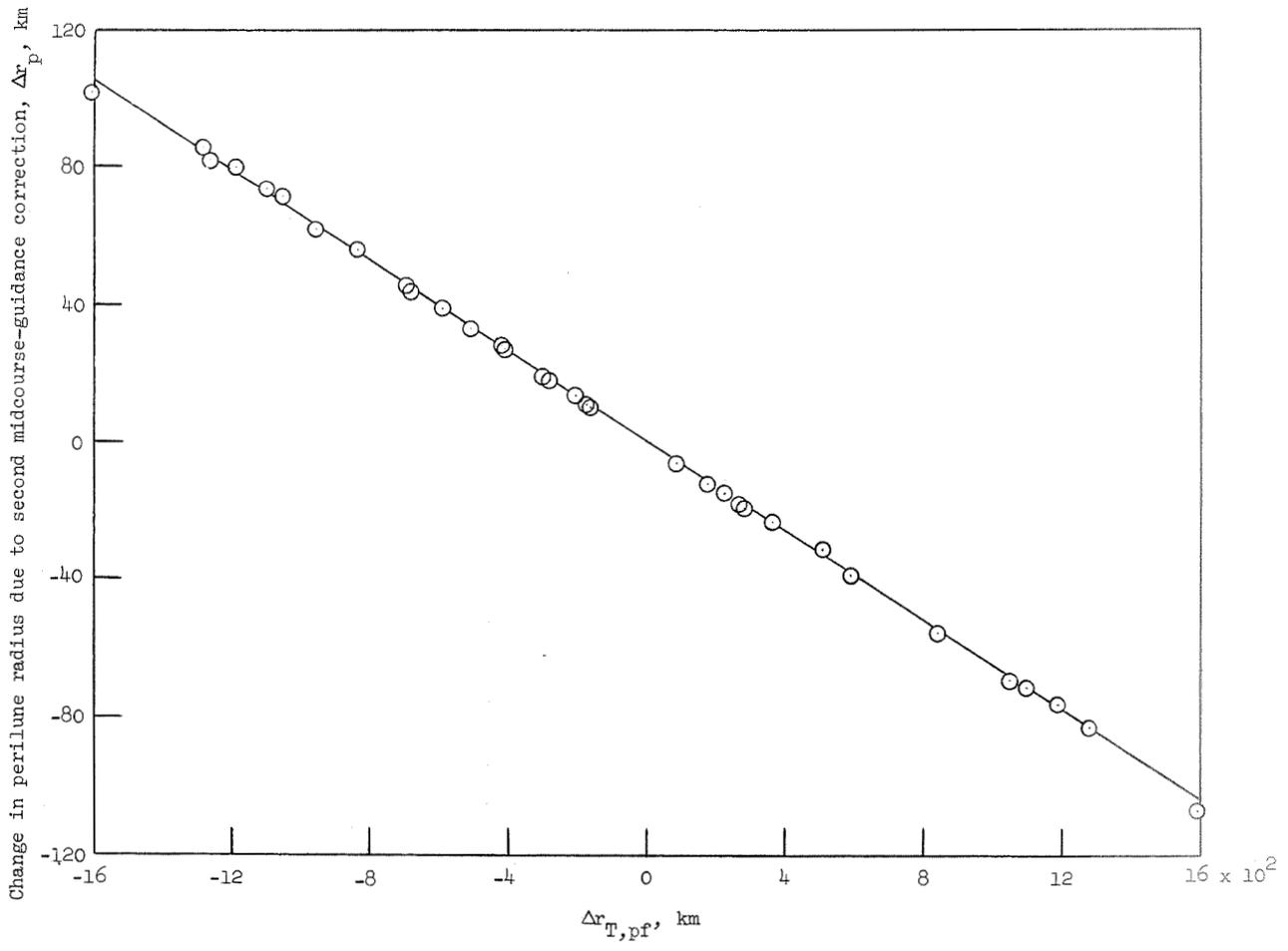


Figure 18.- Effect of second midcourse maneuver on perilune radius. $T_{pf} = 9.5$ hours.

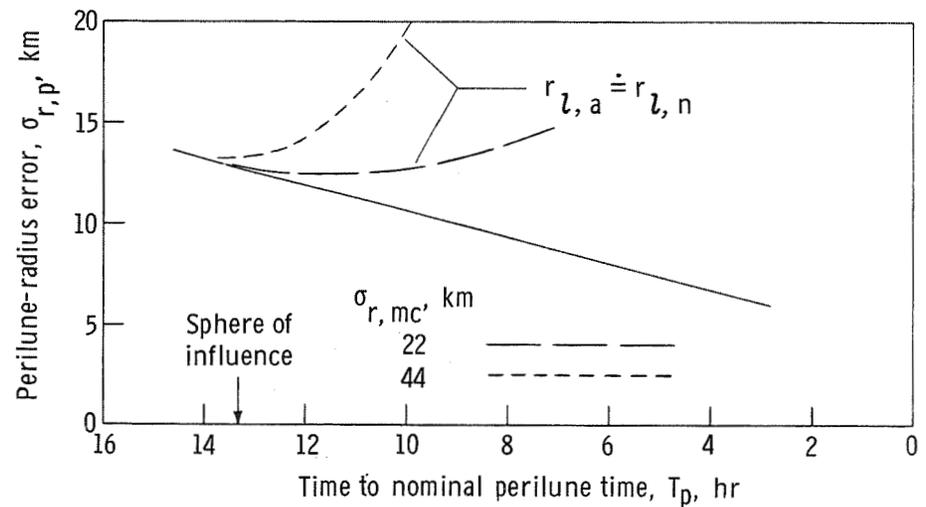


Figure 19.- Approach-guidance accuracy for trajectories employing exact midcourse-guidance pointing angles (refs. 1 and 2). $(\sigma_{\Delta V})_{\text{cutoff}} = 0.2$ m/sec; $\sigma_{\alpha} = 10$ arc seconds; $\sigma_{\theta} = 10$ arc seconds. (For the solid curve, the magnitude of $\sigma_{r,mc}$ is insignificant.)

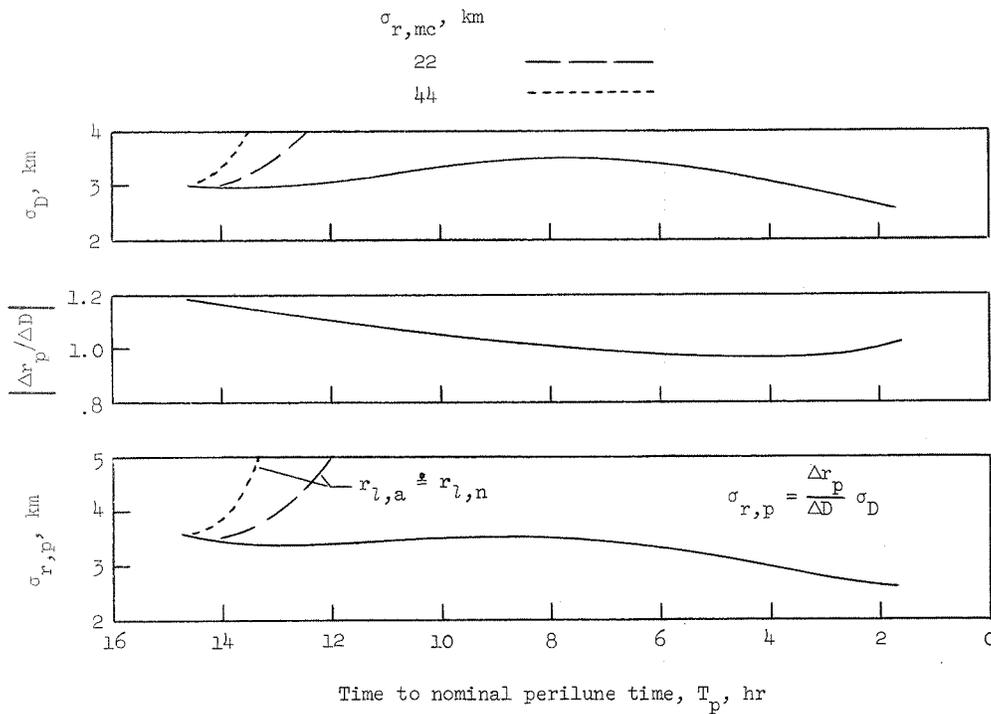


Figure 20.- Effect of approach measurement error on guidance accuracy as determined in reference 2. $\sigma_{\alpha} = 10$ arc seconds; $\sigma_{\theta} = 10$ arc seconds. (For the solid curves, the magnitude of $\sigma_{r,mc}$ is insignificant.)

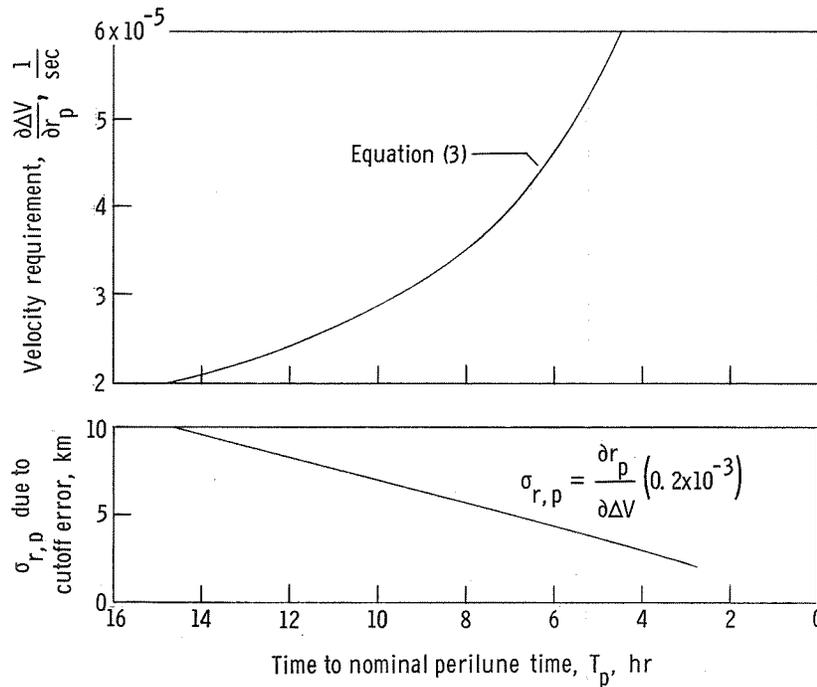


Figure 21.- Effect of velocity-cutoff error on approach-guidance accuracy. $(\sigma_{\Delta V})_{\text{cutoff}} = 0.2$ m/sec.

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